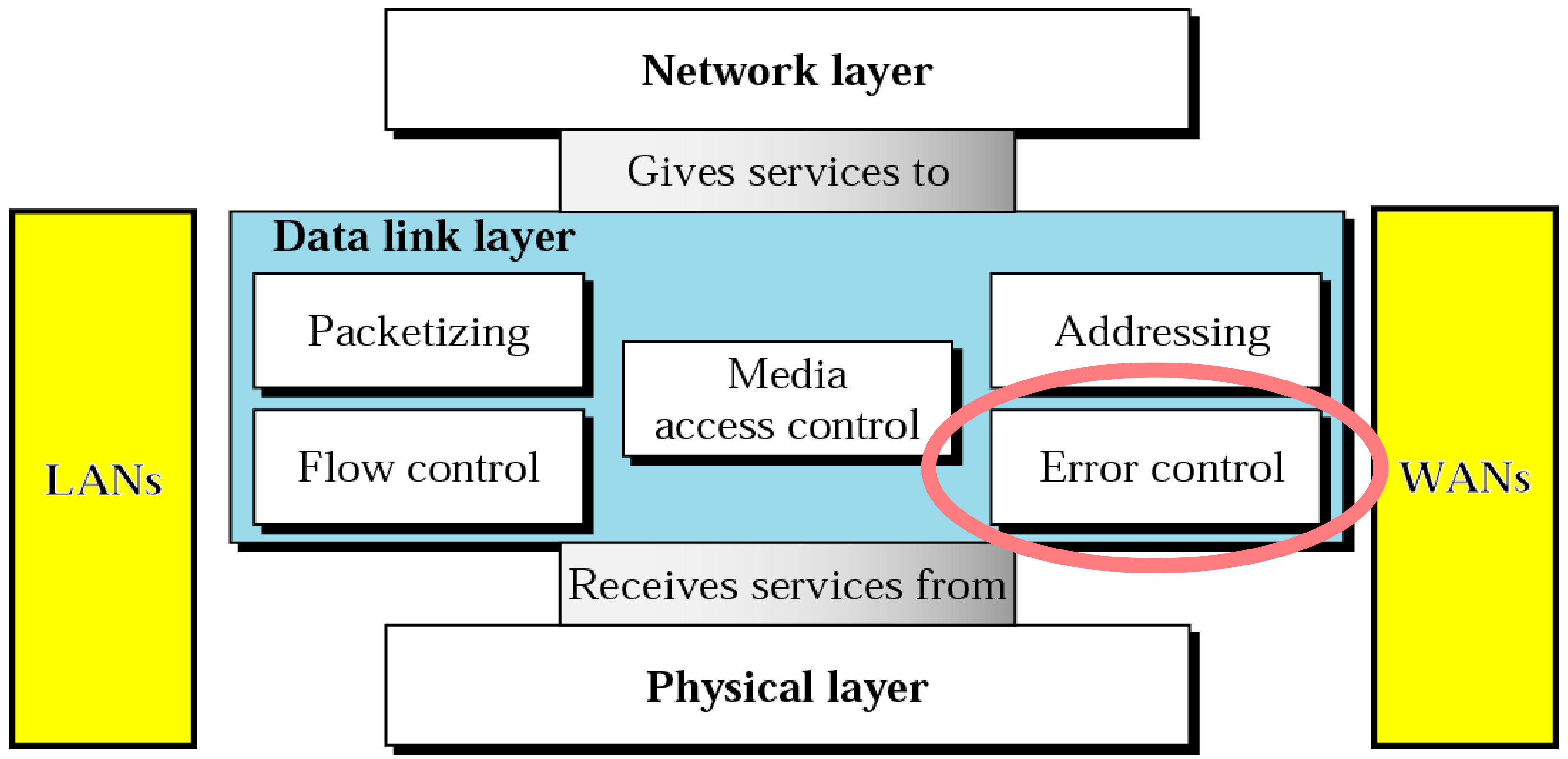
Error Detection and Correction

Chapter-10

Outline

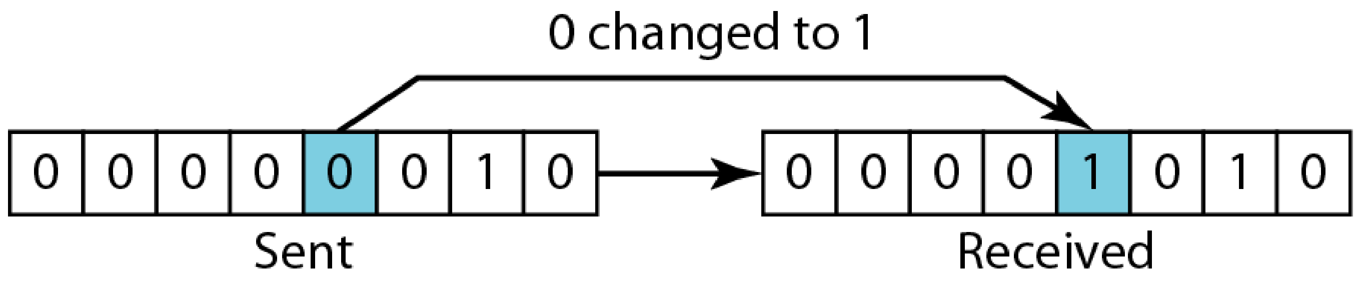
- Overview of Data Link Layer
- Types of errors
- Redundancy
- Correction vs. detection
- Coding

Data Link Layer

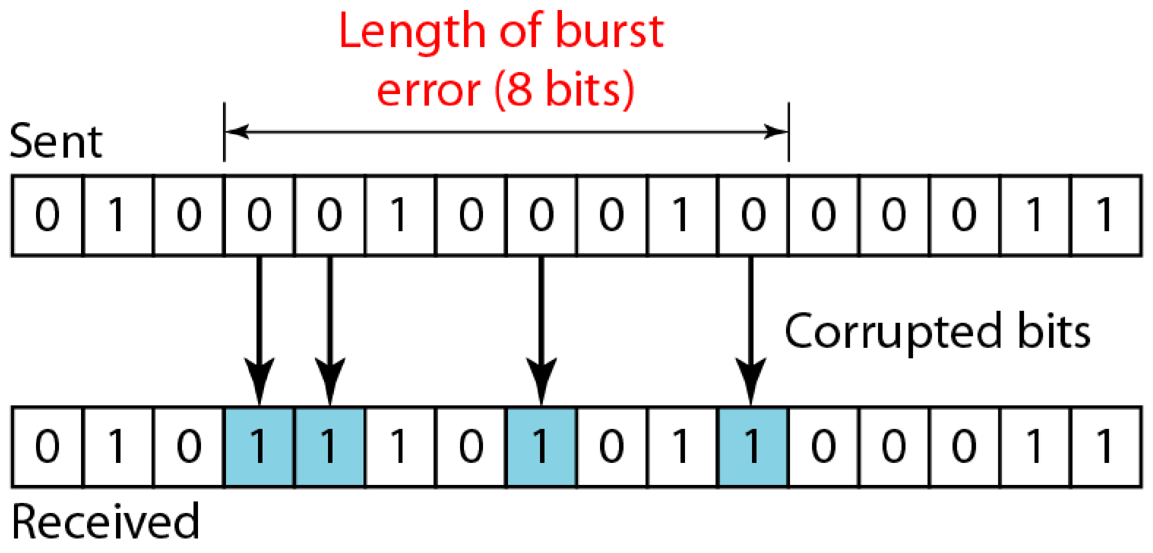


Types of Errors

- Single-bit errors

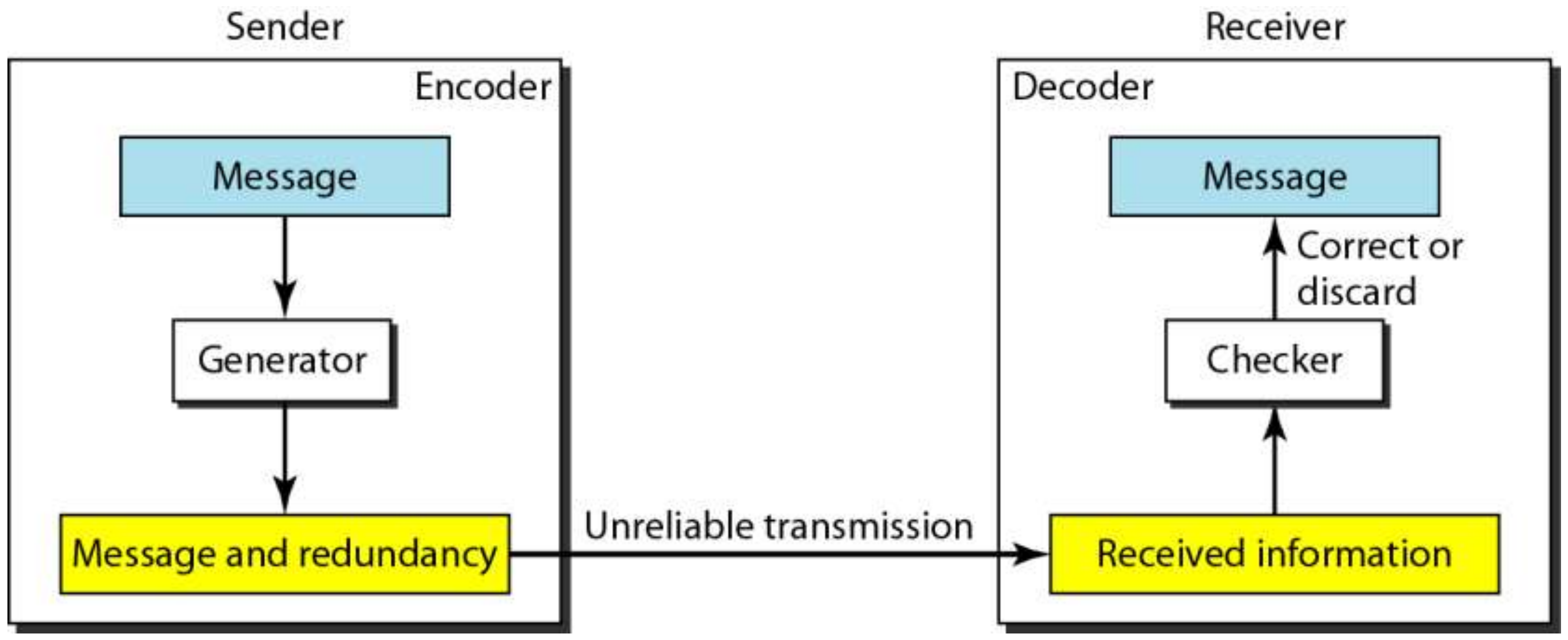


- Burst errors



Redundancy

- To detect or correct errors, redundant bits of data must be added



Detection VS Correction

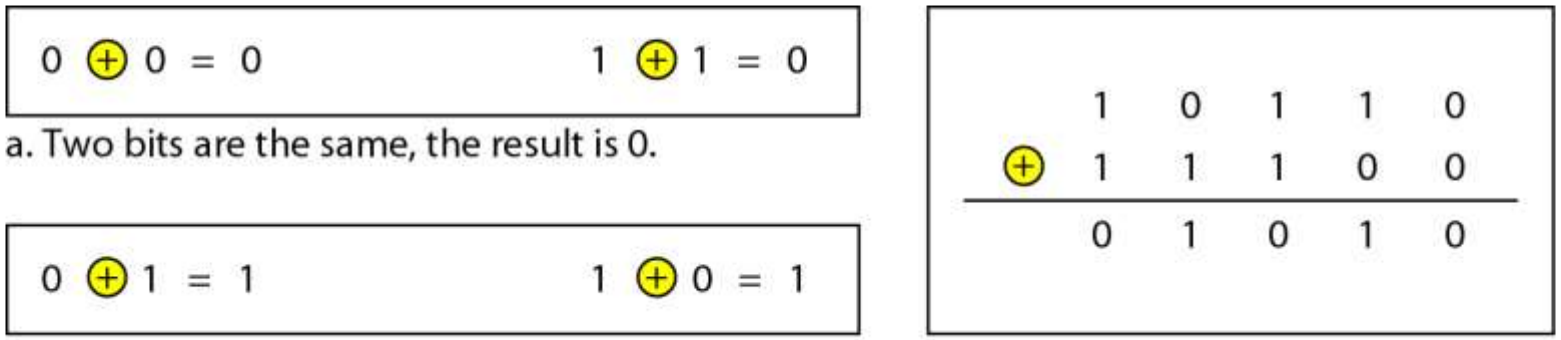
- In error detection, we are only looking to see if any error has occurred. The answer is a simple yes or no. We are not even interested in the number of corrupted bits. A single-bit error is the same for us as a burst error.
- In error correction, we need to know the exact number of bits that are corrupted and, more importantly, their location in the message. The number of errors and the size of the message are important factors. If we need to correct a single error in an 8-bit data unit, we need to consider eight possible error locations; if we need to correct two errors in a data unit of the same size, we need to consider 28 (permutation of 8 by 2) possibilities. You can imagine the receiver's difficulty in finding 10 errors in a data unit of 1000 bits.

Coding

- Process of adding redundancy for error detection or correction
- Two types:
  - Block codes
    - Divides the data to be sent into a set of blocks
    - Extra information attached to each block
    - Memoryless
  - Convolutional codes
    - Treats data as a series of bits, and computes a code over a continuous series
    - The code computed for a set of bits depends on the current and previous input

XOR Operation

- Main operation for computing error detection/correction codes
- Similar to modulo-2 addition



a. Two bits are the same, the result is 0.

b. Two bits are different, the result is 1.

c. Result of XORing two patterns

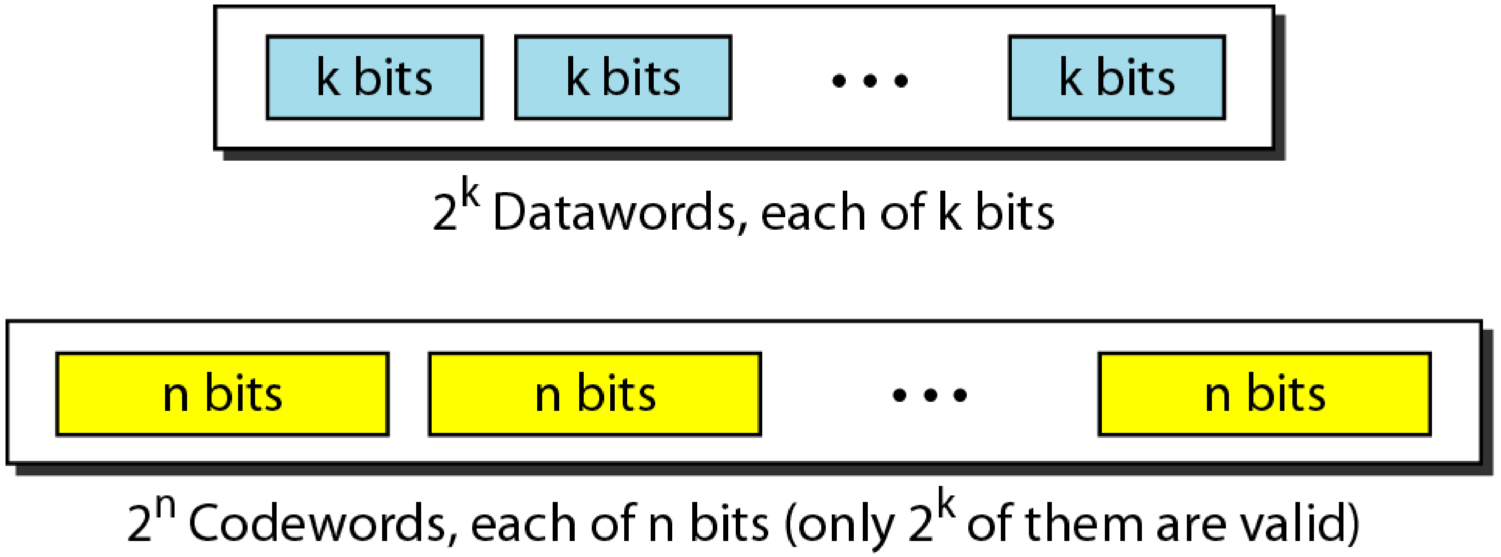
10-2 BLOCK CODING

In block coding, we divide our message into blocks, each of k bits, called datawords. We add r redundant bits to each block to make the length n = k + r. The resulting n-bit blocks are called codewords.

Topics discussed in this section:

Error Detection
Error Correction
Hamming Distance
Minimum Hamming Distance

Figure 10.5 *Datawords and codewords in block coding*


k bits
k bits
• • •
k bits
2^k Datawords, each of k bits
n bits
n bits
• • •
n bits
2^n Codewords, each of n bits (only 2^k of them are valid)

Example 10.1

The 4B/5B block coding discussed in Chapter 4 is a good example of this type of coding. In this coding scheme, $k = 4$ and $n = 5$. As we saw, we have $2^k = 16$ datawords and $2^n = 32$ codewords. We saw that 16 out of 32 codewords are used for message transfer and the rest are either used for other purposes or unused.

Example: *4B/5B Block Coding*

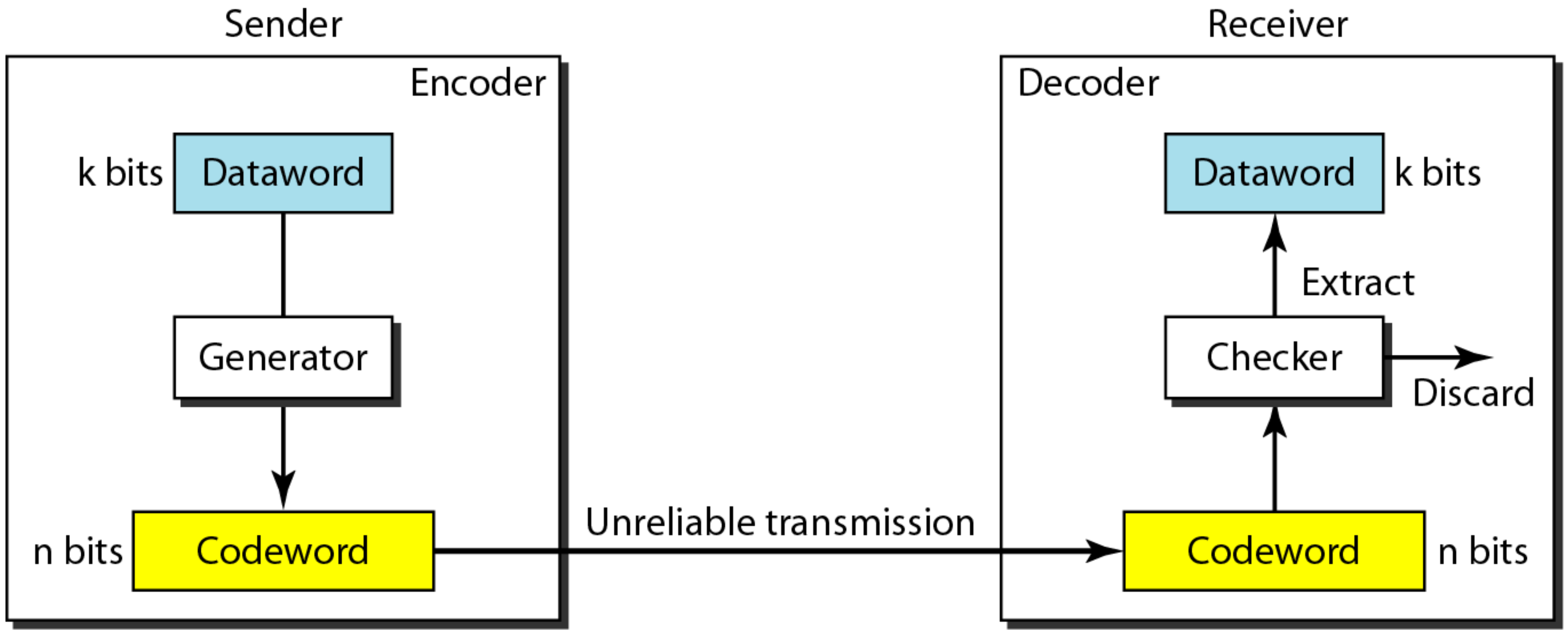
Data	Code	Data	Code
0000	11110	1000	10010
0001	01001	1001	10011
0010	10100	1010	10110
0011	10101	1011	10111
0100	01010	1100	11010
0101	01011	1101	11011
0110	01110	1110	11100
0111	01111	1111	11101

k = 5
r = ?
n = 4

Error Detection

- Enough redundancy is added to detect an error.
- The receiver knows an error occurred but does not know which bit(s) is(are) in error.
- Has less overhead than error correction.

Figure 10.6 *Process of error detection in block coding*



Example 10.2

Let us assume that k = 2 and n = 3. Table 10.1 shows the list of datawords and codewords. Later, we will see how to derive a codeword from a dataword.

Table 10.1 *A code for error detection (Example 10.2)*

Datawords	*Codewords*
00	000
01	011
10	101
11	110

Example 10.2 (continued)

Assume the sender encodes the dataword 01 as 011 and sends it to the receiver. Consider the following cases:

1. The receiver receives 011. It is a valid codeword. The receiver extracts the dataword 01 from it.

2. *The codeword is corrupted during transmission, and 111 is received. This is not a valid codeword and is discarded.*
3. *The codeword is corrupted during transmission, and 000 is received. This is a valid codeword. The receiver incorrectly extracts the dataword 00. Two corrupted bits have made the error undetectable.*

Note

An error-detecting code can detect only the types of errors for which it is designed; other types of errors may remain undetected.

Hamming Distance

Note

The Hamming distance between two words is the number of differences between corresponding bits.

Example 10.4

Let us find the Hamming distance between two pairs of words.

1. ***The Hamming distance d(000, 011) is 2 because***

$$\boxed{000 \oplus 011 \text{ is } 011 \text{ (two 1s)}}$$

2. ***The Hamming distance d(10101, 11110) is 3 because***

$$\boxed{10101 \oplus 11110 \text{ is } 01011 \text{ (three 1s)}}$$

Note

The minimum Hamming distance is the smallest Hamming distance between all possible pairs in a set of words.

Example 10.5

Find the minimum Hamming distance of the coding scheme in Table 10.1.

Solution

We first find all Hamming distances.

$d(000, 011) = 2$	$d(000, 101) = 2$	$d(000, 110) = 2$	$d(011, 101) = 2$
$d(011, 110) = 2$	$d(101, 110) = 2$		

The d_{min} in this case is 2.

Example 10.6

Find the minimum Hamming distance of the coding scheme in Table 10.2.

Dataword	*Codeword*
00	00000
01	01011
10	10101
11	11110

Solution

We first find all the Hamming distances.

$d(00000, 01011) = 3$ $d(00000, 10101) = 3$ $d(00000, 11110) = 4$
$d(01011, 10101) = 4$ $d(01011, 11110) = 3$ $d(10101, 11110) = 3$

The d_{min} in this case is 3.

Note

To guarantee the detection of up to s errors in all cases, the minimum Hamming distance in a block code must be $d_{min} = s + 1$.

Example 10.7

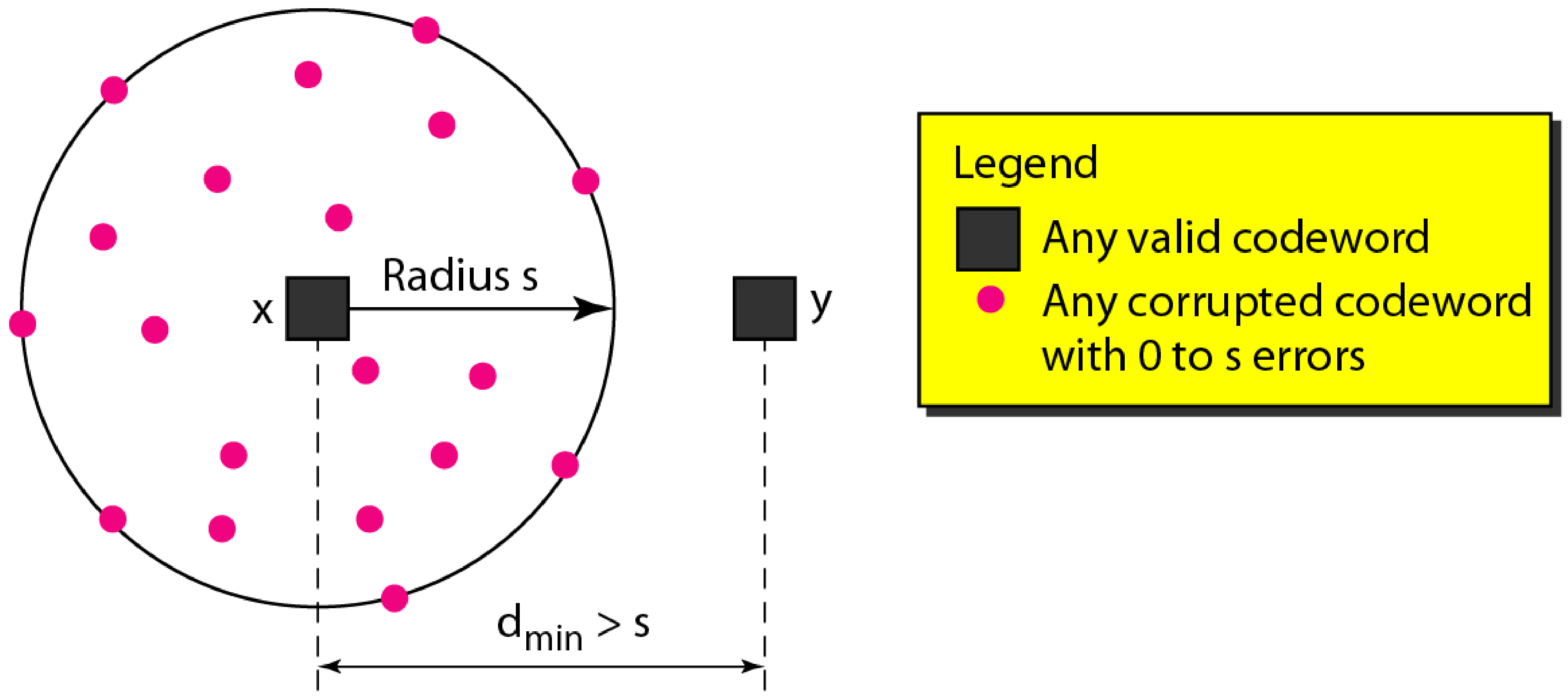
The minimum Hamming distance for our first code scheme (Table 10.1) is 2. This code guarantees detection of only a single error. For example, if the third codeword (101) is sent and one error occurs, the received codeword does not match any valid codeword. If two errors occur, however, the received codeword may match a valid codeword and the errors are not detected.

Example 10.8

Our second block code scheme (Table 10.2) has d_{min} = 3. This code can detect up to two errors. Again, we see that when any of the valid codewords is sent, two errors create a codeword which is not in the table of valid codewords. The receiver cannot be fooled.

However, some combinations of three errors change a valid codeword to another valid codeword. The receiver accepts the received codeword and the errors are undetected.

Figure 10.8 *Geometric concept for finding d_{min} in error detection*



Note

To guarantee correction of up to *t* errors in all cases, the minimum Hamming distance in a block code must be $d_{min} = 2t + 1$.

10-3 LINEAR BLOCK CODES

Almost all block codes used today belong to a subset called linear block codes. A linear block code is a code in which the exclusive OR (addition modulo-2) of two valid codewords creates another valid codeword.

Topics discussed in this section:

Minimum Distance for Linear Block Codes
Some Linear Block Codes

Note

In a linear block code, the exclusive OR (XOR) of any two valid codewords creates another valid codeword.

Example 10.10

Let us see if the two codes we defined in Table 10.1 and Table 10.2 belong to the class of linear block codes.

1. ***The scheme in Table 10.1 is a linear block code because the result of XORing any codeword with any other codeword is a valid codeword. For example, the XORing of the second and third codewords creates the fourth one.***

2. ***The scheme in Table 10.2 is also a linear block code. We can create all four codewords by XORing two other codewords.***

Example 10.11

In our first code (Table 10.1), the numbers of 1s in the nonzero codewords are 2, 2, and 2. So the minimum Hamming distance is $d_{min} = 2$. In our second code (Table 10.2), the numbers of 1s in the nonzero codewords are 3, 3, and 4. So in this code we have $d_{min} = 3$.

Common Detection Methods

- Parity check
- Cyclic Redundancy Check
- Checksum

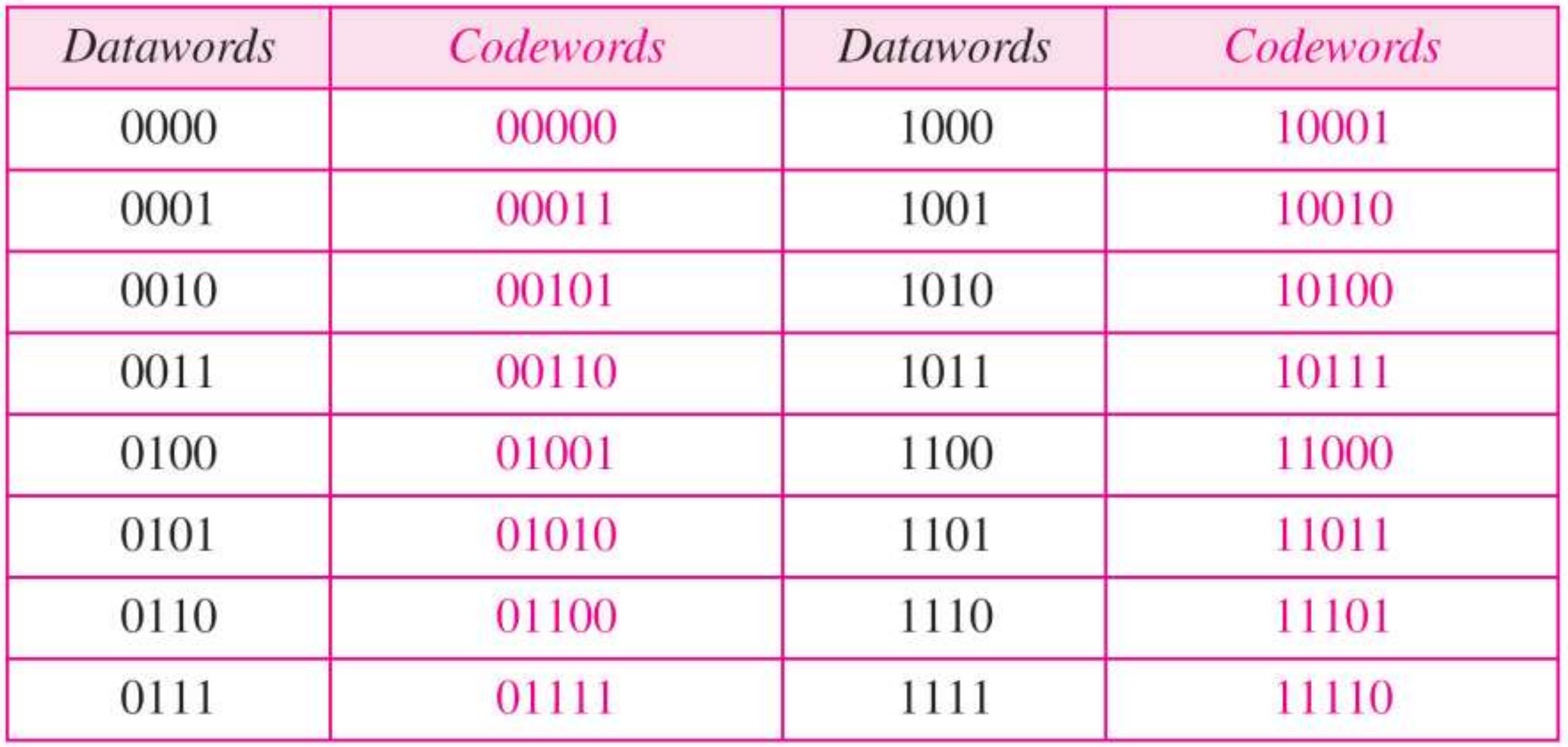
Parity-check code

- ***The most familiar error-detecting code is the parity-check code.***
- ***This code is a linear block code.***
- ***In this code, a k-bit dataword is changed to an n-bit codeword where n = k + 1.***
- ***The extra bit, called the parity bit, is selected to make the total number of 1s in the codeword even.***
- ***Although some implementations specify an odd number of 1s, we discuss the even case.***

Note

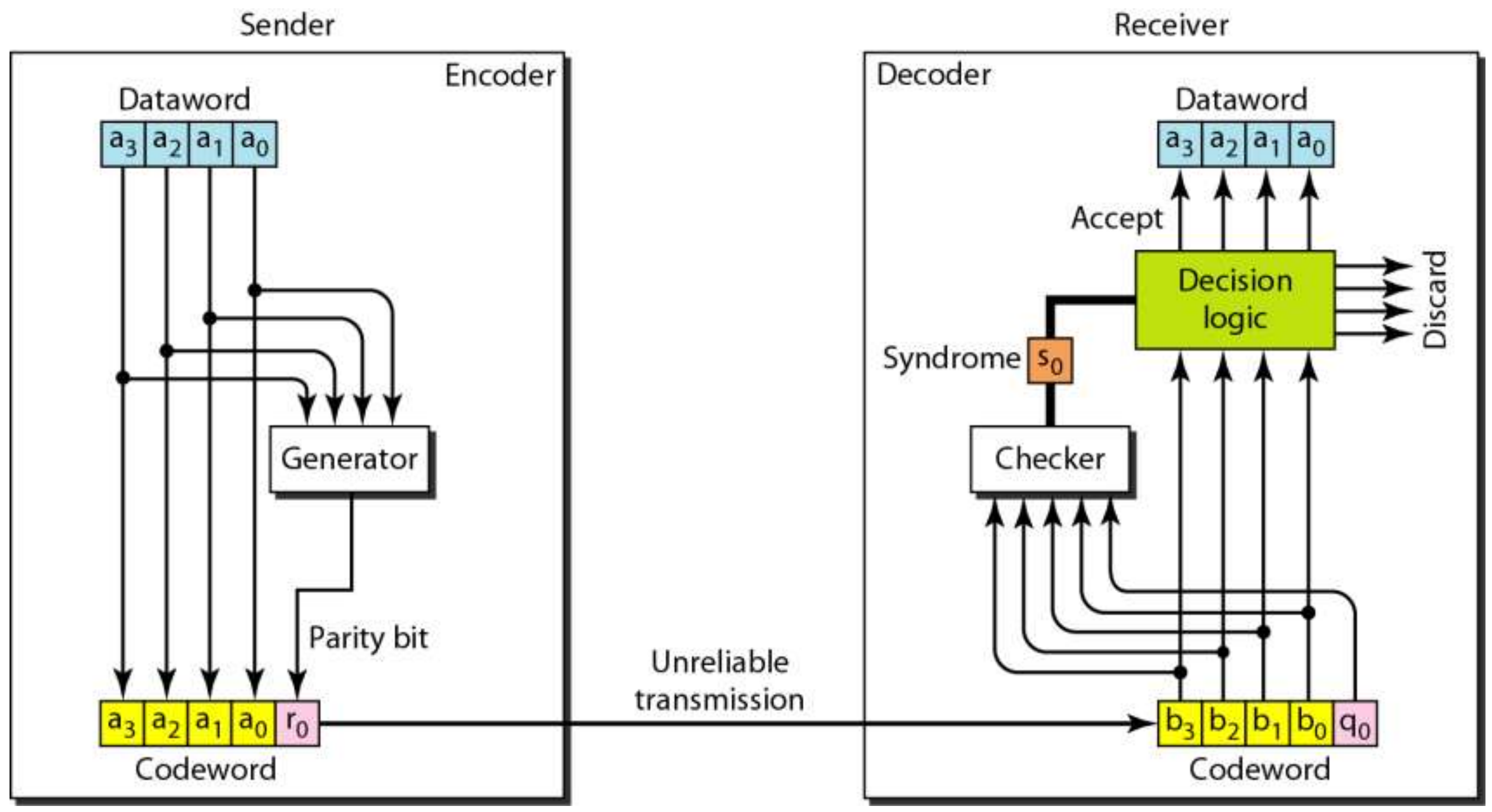
A simple parity-check code is a single-bit error-detecting code in which $n = k + 1$ with $d_{\min} = 2$.
Even parity (ensures that a codeword has an even number of 1's) and odd parity (ensures that there are an odd number of 1's in the codeword)

Table 10.3 ***Simple parity-check code C(5, 4)***



Datawords	*Codewords*	*Datawords*	*Codewords*
0000	00000	1000	10001
0001	00011	1001	10010
0010	00101	1010	10100
0011	00110	1011	10111
0100	01001	1100	11000
0101	01010	1101	11011
0110	01100	1110	11101
0111	01111	1111	11110

Figure 10.10 *Encoder and decoder for simple parity-check code*



Example 10.12

Let us look at some transmission scenarios. Assume the sender sends the dataword 1011. The codeword created from this dataword is 10111, which is sent to the receiver. We examine five cases:

1. ***No error occurs; the received codeword is 10111. The syndrome is 0. The dataword 1011 is created.***
2. ***One single-bit error changes a_1. The received codeword is 10011. The syndrome is 1. No dataword is created.***
3. ***One single-bit error changes r_0. The received codeword is 10110. The syndrome is 1. No dataword is created.***

Example 10.12 (continued)

4. An error changes r_0 and a second error changes a_3 . The received codeword is 00110. The syndrome is 0. The dataword 0011 is created at the receiver. Note that here the dataword is wrongly created due to the syndrome value.

5. Three bits—a_3, a_2, and a_1—are changed by errors. The received codeword is 01011. The syndrome is 1. The dataword is not created. This shows that the simple parity check, guaranteed to detect one single error, can also find any odd number of errors.

Note

A simple parity-check code can detect an odd number of errors.

10-4 CYCLIC CODES

***Cyclic codes** are special linear block codes with one extra property. In a cyclic code, if a codeword is cyclically shifted (rotated), the result is another codeword.*

Topics discussed in this section:

Cyclic Redundancy Check
Polynomials

Table 10.6 ***A CRC code with C(7, 4)***

Dataword	*Codeword*	*Dataword*	*Codeword*
0000	0000000	1000	1000101
0001	0001011	1001	1001110
0010	0010110	1010	1010011
0011	0011101	1011	1011000
0100	0100111	1100	1100010
0101	0101100	1101	1101001
0110	0110001	1110	1110100
0111	0111010	1111	1111111

Figure 10.14 *CRC encoder and decoder*

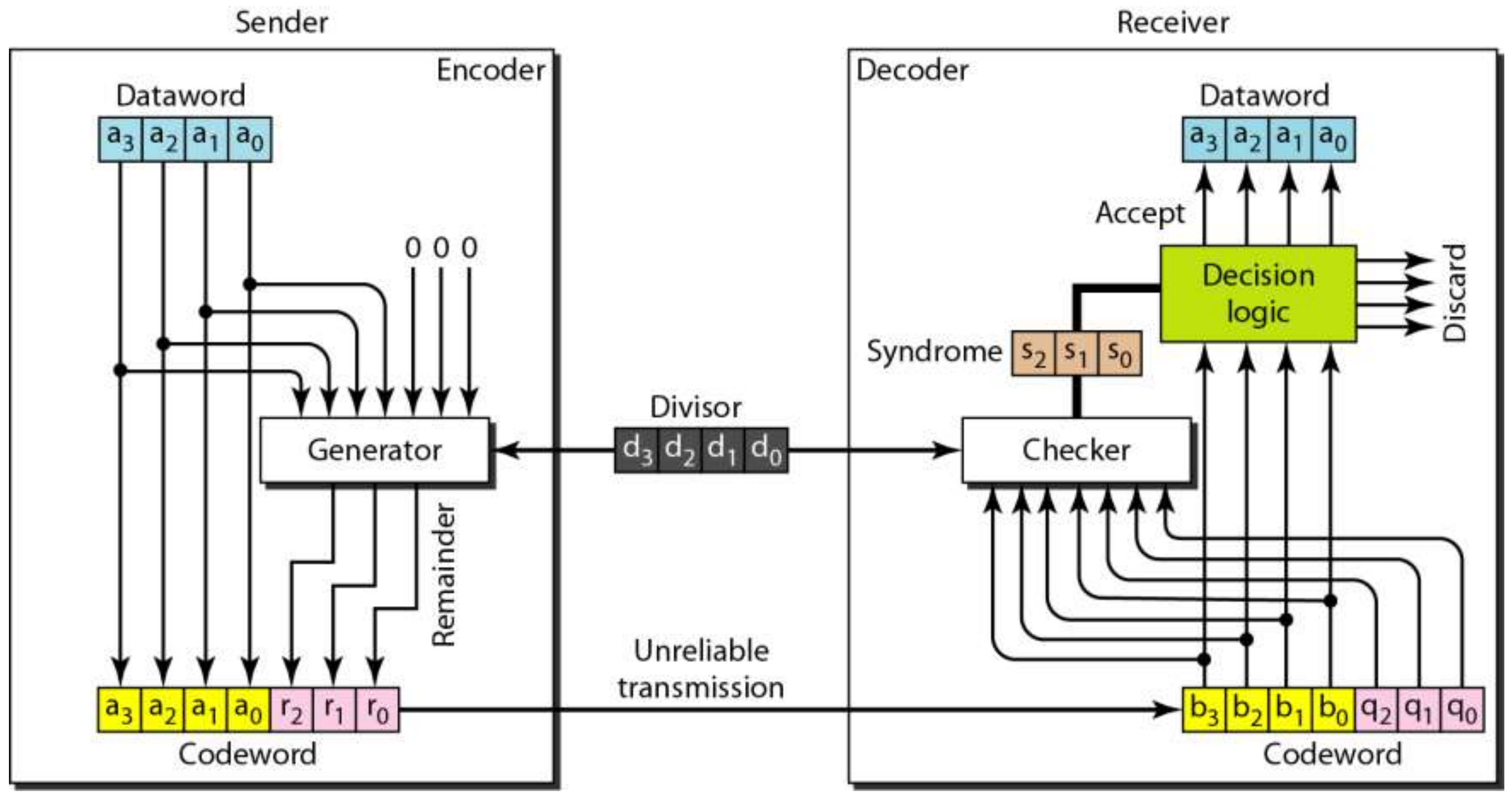


Figure 10.15 *Division in CRC encoder*

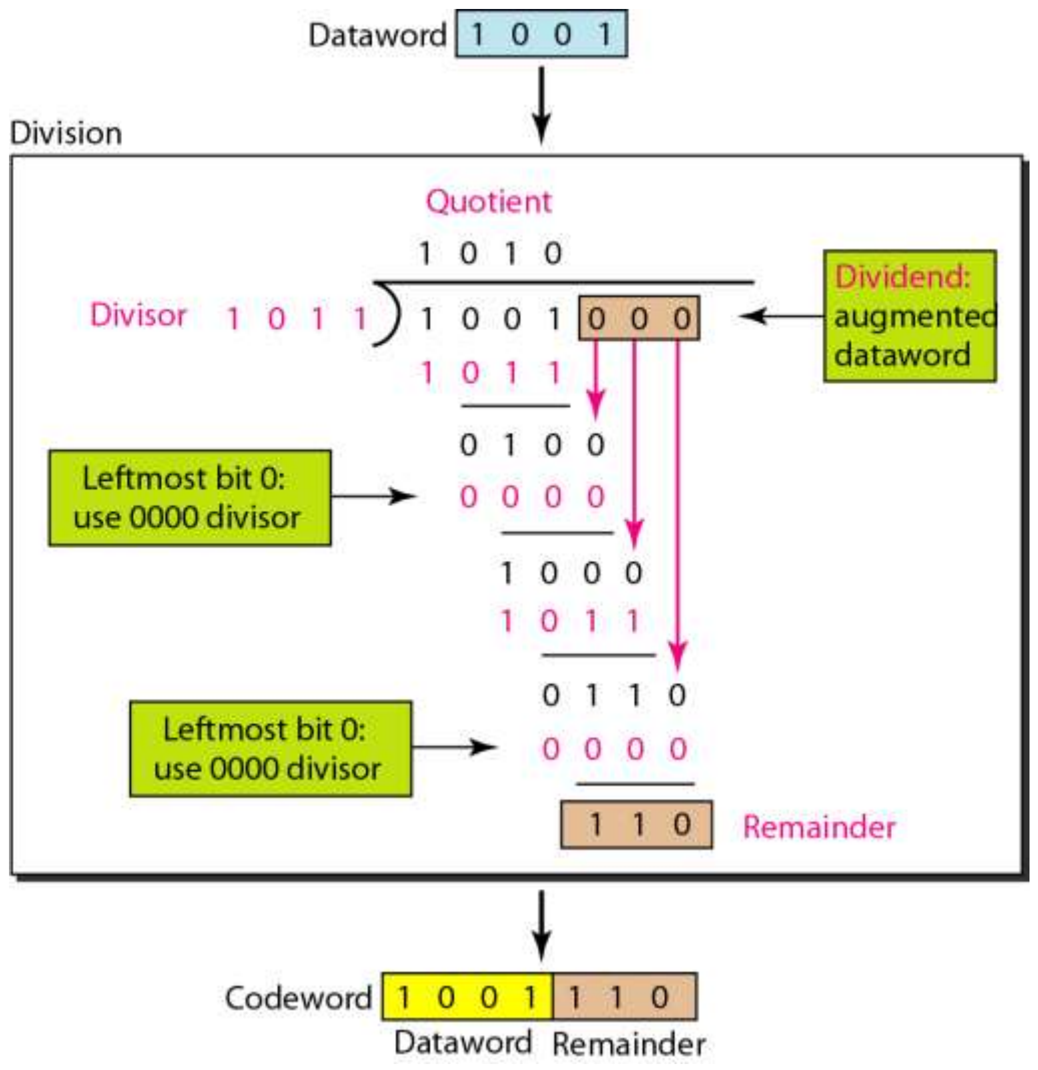
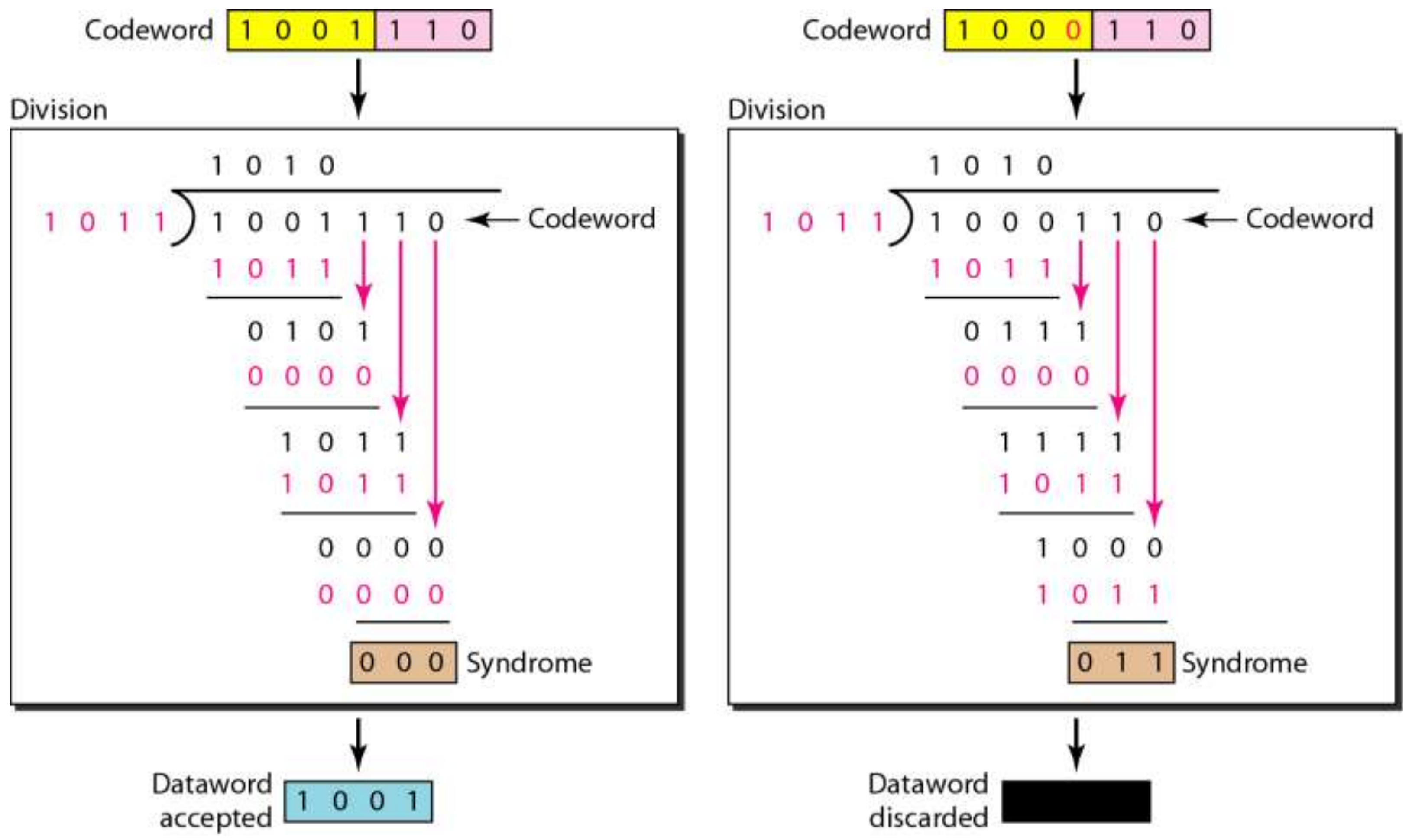


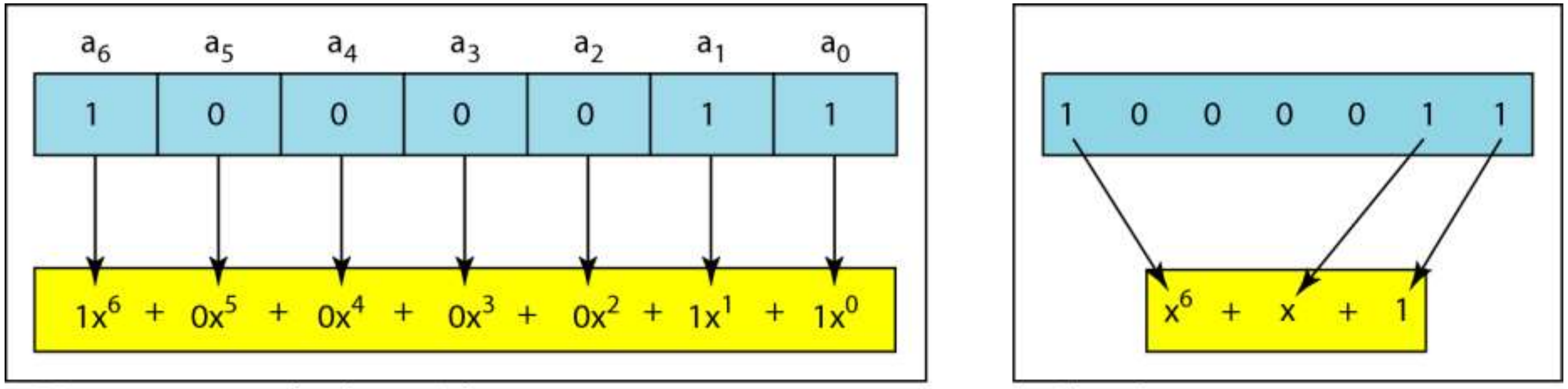
Figure 10.16 *Division in the CRC decoder for two cases*



Using Polynomials

- We can use a polynomial to represent a binary word.
- Each bit from right to left is mapped onto a power term.
- The rightmost bit represents the “0” power term. The bit next to it the “1” power term, etc.
- If the bit is of value zero, the power term is deleted from the expression.

Figure 10.21 *A polynomial to represent a binary word*

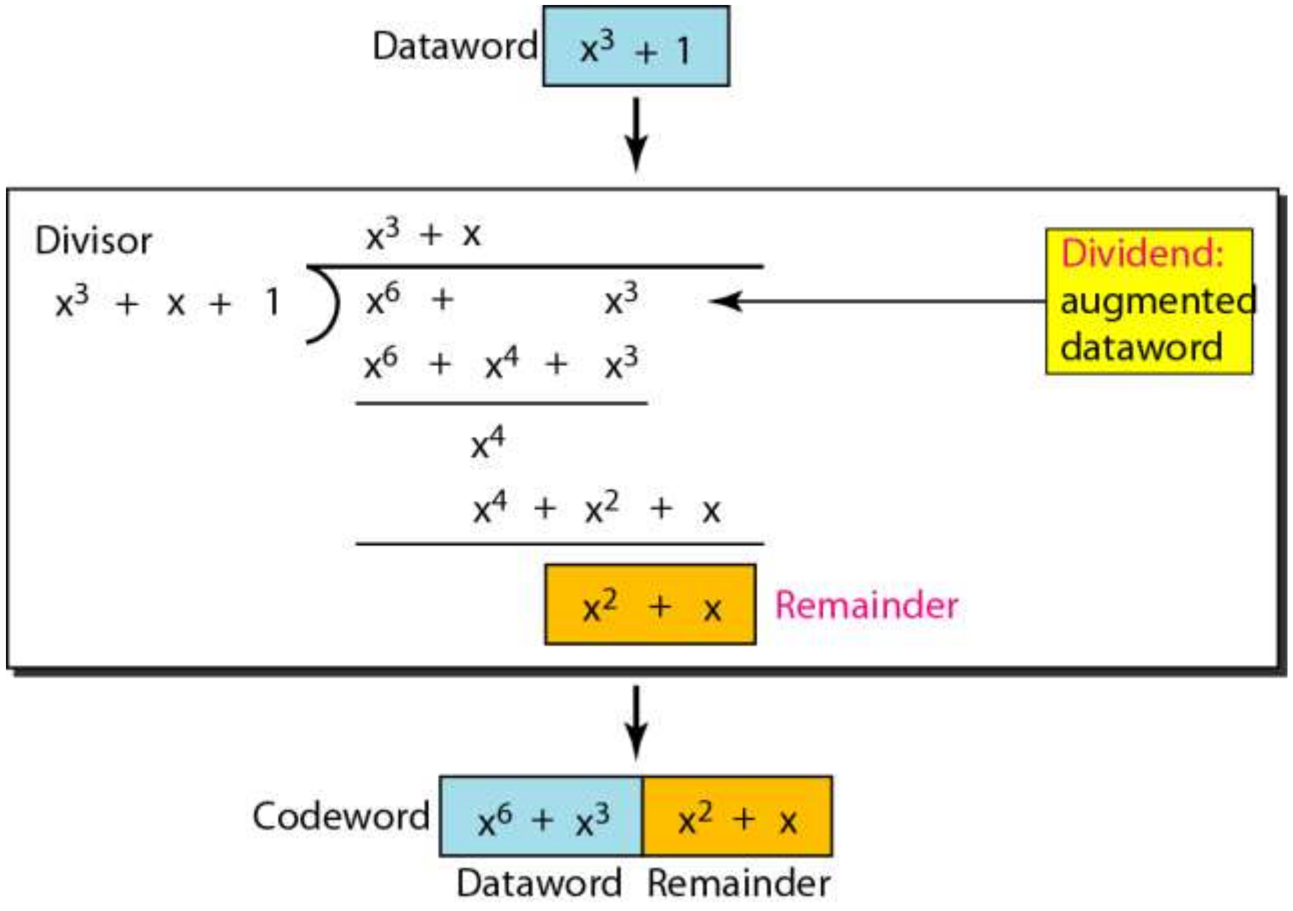


a_6 a_5 a_4 a_3 a_2 a_1 a_0
1 0 0 0 0 1 1
$1x^6 + 0x^5 + 0x^4 + 0x^3 + 0x^2 + 1x^1 + 1x^0$
1 0 0 0 0 1 1
$x^6 + x + 1$

a. Binary pattern and polynomial

b. Short form

Figure 10.22 *CRC division using polynomials*



10-5 CHECKSUM

The last error detection method we discuss here is called the checksum. The checksum is used in the Internet by several protocols although not at the data link layer. However, we briefly discuss it here to complete our discussion on error checking

Topics discussed in this section:

Idea
One's Complement
Internet Checksum

Example 10.18

Suppose our data is a list of five 4-bit numbers that we want to send to a destination. In addition to sending these numbers, we send the sum of the numbers. For example, if the set of numbers is (7, 11, 12, 0, 6), we send (7, 11, 12, 0, 6, 36), where 36 is the sum of the original numbers. The receiver adds the five numbers and compares the result with the sum. If the two are the same, the receiver assumes no error, accepts the five numbers, and discards the sum. Otherwise, there is an error somewhere and the data are not accepted.

Example 10.19

We can make the job of the receiver easier if we send the negative (complement) of the sum, called the checksum. In this case, we send (7, 11, 12, 0, 6, −36). The receiver can add all the numbers received (including the checksum). If the result is 0, it assumes no error; otherwise, there is an error.

Example 10.20

How can we represent the number 21 in one's complement arithmetic using only four bits?

Solution

The number 21 in binary is 10101 (it needs five bits). We can wrap the leftmost bit and add it to the four rightmost bits. We have (0101 + 1) = 0110 or 6.

Example 10.21

How can we represent the number −6 in one's complement arithmetic using only four bits?

Solution

In one's complement arithmetic, the negative or complement of a number is found by inverting all bits. Positive 6 is 0110; negative 6 is 1001. If we consider only unsigned numbers, this is 9. In other words, the complement of 6 is 9. Another way to find the complement of a number in one's complement arithmetic is to subtract the number from $2^n - 1$ (16 − 1 in this case).

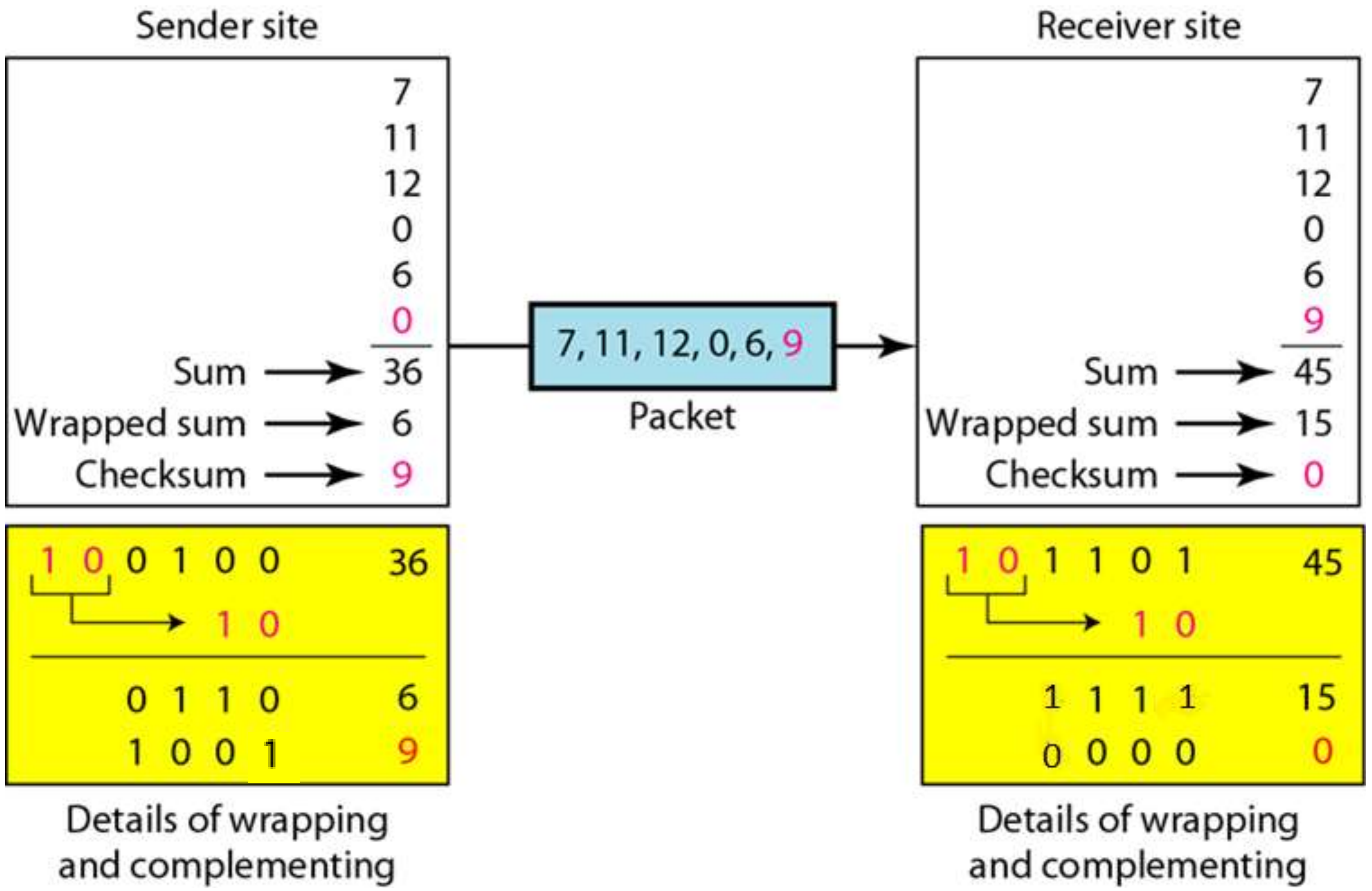
Example 10.22

Let us redo Exercise 10.19 using one's complement arithmetic. Figure 10.24 shows the process at the sender and at the receiver. The sender initializes the checksum to 0 and adds all data items and the checksum (the checksum is considered as one data item and is shown in color). The result is 36. However, 36 cannot be expressed in 4 bits. The extra two bits are wrapped and added with the sum to create the wrapped sum value 6. In the figure, we have shown the details in binary. The sum is then complemented, resulting in the checksum value 9 (15 − 6 = 9). The sender now sends six data items to the receiver including the checksum 9.

Example 10.22 (continued)

The receiver follows the same procedure as the sender. It adds all data items (including the checksum); the result is 45. The sum is wrapped and becomes 15. The wrapped sum is complemented and becomes 0. Since the value of the checksum is 0, this means that the data is not corrupted. The receiver drops the checksum and keeps the other data items. If the checksum is not zero, the entire packet is dropped.

Figure 10.24 *Example 10.22*



1

Internet Checksum

Traditionally, the Internet has used a 16-bit checksum. The sender and the receiver follow the steps depicted in below.
The sender or the receiver uses five steps.

Note

Sender site:

1. The message is divided into 16-bit words.
2. The value of the checksum word is set to 0.
3. All words including the checksum are added using one’s complement addition.
4. The sum is complemented and becomes the checksum.
5. The checksum is sent with the data.

Note

Receiver site:

1. The message (including checksum) is divided into 16-bit words.
2. All words are added using one's complement addition.
3. The sum is complemented and becomes the new checksum.
4. If the value of checksum is 0, the message is accepted; otherwise, it is rejected.

Book Reference

- Data Communications and Networking 5th Edition- Behrouz A. Forouzan