MADALINE

XOR

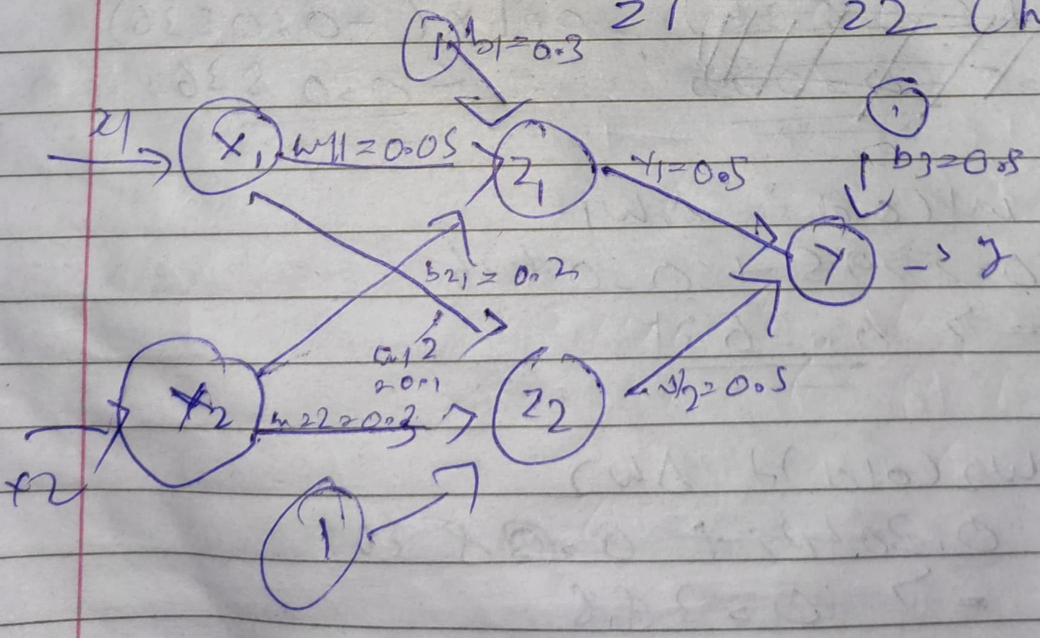
x_1	x_2	t
1	1	-1
1	-1	1
-1	1	1
-1	-1	-1

Initialize weights as $[w_{11}, w_{21}, b_1]$
$= [0.05, 0.2, 0.3]$

$[w_{12}, w_{22}, b_2] = [0.1, 0.2, 0.15]$
$[v_1, v_2, b_3] = [0.5, 0.5, 0.5]$

we know that,
$y = x_1 \bar{x}_2 + x_2 \bar{x}_1$

z_1 z_2 (hidden layer)



for first input sample

$x_1 = 1 \quad x_2 = 1 \quad t = -1 \quad \alpha = 0.5$

calculate net input to hidden units

$$z_1 = b_1 + w_{11}x_1 + x_2 w_{21}$$
$$= 0.3 + 1\times0.05 + 1\times0.2$$
$$= \underline{0.55}$$

$$z_2 = b_2 + x_1 w_{12} + x_2 w_{22}$$
$$= 0.15 + 1\times0.1 + 1\times0.2$$
$$= \underline{0.45}$$

Calculate output z_1, z_2 by applying activation function given by

$$f(z_{in}) = \begin{cases} 1, & z_{in} \geq 0 \\ -1, & z < 0 \end{cases}$$

hence,

$$z_1 = f(z_{in}) = f(0.55) = 1$$
$$z_2 = f(z_{in2}) = f(0.45) = 1$$

Apply the activation function over net input 'y_{in}' to calculate op y

$$y = f(y_{in}) = f(1.5) = \underline{1}$$

Since $t \neq y$, apply weight updation.
Also since $t = -1$, weights are updated on z_1 & z_2 that have positive net input.
Since, here both net inputs z_{in1} & z_{in2} are positive, updating weights & bias are both hidden units.

$w_{ij}(new) = w_{ij}(old) + \alpha(t - z_{inj}) x_i$
$b_j(new) = b_j(old) + \alpha(t - z_{inj})$

$w_{11} = w_{11}(old) + \alpha(t - x_{inj}) x_1$
$= 0.05 + 0.5(-1 - 0.55) \times 1$
$= -0.725$

$w_{12} \Rightarrow 0.1 + 0.5(-1 - 0.45) \times 1$
$\Rightarrow -0.625$

$b_1(n) = b_1(old) + \alpha(t - z_{in1})$
$= 0.3 + 0.5(-1 - 0.55)$
$= -0.475$

$w_{21} = 0.2 + 0.5(-1 - 0.55) \times 1$
$= -0.575$

$w_{22} = 0.2 + 0.5(-1 - 0.45) \times 1$
$= -0.525$

$b_2(n) = 0.15 + 0.5(-1 - 0.45)$
$= -0.575$

for 2nd sample, $x_1 = 1$, $x_2 = -1$, $t = 1$

$z_{in} = b_1 + x_1 w_{11} + x_2 w_{21}$
$= -0.475 + 1 \times (-0.725) + (-1)(-0.575)$
$= -0.625$

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$z_{in2} = b_2 + x_1 w_{12} + x_2 w_{22}$

$= -0.575 + 1(-0.625) + (-1)(-0.525)$

$= -0.675$

$z_1 = f(z_1) = f(-0.625) = -1$

$z_2 = f(z_2) = f(-0.675) = -1$

Apply net of y,

$y = f(y_{in}) = f(-0.5) = -1$

$t \neq y$, apply weight updation since $t = 1$, weights are updated on z_1 & z_2 where net input is close to 0,

$\therefore$ apply weight updation on z_1

$w_{11}(n) = w_{11}(old) + \alpha(t - z_{in})x_1$

$= -0.725 + 0.5(1(-0.625))\times 1$

$= 0.0875$

$w_{21} = -0.575 + 0.5(1 - (-0.625))\times -1$

$= -1.39$

$b_1(n) = 0.3 + 0.5(1 - (-0.625))$

$= 0.34$

$w_{12}(n) = w_{12}(old) = -0.625$

$w_{22}(n) = -0.525, \quad b_2(n) = -0.575$

epoch 1

x_1	x_2	t	x_{in}	z_{in}	x_1	x_2	y_{in}	y	w_{11}	w_{21}
1	1	-1	0.55	0.45	1	1	1.5	1	0.725	-0.88
1	-1	1	-0.675	-0.675	-1	-1	0.5	-1	-0.825	-1.39
-1	1	1	-0.475	-0.475	-1	-1	-0.5	-1	0.0875	-1.39
-1	-1	-1	1.312	1.3125	1	1	1.5	1	1.8065	-0.069

b_1	w_{12}	w_{22}	b_2
-0.475	-0.625	-0.525	-0.575
0.34	-0.626	-0.525	-0.575
0.34	-1.3625	0.2125	0.1625
-0.98	-0.207	1.369	-0.994