Set Theory Introduction

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Set theory is the study of sets. Which beg the question, what is a set? A set is a collection of distinguishable elements. The elements are called either *members* or *elements*.
For example, "Animals" could be a set. We could arbitrarily define Animals as

$$\{donkeys, crows, frogs\}$$

If we were to say are "tables an animal" we could both point to the linguistic definition (A table is definitely not an animal) or we could go to the set itself and say that a table is not a member of that set
This could be defined as follows:

$$\text{crows} \in \text{animals}$$

$$\text{table} \notin \text{animals}$$

More broadly, in math, every number is a part of the set. The most common ones are listed here:
$\mathbb{R}$: or the set of real numbers
$\mathbb{N}$: or the set of natural numbers
$\mathbb{Z}$: or the set of Integers
$\mathbb{C}$: or the set of Complex numbers
$\mathbb{Q}$: or the set of Natural numbers
$\mathbb{P}$: or the set of Irrational numbers (not used as often)
$\mathbb{W}$: or the set of Whole numbers

Similiar to how all squares are rectangles, we can say "All Whole Numbers are Integers" or "Whole Numbers are a subset of Integers"

$$\mathbb{W} \subset \mathbb{Z}$$

Theorem 1. *This is a theorem*

Proof. For the sake of contradiction suppose $\sqrt{2}$ is rational. Write $\sqrt{2} = a/b$ with a, b positive integers with gcd 1. Then $2 = a^2/b^2$, so $a = 2k$ is even. Then $2 = 4k^2/b^2$ so that $b = 2k^2$, implying b is even. This contradicts that a, b have gcd 1. □

Some random facts in a list:

- Compared with the "itemize" environment in LaTeX, itemize* has smaller separation between bullet points.
- The nth Catalan number is $C_n \stackrel{\text{def}}{=} \frac{1}{n+1}\binom{2n}{n}$.
- If $\pi(x)$ is the number of primes less than or equal to x, then

$$\lim_{x\to\infty} \frac{\pi(x)}{x/\ln(x)} = 1.$$

- $$\sum_{\substack{1\le i\le 2n \\ i \text{ even}}} i = \frac{2n(n+1)}{2}.$$

Have fun on your problem sets.