AERO 4630 - Aerospace Structural Dynamics

Project 1

Assigned: Friday, January 25 2019
Due: Friday February 8 2019 at 17:00, uploaded as PDF on Canvas

Office Hours: Davis 335, Wednesdays 1300-1400 hrs

Problem 1: Method of manufactured solutions

A common way to verify that your code is working is to start with a known solution and then try to obtain that solution using your code. This is called the *method of manufactured solutions*. Let's look at the original Poisson equation

$$-\nabla^2 u(x) = f(x), \quad x \in \Omega \tag{1.1}$$
$$u(x) = u_D(x), \quad x \in \partial\Omega \tag{1.2}$$

In the class, we did the example where

$$u_D = 1 + x_1^2 + 2x_2^2, \quad f(x) = 6.0 \tag{1.3}$$

The solution to the Laplace equation in this case is

$$u(x) = 1 + x_1^2 + 2x_2^2 \tag{1.4}$$

We can verify this by computing the laplacian of $u(x)$ as

$$\frac{\partial^2 u}{\partial x_1^2} + \frac{\partial^2 u}{\partial x_2^2} = 2 + 2 \times 2 = 6 = -f(x) \tag{1.5}$$

Let's try this out for more cases.

(1a) Try $u_{known}(x) = u_D(x) = x_1^3$ and $f(x) = -6x_1$. First show that this satisfies the Poisson equation. Next, Edit the python script and show results generated by Paraview. Report the errors.

(1b) Repeat the above with $u_{known}(x) = u_D(x) = x_2^3$ and $f(x) = -6x_2$.

Problem 2: Improving accuracy

Let's go back to the above problem and take

$$u_{known}(x) = u_D(x) = x_1^3, \quad f(x) = -6x_1 \tag{2.1}$$

You must have noticed error of the order to ~ 0.001 or more. Let's try to improve this.

(2a) One way of improving the error is to make your mesh higher resolution. Try increasing the mesh from 8×8 to 16×16. Has the error reduced? Plot the L2 error norm and max error for increasing number of elements: $4, 8, 16, 32$ and 64. Include the plots in your submission.

(2b) Another way to improve is to change the shape function. So far we have been dealing with Lagrange Polynomial of order 1. These are basically linear functions. If the solution u_{known} is cubic, the linear function might not do a good job approximating it. Increase the order of the polynomial. Plot the L2 error norm and max error for increasing order: $1, 2, 3, 4$ and 5 These simulations will take time... be patient.