AERO 4630 - Aerospace Structural Dynamics

Project 5

Assigned: Wednesday April 10 2019
Due: Friday April 24 2019 at 17:00, uploaded as PDF on Canvas

Office Hours: Davis 335, Wednesdays 1300-1400 hrs

Problem 1: Composite Beams

Let's look at a composite beam as shown in the figure below.

The overall dimensions are $L = 3m$, $W = 0.1m$, $H = 0.1m$. The beam is made of three layers of equal width. The top and bottom layers are made of steel $E = 200GPa$, $\nu = 0.3$, $\rho = 7960kg/m^3$. The middle layer is softer and made of copper $E = 100GPa$, $\nu = 0.3$, $\rho = 8960kg/m^3$. The left end $x = 0$ is clamped. We are first applying a traction of $T = 10^5 N/m^2$ on the entire right *face* $x = L$ in the $-y$ direction. Once the beam is bent, we are letting it go making the beam vibrate.

(1a) Plot the displacement profile (in meters) as a function of time for points $(L, W/2, H/2)$ over at least 10 vibration periods and compute the frequency of oscillation. *Hint*: Choose appropriate non-dimensionalization method.

(1b) Let's make the middle layer thicker $3W/5$ and outer steel layers thinner $W/5$. Repeat the above analysis. Did the frequency and overall amplitude increase or decrease?

Problem 2: Vibrations of a composite plate

Consider a thin plate ($L = 1m$, $W = 1m$, $H = 0.01m$) clamped at all side faces $x = 0$, $x = L$, $y = 0$ and $y = W$. The plate is made of two different materials. The middle patch $0.4L \leq x \leq 0.6L$, $0.4W \leq y \leq 0.6W$ is made of Copper while the remaining parts are made of steel. You can use the properties in the previous question for E, ρ and ν. We are first applying a traction of $T = 10^5 N/m^2$ in a small middle part $0.49L \leq x \leq 0.51L, 0.49W \leq y \leq 0.51W, z = H$ in the downward $(-y)$ direction. Once the plate is deformed, we are letting it go making it vibrate.

(2a) Plot the displacement profile (in meters) as a function of time for points $(L/2, W/2, H)$, $(L/4, W/2, H)$, $(3L/4, W/2, H)$, $(L/2, W/4, H)$ and $(L/2, 3W/4, H)$ over at least 10 vibration periods and compute the frequency of oscillation for each point.

(2b) Let's vary the middle patch to be $(0.35L \leq x \leq 0.55L, 0.35W \leq y \leq 0.55W)$ and $(0.3L \leq x \leq 0.6L, 0.3W \leq y \leq 0.6W)$. Repeat the above analysis. Did the frequency and overall amplitude of the points increase or decrease?

