AERO 4630 - Aerospace Structural Dynamics
Project 3

Assigned: Friday, February 22 2019
Due: Friday March 08 2019 at 17:00, uploaded as PDF on Canvas

Office Hours: Davis 335, Wednesdays 1300-1400 hrs

Problem 1: Clamped beam with initial force

Let's look at the same problem of clamped beam we did in the last project. We are looking at a beam of length $L = 1m$, height $H = 0.2m$ and width $W = 0.2m$. The properties are given by $E = 200 GPa$, $\nu = 0.3$ and $\rho = 7800 kgm^{-3}$. The beam is clamped at ends $x = 0$ and $x = L$.

(1a) The governing equation is

$$\text{div}\, \boldsymbol{\sigma} + \mathbf{f} = \rho \ddot{\mathbf{u}} \tag{1a.1}$$

where we are solving for displacement vector $\mathbf{u}$ at every point as a function of time. The stress tensor is given through our usual linear elasticity laws

$$\boldsymbol{\sigma} = \lambda\, \text{tr}(\varepsilon)\, \mathbf{I} + \mu \left(\varepsilon + \varepsilon^T\right) \tag{1a.2}$$

where λ and μ are the Lame' parameters given in terms of E and ν. The strain tensor ε is defined as

$$\varepsilon = \frac{1}{2}\left(\text{grad}\, \mathbf{u} + (\text{grad}\, u)^T\right) \tag{1a.3}$$

First, re-write the equations in the non-dimensional form. Just like we did in class, you can non-dimensionalize x, y and z as

$$\tilde{x} = \frac{x}{L}, \quad \tilde{y} = \frac{y}{L}, \quad \tilde{z} = \frac{z}{L} \tag{1a.4}$$

and time as

$$\tilde{t} = t/t_{char}, \quad t_{char} = L/c, \quad c = \sqrt{\frac{E}{\rho}} \tag{1a.5}$$

Derive the non-dimensional equation and obtain its weak form like we did in class.

(1b) Now as a first step, we are imposing a force $F = 100N$ at the top face $y = W$ on a small patch $0.48L < x < 0.52L$, $0.3H < z < 0.7H$. We are allowing the beam to deform quasi-statically (meaning, no time dependence for this one). Obtain the deflection of the beam and include the deformed profile (use warp by vector filter).

(1c) Now we are going to let go of this force. We know that the beam should vibrate. Use the deflection obtained earlier as the initial condition and obtain the vertical vibrations of the beam. What's the time period of the oscillation?

(1d) Change the width W and height H of the beam to $W_{new} = \alpha W$ and $H_{new} = \beta H$, where $0 < \alpha < 1$ and $0 < \beta < 1$. Plot the natural frequency of the system as a function of α, keeping β fixed. Repeat this for β, keeping α fixed.