Differential Privacy

27 April 2021 09:05

Lots of data, lots of "analysis" one can do.

Eg:
{ Understand causality b/w smoking and lung disease. }

↓ Population wide studies.

Q:
{ How to conduct useful population-wide studies / "data analysis" without compromising individual privacy }

↓

Means users are incentivized to join the study

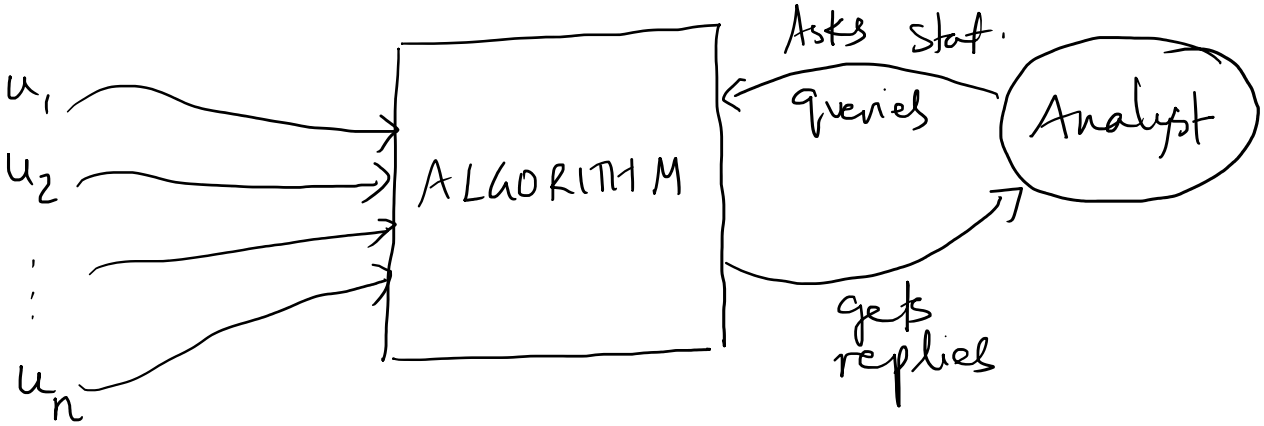
[Collectively we can learn something, but individually don't lose anything]

{ different from "CRYPTOGRAPHY" }
Which is like a vault + key.

{ Here we want to contribute data but the analyst learns ... "..." }

but the analyst learns nothing "private"

INITIAL ATTEMPT AT MODELING PRIVACY



Attempt (1)

Want the analyst to not learn anything new about any individual.

Natural Issue

You will learn something new from the answer of the query.

Example

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Imagine each $u_i = (\#\text{left feet}, \#\text{right feet})$

- Analyst Asks $Avg(\#\text{left ft} - \#\text{right ft})$
- Reply (likely) = 0

⇒ Analyst "learns" that u_1 has equal # of left & right ft.

(Maybe need few more queries to get $Min(\#\text{left ft})$, $Min(\#\text{right ft})$, etc.

- Why is this definition vacuous?

A: what we learnt is something "global". Nothing specific about the particular individual

Q: How do we capture "Individual Privacy"?

[The earlier definitions of privacy is broken]

DMNS - Dwork, McSherry, Nissim, Smith

[Differential Privacy]

[What if the Algorithm guarantees that the answer to the query is "almost" the same regardless of whether u_i (or any fixed individual) is present in the input or not?]

The analyst can't even distinguish if u_i was part of the study or not, so how can he/she learn anything about u_i's data?

The Differential Privacy Model

Input: Database X of 'n' rows, each corr. to a user; fn f.

Algo takes X and outputs $\tilde{f}(X)$

[it can be scalar output, vector output, etc]

f is the statistical query

$\tilde{f}$ is the output.

$\tilde{f}$ is the output.

$\tilde{f}(x)$ can be some "noisy / approximate" response to $f(x)$.

If x and x' are two databases which differ in a single row, then want

$$\boxed{\tilde{f}(x) \text{ "}\approx\text{" } \tilde{f}(x')} \quad (1)$$

↑ guarantees privacy.

and

(2) $|f(x) - \tilde{f}(x)|$ is "small" for all x

↑ guarantees utility of study.

Need to formalize "$\approx$" meaning and "small"

{ Just (1) is easy to satisfy:
Output $\tilde{f}(x) = 0$ always.
Full privacy, No utility }

{ How to get both together? }

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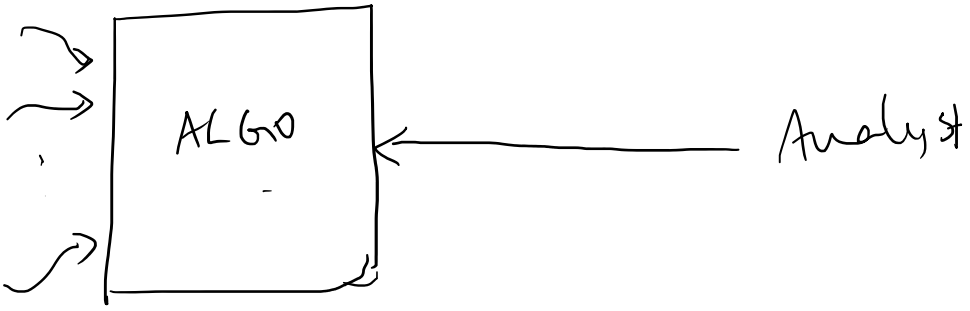
There are many simple queries for which we need to introduce noise, else we break privacy.

Ex ①



All Microsoft Employees SALARY details

Analyst asks " count # people with Salary ≥ 2M $ "

Sps Algo replies (1).

{ Then analyst can learn that CEO's salary ≥ 2M $. }

Output will be 0 if CEO doesn't take part in survey.

take part in survey.

⇒ CEO's privacy is compromised acc. our definition

Is this just a 1 vs 0 issue ?

Perhaps not.

Sps Answer = 1000.

and tomorrow, the answer is 1001.

Maybe Microsoft hired a new employee,

⇒ {likely that this person has salary ≥ 2M $.}

we may learn something about the new individual.

So our model / definition captures these issues well

⇒ we must add some noise to $f(x)$

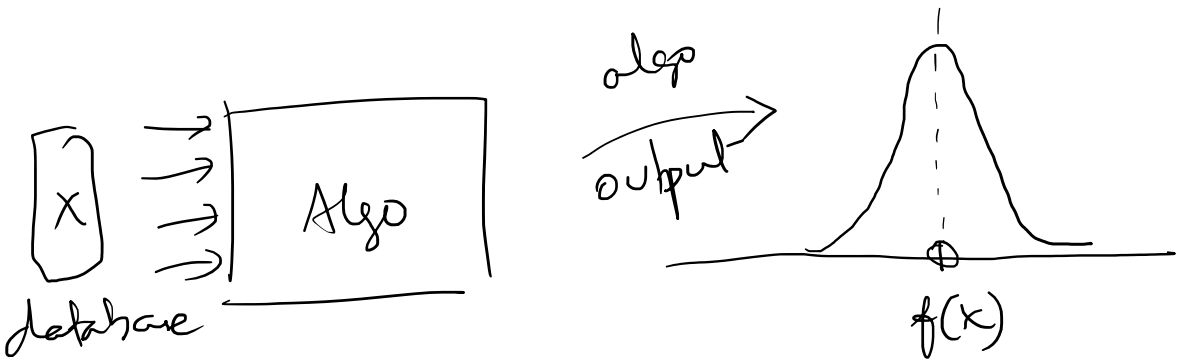
So that

$\tilde{f}(x) \approx \tilde{f}(x')$ for all

$f(x) \approx f(x')$ for all neighboring datasets.

Model formally

is randomized, adds noise acc. distribution



Distribution of $\tilde{f}(x)$.

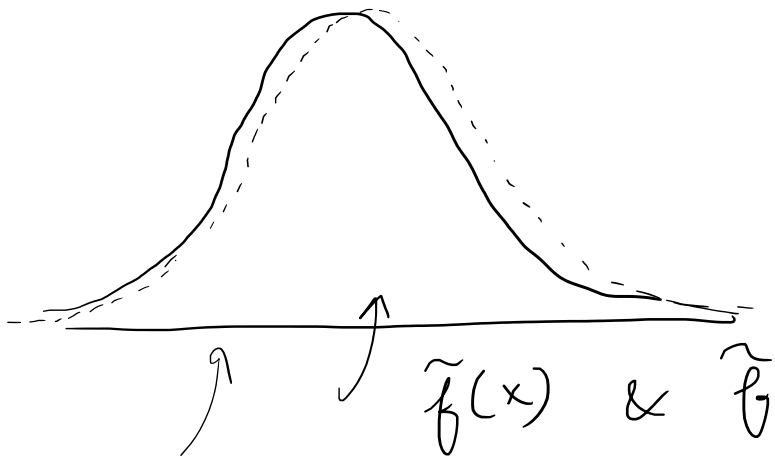
Error of Algo

$$\underset{\text{random choices}}{E}\left[\left(\tilde{f}(x) - f(x)\right)^2\right]$$

Privacy Requirement :-

$\forall\ x, x'$ differing in one row, want distributions to be nearly identical.

nearly identical.



$\tilde{f}(x)$ & $\tilde{f}(x')$ distribution.

$\forall\ x, x'$, and for all subsets R of possible outputs of $\tilde{f}$,

$$\boxed{\Pr\left[\tilde{f}(x) \in R\right] \leq e^{\epsilon} \Pr\left[\tilde{f}(x') \in R\right]}$$

e^{ϵ} like $(1+\epsilon)$

ϵ - Differential Privacy.

Goal:

[How to answer queries with ϵ - DP, with MIN. ERROR?]

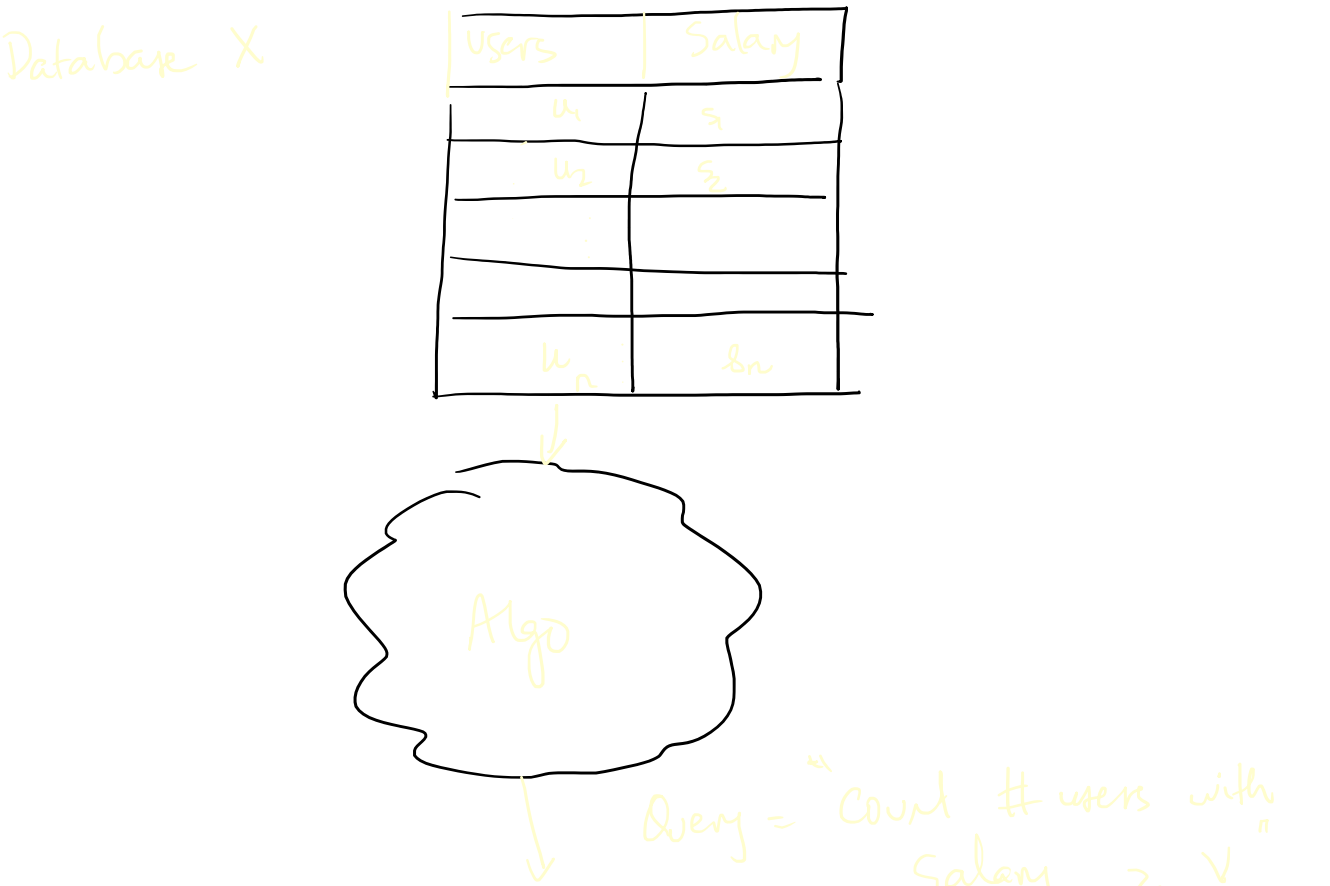
↓

TOMORROW

Such a scheme for "counting" queries

Counting Queries

30 April 2021 11:59



where R is some suitable noise.

eg $R \sim N(0, \sigma^2)$ for suitable σ will give privacy with low error for suitable parameters

Our Algo [Technical Difference]

Add noise z w.p $\propto e^{-\frac{|z|}{\sigma}}$

(in contrast gaussian noise has prob $\propto e^{-z^2/\sigma^2}$)

"LAPLACIAN DISTRIBUTION"

$$f_R(z) = \frac{1}{2\sigma} e^{-\frac{|z|}{\sigma}}$$

Check ① $\int_{-\infty}^{\infty} f_R(z)\, dz = 1$

② $\int_{-\infty}^{\infty} z\, f_R(z)\, dz = E[R] = 0$

③ $\int_{-\infty}^{\infty} z^2 f_R(z)\, dz = E[R^2] = 2\sigma^2$

$\int_{-\infty}^{\infty} f_R(z)\, dz = 1$

eg, Integration by parts

If we add noise acc. Laplacian(σ), what is the squared error like?

$$\tilde{f}(x) = f(x) + R$$

$$\Rightarrow \text{error} = E\left[\left(\tilde{f}(x) - f(x)\right)^2 \right]$$

$$= E\left[R^2 \right] \quad \text{where } R \sim \text{Lap}(\sigma)$$

$$= 2\sigma^2$$

Want to set σ sufficiently large to get desired privacy.

$\forall$ x, x' differing in a row, and any subset S of output values

$$\left\{ \begin{array}{l} \text{want} \\ \Pr\left[\tilde{f}(x) \in S \right] \leq e^{\varepsilon} \Pr\left[\tilde{f}(x') \in S \right] \end{array} \right.$$

Privacy Requirement

We'll instead set noise such that

We'll infect set noise such that the pdf of Algo output for X and X' are very similar.

Fix an output value 't'.

Let $f_{Alg}(X, t)$ = PDF of Alg outputting t on input X

$$= \frac{1}{2\sigma} \exp\left(-\frac{|f(X) - t|}{\sigma}\right)$$

Similarly,

$$f_{Alg}(X', t) = \frac{1}{2\sigma} \exp\left(-\frac{|f(X') - t|}{\sigma}\right)$$

$$\Rightarrow \frac{f(X, t)}{f(X', t)} \leq e^{-\frac{|f(X) - f(X')|}{\sigma}}$$

$$\leq e^{-\frac{1}{\sigma}}$$

So we can set $\sigma = \frac{1}{\varepsilon}$ 

$\forall x, x', t$ (neighboring databases)

$$\frac{f(x,t)}{f(x',t)} \leq e^{\varepsilon}$$ ← Privacy

And $E\left[\left(\tilde{f}(x) - f(x)\right)^2\right] \leq \frac{2}{\varepsilon^2}$.

↑ Utility

Only thing we used in proof is how much f can change from $x \rightsquigarrow x'$.

↓ SENSITIVITY of fn.

$$\Delta_f = \max_{x, x' \text{ differing in one row}} |f(x) - f(x')|$$

Noise will simply depend on Δ_f by setting σ appropriately to ensure ϵ-Privacy.

SUMMARY

Simple scheme which works not just for counting queries, but for any low-sensitivity function

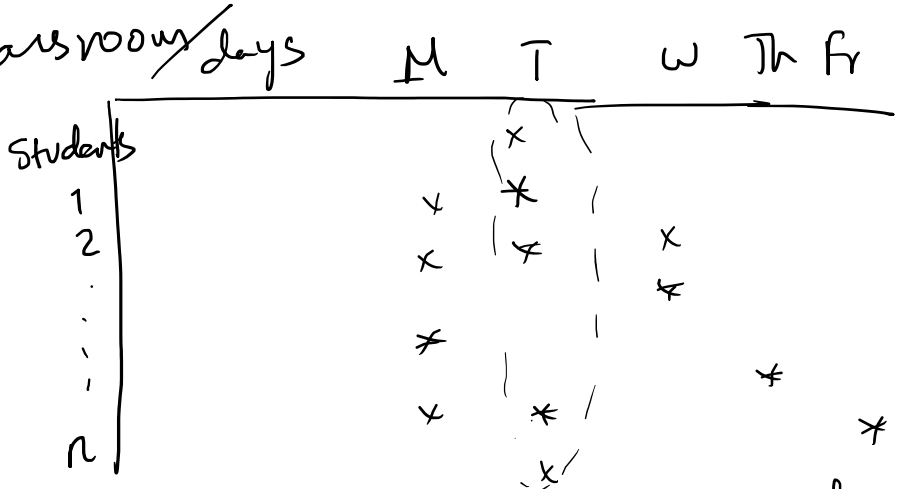
any low-sensitivity function

Algo is called "Laplace Mechanism"

Similarly, Gaussian Mechanism also exists (w/ gaussian noise).

In above example, query answer was a numerical value. What if its not?

Example



Each student has a preference for when to conduct exam.

We want to select "most popular day" in a differentially private manner.

Q1. - Can't simply add noise, (Meaningless)

Q1:- Count simply add noise, (Meaningless)

Q2:- How to measure utility of a scheme.

Privacy is easy to extend

$$\frac{Pr[\text{Alg selects Monday for } X]}{Pr[\text{" " " } X']} \leq e^{\varepsilon}$$

Similarly for each other day.

How to get utility?

{ Want output to be a popular day if not the most popular day. }

Idea

For each possible output (days in our example)

Day	1	2	3	4	5
	n_1	n_2	n_3	n_4	n_5

Compute # people who prefer

Output day 'i' as answer with prob $\propto e^{\varepsilon \cdot n_i}$

$$\frac{}{\sum e^{\varepsilon n_i}}$$

$\Rightarrow$ Intuitively popular days are more likely to be output.

Privacy + Error Analysis

For any day i and inputs x and x'

$$\frac{\Pr[\text{Alg selects } i \text{ for } x]}{\Pr[\text{Alg selects } i \text{ for } x']}$$

$$= \frac{e^{\varepsilon n_i(x)}}{\sum_j e^{\varepsilon n_j(x)}} \cdot \frac{\sum_j e^{\varepsilon n_j(x')}}{e^{\varepsilon n_i(x')}}$$

$$= e^{\varepsilon(n_i(x) - n_i(x'))} \cdot \frac{\sum_j e^{\varepsilon n_j(x')}}{\sum_j e^{\varepsilon n_j(x)}}$$

Since x and x' differ in a single row,

each $-1 \leq n_i(x) - n_i(x') \leq 1$

for all i.

Overall, $\frac{\Pr[\text{Alg output } i \text{ on } X]}{\Pr[\text{Alg output } i \text{ on } X']} \le e^{\varepsilon} \cdot e^{\varepsilon} = e^{2\varepsilon}$.

Satisfies 2ε- Differential Privacy.

What about error?

Let database has n people and suppose $n_1 = OPT$ is the day with largest count.

Ideally: Want Alg to output a day with count close to n_1

Let's calculate

$\Pr[\text{Alg outputs a day with count} \le n_1 - t]$

Let's fix a day i with count $\le n_1 - t$

$\Pr[\text{Alg outputs this day}] = \frac{e^{\varepsilon n_i}}{\sum_j e^{\varepsilon n_j}}$

$$\leq \frac{e^{\varepsilon(n_1 - t)}}{e^{\varepsilon n_1}} \leq e^{-\varepsilon t}$$

So for $t = \frac{\log(n/\delta)}{\varepsilon}$

This probability is $\leq \delta/n$

$\Rightarrow$ Union bound over all bad days gives

$\Pr[\text{outputting a bad day}] \leq \delta$

$\Rightarrow$

① with good probability, Algo chooses a day with score $\geq$ OPT $- \frac{\log(n/\delta)}{\varepsilon}$

[very useful if OPT $\gg \frac{\log n}{\varepsilon}$]

AND

② Preserves 2ε- Privacy of users.

EXPONENTIAL MECHANISM.