Unit 2 - Relations

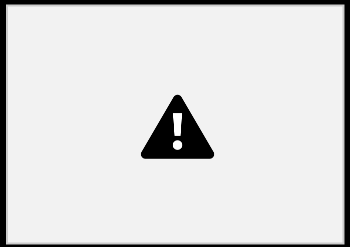
Definition :

“ a is related to b by R”

“a is note related to b by R”

R

S




Example:

Properties of Relation

Reflexive Relation

Non Reflexive relation

Irreflexive Relation

Irreflexive Relation….

(reflexive)

(irreflexive)

(irreflexive)

(reflexive)

Symmetric Relation

Asymmetric

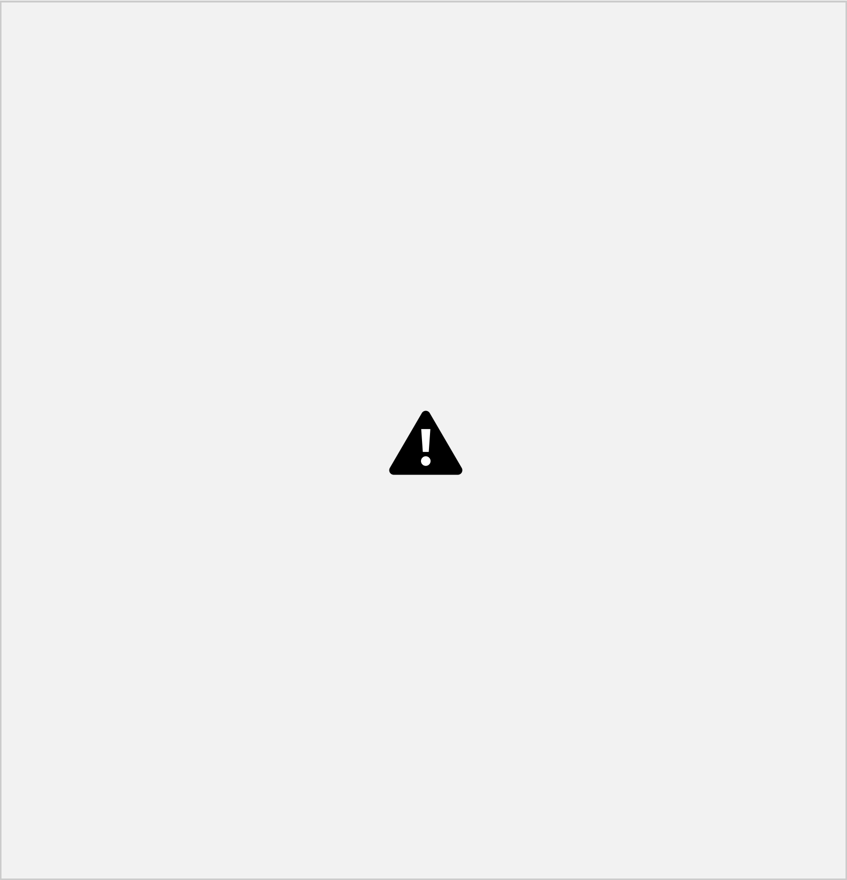
relation. (symmetric)

(symmetric)

Asymmetric Relation

Antisymmetric Relation

$(3,4),(4,3),$



Transitive Relation

Equivalence Relation (RST)

i)

ii)

iii)

Partial Ordered Relations (RAT)

Computer Recognition

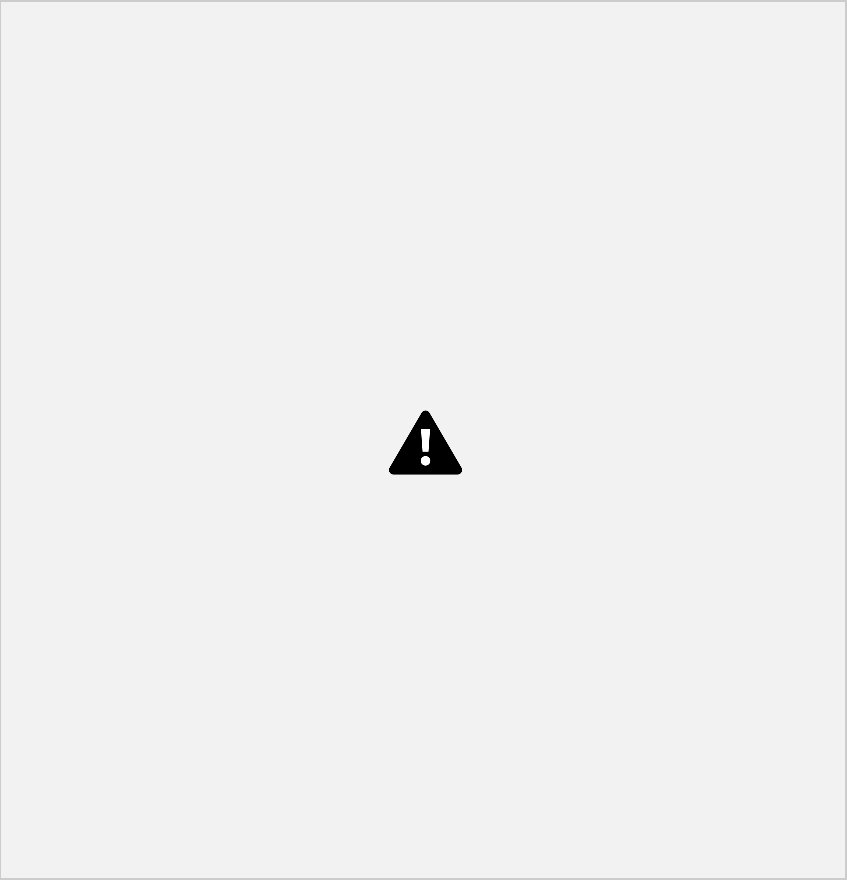
REPRESENTATION OF RELATION FOR COMPUTER RECOGNITION

Tools for representation of a relation

1.

2.

Relation Matrix (Zero-One Matrix) :



Relation Matrix (Zero-One Matrix)….

“Relation matrix”

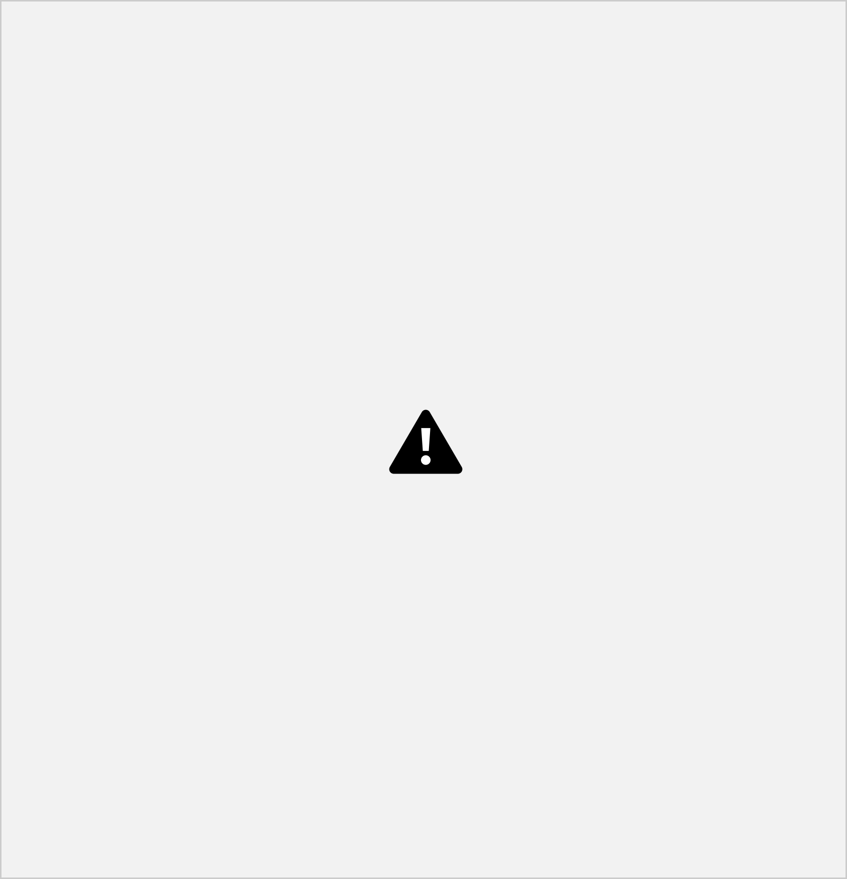
“ Zero-One Matrix ”

Rows of the matrix corresponds to the elements in set A and columns corresponds to the elements in set B

Relation Matrix (Zero-One Matrix)….

	p	q
0		
1		
2		

M(R)=



Directed Graphs (Digraphs) :

Vertex Set

Edge set

Directed Graphs (Digraphs) : Example :

Directed Graphs (Digraphs) :…..

<u>Isolated Vertex</u>

<u>Self-loop</u>

<u>In-degree</u>

<u>Out</u>

<u>degree</u>

Problems :

1.

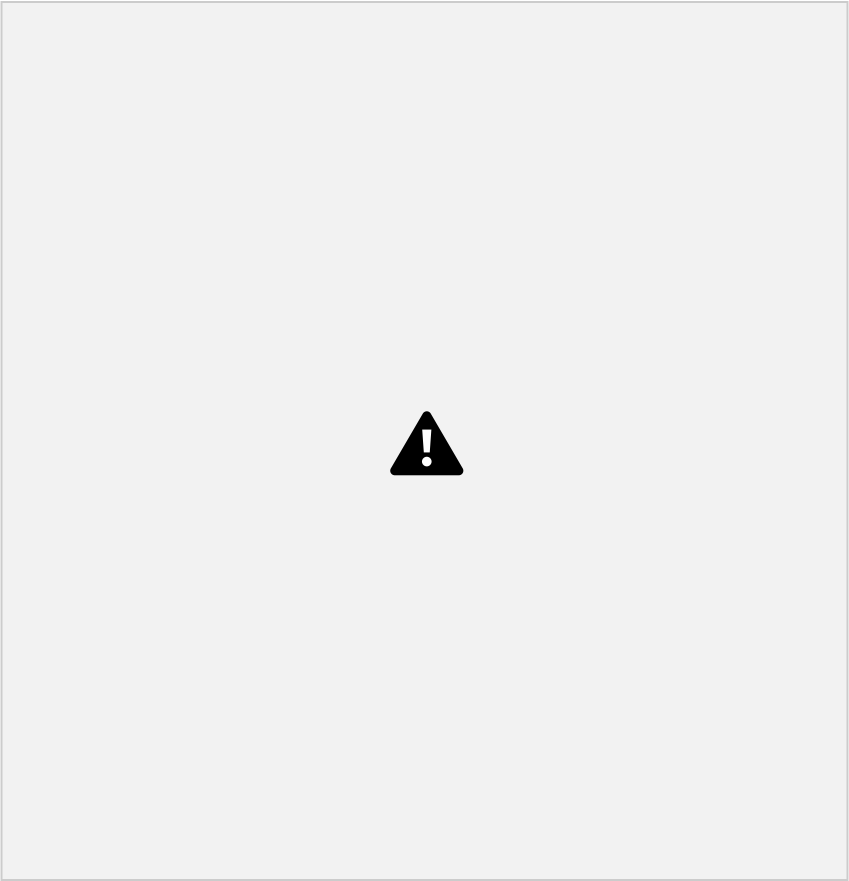
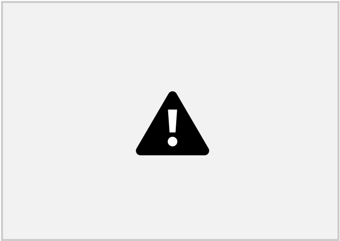
1

Out-Degree	4	2	1	1

In-Degree	1	2	3	3

Problems :….

Problems….


Representation of properties of relation using Zero-One matrix

and digraph

Reflexive Relation :

1		
	1	
		1

Diagonal elements should be 1 i.e Mij = 1 (i=j)

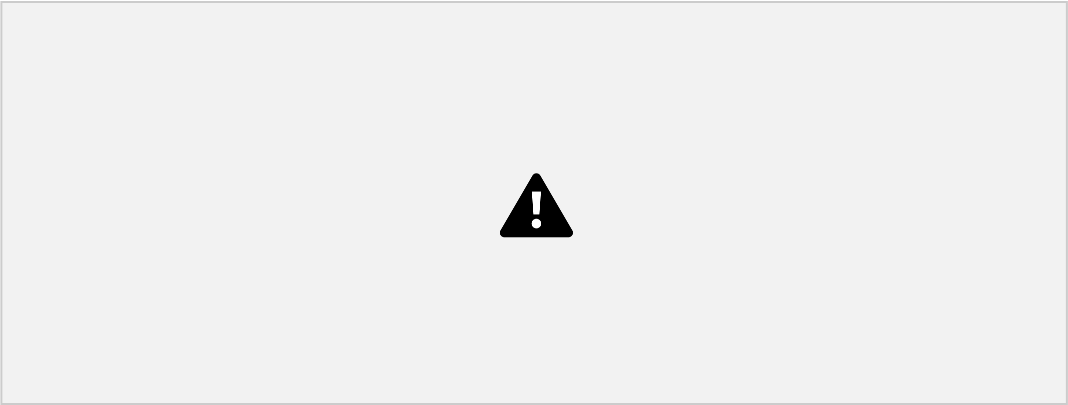
Each vertex should have self loop

Irreflexive Relation :

None of the diagonal elements should be 1 i.e mij $\neq$ 1(i=j)

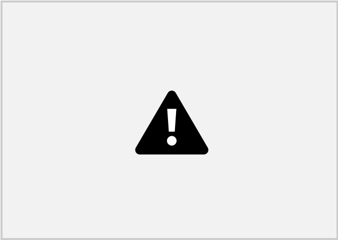
None of the vertex should have self loop

Symmetric Relation:

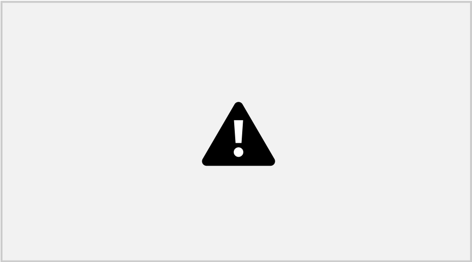


If $m_{ij} = 1$ then $m_{ji} = 1$

There should arrows in both the direction




Asymmetric Relation :

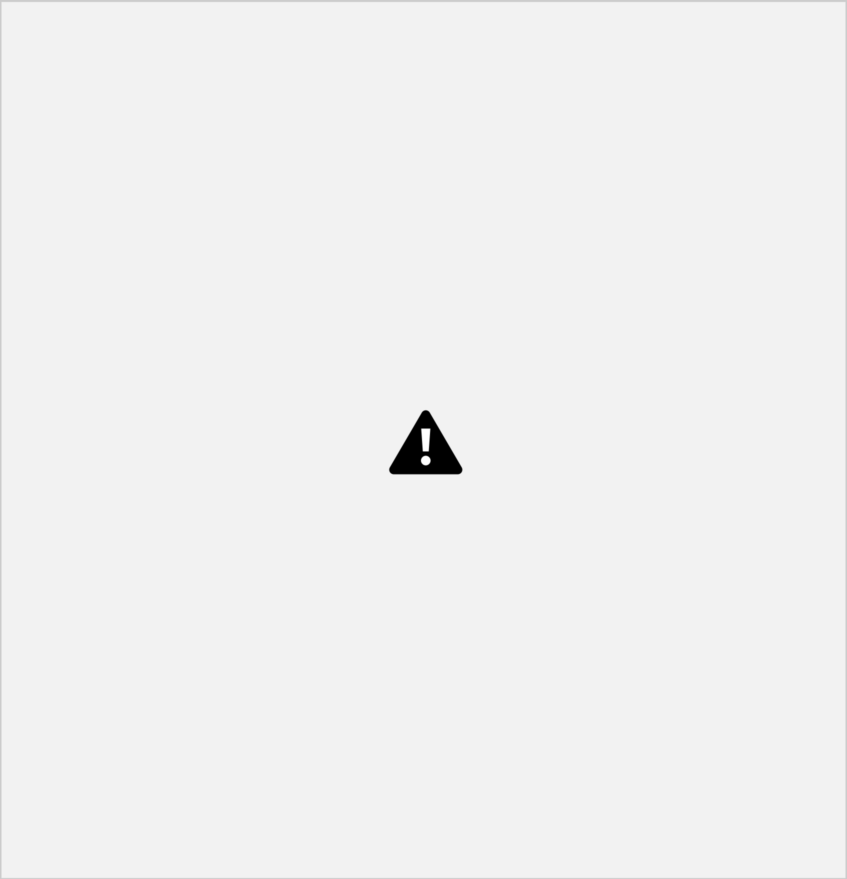


If $mij = 1$ then $mij \neq 1$

None of the pair of vertex should have bi-directional arrows

If mij = 1 then mji = 0 but mij = 1.(i=j)

None of the pair of vertex should have bi-directional arrows but any vertex can have self loop

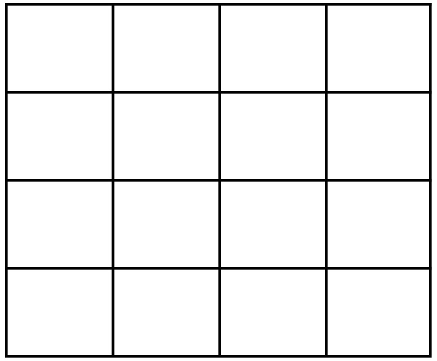


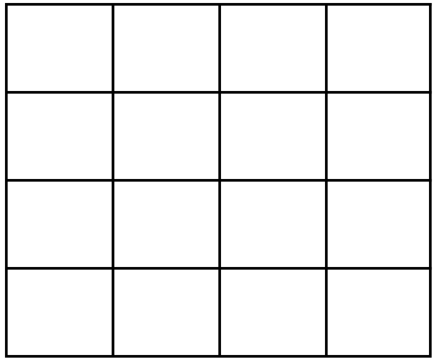
Transitive Relation :

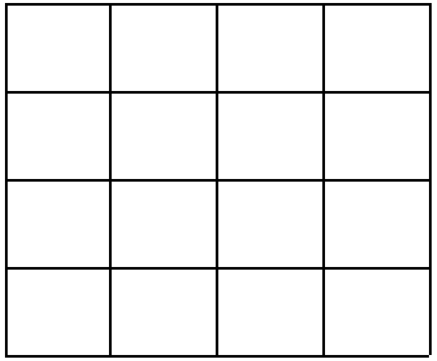
If mik = 1 and mij =1 then mij = 1

If there is a path of length greater than 1 from vertex a to b, then there is path of length 1 from a to b

Problems :







Operations on Relations :

Union of Relations : $(R_1 \cup R_2)$

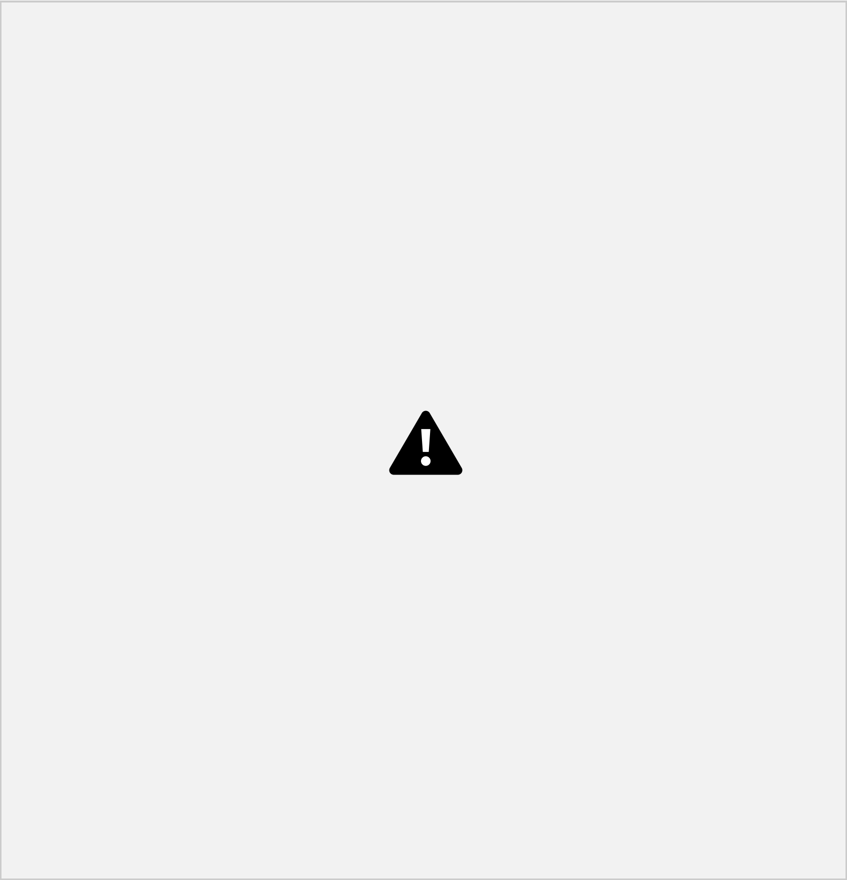
Intersection of Relations : $(R_1 \cap R_2)$

Complement of a Relation: $\overline{R}$

Converse of a Relation : R^c

Problems :

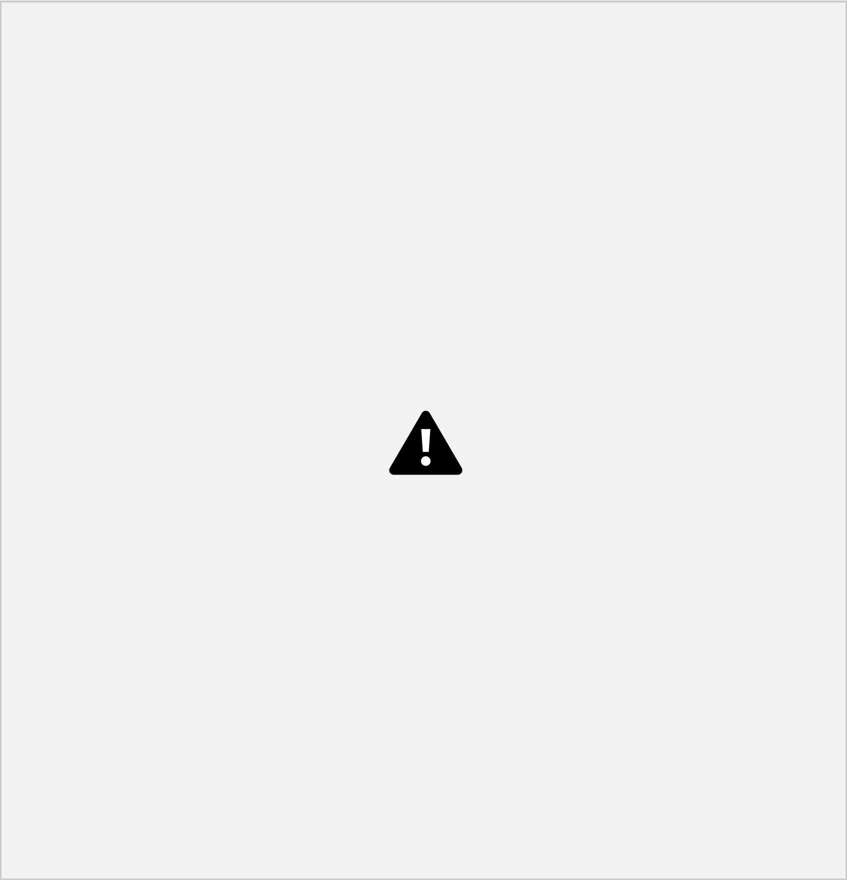
Problem….



Problems…

Solution

Problems..



Composition of Relations : ∘

-

- **product or the composition of R and S**

-

-

Composition …..

ems:

Composition …..

Problems:

a)

b)

Solution:

Problems:…

Solution :

Problems:…

Equivalence Relation , equivalence class and Partition

Equivalence Relation :

i)

ii)

iii)

Equivalence Class :

Partition of a Set :

Fundamental Theorem on Equivalence relations

1)

2)

Problems :

Solution :

Problems :

Partial Order and Hasse Diagrams(simplified Graphs)

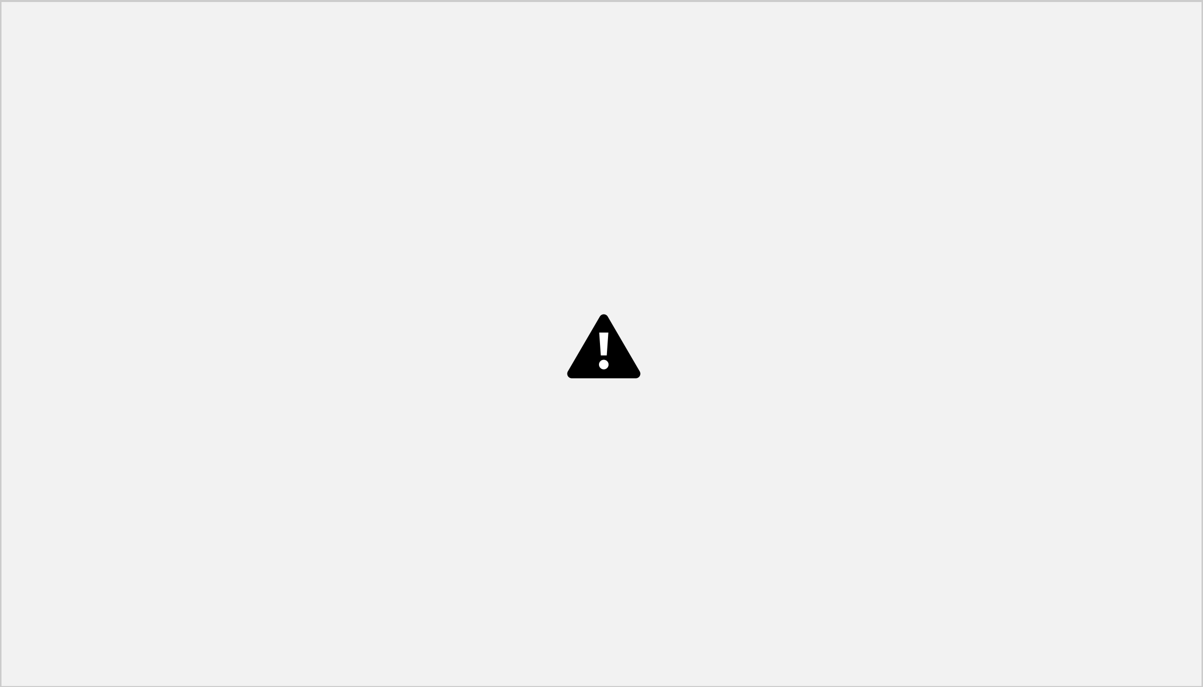
i)

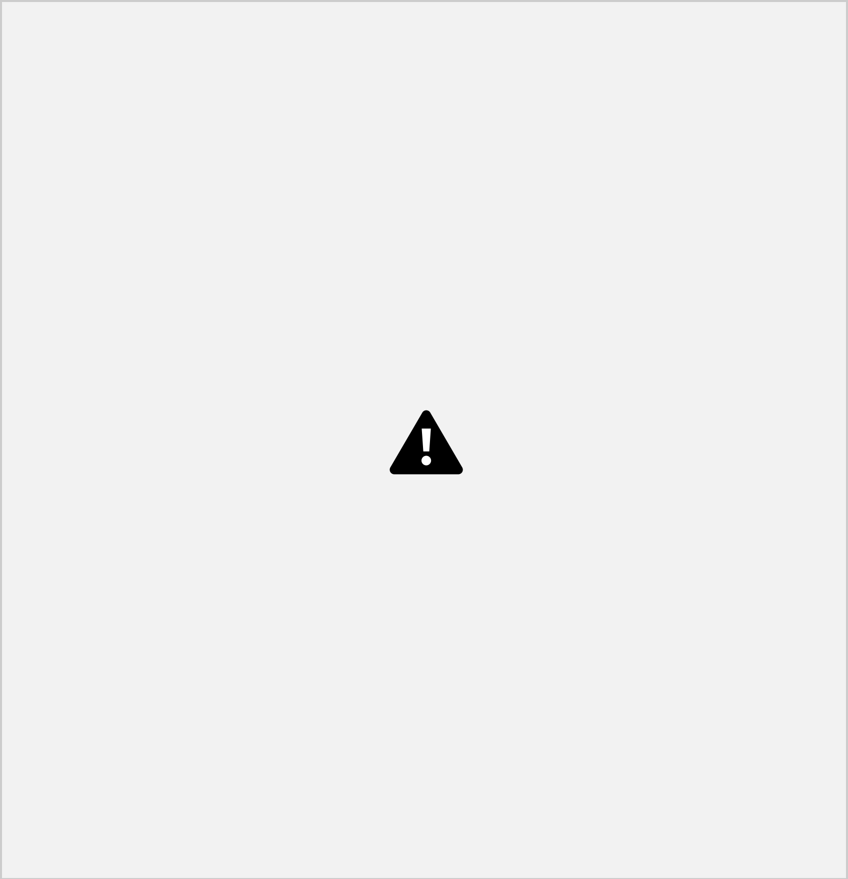
ii)

iii)

Hasse Diagram :

1.



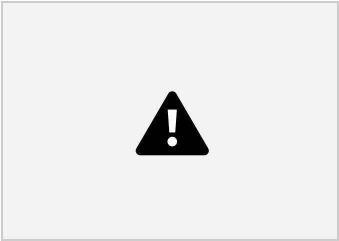


Problems :

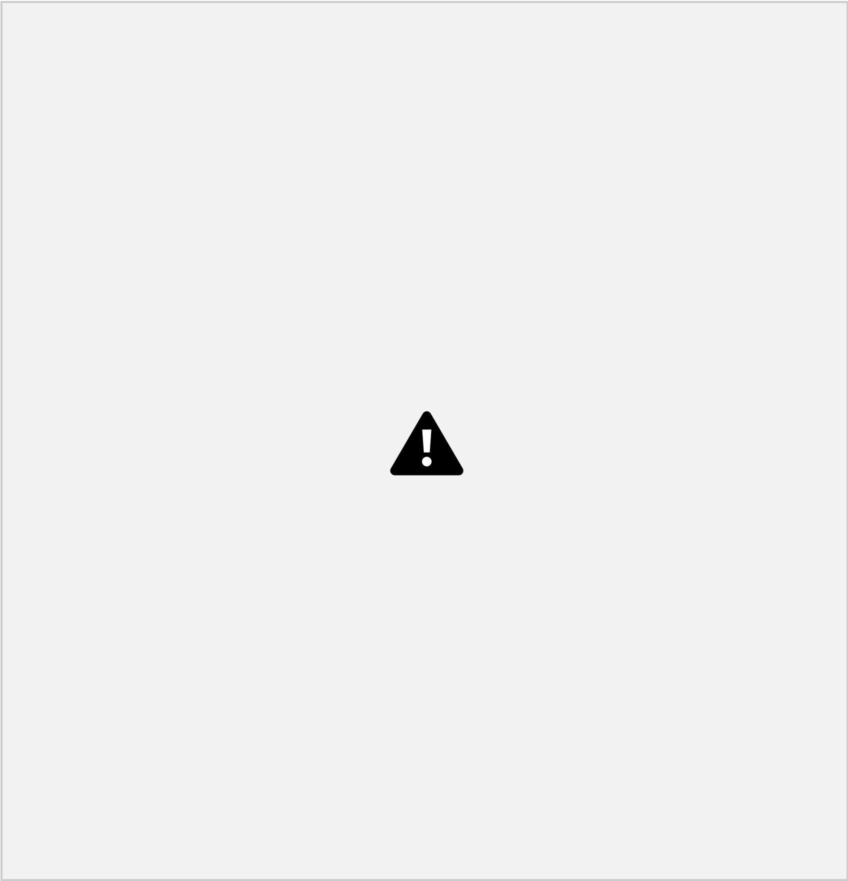
Solution :

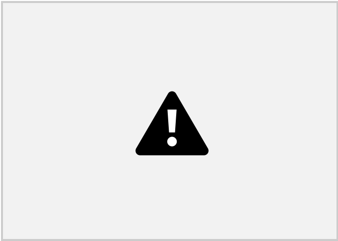
Problems:…

2.



Solution 2:

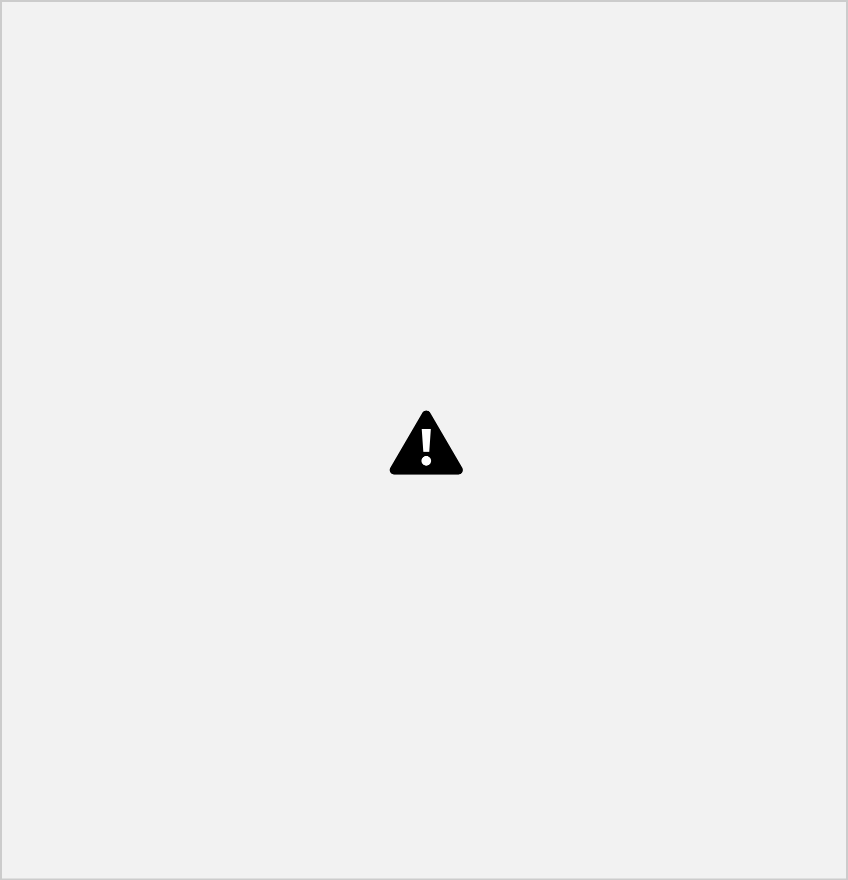




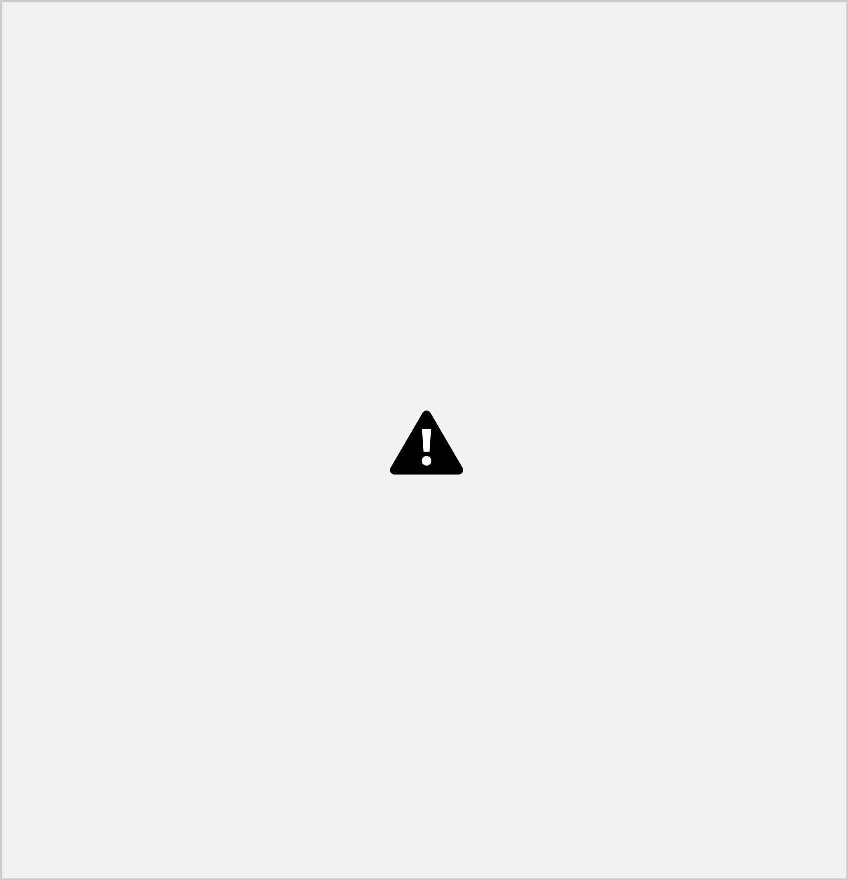
Problems:…

3.

Problems:…



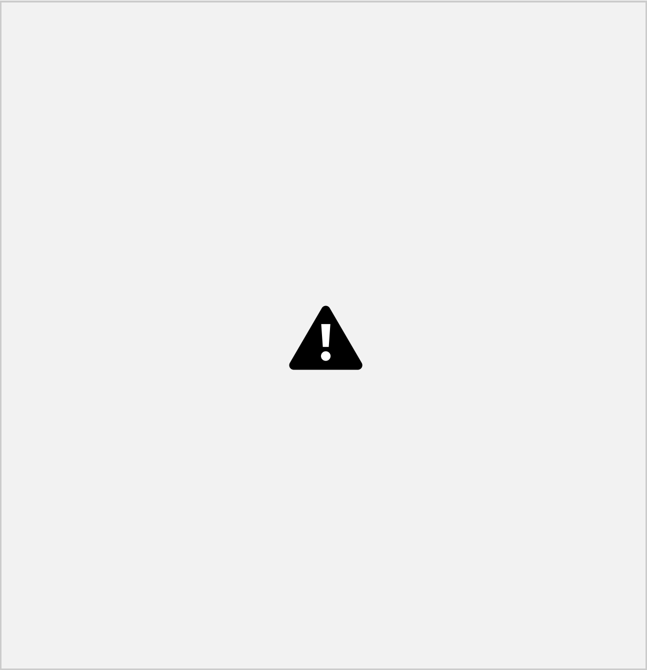
Solution 4:

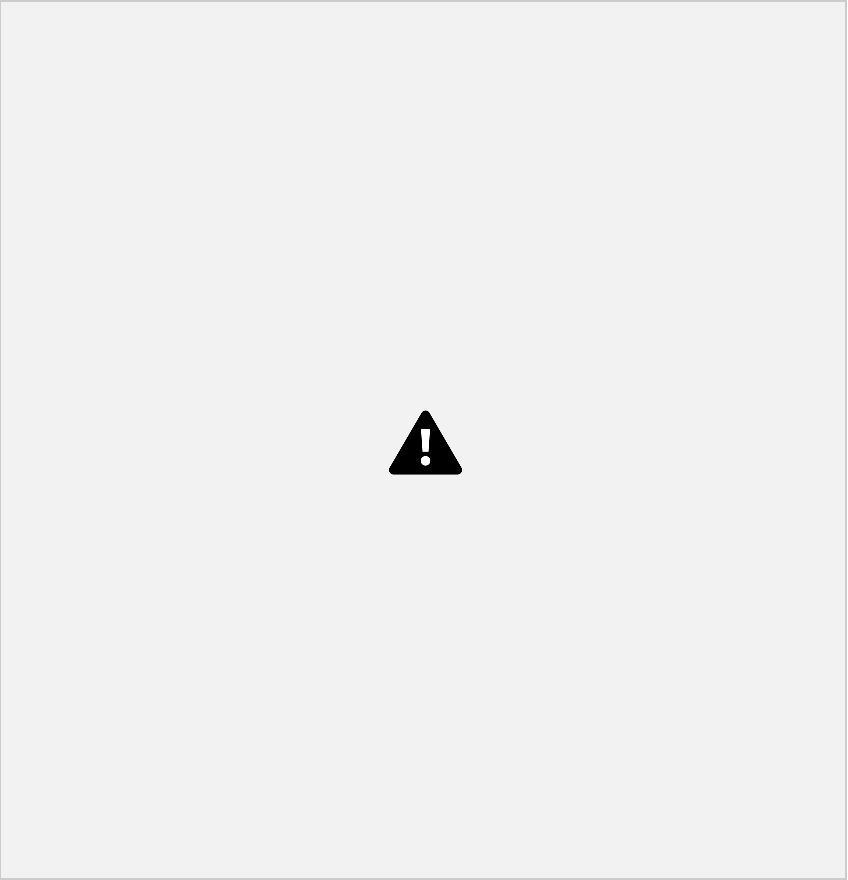


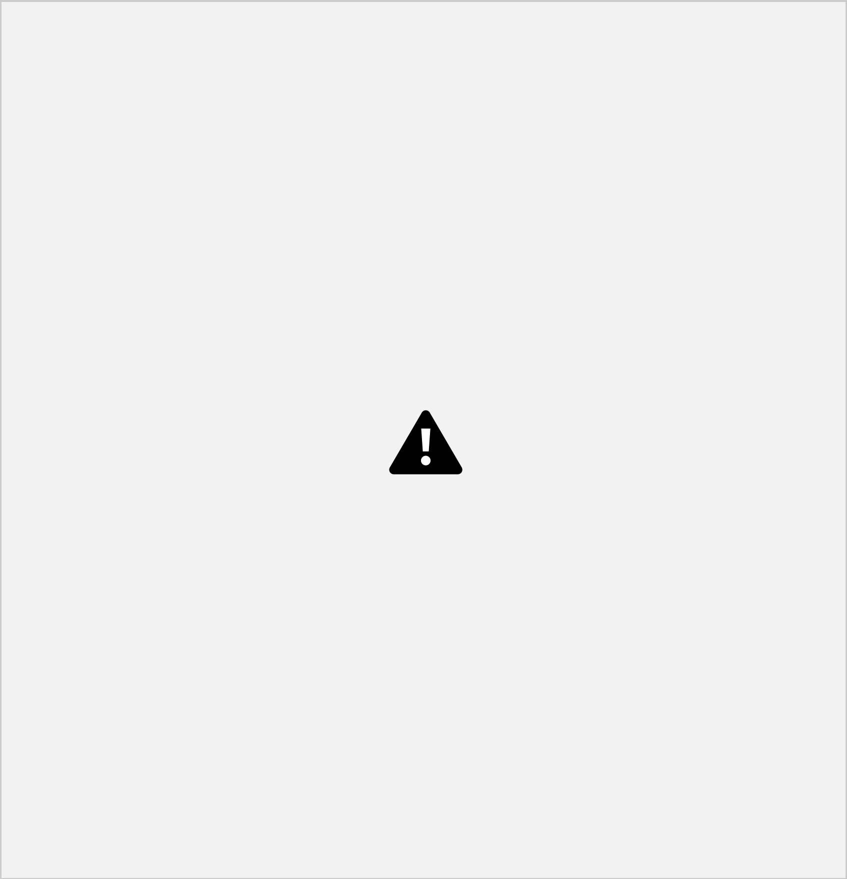

Problems:…

Problems : ..

Solution 6:







Total Order

Note : “Every Total order is a partial order, but not every partial order is a Total order”

Whether the relation “ $\leq$ ” on a set of natural numbers N is total order or not ? (N, $\leq$)