14EC505

Hall Ticket Number:

Y	1	4	A	E	C			

III/IV B.Tech (Regular) DEGREE EXAMINATION

November, 2016 **Electronics and Communication Engineering**

Fifth Semester **Digital Communications**

Time: Three Hours **Maximum :** 60 Marks

Answer Question No.1 compulsorily. (1X12 = 12 Marks)

Answer ONE question from each unit. (4X12=48 Marks)

1. Answer all questions (1X12=12 Marks)

a) Draw the Block diagram of Digital Communication System
b) What are the draw backs of Quantization Process?
c) List the properties of Matched filter.
d) Draw the signal space diagram of BFSK
e) Compare Baseband and Pass band modulation
f) Draw the Power Spectral Density of BPSK and find Bandwidth.
g) State Shannon three laws.
h) Define mutual information.
i) List the properties of Entropy.
j) Explain Properties of Cyclic codes.
k) Define Constraint length and Code rate.
l) List the properties of PN Sequence

UNIT I

2. a) Draw and explain the functional block diagram of a Delta modulation system. 6 M
   b) A DM system is designed to operate at three times the Nyquist rate for a signal with 3 KHz Bandwidth. The quantizing step size is 250 mV 6 M
      i) Determine the maximum amplitude of a 1 KHz input sinusoid for which the delta modulator does not show slope overload
      (ii) Determine the post filtered output SNR for the signal of part (i)

(OR)

3. a) Explain Duobinary signaling with an example 6 M
   b) Explain in detail about Intersymbol Interference 6 M

UNIT II

4. a) Explain the generation and reception of BPSK signal. 6 M
   b) The bit stream b(t) is to be transmitted using DPSK. If b(t) is 001010011010, determine DPSK modulated and demodulated data. 6 M

(OR)

5. a) Derive probability of bit error for BFSK and BPSK 6M
   b) Explain Gram Schmidt Orthogonalisation procedure. 6 M

UNIT III

6. a) Consider five symbols given by the probabilities 1/2,1/4, 1/8, 1/16, 1/16.
      (i) Calculate H
      (ii) Use Shannon-Fano algorithm to develop an efficient code and for that code, Calculate the average number of bits/symbol. Compare with H 8 M
   b) State and prove the properties of Mutual Information 4 M

(OR)

7. a) State and explain information capacity theorem 6M
   b) Consider a telegraph source having two symbols, dot and dash. The dot duration is 0.2 s. The dash duration is 3 times that of the dot duration. The probability of the dot's occurring is twice that of the dash, and the time between symbols is 0.2 s. Calculate the information and entropy of the telegraph source. 6M

EC – 6

UNIT IV

8. a) Explain encoding procedure of Linear Block codes. 6 M
   b) Draw the convolutional encoder and its state diagram for K=3, rate 1/3 code, $g^{(1)}(x) = 1+x^2$, $g^{(2)}(x) = 1+x$ and $g^{(3)}(x) = 1+x+x^2$ 6 M

(OR)

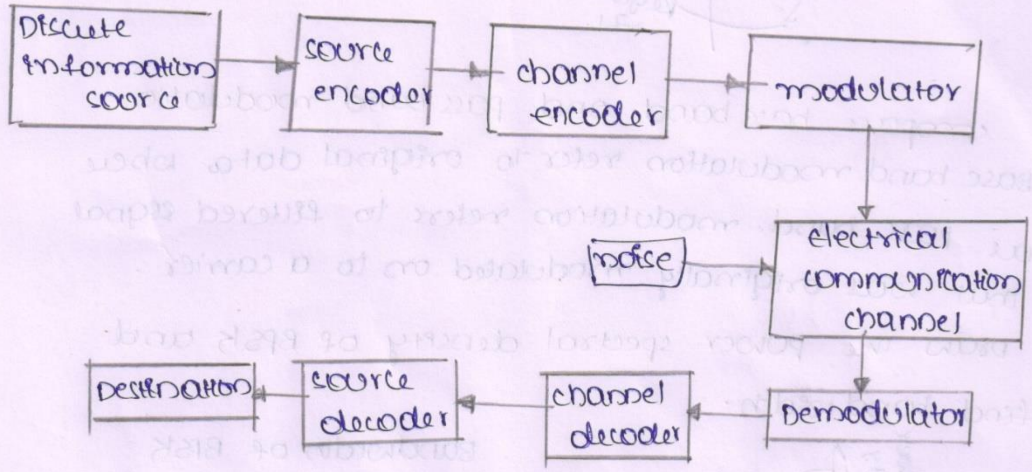
9. a) Draw the block diagram of direct sequence spread spectrum technique and explain in detail. 6 M
   b) Explain the concept of acquisition and tracking in spread spectrum modulation 6 M

Scheme of Evaluation   Subject: Digital Communication

3/4 - B.Tech - I Sem Regular Examination   Code: 14EC505

I Answer All questions.   (1)

a) Draw the block diagram of Digital communication system.



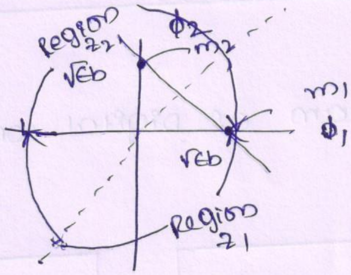
b) What are the drawbacks of quantization process.

In quantization process, the quantizer compares the discrete input $x(nT_s)$ with its fixed digital levels. It assigns any one of the digital level to $x(nT_s)$ with its fixed digital levels which results in minimum error. This error is called quantization error. This is the drawback in quantization process.

c) List the properties of matched filter

The peak pulse signal to noise ratio of matched filter depends only on the ratio of signal energy to the power spectral density of white noise at the filter input $\eta_{max} = 2E/N_0$.

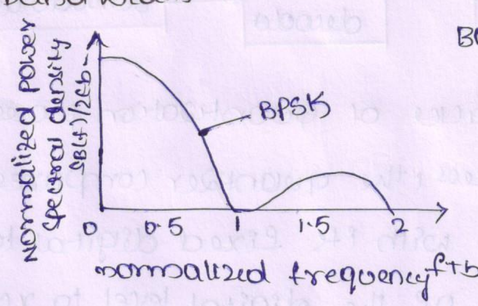
d) Draw the signal space diagram of BFSK.



e) compare base band and pass band modulation.

Base band modulation refer to original data where as pass band modulation refers to filtered signal that was originally modulated on to a carrier.

f) Draw the power spectral density of BPSK and find band width.



Bandwidth of BPSK signal will be

$B \cdot W = f_c + f_b - (f_c - f_b)$

$B \cdot W = 2f_b$

g) state shannon three laws.

Given A DMS of entropy H(S), the average code word $\bar{L}$ for any distortionless source encoding is bounded as $\bar{L} \geq H(S)$ — source coding theorem.

ii) channel coding theorem: For a DMS there exists a coding scheme which can reconstruct the transmitted signal with small probability of error $\Longleftrightarrow H(S)/T_S \leq C/T_C$.

iii) Information capacity theorem: The information capacity of continuous channel of bandwidth B Hz perturbed by AWGN of PSD $N_0/2$ is limited in B.W 2B is given by

$C = B\log_2(1 + P/N_0 B)$ bits/second

h) Define Mutual Information.

since the entropy $H(x)$ represents uncertainty remaining about the channel input before observing the channel output, and conditional entropy $H(x/y)$ represents our uncertainty about the channel input after observing the channel output, it follows that difference $H(x) - H(x/y)$ must represent our uncertainty about the channel input that is resolved by observing the channel output. This important quantity is called Mutual Information.

$$\therefore I(x, y) = H(x) - H(x/y)$$

similarly $I(y, x) = H(y) - H(y/x)$

i) List the properties of entropy.

The property of entropy is

$0 \le H(S) \le \log_2 K$ where K is the radix (number of symbols) of the alphabet S of the source.

j) explain the properties of cyclic codes.

1. Linearity property: The sum of any two code words in the code is also code word.
2. cyclic property: Any cyclic shift of a code word in the code is also code word.

k) Define constraint length and code rate.

The no. of memory shifts required to influence the output bit in convolutional encoder is called as constraint length (K).

The ratio k/n is called the code rate.

$\therefore r = k/n$.

1) List the properties of PN sequence

1) balance property.

2) run property.

3) Auto correlation property.

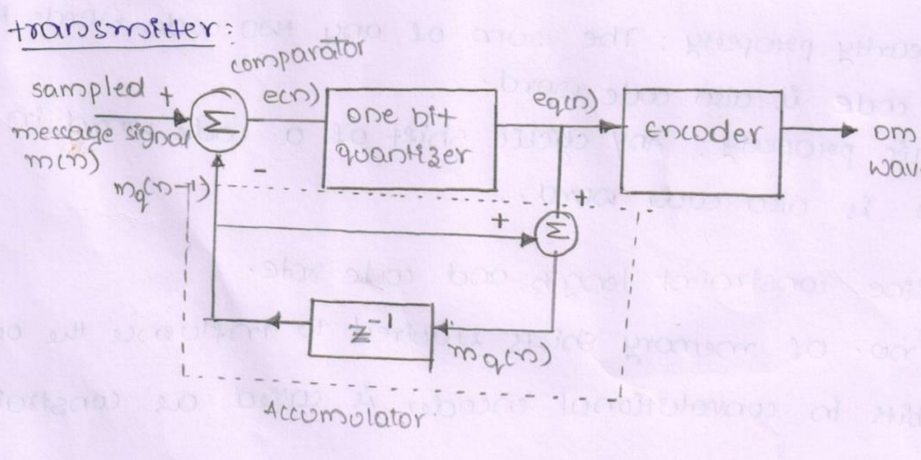
Unit-I

2) a) Draw and explain the functional block diagram of delta modulation system.

Delta modulation transmits only one bit per sample. Here the present sample value is compared with the previous sample value i.e this results whether the amplitude is increased or decreased is transmitted.

The functional block diagram of delta modulator is shown in below.

transmitter:



The z^{-1} inside the accumulator represents a unit delay, that is delay equal to one sampling period.

The comparator computes the difference between its two inputs. i.e $e(n) = m(n) - m_q(n-1)$

Here the quantizer consists of hard limiter with an input-output relation i-e scaled version of signum function.

$$e_q = \Delta \, sgn(e(n))$$

Depending on the sign of $e_q(n)$, one bit quantizer generates an output of $+\Delta$ or $-\Delta$. If stepsize is $+\Delta$.

The quantizer output is then applied to an accumulator producing the result.

$$m_q(n) = \Delta \sum_{i=1}^{n} sgn(e(i))$$

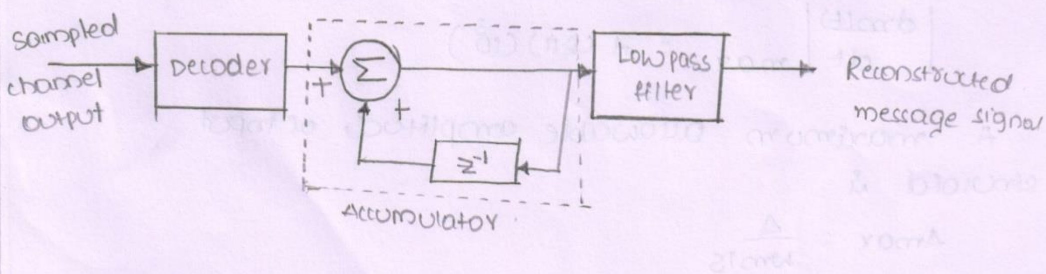
$$= \sum_{i=1}^{n} e_q(i)$$

At sampling instant nT_s, the accumulator increments the approximation by a step Δ in +ve or -ve direction, depending on the sign of error sample $e(n)$.

If the input sample $m(n)$ is greater than $m_q(n)$, a +ve increment $+\Delta$ is applied to the approximation.

If input sample $m(n)$ is less than $m_q(n)$, '$-\Delta$' is applied to the approximation.

Receiver.



At the receiver end, the accumulator and low pass filter are used. The accumulator generates the stair case approximated signal output and is delayed by one sampling period 'Ts'. It is then added to the input signal. If input is binary '1' then it adds $+\Delta$ step to the previous output. If input is binary '0' then one step 'Δ' is subtracted from the delayed signal.

Here the low pass filter has the cut off frequency equal to highest frequency in $m(t)$. This low pass filter smoothens the stair case signal to reconstruct original message signal $m(t)$.

b) A DM system is designed to operate at three time the Nyquist rate for signal with 3KHz Bandwidth. The quantizing step size is 250mV.

i) Determine the maximum amplitude of 1KHz input sinusoid for which the delta modulator does not show slope overload

ii) Determine the post filtered output SNR for the signal of part (i).

We have $m(t) = A\cos w_m t$

$$= A\cos 2\pi f_m t$$

$$= A\cos 2\pi(10^3 t)$$

$$\left|\frac{dm(t)}{dt}\right|_{max} = A(2\pi)(10^3)$$

A maximum allowable amplitude of input sinusoid is

$$A_{max} = \frac{\Delta}{w_m T_S}$$

$$A_{max} = \frac{\Delta f_s}{w_m}$$

$\cancel{f_s} = \frac{A_{max}\, \cancel{2\pi} f_m}{\Delta}$ $\quad A_{max} = \frac{\Delta f_s}{2\pi f_m}$

$= \frac{A_{max}\, 2\pi f_m}{\Delta}$ $\quad = \frac{250 \times 10^{-3} \times 6 \times 3 \times 10^{3}}{2\pi \times 10^{3}}$

$= \cancel{25}$

$= \cancel{71.2mv}\ 716.19mV$

ii) Assumming that cut off fequency of low pass filter is f_m

$(SNR)_0 = (S/Nq)_0 = \frac{3 f_s^3}{8\pi^2 f_m^2 f_M}$ $\quad f_M = f_m$

$= \frac{3 \times [3 \times 6 \times 10^3]^3}{8\pi^2 (10^3)^3}$

$= 221.6 = 23.5db$

(OR)

3\. a) Explain duo binary signalling with an example.

Duo binary signalling:-

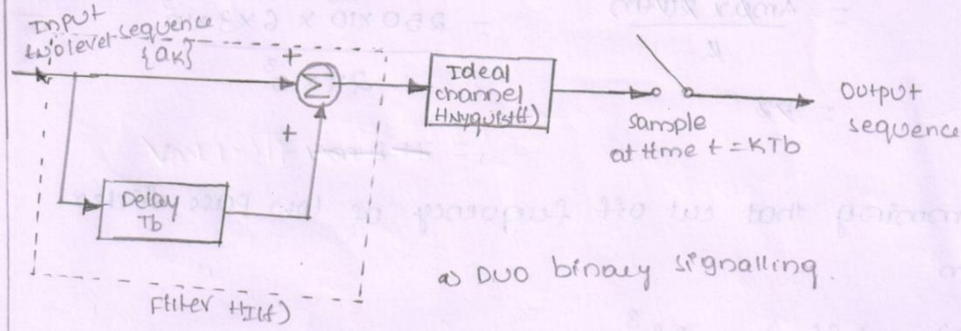
Duo binary signalling is also called as class I partial response. Here 'duo' means doubling of the transmission capacity of the binary system.

consider a binary input sequence $\{b_k\}$ consisting of uncorrelated binary symbols 1 and 0 is each one having duration T_b.

This sequence is applied to pulse Amplitude Modulation produ-cing a two level sequence of short pulses.

Here $a_k = \begin{cases} +1 & \text{if symbol } b_k \text{ is } 1 \\ -1 & \text{if symbol } b_k \text{ is } 0 \end{cases}$

This sequence is applied to duobinary encoder, It is converted in to a three level output namely -2, 0 and +2.



a) Duo binary signalling.

For this transformation, the two level sequence {ak} first passed through simple filter consists of delay element & summer.

For every unit impulse applied to input of the filter, we get two unit impulses spaced 'Tb' seconds at the filter output.

The duo coder output C_K as the sum of present input a_K and previous value a_{K-1}

$$C_K = a_K + a_{K-1}.$$

The frequency response of simple delay filter is

$1 + \exp(-j2\pi f T_b)$.

∴ The overal frequency response of filter connected in cascade with ideal Nyquist channel is

$$H_I(f) = H_{Nyquist}(f)\left(1+\exp(-j2\pi f T_b)\right)$$

$$= H_{Nyquist}(f)\left(\exp(j\pi f T_b) + \exp(-j2\pi f T_b)\right)\exp(-j\pi f T_b)$$

$$= 2H_{Nyquist}(f)\cos(\pi f T_b)\exp(-j\pi f T_b)$$

$$H_{Nyquist}(f) = \begin{cases} 1 & |f| \le 1/2T_b \\ 0, & \text{otherwise} \end{cases}$$

The overall frequency response of duo binary signaling scheme is

$$H_I(f) = \begin{cases} 2\cos(\pi f T_b)\exp(-j\pi f T_b) & |f| \le 1/2T_b \\ 0 & \text{otherwise.} \end{cases}$$

The impulse response of $H_I(f)$ consists of two sinc pulses with spaced T_b seconds.

$$h_I(t) = \frac{\sin(\pi t/T_b)}{\pi t/T_b} + \frac{\sin(\pi(t-T_b)/T_b)}{\pi(t-T_b)/T_b}$$

$$= \frac{\sin(\pi t/T_b)}{\pi t/T_b} - \frac{\sin(\pi t/T_b)}{\pi(t-T_b)/T_b}$$

$$= \frac{T_b^2 \sin(\pi t/T_b)}{\pi t(T_b-t)}$$

The original two level sequence $\{a_k\}$ may be detected from duo binary sequence.

Let $\hat{a}_k$ represent estimate the original pulse a_k then subtracting the previous estimate $\hat{a}_{k-1}$ from C_k, we get

$$\hat{a}_k = C_k - \hat{a}_{k-1}$$

<u>example</u> :-

Binary sequence (b_k)	0	0	1	0	1	1	0.
Two level sequence (a_k)	-1	-1	1	-1	1	1	-1
delay sequence a_{k-1}		-1	-1	1	-1	1	1
Duo binary coder output C_k		-2	0	0	0	2	0
$\hat{a}_k$		0	1	0	1	1	0

b) Explain in detail about Inter Symbol Interference.

Inter symbol Interference arises when communication channel is dispersive.

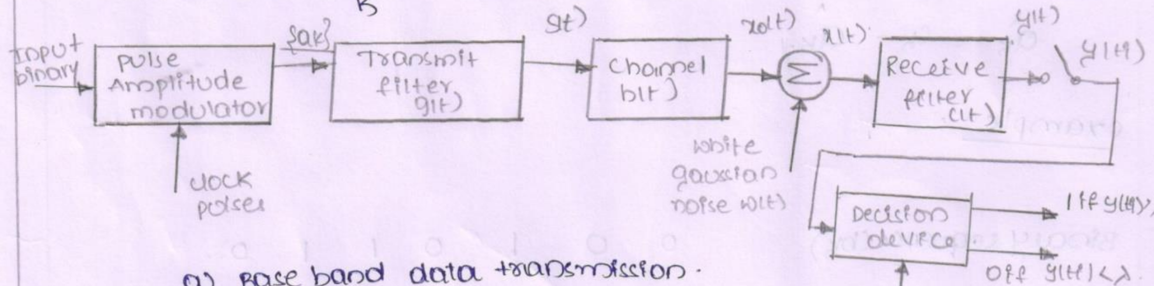
consider a base band binary PAM system which is shown in below. The incoming binary sequence $\{b_k\}$ consists of symbols 1 and 0 & each of duration is T_b.

The pulse amplitude modulator modifies this binary sequence in to new sequence of short pulses.

i.e $a_k = \begin{cases} +1 & \text{if symbol } b_k \text{ is } 1 \\ -1 & \text{if symbol } b_k \text{ is } 0 \end{cases}$

The output of PAM is applied to transmit filter of impulse response g(t), producing a transmitted signal.

$$s(t) = \sum_k a_k\, g(t - kT_b)$$



a) Base band data transmission.

Here the signal s(t) is changed through channel of impulse response h(t) and also

The noisy signal x(t) is then passed through a receive filter of impulse response c(t).

The resulting o/p y(t) is sampled synchronously with the transmitter.

The sequence of samples is obtained & used to contribution of i^{th} transmitted bit.

The second term represents the residual effect of all other transmitted bits on the decoding of i^{th} bit. This term is called Inter symbol Interference.

In the absence of both ISI & noise,

$$y(t_i) = \mu a_i .$$

Unit-II

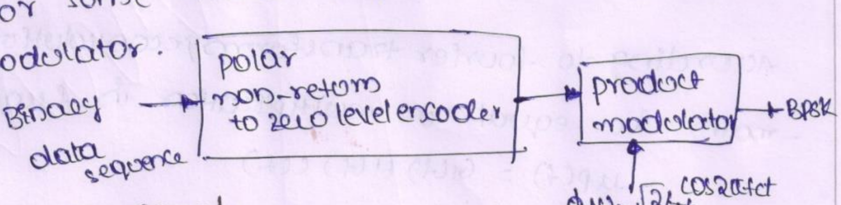
4a) Explain the generation and reception of BPSK signal.

Generation of BPSK signal:-

The BPSK signal can be generated by applying carrier signal to product modulator.

The base band signal b(t) is applied as modulating signal to product modulator.

The NRZ level encoder converts the binary data sequence in to bipolar NRZ signal. The resulting binary wave and sinusoidal carrier $\phi_1(t)$ whose frequency $f_c = n_c/T_b$) for some fixed integer n_c are applied to product modulator.



Detection of BPSK signal:-

To detect the original binary sequence of 1's and 0's, we apply the noisy PSK signal x(t) to a correlator. The correlator output x_1 is compared with a threshold of zero volts.

reconstruct the original data sequence.

Here amplitude of each sample is compared to a threshold λ.

If threshold λ is exceeded, decision is made in favour of symbol '1'.

If threshold λ is not exceeded, a decision is made in favour of symbol '0'. If sample amplitude equals the threshold, the receiver simply makes a random guess.

The receive filter output is written as

$$y(t) = \mu \sum_k a_k p(t-kT_b) + n(t)$$ where $\mu \Rightarrow$ scaling factor.

$p(t-kT_b)$ represents effect of transmission delay through the system.

The scaled pulse $\mu p(t)$ is obtained by double convoluting impulse response $g(t)$ of the transmit filter, the impulse response $h(t)$ of the channel i.e impulse response $c(t)$ of the receive filter.

$$\mu p(t) = g(t) * h(t) * c(t)$$

Assume $p(0) = 1$

According to fourier transform, convolution in time domain is equal to multiplication in frequency domain

$$\mu P(f) = G(f)\, H(f)\, C(f)$$

The receiver filter o/p $y(t)$ is sampled at time $t_i = iT_b$

$$y(t_i) = \mu \sum_{k=-\infty}^{\infty} a_k p[(i-k)T_b] + n(t_i)$$

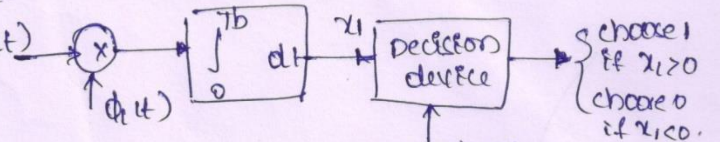
$$= \mu a_i + \mu \sum_{k=-\infty}^{\infty} a_k p((i-k)T_b) + n(t_i)$$

from above eq. the first term μa_i represents the

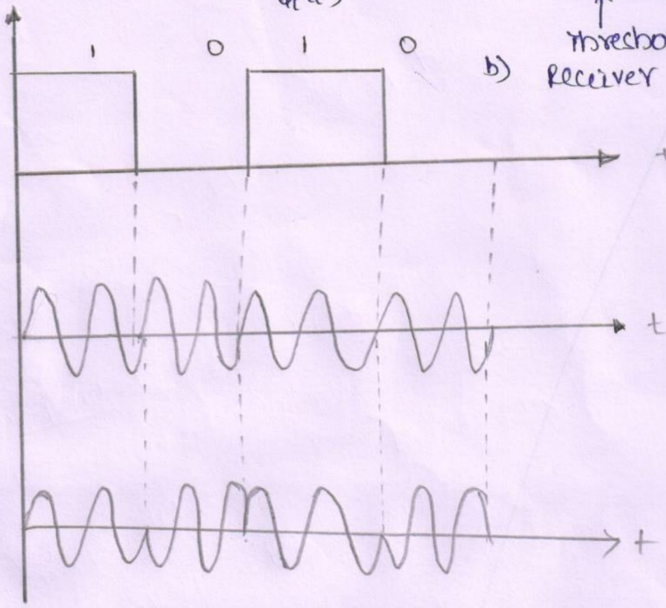
If $x_1 > 0$, the receiver decides in favour of symbol 1.

If $x_1 < 0$, it decides in favour of symbol 0.

If $x_1 = 0$, the receiver makes a random ~~gx~~ guess in favour of 0 or 1.



b) Receiver.



b) The bit stream b(t) is to be transmitted using DPSK. If b(t) is 001010011010. Determine DPSK modulated and demodulated data.

Binary data b(t)		0	0	1	0	1	0	0	1	1	0	1	0.
Differentially encoded data	1	0	1	1	0	0	1	0	0	0	1	1	0
phase of DPSK	0	π	0	0	π	π	0	π	π	π	0	0	π
shifted differentially encoder data $\{d_{k-1}\}$		1	0	1	1	0	0	1	0	0	0	1	1
		0	0	1	0	1	0	0	1	1	0	1	0.

phase of the shifted DPSK   0 π 0 0 π π 0 π π π 0 0.

Detected binary sequence   ± − + − + − − + + − + −

0 0 1 0 1 0 0 1 1 0 1 0.

Detected binary sequence   0 0 1 0 1 0 0 1 1 0 1 0

(or)

5a) Derive probability of bit error for BFSK and BPSK.

<u>error probability of BPSK</u>:

If the received signal point falls in region Z_1, the receiver decides the signal $s_1(t)$ (i.e binary 1) was transmitted.

If the received signal point falls in region Z_2, the receiver decides the signal $s_2(t)$ (i.e binary 0) was transmitted.

Two kinds of error decisions are there, first one is signal $s_2(t)$ is transmitted, but the noise is such that the received signal point falls inside region Z_1 and so receiver decides infavour of signal $s_1(t)$.

similarly signal $s_1(t)$ is transmitted but the noise is such that the received signal point falls inside region Z_2 and so the receiver decides infavour of signal $s_2(t)$.

To calculate the probability of error, we described

$$Z_1 : 0 < x_1 < \infty.$$

where x_1 is observable element.

The received signal $s_1(t)$ is described by

$y = x_1 - x_2$.

The conditional mean of random variable Y, given that symbol '1' was transmitted is

$$E(Y/1) = E(x_1/1) - E(x_2/1)$$
$$= +\sqrt{E_b}$$

on other hand, given that symbol '0' was transmitted, the random variables x_1 and x_2 have mean values equal to zero i.e $\sqrt{E_b}$.

The condition mean of random variable 'y', given that symbol '0' was transmitted is

$$E(Y/0) = E(x_1/0) - E(x_2/0)$$
$$= -\sqrt{E_b}$$

The i.e variance $var(Y) = var(x_1) + var(x_2)$
$$= N_0.$$

we know that symbol '0' was transmitted. The conditional probability density function of random variable 'y' is then given by

$$f_Y(Y/0) = \frac{1}{\sqrt{2\pi N_0}} \exp\left[-\frac{(y+\sqrt{E_b})^2}{2N_0}\right]$$

Here the conditional probability of error given that symbol '0' was transmitted is

$$P_{10} = P(y > 0 \mid \text{symbol 0 was sent})$$
$$= \int_0^\infty f_Y(y/0)\, dy$$
$$= \frac{1}{\sqrt{2\pi N_0}} \int_0^\infty \exp\left(-\frac{(y+\sqrt{E_b})^2}{2N_0}\right) dy$$

$$x_1 = \int_0^{T_b} x(t)\, \phi_1(t)\, dt$$

The conditional probability density function of random variable x_1, given that symbol 0

$$f_{x_1}(x_1/0) = \frac{1}{\sqrt{\pi N_0}} \exp\left[\frac{-1}{N_0}(x_1 - s_{21})^2\right]$$

$$= \frac{1}{\sqrt{\pi N_0}} \exp\left[\frac{-1}{N_0}(x_1 + \sqrt{E_b})^2\right]$$

The conditional probability of the receiver deciding in favour of symbol 1, given that symbol '0' was transmitted.

$$P_{10} = \int_0^{\infty} f_{x_1}(x_1/0)\, dx_1$$

$$= \frac{1}{\sqrt{\pi N_0}} \int_0^{\infty} \exp\left[\frac{-1}{N_0}(x_1 + \sqrt{E_b})^2\right] dx_1$$

$$z = \frac{1}{\sqrt{N_0}}(x_1 + \sqrt{E_b})$$

ie $$P_{10} = \frac{1}{\sqrt{\pi}} \int_{\sqrt{E_b/N_0}}^{\infty} \exp(-z^2)\, dz$$

$$= \frac{1}{2}\, \mathrm{erfc}\sqrt{E_b/N_0}$$

The average probability of symbol error or bit error rate for coherent binary PSK is

$$P_e = \frac{1}{2}\, \mathrm{erfc}\sqrt{E_b/N_0}.$$

Error probability of BFSK:-

The observation vector 'x' has two elements x_1 and x_2 are defined as

$$x_1 = \int_0^{T_b} x(t)\, \phi_1(t)\, dt$$

$$x_2 = \int_0^{T_b} x(t)\, \phi_2(t)\, dt$$

Here the gaussian random variable y is equal to

$$\frac{y+\sqrt{Eb}}{\sqrt{2N_0}} = z$$

changing the variable of integration from y to z

$$P_{10} = \frac{1}{\sqrt{\pi}} \int_{\sqrt{Eb/2N_0}}^{\infty} \exp(-z^2)\, dz$$

$$= \tfrac{1}{2}\, \mathrm{erfc}\left(\sqrt{Eb/2N_0}\right)$$

$$\Rightarrow P_e = \tfrac{1}{2}\, \mathrm{erfc}\left(\sqrt{Eb/2N_0}\right)$$

b) explain Gram schmidt orthogonalisation procedure.

<u>purpose:</u>

Gram-schmidt orthogonalization procedure is the tool to get the orthonormal basic functions $\phi_i(t)$.

To derive an expression for $\phi_j(t)$.

we have the set of 'M' energy signals $s_1(t), s_2(t) \ldots s_M(t)$ for $s_1(t)$ the basic function is defined by

$$\phi_1(t) = s_1(t)/\sqrt{E_1} \rightarrow (1)$$

Here 'E_1' is the energy of $s_1(t)$. From eq (1) we can write

$$s_1(t) = \sqrt{E_1}\,\phi_1(t) \rightarrow (2)$$

From eq for N=1.

$$s_1(t) = s_{11}\,\phi_1(t) \rightarrow (3)$$

From above two eq's $s_{11} = \sqrt{E_1}$, i.e. $\phi_1(t)$ has unit energy, the co-efficients $s_{21}(t)$ will be given as

$$s_{21} = \int_0^T s_2(t)\,\phi_1(t)\, dt$$

Let $g_2(t)$ be a new function which is given as

$$g_2(t) = s_2(t) - s_{21}\phi_1(t)$$

The function is orthogonal to $\phi_1(t)$ over an interval 0 to T. The second basic function is defined as

$$\phi_2(t) = \frac{g_2(t)}{\sqrt{E g_2}}$$

Here $E g_2 = \int_0^T g_2^2(t)\,dt$ is the energy of $g_2(t)$

i) To prove that $\phi_2(t)$ has unit energy.

Energy of $\phi_2(t)$ will be

$$\int_0^T \phi_2^2(t)\,dt = \int_0^T \left[\frac{g_2(t)}{\sqrt{E g_2}}\right]^2 dt$$

$$= 1/E g_2 \int_0^T g_2^2(t)\,dt.$$

$$= \frac{1}{E g_2} E g_2 = 1.$$

Thus $\phi_2(t)$ has unit energy.

ii) To prove that $\phi_1(t)$ & $\phi_2(t)$ are orthonormal.

consider $\int_0^T \phi_1(t)\phi_2(t)\,dt = \int_0^T \frac{s_1(t)}{\sqrt{E_1}} \cdot \frac{g_2(t)}{\sqrt{E g_2}}\,dt.$

$$= \frac{1}{\sqrt{E g_1 E g_2}} \int_0^T s_1(t)\, g_2(t)\,dt.$$

putting for $g_2(t)$ in to above eq.

$$\int_0^T \phi_1(t)\phi_2(t)\,dt = \frac{1}{\sqrt{E_1 E g_2}} \int_0^T s_1(t)\left(s_2(t) - s_{21}\phi_1(t)\right)dt.$$

$$= \frac{1}{\sqrt{E_1 E g_2}} \left(\int_0^T s_1(t) s_2(t)\,dt - \int_0^T s_{21} s_1(t)\phi_1(t)\,dt \right]$$

putting for $s_{21}(t)$ ~~from~~ in above equation

$$\int_0^T \phi_1(t)\, \phi_2(t)\, dt = \frac{1}{\sqrt{E_1 E g_2}} \left(\int_0^T s_1(t)\, s_2(t)\, dt - \int_0^T \int_0^T s_2(t)\, \phi_1(t)\, s_1(t)\, \phi_1(t)\, dt \right)$$

$$\int_0^T \phi_1(t)\, \phi_2(t)\, dt = 0$$

Thus two basis functions are orthonormal.

iii) Generalized equations for ortho normal basis functions.

The generalized eq's for orthonormal basis function $\phi_i(t)$ as

$$\phi_i(t) = \frac{g_i(t)}{\sqrt{E g_i}} \quad , i = 1, 2 \dots N$$

$$g_i(t) = s_i(t) - \sum_{j=1}^{i-1} s_{ij}\, \phi_j(t)$$

And the co-efficients s_{ij} are

$$s_{ij} = \int_0^T s_i(t)\, \phi_j(t)\, dt \quad j = 1, 2 \dots i-1.$$

unit - III.

6) a) consider five symbols given by the probabilities $1/2$, $1/4$, $1/8$, $1/16$, $1/16$

i) calculate H.

ii) use shannon - fano algorithm to develop an efficient code C_e for that code calculate the average NO. of bits / symbol. compare with H.

i) Entropy

$$H = \sum_{k=0}^{K-1} P_k \log_2 (1/P_k)$$

$$= \sum_{k=0}^{4} P_k \log_2 (1/P_k)$$

$$= P_0 \log_2 (1/P_0) + P_1 \log_2 (1/P_1) + P_2 \log_2 (\frac{1}{P_2}) + P_3 \log_2 (1/P_3) + P_4 \log_2 (1/P_4)$$

$$= 1/2 \log_2 (1/1/2) + 1/4 \log_2 (1/1/4) + 1/8 \log_2 (1/1/8) + 1/16 \log_2 (1/1/16) + 1/16 \log_2 (\frac{1}{1/16})$$

$= 0.5 + 0.5 + 0.375 + 0.25 + 0.25 = 1.875$ bits / symbol

ii)

symbol	step 1	step-2.	step 3	step 4.
S_0	1/2	0		
S_1	1/4	1	0	
S_2	1/8	1	1	0
S_3	1/16	1	1	1	0
S_4	1/16.	1	1	1	1

code word.	length l_K
0	1
1 0	2
1 1 0	3
1 1 1 0	4
1 1 1 1	4

The average no. of bits per message is given by

$$\bar{L} = \sum_{k=0}^{K-1} P_K l_K. = \sum_{k=0}^{4} P_K l_K =$$

$$= P_0 l_0 + P_1 l_1 + P_2 l_2 + P_3 l_3 + P_4 l_4$$

$$= 1/2 (1) + 1/4 (2) + 1/8 (3) + 1/16 (4) + 1/16 (4)$$

$= 1.875$ bits.

b) State & prove the properties of Mutual Information.

i) The mutual information of channel is symmetric

i.e $I(x, y) = I(y, x)$.

To prove this property, we use first entropy formula

$$H(x) = \sum_{j=0}^{J-1} P(x_j) \log_2 (1/P(x_j))$$

$$= \sum_{j=0}^{J-1} P(x_j) \log_2 \left[\frac{1}{P(x_j)}\right] \sum_{k=0}^{K-1} P(y_k/x_j)$$

$$= \sum_{j=0}^{J-1} \sum_{k=0}^{K-1} P(y_k/x_j) P(x_j) \log_2 \left(\frac{1}{P(x_j)}\right)$$

$$= \sum_{j=0}^{J-1} \sum_{k=0}^{K-1} P(x_j y_k) \log_2 \left(\frac{1}{P(x_j)}\right).$$

then $$I(x, y) = \sum_{j=0}^{J-1} \sum_{k=0}^{K-1} P(x_j y_k) \log_2 \left(\frac{P(x_j/y_k)}{P(x_j)}\right)$$

from bayes rule

$$\frac{P(x_j/y_k)}{P(x_j)} = \frac{P(y_k/x_j)}{P(y_k)}$$

$$\therefore I(x, y) = \sum_{k=0}^{K-1} \sum_{j=0}^{J-1} P(x_j y_k) \log_2 \left(\frac{P(y_k/x_j)}{P(y_k)}\right)$$

$$= I(y, x)$$

$$\therefore I(x, y) = I(y, x).$$

ii) The mutual information is always non negative.

i.e $I(x, y) \geq 0$

To prove this property

$$= \sum_{j=0}^{J-1} P(x_j) \log_2 \left[\frac{1}{P(x_j)}\right] + \sum_{k=0}^{K-1} P(y_k) \log_2 \left[\frac{1}{P(y_k)}\right]$$

$$= H(x) + H(y)$$

$$\therefore H(x, y) = -I(x, y) + H(x) + H(y)$$

$$I(x, y) = H(x) + H(y) - H(x, y)$$

(OR)

7) a) state and explain Information capacity theorem.

consider a zero mean stationary process $x(t)$ that is bandlimited to 'B' Hertz.

Let X_K, $K = 1, 2 \ldots K$ denote the continuous random variables obtained by uniform sampling process $x(t)$ at the nyquist rate of $2B$ samples per second. The samples are transmitted in 'T' seconds over a noisy channel, also band limited to B Hertz.

Hence, the no. of samples 'K' is given by

$$K = 2BT$$

Assume channel output is affected by additive white gaussian noise of zero mean & power spectral density $N_0/2$.

The noise is bandlimited to 'B' Hertz.

Let the continuous random variable Y_K, 13

$$P(x_j/y_k) = \frac{P(x_j, y_k)}{P(y_k)} \rightarrow (1)$$

$$\therefore \quad I(X;Y) = \sum_{j=0}^{J-1} \sum_{k=0}^{K-1} P(x_j y_k) \log_2 \left(\frac{P(x_j y_k)}{P(x_j) P(y_k)} \right)$$

According to fundamental inequality

$$I(X;Y) \geq 0.$$

with equality if & only if

$$P(x_j y_k) = P(x_j) P(y_k)$$

Property 3 :-

The mutual information of channel ie channel is related to joint entropy of channel i/p & channel o/p given by

$$I(X;Y) = H(X) + H(Y) - H(X,Y)$$

where the joint entropy $H(X,Y)$ is given by

$$H(X,Y) = \sum_{j=0}^{J-1} \sum_{k=0}^{K-1} P(x_j y_k) \log_2 \left(\frac{1}{P(x_j, y_k)} \right)$$

$$H(X,Y) = \sum_{j=0}^{J-1} \sum_{k=0}^{K-1} P(x_j y_k) \log_2 \left[\frac{P(x_j) P(y_k)}{P(x_j, y_k)} \right]$$

$$+ \sum_{j=0}^{J-1} \sum_{k=0}^{K-1} P(x_j y_k) \log_2 \left[\frac{1}{P(x_j) P(y_k)} \right]$$

the second term

$$\sum_{j=0}^{J-1} \sum_{k=0}^{K-1} P(x_j y_k) \log_2 \left[\frac{1}{P(x_j) P(y_k)} \right]$$

$$= \sum_{j=0}^{J-1} \log_2 \left(\frac{1}{P(x_j)} \right) \sum_{k=0}^{K-1} P(x_j y_k) + \sum_{k=0}^{K-1} \log_2 \left(\frac{1}{P(y_k)} \right) \sum_{j=0}^{J-1} P(x_j y_k)$$

since X_K & N_K are independent random variables, the

their sum equals Y_K.

∴ The conditional differential entropy of Y_K,

given X_K, is equal to the differential entropy of N_K.

$$h(Y_K/X_K) = h(N_K)$$

we may rewrite $I(X_K; Y_K) = h(Y_K) - h(N_K) \rightarrow ①$

The variance of sample Y_K of the received signal equals

$P+\sigma^2$ - Hence differential entropy of Y_K is

$$h(Y_K) = \frac{1}{2}\log_2(2\pi e(P+\sigma^2)) \rightarrow ②$$

The variance of noise sample N_K equals σ^2

$$h(N_K) = \frac{1}{2}\log_2(2\pi e\sigma^2) \rightarrow ③$$

substituting ② & ③ in to eq ①.

$$C = \frac{1}{2}\log_2(1+P/\sigma^2) \text{ bits per transmission}$$

The information capacity per unit time is

$$C = B\log_2(1+P/N_0B) \text{ bits/sec.}$$

'P' is the average transmitted power.

b) consider a telegraph source having two symbols, dot & dash. The dot duration is 0.2s. The dash duration is 3 times that of dot duration.

The ~~p~~

$k = 1,2 \ldots K$ denote samples of received signal.

$$Y_K = X_K + N_K.$$

The Noise sample N_K is gaussian with zero mean & variance is given by

$$\sigma^2 = N_0 B.$$

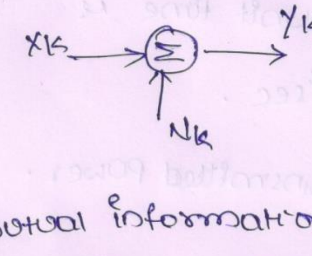
we assume that the samples Y_K, $k = 1,2 \ldots K$ are statis-tically Independent.

The transmitter is power limited.

$$E(X_K^2) = P, \quad k = 1,2 \ldots K. \rightarrow (1)$$

'P' is the average transmitted power.

The information capacity of the channel is defined as maximum of mutual Information between the channel input & channel output Y_K over all distributions on the input X, that satisfies the power constraint of eq(1)



The mutual information b/w X_K & Y_K.

we may then information capacity of the channel is

$$C = \max\{I(X_K; Y_K) : E(X_K^2) = P\}$$

so write $I(X_K\, Y_K) = h(Y_K) - h(Y_K/X_K)$

The probability of the dot's occurring is twice that of dash & time between symbols is 0.25. calculate Information & entropy of telegraph source.

Given that

$P(dot) = 2P(dash)$

$P(dot) + P(dash) = 3P(dash) = 1.$

From above two equations

$P(dash) = 1/3$

$P(dot) = 2/3$ Information $= \log_2(1/P_K)$

Information of P (dash) $= \log_2(1/1/3)$

$= \log_2 3$

$= 1.584$ bits

Information of P(dot) $= \log_2(1/2/3)$

$= \log_2(3/2)$

$= 0.584$

entropy.

$H(X) = \sum_{k=0}^{K-1} P_K \log_2(1/P_K)$

$= \sum_{k=0}^{1} P_K \log_2(1/P_K)$

$= P_0 \log_2(1/P_0) + P_1 \log_2(1/P_1)$

$= \frac{1}{3}\log_2(1/\frac{1}{3}) + \frac{2}{3}\log_2(1/\frac{2}{3})$

$= 0.528 + 0.389 = 0.917$

Unit - IV

8) a) Explain encoding procedure of linear block codes.

A code is said to be linear if any any two code words in the code can be added in modulo 2-arithmetic to produce a third code word in the code.

consider (n,k) linear block code, in which 'n' is the code word output i.e k is the no. of message bits. (n-k) bits are the redundant bits.

Let $m_0, m_1 \cdots m_{k-1}$ gives a block of k message bits. Thus we may have 2^k message blocks.

The sequence of message bits applied to linear block encoder. producing n- bit code word.

Here the code word is devided in to two parts one of which is occupied by the message bits and other by parity bits.

$b_0\ b_1 \cdots b_{n-k-1}$	$m_0\ m_1 \cdots m_{k-1}$
$\underbrace{\qquad\qquad}_{\text{parity bits}}$	$\underbrace{\qquad\qquad}_{\text{message bits}}$

Here the message, parity and code vector are follows.

$$m = [m_0\ m_1 \cdots m_{k-1}]$$

$$b = [b_0, b_1 \cdots b_{n-k-1}]$$

$$c = [c_0, c_1 \cdots c_{n-1}].$$

The parity bits in compact form

$$b = mP.$$

where 'P' is the $(K \times n-k)$ co-efficent matrix is defined by

$$P = \begin{bmatrix} P_{00} & P_{01} & \cdots & P_{0,n-k-1} \\ P_{10} & P_{11} & \cdots & P_{1,n-k-1} \\ \vdots & & & \\ P_{k-1,0} & P_{k-1,1} & \cdots & P_{k-1,n-k-1} \end{bmatrix}$$

Here 'c' may be expressed as partitioned row vector in terms of vectors m & b

$$c = [b : m]$$

$$c = [mP : m]$$

$$c = m[P : I_k]$$

where I_K is the K by K identity matrix

$$I_K = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & & 1 \end{bmatrix}$$

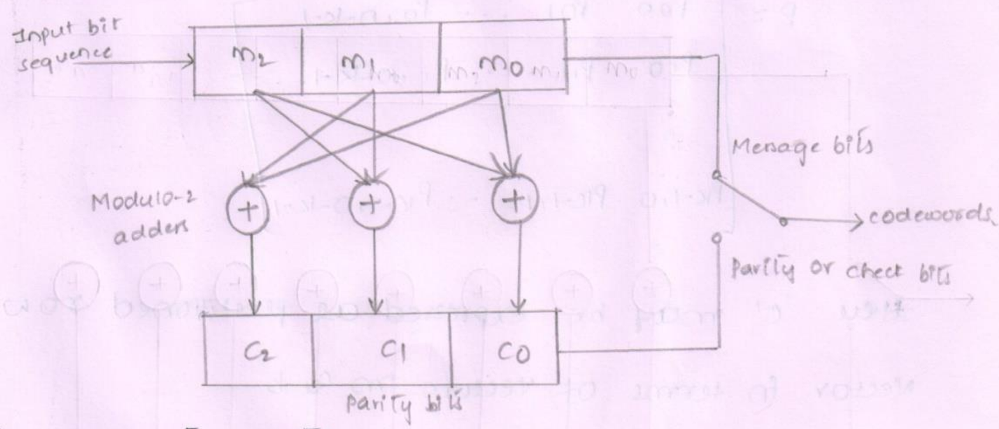
defined the K by n generator matrix

$$G = [P : I_K]$$

$\therefore$ using the definition of generator matrix

$$c = mG.$$

encoder diagram of (n,k) linear block code is shown in below.



For generator matrix

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

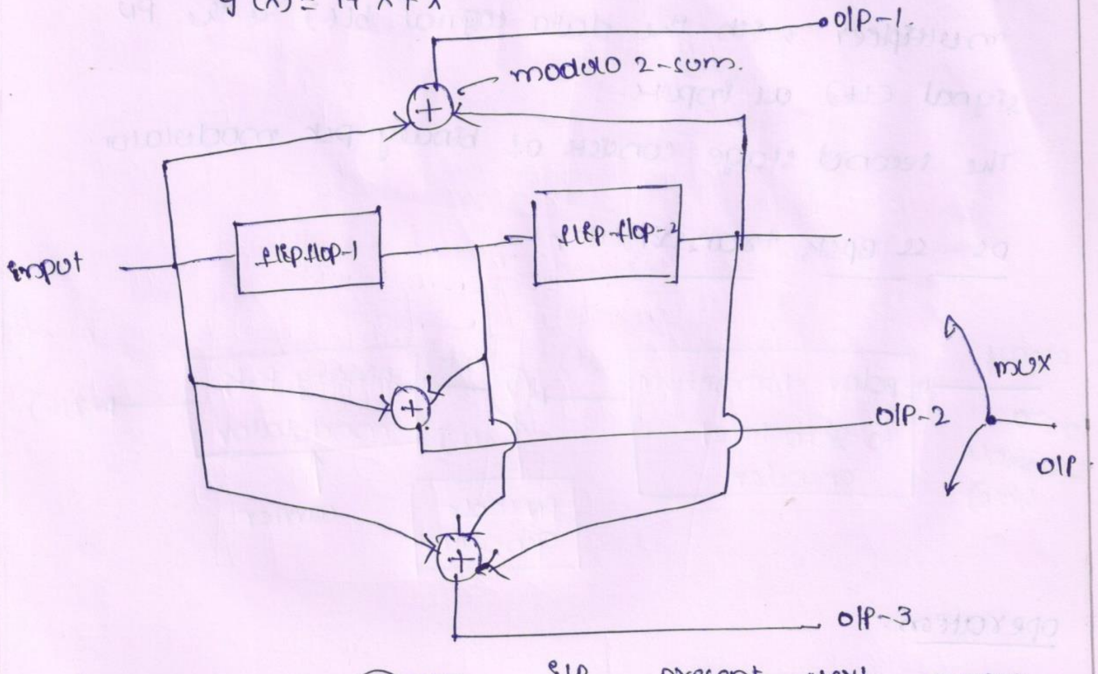
b) Draw the convolutional encoder & its state diagram for $K=3$, rate $1/3$ code, $g^{(1)}(x) = 1+x^2$, $g^{(2)}(x) = 1+x$ & $g^{(3)}(x) = 1+x+x^2$

Given $K=3$, rate $= 1/3$

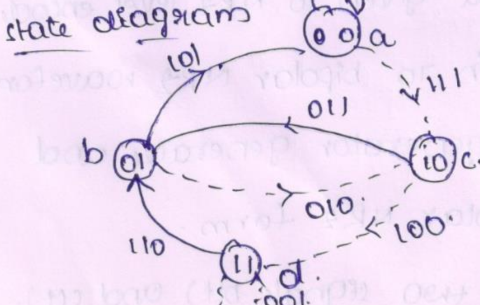
$$g^1(x) = 1+x^2$$

$$g^2(x) = 1+x$$

$$g^3(x) = 1+x+x^2$$



state diagram



i/p	present state	Next state	output
0	00 } a	00	000
1	00	10	111
0	01 } b	00	101
1	01	10	010
0	10 } c	01	011
1	10	11	100
0	11 } d	01	110
1	11	11	001

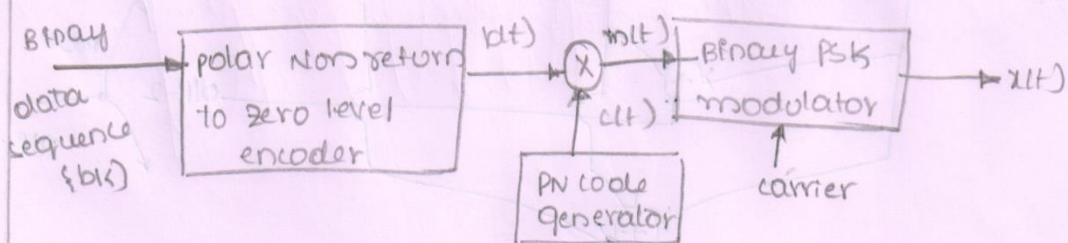
9 a) Draw the block diagram of direct sequence spread spectrum technique and explain in detail.

In the transmitter incoming binary sequence {bk} is to polar NRZ waveform b(t), which is followed by two stages of modulation.

The first stage consists of product modulator or multiplier with the data signal b(t) & the PN signal c(t) as inputs.

The second stage consists of binary PSK modulator.

DS - SS BPSK transmitter :



operation:

The binary data sequence is given to NRZ level encoder.

This encoder converts bk in to bipolar NRZ waveform.

The pseudo-noise sequence generator generates and encodes this sequence in bipolar NRZ form.

The multiplier multiplies the two signals b(t) and c(t).

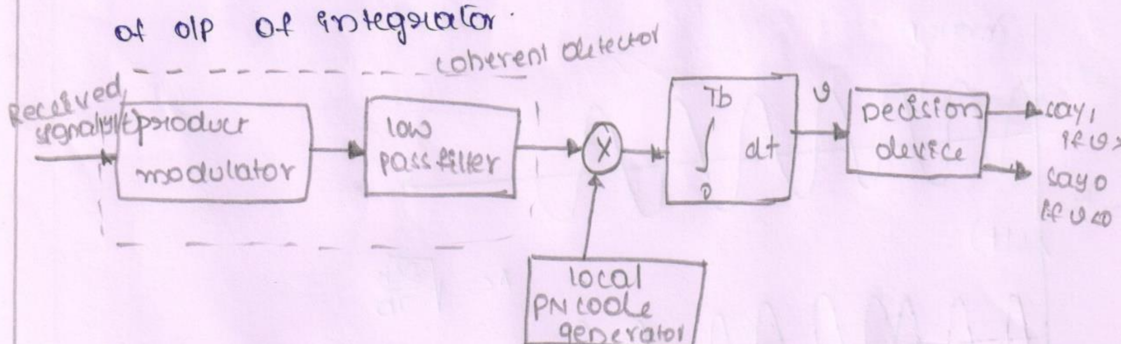
The output of multiplier is then applied the low pass filter. The bandwidth of low pass filter is equal to message signal m(t). This stage is BPSK detector.

This signal is applied to the second modulator which despreads the signal.

The local pseudo-noise signal is exact replica of that used in transmitter.

The integrator integrates the product of detected message signal & pseudo-noise signal over one bit period Tb.

The decision is then taken depending on the polarity of o/p of integrator.



b) Reciver

In methamatical form

$$y(t) = x(t) + j(t)$$

$$= c(t)s(t) + j(t)$$

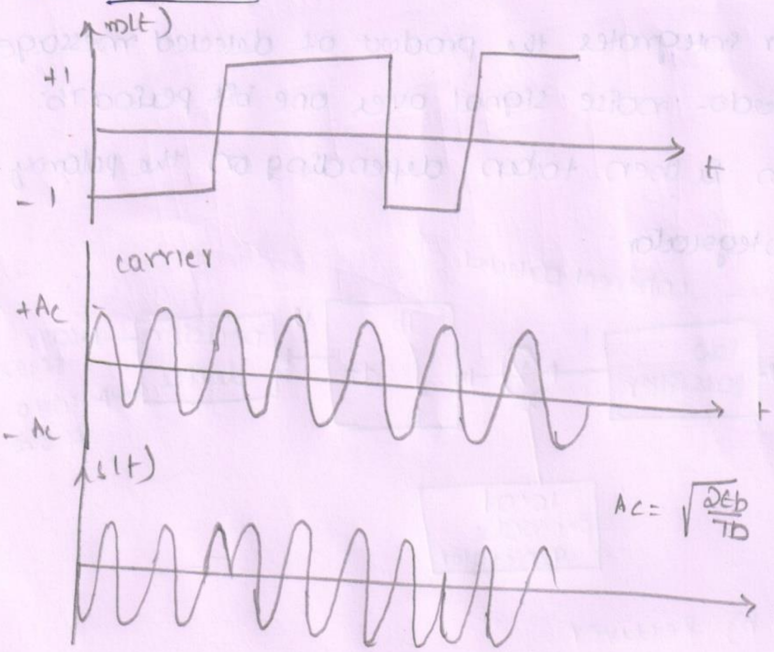
In the reciever, the received signal y(t) is first multiplied by the PN signal c(t)

The output of multiplier is direct sequence spread signal m(t)

This signal is given as modulating signal to BPSK transmitter.

The direct sequence BPSK signal is generated at the output i.e x(t)

(c) Waveform.



Receiver :-

There are two stages of modulation

i) The received signal y(t) is applied to the multiplier which is also supplied with locally generated coherent carrier.

$$u(t) = c(t)\, y(t)$$
$$= c^2(t)\, s(t) + c(t)\, j(t)$$
$$= s(t) + c(t)\, j(t)$$

The coherent detector input $u(t)$ consists of binary PSK signal.

b) Explain about acquisition and tracking in detail.

Tracking:

once acquisition or coarse synchronization is completed. tracking or fine synchronization takes place. tracking code loops can be classified as coherent or non coherent.

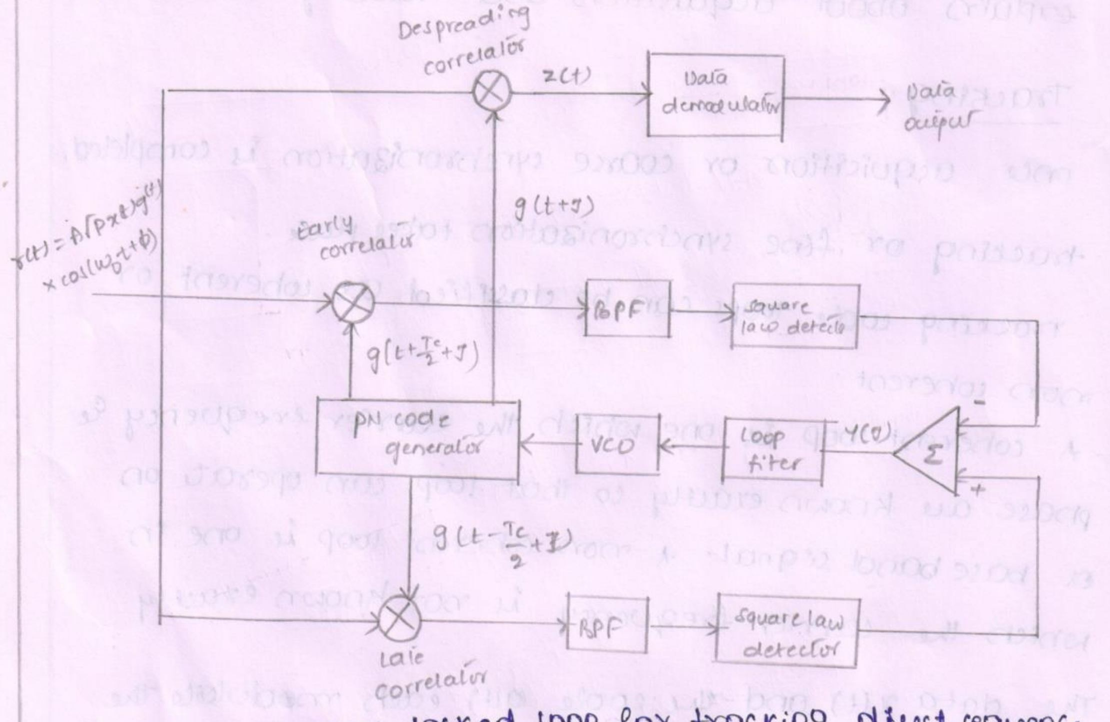
A coherent loop is one which the carrier frequency & phase are known exactly so that loop can operate on a base band signal. A non coherent loop is one in which the carrier frequency is not known exactly

The data $x(t)$ and the code $g(t)$ each modulate the carrier wave using BPSK i.e as before in the absence of noise & interference, the received wave form can be expressed as $r(t) = A\sqrt{2P}\, x(t)\, g(t) \cos(\omega_0 t + \phi)$

where constant 'A' is system gain

'ϕ' is random phase angle in the range $(0, 2\pi)$.

The locally generated code of the tracking loop is offset in phase from the incoming $g(t)$ by a time τ, where $\tau < T_c/2$. The loop provides fine synchronization by first generating two PN sequences $g(t+T_c/2+\tau)$ & $g(t-T_c/2+\tau)$ delayed from each other by one chip. The two bandpass filters are designed to pass the data



a) Delay locked loop for tracking direct sequence signals.

& to average the product $g(t)$ & the two PN sequences $g(t \pm T_c/2 + \tau)$.

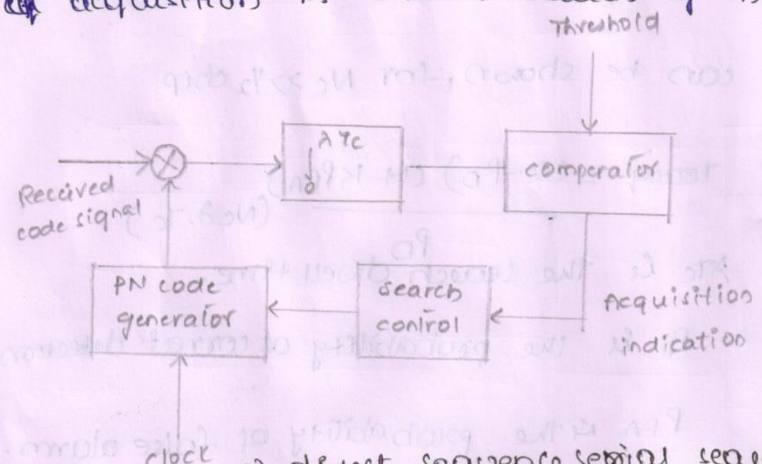
The O/P of each envelope detector is given

$$E_0 \approx E\{|g(t)\, g(t \pm T_c/2 + \tau)|\} = |R_g(\tau \pm T_c/2)|$$

The feed back signal $y(T)$ instructs the VCO to increase frequency, there by forcing T' to decrease, when T is -ve $y(T)$ instructs the VCO to decrease. When T' is small number $g(t)\,g(t+T) \simeq 1$ giving the despread signal $z(t)$.

Acquisition :-

The acquisition is one of searching throughout



a) direct sequence serial search.

the region of time & frequency to synchronize the received spread spectrum signal.

In a stepped serial acquisition scheme for a DS system. The timing epoch of the local PN code is set. i.e. locally generated PN signal is correlated with incoming PN signal.

At fixed examination intervals of λT_c where $\lambda >> 1$ the o/p signal is compared to present threshold. If the o/p is below the threshold, the phase of locally generated carrier code signal is incremented by a fraction of chip.

When the threshold exceeded, the PN code is assumed to have been acquired.

The mean acquisition time of serial DS search system can be shown, for $N_C >> 1/2$ chip

$$\overline{T_{acq}} = \frac{(2-P_D)(1+KP_{FA})}{P_D}(N_C \lambda T_C)$$

λT_C is the search dwell time.

P_D is the probability of correct detection

P_{FA} is the probability of false alarm.

b) Frequency hopping serial search.