The Formation of Exhaust Condensation Trails by Jet Aircraft

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ABSTRACT

This paper defines the meteorological state of the atmosphere which will give rise to the formation of condensation trails (contrails) as the exhaust from an aircraft engine mixes with and saturates the environment. Three basic assumptions were made with regard to the formation of visible contrails: (1) contrails are composed of ice crystals; (2) water vapor cannot be transformed into ice without first passing through the liquid phase, thus necessitating an intermediate state of saturation with respect to water; (3) a minimum visible water content of 0.004 gm/m^3 is required for a faint trail and 0.01 gm/m^3 for a distinct trail. This last requirement proved of no importance in determining whether or not a trail would form, but did affect its persistence.

Curves were constructed showing the critical temperature for the formation of a visible trail as a function of the pressure and relative humidity of the environment and the amount of air entrained into the exhaust. It is shown that these curves are applicable to any aircraft which has the same water to heat ratio in its exhaust as the case discussed in this report. In general this ratio is fairly constant regardless of the type of airplane, control settings, or fuel. The major exception occurs with aircraft powered by reciprocating engines in which case a considerable portion of the heat produced may be dissipated outside of the trail. A separate, but similar, study would be necessary for each aircraft with a significantly different proportion of such heat loss.

1. INTRODUCTION

THE fuel used in airplanes—both conventional and jet—is a hydrocarbon, which upon combustion results in the addition of water vapor and heat to the wake of the aircraft. The added moisture (ΔW) tends to raise the relative humidity of the affected environment, while the added heat (ΔH) tends to lower it. The resulting relative humidity in the trail is dependent on the amount of heat (ΔH) and water (ΔW) added by the exhaust, the ratio (N) of entrained environment air to exhaust gas, and the initial pressure (P), temperature (T), and relative humidity (f) of the environment. The critical temperature (T_c) for the formation of a saturated wake (contrail) is therefore a function of $\Delta W, \Delta H, N, P,$ and f. The heat and moisture liberated during combustion are very nearly constant for all the fuels used in aircraft [1, 2], so that T_c can be considered a function of $N, P,$ and f. Since the initial pressure and relative humidity are those of the environment, and since N varies from zero to infinity for every case, the critical temperature curves thus obtained are universally applicable to all aircraft, regardless of type, fuel, control settings, etc., as long as nearly all of the heat and moisture produced by the combustion of the fuel is dissipated only into the exhaust wake or trail (true for jet aircraft but not for propeller-driven ones).

Although N varies continuously from zero to infinity, the rate at which this occurs is dependent on a number of factors. Just before the exhaust gas first strikes the atmosphere, only the unburned air which has passed through the combustion chamber is mixed with it. Immediately after it strikes the atmosphere, however, the entrainment of air into the trail sets in, continuing until at some distance behind the aircraft the mixture is made up almost entirely of the environment. The relationship of N to distance behind the airplane is affected by the type of aircraft, its control settings, the stability and density of the surrounding atmosphere, and the radial distance from the axis of the trail. It would be possible to carry out individual studies for various types of aircraft under various conditions of atmospheric stability in order to obtain the exact relation between N and distance behind the airplane. This would enable the translation of the formation and dissipation points of the trail from values of N into terms of distance, thus giving the length of the trail.

2. COMBUSTION OF THE FUEL

For each gram of fuel burned by the jet aircraft, there are produced and added to the wake environment approximately 12 grams of exhaust gases, 1.4 grams of water vapor, and 10,000 calories of heat [1]. Each gram of exhaust gas mixes with

N grams of the surrounding air. Thus the increase in temperature (ΔT) of the affected environment is $10{,}000/(12N \times 0.24)$°C, where 0.24 is the specific heat of air, and the increase in mixing ratio (Δw) is $(1.4 \times 1{,}000)/12\ N$ gm/kg. The ratio of the moisture increase to temperature increase, $(\Delta w)/(\Delta T)$, is approximately 0.0336 gm/kg per degree Centigrade. This ratio is independent of the value of N.

Thus an aircraft flying through the atmosphere will raise the mixing ratio of the affected part of the environment (wake) 0.0336 gm/kg for each degree of temperature increase. Depending upon the initial pressure, temperature, and relative humidity of the atmosphere, and upon the value of N, this affected part of the atmosphere may be left either saturated or unsaturated after the passage of the aircraft.

3. Maintaining Saturation in a Saturated Atmosphere Undergoing Heating

Tables were set up showing the increase in mixing ratio (per degree of temperature rise) required to maintain saturation in initially saturated air which is undergoing heating. The calculations were carried out for each 100-mb level from 1,000 to 100-mb, for 5C° temperature arguments down to -80°C, and for ΔT-values of 60, 40, 20, 10, 5, 2, 1, and ½C°, which correspond to N-values of approximately 58, 87, 175, 350, 700, 1,750, 3,500 and 7,000 respectively. Since $\Delta T = 10{,}000/(12N \times 0.24)$, the amount of temperature increase depends upon, and can be used in place of, N in all calculations. An extract from these tables is presented here (TABLE I) to show the influence of the various factors involved. Wherever the TABLE shows values less than .0336 gm/kg/C°, the passage of an aircraft would cause an initially saturated atmosphere to become supersaturated with the consequent production of water droplets. Where the values are greater than .0336, the aircraft would leave an initially saturated environment in a sub-saturated state, and could even cause evaporation in a cloud in its path ("negative contrails" or "distrails") [3, 4, 5].

TABLE I. REQUIRED INCREASE IN MIXING-RATIO PER DEGREE TEMPERATURE RISE (GM/KGM/C°) TO MAINTAIN SATURATION WITH RESPECT TO WATER IN A SATURATED ATMOSPHERE UNDERGOING HEATING

T (°C)	−40	−50	−60	−70	−80
ΔT (°C)			*1000 mb*		
5	.0154*	.0059*	.0020*	.0006*	.0002*
40	.0926	.0436	.0192*	.0078*	.0029*
			500 mb		
5	.0308*	.0117*	.0040*	.0013*	.0004*
40	.1864	.0875	.0384	.0156*	.0058*
			100 mb		
5	.1544	.0587	.0201*	.0062*	.0017*
40	.9822	.4466	.1935	.0778	.0291*

* Indicates conditions under which an initially saturated atmosphere will be supersaturated by the passage of an aircraft.

From TABLE I it is evident that supersaturation of a *saturated* environment is aided by cold temperatures, high pressures, and increased mixing (small values of ΔT) between the environment and exhaust. As shown in the next section, not all of these conditions apply to an environment initially subsaturated.

4. Saturation of Initially Unsaturated Air Undergoing Heating

If contrails are to form in an environment that is not initially saturated, part of the water from the aircraft exhaust must be used to saturate the air with respect to water at its initial temperature, and the remainder† to maintain saturation during the temperature rise caused by the heat from the exhaust. If we let w be the initial mixing ratio of the environment, w_{s_w} the saturation mixing ratio, and f_w the relative humidity (all with respect to water), we can write $w = w_{s_w}(f_w/100)$. Then $\delta w = w_{s_w}(1 - f_w/100)$, where δw is the amount of water which must be furnished by the exhaust to raise the relative humidity of the affected environment to 100 per cent at the initial temperature and pressure. This gives

$$\Delta w = \frac{1.4 \times 1000}{12N} - w_{s_w}(1 - f_w/100),$$

where Δw is the amount of water available to maintain saturation in the environment as it is heated by the exhaust. The ratio $(\Delta w)/(\Delta T)$ now becomes $0.0336 - (1/\Delta T)w_{s_w}(1 - f_w/100)$ where $(\Delta w/\Delta T)$ is the moisture (gm/kg per Centigrade degree temperature rise) available for maintaining saturation during the passage of the aircraft. For example, with a 5C° temperature rise of the environment (ΔT = 5C°)

† As shown in SECTION 6, this remainder may be reduced still further due to the visible-water requirement.

and an initial relative humidity of 60 percent, the value of $(\Delta w/\Delta T)$ becomes $0.0336 - .08w_{sw}$.

Tables were set up showing the values of $(\Delta w/\Delta T)$ available for maintaining saturation for initial relative humidities of 100, 90, 60 and 0 percent respectively. These calculations were carried out for the pressure, temperature, and ΔT values given in SECTION 3. An extract of these results is shown in TABLE II. It is seen

TABLE II. MOISTURE (GM/KGM/C°) AVAILABLE FROM FUEL FOR MAINTAINING SATURATION WITH RESPECT TO WATER IN AN ATMOSPHERE UNDERGOING HEATING

	T (°C)	−40	−50	−60	−70	−80
ΔT (°C)	f_w (%)	*1000 mb*				
5	100	.0336	.0336	.0336	.0336	.0336
	0	.0102	.0257	.0312	.0330	.0335
40	100	.0336	.0336	.0336	.0336	.0336
	0	.0307	.0326	.0333	.0335	.0336
		500 mb				
5	100	.0336	.0336	.0336	.0336	.0336
	0	0	.0178	.0288	.0324	.0333
40	100	.0336	.0336	.0336	.0336	.0336
	0	.0278	.0316	.0330	.0334	.0336
		100 mb				
5	100	.0336	.0336	.0336	.0336	.0336
	0	0	0	.0098	.0273	.0321
40	100	.0336	.0336	.0336	.0336	.0336
	0	.0044	.0237	.0306	.0328	.0334

that in the case of an initially subsaturated environment the amount of moisture available to maintain saturation (per degree of temperature increase) decreases with low pressure and relative humidity, high temperature, and with increased mixing between the exhaust and environment.

5. DETERMINATION OF THE CRITICAL TEMPERATURE FOR FORMATION OF A TRAIL SATURATED WITH RESPECT TO WATER

The above sets of data make it possible to determine the critical temperatures which will give rise to a saturated trail for various initial conditions of pressure and relative humidity, and for different ratios of mixing between the environment and the exhaust. From the tables described in SECTION 4 (see TABLE II), curves were constructed showing the amount of moisture *available* from the exhaust (gm/kg/C°) after the relative humidity of the environment has been raised to 100 percent at its initial temperature. From the tables of SECTION 3 (see TABLE I), curves were drawn showing the amount of moisture *necessary* to maintain saturation in saturated air undergoing heating. The intersections of the two sets of curves determine, for a given value of pressure and mixing, the critical initial temperature of the environment which must exist for the formation of a trail saturated with respect to water. A warmer temperature would leave the trail subsaturated, as the environment would require more moisture than was available from the fuel. A colder temperature would lead to supersaturation. FIGURE 1 shows how the critical temperatures were obtained for the case of $P = 500$ mb and $\Delta T = 5\text{C}°$ ($N \doteq 700$). Curves were plotted of critical temperature as a function of N and relative humidity for each 100-mb level from 1,000 to 100 mb (solid curves in FIGS. 2 and 3).

6. PRODUCTION OF VISIBLE WATER IN CONTRAILS

As mentioned in SECTION 4, it is not only necessary for the water vapor from the fuel combustion to raise the relative humidity of the entrained environment to 100 percent with respect to water,

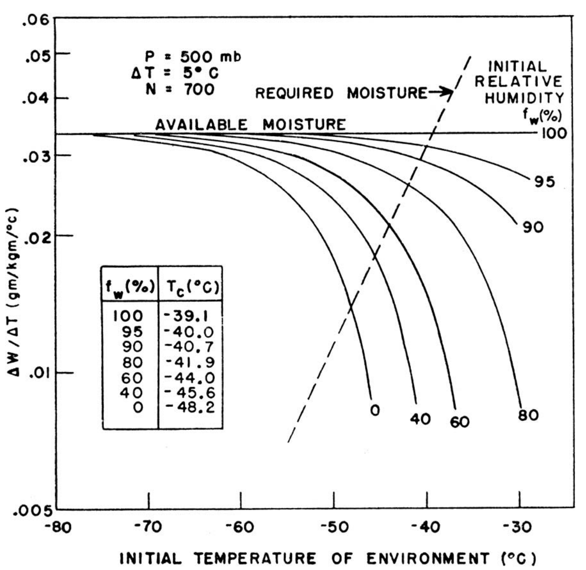

FIG. 1. Critical temperatures (T_c) for formation of a trail saturated with respect to water, for the case of $P = 500$ mb and $N \doteq 700$. (Solid lines show moisture available from exhaust after environment is saturated at its initial temperature and dashed line shows moisture required to maintain saturation in a saturated atmosphere undergoing heating of 5C°.)

but also some *visible* water must be produced. The results of many observations have indicated that in the great majority of cases the visible water making up the contrail is in the form of ice crystals. This fact has been determined from the visual appearance of the trails, from the fact that aircraft flying through such trails only rarely pick up ice [6], from actual photographs of the ice crystals in contrails [7], and from halos and other solar phenomena produced by the trail [8, 9]. This is not surprising since the critical temperatures of contrail formation were in all cases colder than $-29°C$, and for high altitudes and low humidities much colder than this. Thus, even though the vapor first passed through the liquid stage, the highly supercooled water would be expected to freeze almost instantaneously. Freezing nuclei necessary for this phase transition, if not already available at the flight level, would be plentifully supplied by the exhaust itself.

As soon as the transformation of the liquid to the solid water began, the contrail, which had been at approximately 100 percent relative humidity with respect to water, would become

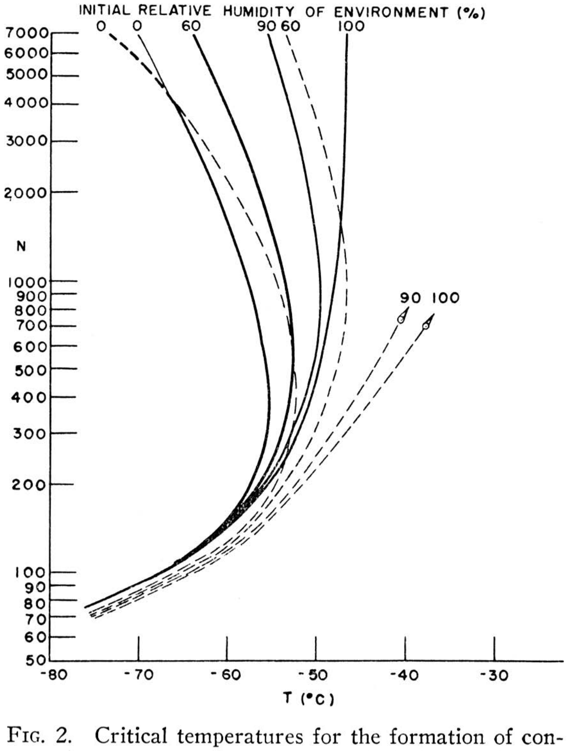

FIG. 2. Critical temperatures for the formation of contrails, for the case of $P = 200$ mb. (Solid lines indicate a final relative humidity in the trail of 100 percent with respect to water, dashed lines indicate a final relative humidity of 100 percent with respect to ice plus a visible water content of .004 gm/m³.)

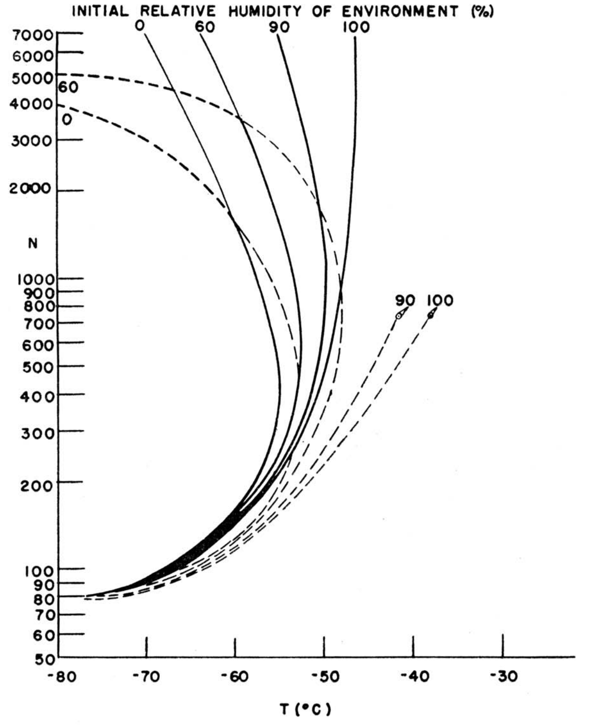

FIG. 3. Critical temperatures for the formation of contrails, for the case of $P = 200$ mb. (Solid lines indicate a final relative humidity in the trail of 100 percent with respect to water, dashed lines indicate a final relative humidity of 100 percent with respect to ice plus a visible water content of .01 gm/m³.)

highly supersaturated with respect to ice. The excess water vapor would immediately begin to sublime onto the ice particles, resulting in the production of larger and larger crystals until a final relative humidity of approximately 100 percent with respect to ice was reached in the trail. The resulting concentration of visible water would be dependent upon the initial meteorological conditions in the atmosphere, and upon the amount of mixing between the exhaust and the environment. It might or might not be of sufficient density to make a visible trail.‡ Various methods could be devised to determine the exact conditions

‡ In determining the critical temperatures for contrail formation, it is the least density of solid water required to produce a visible trail which is of interest. This value has never been measured directly, but has been estimated by A. Goldie and A. Parker [10, 11] to be of the order of .002 to .004 gm/m³. In a verbal communication, H. Weickmann and H. aufm Kampe of the Signal Corps Enginering Laboratories have estimated that a reasonable value would be in the vicinity of .004 to .01 gm/m³. Because of the lack of exact knowledge on this subject, it was decided to compare the effect on the critical temperature produced by the requirement of two different solid-water contents—.004 and .01 gm/m³—and arbitrarily to label such trails as *faint* and *distinct*, respectively.

under which a trail saturated with respect to water would furnish sufficient excess water to make the trail visible at saturation with respect to ice. The method used here was to determine and compare the T_c-values for both conditions. The colder of the two temperatures would be the true critical temperature which would fulfill both criteria.

7. Determination of the Critical Temperature for the Existence of a Visible Trail Saturated with Respect to Ice

The methods used in calculating these curves were similar to the ones outlined in Sections 3, 4, and 5, except for two fundamental differences. First, the initial relative humidity of the environment is given with respect to water, while the final relative humidity in the trail is given with respect to ice. Thus, using the notation of Section 4,

$$\delta w = w_{s_i} - w = w_{s_i} - w_{s_w}\frac{f_w}{100},$$

$$\Delta w = \frac{1.4 \times 1000}{12N} - \left(w_{s_i} - w_{s_w}\frac{f_w}{100}\right),$$

and

$$\frac{\Delta w}{\Delta T} = .0336 - \frac{1}{\Delta T}\left(w_{s_i} - w_{s_w}\frac{f_w}{100}\right),$$

whereas in Section 4,

$$\frac{\Delta w}{\Delta T} = .0336 - \frac{1}{\Delta T}\left(w_{s_w} - w_{s_w}\frac{f_w}{100}\right).$$

Thus, for a value of $T = 5°C$ and $f_w = 60\%$,

$$\frac{\Delta w}{\Delta T} = .0336 - .2w_{s_i} + .12w_{s_w}.$$

The excess moisture available after the entrained environment has been brought to ice saturation at its initial temperature is, of course, greater than if the trail had been brought to water saturation.

The second major difference is the required production of a specified quantity of solid water. Thus to the moisture required to maintain saturation with respect to ice during the temperature rise caused by the exhaust, must be added sufficient moisture to satisfy the visible water requirements of .004 and .01 gm/m^3 respectively.

The intersections of the curves of required and available moisture define the values of T_c for the existence of a trail saturated with respect to ice and with a visible water content of the specified amount. A warmer temperature would result in the production of less than the required amount of ice, while a colder temperature would result in the production of more than was required. As in Section 5, curves were plotted of critical temperature as a function of N and relative humidity for each 100-mb level from 1000 to 100 mb (dashed curves, Figs. 2 and 3).

8. The Critical Temperature for the Formation of Visible Contrails

Depending upon the initial temperature, pressure, and relative humidity of the environment, and upon the ratio of the entrained environment to the exhaust, the temperature required to form a trail saturated with respect to water may be colder or warmer than the temperature required to form a trail saturated with respect to ice but with a visible water content of .004 or .01 gm/m^3 (see Figs. 2 and 3). Since both requirements must be satisfied, the colder of the two temperatures in each case is the critical temperature which must exist for the formation of a visible contrail. Therefore, in determining the critical temperature, composite graphs showing both sets of curves must be used. A complete set of such graphs was constructed for each 100-mb level from 1,000 to 100 mb, for relative humidities with respect to water of 0, 60, 90, and 100 percent, for temperatures down to −80°C, and ratios of entrained environment to exhaust gas from 58:1 to 7,000:1. Figures 2 and 3 show the composite graphs for the 200-mb level.

As indicated in Figures 2 and 3, every curve is double-valued with respect to temperature except for the case of an initially saturated environment involving no requirement for the production of visible water (such as the solid curves labeled 100 percent). Thus, when the exhaust gas first strikes the atmosphere, the required temperature for contrail formation is extremely low, and the contrail may not form. Further mixing between the exhaust and the environment, however, raises the critical temperature until a maximum value is reached. This portion of the curve can be considered as the region of contrail formation, since if at any time the critical temperature exceeds the actual temperature, a contrail will result. In all cases, the entire formation-region is colder for the water trails than for the corresponding ice trails. Therefore, the critical temperature for the formation of water trails also satisfies the requirements for both faint and distinct ice trails. After a trail has once formed, and has been completely transformed into ice, it will not dissipate until further mixing reduces the critical temperature for the existence of the visible ice trail below the

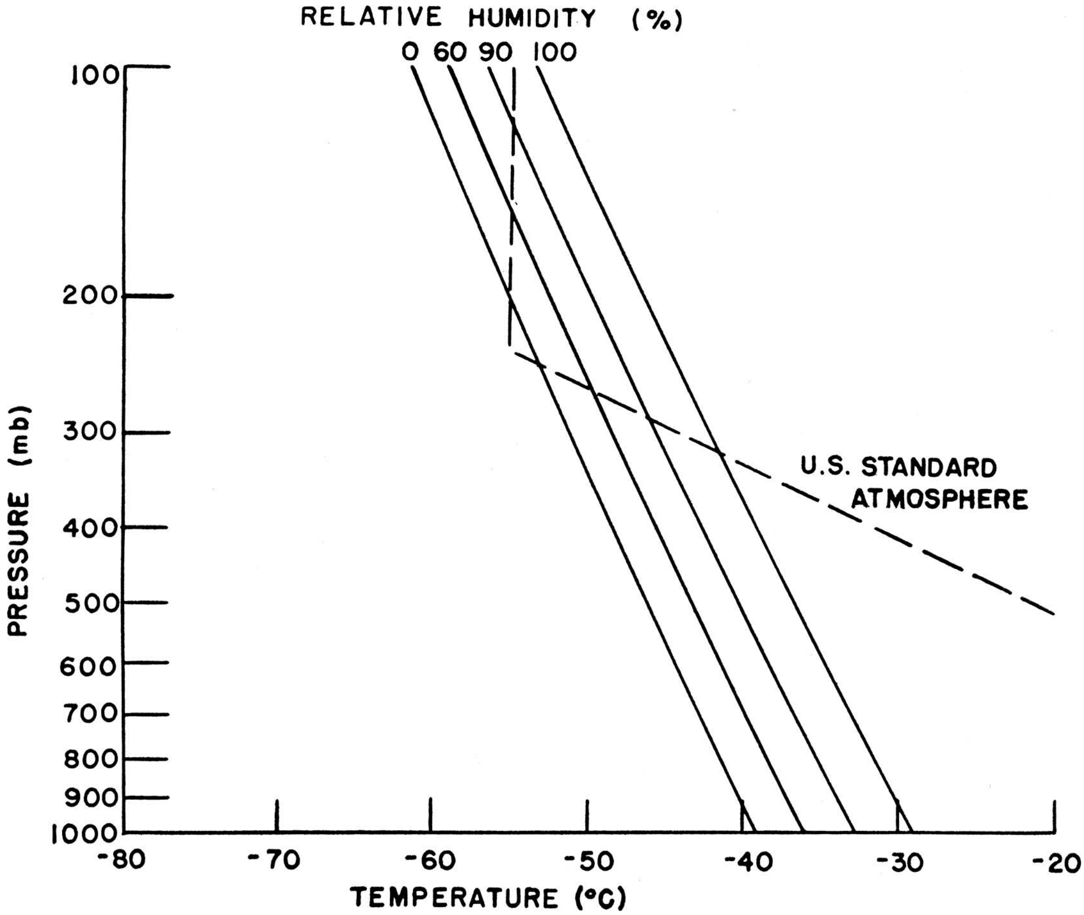

FIG. 4. A graph of the required relative humidity for contrail formation as a function of the pressure and temperature of the environment.

actual temperature of the environment. Thus in every case the point of formation of a contrail lies on the water curve, and the point of dissipation on the appropriate ice curve.

For an example of how the composite graphs can be used, suppose a flight is planned at 200 mb (approximately 39,000 feet). If the temperature and relative humidity at this level are $-60°C$ and 60 percent respectively, a distinct trail will form at a point behind the aircraft where the environment/exhaust ratio is 150:1. The trail will persist until, with further entrainment, the ratio becomes 3700:1 (FIG. 3), at which point the contrail can no longer be considered distinct according to the definition used in this report. A faint trail will continue, however, until the ratio is well above 7,000:1 (FIG. 2). If, with the same pressure and relative humidity, the temperature of the environment were $-52°C$, no contrail would form, as the wake would never reach saturation with respect to water.

If it is desired to know only whether or not a visible contrail will occur, and not the point at which it will form or dissipate, a much simpler graph can be used. The maximum temperature for contrail formation was selected from each of the composite curves and plotted as a function of the pressure and relative humidity (FIG. 4). This is the warmest temperature at which a contrail can form under the prescribed conditions. The curves in FIGURE 4 can also be considered as defining the minimum initial relative humidity (with respect to water) of the environment necessary to give rise to contrail formation at a given temperature and pressure.

In making a forecast, the pressure and temperature for the point are entered on the graph and the relative humidity required for the formation of a contrail is found. If the point entered lies to the right of the 100 percent line, it is extremely unlikely that trails will form, as the atmosphere is rarely supersaturated with respect to water. If the point lies to the left of the 0 percent line, contrails will form even in completely dry air. At any in-between point, the formation of trails depends on whether the actual relative humidity of the environment is greater than the required value as given by the graph. Using the NACA Standard Atmosphere as an example, trails are very unlikely to form below 320 mb, but will always form between 240 and 205 mb. Between 320 and 240 mb, and above 205 mb, it is necessary to know the actual relative humidity or to assume a reasonable value. Arctic air-masses are much colder at low levels than is the Standard Atmosphere, and give rise to the possibility of contrail formation even as low as the surface of the earth. In tropical air-masses, on the other hand, the intense cold aloft increases the probability of contrails at very high levels. For routine use, the curves of FIGURE 4 should be made into a transparent overlay designed to fit the scale of the adiabatic charts in current use.

Relative humidity measurements as obtained from radiosonde observations at low temperatures are both inaccurate and few in number. In the complete absence of data, the relative humidity with respect to water in the upper troposphere can generally be assumed to be in the vicinity of 40 to 90 percent, and in the stratosphere between 0 and 60 percent [13]. As shown in FIGURE 4, this is equivalent to an error in the critical temperature of only 3 or 4 degrees. Furthermore, the above estimates can be refined by noting the source of the upper flow, and the presence or absence of cirrus clouds.

9. CONCLUSIONS

The assumptions upon which the calculations in this paper were based are threefold: (*1*) the wake behind the aircraft must somewhere reach saturation with respect to water before any vapor can be transformed into visible water; (*2*) after the water droplets have formed, immediate freezing will occur, and the excess vapor in the trail will deposit onto the ice crystals until the relative humidity in the trail falls to 100 percent with respect to ice; (*3*) an ice crystal content of .004 gm/m^3 is required for a faint trail and .01 gm/m^3 for a distinct trail. It was found that the last requirement had no effect on the formation of the trail, but did affect its dissipation.

FIGURE 4 shows the minimum relative humidity at which visible contrails can occur for any given pressure and temperature. A drier environment will not support the formation of contrails, while in a more humid environment, trails will form, and their length will vary directly with the difference between the actual and required relative humidities.

Composite graphs, such as FIGURES 2 and 3, can be used qualitatively to obtain the distance behind the aircraft at which the contrails will form and dissipate. Since the curves are given in terms of mixing between the environment and the exhaust, this parameter must be translated into terms of distance, or time. This relationship is dependent on the speed of the aircraft, the rate of burning fuel, the radial distance of the point from the axis of the trail, and the density and stability of the atmosphere. Until such studies are available, only qualitative forecasts can be made as to the duration of the contrails. As experience is gained with specific types of aircraft, however, fairly accurate forecasts should be practicable not only as to whether contrails will occur, but also as to their density and persistence.

10. ACKNOWLEDGMENTS

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