Blending Problem's Solution using CPLEX (LP)

Problem Statement

A firm faces the problem of blending three raw materials into two final products.

The required numerical information is provided in the following table:

Raw Materials	Final Products		Amount available (in tons)	Unit Cost ($)
	1	2		
1	[0.4; 0.6]	[0.5; 0.6]	2,000	1.00
2	[0.1; 0.2]	[0.1; 0.4]	1,000	1.50
3	[0.2; 0.5]	[0.2; 0.3]	500	3.00
Quantity required (in tons)	600	700		
Unit selling price ($)	10	8		

Model Formulation

Decision Variables:

Q_{ij} =Quantity of i^{th} raw material in j^{th} product

The Model:

Maximize:

$$Z = [10 (Q_{11} + Q_{21} + Q_{31}) + 8 (Q_{12} + Q_{22} + + Q_{32})] – [1(Q_{11} + Q_{12})+ 1.5(Q_{21} + Q_{22}) + 3(Q_{31} + Q_{32})]$$

Subject to:

$Q_{11} + Q_{12} \leq 2000$

$Q_{21} + Q_{22} \leq 1000$

$Q_{31} + Q_{32} \leq 500$

$Q_{11} + Q_{21} + Q_{31} = 600$

$Q_{12} + Q_{22} + Q_{32} = 700$

$Q_{11} \geq 0.4(Q_{11} + Q_{21} + Q_{31})$

$Q_{21} \geq 0.1(Q_{11} + Q_{21} + Q_{31})$

$Q_{31} \geq 0.2(Q_{11} + Q_{21} + Q_{31})$

$Q_{12} \geq 0.5(Q_{12} + Q_{22} + Q_{32})$

$Q_{22} \geq 0.1(Q_{12} + Q_{22} + Q_{32})$

$Q_{32} \geq 0.2(Q_{12} + Q_{22} + Q_{32})$

$Q_{11} \leq 0.6(Q_{11} + Q_{21} + Q_{31})$

$Q_{21} \leq 0.2(Q_{11} + Q_{21} + Q_{31})$

$Q_{31} \leq 0.5(Q_{11} + Q_{21} + Q_{31})$

$Q_{12} \leq 0.6(Q_{12} + Q_{22} + Q_{32})$

$Q_{22} \leq 0.4(Q_{12} + Q_{22} + Q_{32})$

$Q_{32} \leq 0.3(Q_{12} + Q_{22} + Q_{32})$

$Q_{ij} \geq 0 \; \forall \; i, j$

Compact Model Formulation

Index:

- i, k = Raw material
- j = Final Product

Model Parameters:

s_i = Total units of supply of i^{th} raw material

d_j = Demand of the j^{th} product

c_i = Unit cost of i^{th} raw material

p_j =Unit price the j^{th} final product

$\underline{a}_{ij}$ = Smallest proportion of raw material 'i' that is allowed in the final product 'j'

$\overline{a}_{ij}$ = Largest proportion of raw material 'i' that is allowed in the final product 'j'

Decision Variables:

Q_{ij} = Quantity of i^{th} raw material in j^{th} product

The Model

Maximize: $\sum_j p_j \sum_i Q_{ij} - \sum_i c_i \sum_j Q_{ij}$

Subject to:

$\sum_j Q_{ij} \leq s_i$	$\forall\ i$	(Supply or Capacity constraint)
$\sum_i Q_{ij} = d_j$	$\forall\ j$	(Demand Constraint)
$Q_{ij} \geq \underline{a}_{ij} \sum_k Q_{kj}$	$\forall\ i,j$	(Minimum proportion in the final product)
$Q_{ij} \geq \overline{a}_{ij} \sum_k Q_{kj}$	$\forall\ i,j$	(Maximum proportion in the final product)
$Q_{ij} \geq 0$	$\forall\ i,j$	