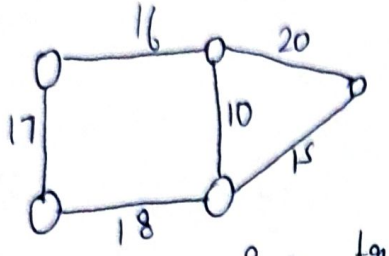
Tutorial - 6

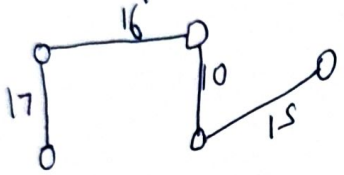
1) Minimum Spanning Tree:

A spanning tree of an undirected graph is a subgraph that is a tree & joined by all vertices. One of those tree which has minimum total cost would be its minimum spanning tree.

Eg:



Minimum cost spanning tree



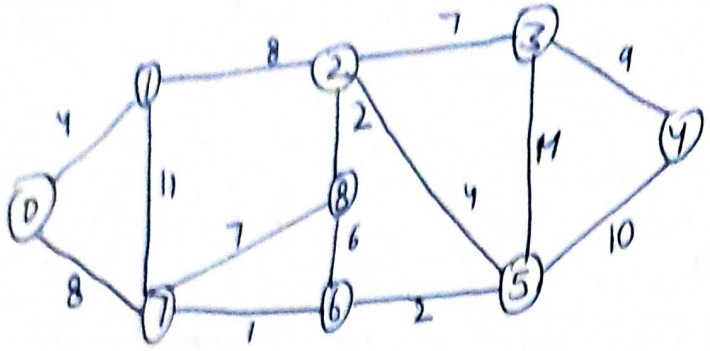
Applications of MST

- It has direct applications in the design of networks including computer networks, telecommunication networks, transportation networks etc

Ans 2)

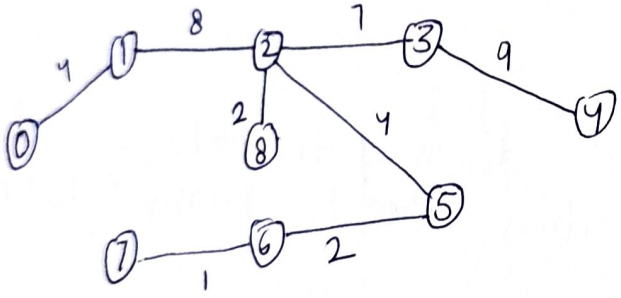
	Prism's Algorithm	Kruskal's Algorithm	Dijkstra's Algo	Bellman Ford's Algo
TC	$O(V^2)$	$O(E \log V)$	$O(V+E \log V)$	$O(VE)$
SC	$O(V+E)$	$O(\|E\|+\|V\|)$	$O(V^2)$	$O(V^2)$

3)



Prism's Algo

0	1	2	3	4	5	6	7	8
[0]	∞	∞	∞	∞	∞	∞	∞	∞
	[4]	[8]					8	
			[7]		[4]	6	7	[2]
						[2]	[11]	
				10				
				[9]				



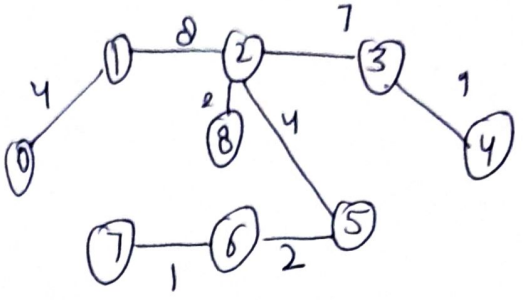
Min weight
= 37

Parent	0	1	2	3	4	5	6	7	8
	-1	~~1~~	~~2~~	~~3~~	-1	~~2~~	-1	~~6~~	~~2~~
		0	1	2		2		6	2
					~~5~~		~~8~~	~~8~~	
					3		5	6	

Parent : -1 0 1 2 3 2 5 6 2

Kruskal's Algo

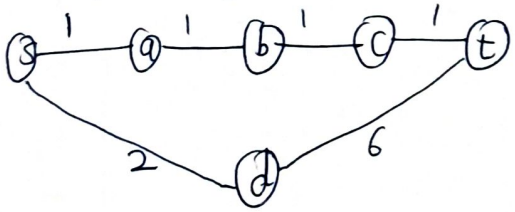
U	V	W	
7	6	1	✓
6	5	2	✓
2	8	2	✓
2	5	4	✓
0	1	4	✓
8	6	6	X
7	8	7	X
2	3	7	✓
1	2	8	✓
0	7	8	X
3	4	9	✓
5	4	10	X
1	7	11	X
3	5	14	X



weight = 37

4) i) If 10 units is added to each edge, the overall weight of the path may change.

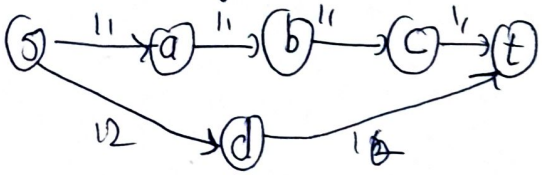
Eg:



Shortest path is $s \to a \to b \to c \to t$

weight = $1+1+1+1 = 4$

now if 10 units weight is added to each edge.

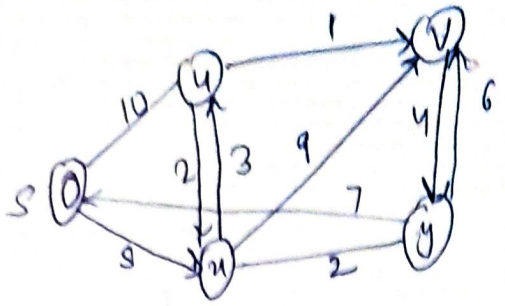


Shortest path changed to $s \to d \to t$

weight = 28

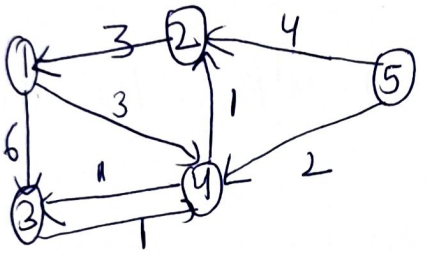
ii) Multiplying the weight of each edge by 10 will have no impact on the shortest path.

5)



s	u	v	x	y
	∞	∞	∞	∞
0			5	
	10	∞		∞
0			5	
	10	11		7
0			5	
	10	11		
0				

6) All pair shortest path algorithm – Floyd Warshall



$$A^0 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 3 & 0 & \infty & \infty & \infty \\ \infty & \infty & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ \infty & 4 & \infty & 2 & 0 \end{bmatrix} \end{array}$$

$$A' = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 3 & 0 & 9 & 6 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ \infty & 4 & \infty & 2 & 0 \end{bmatrix} \end{array}$$

$A^0[2,3] = \infty$

$A^0[2,1] + A^0[1,3] = 3 + 6 = 9$

$9 < \infty$

Similarly

$A^0[2,4] = \infty$

$A^0[2,1] + A^0[1,4] = 3+3 = 6$

$\Rightarrow 6 < \infty$

$A^0[2,5] = \infty$

$A^0[2,1] + A^0[1,5] = 3+\infty$

$$A^2 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 3 & 0 & 9 & 6 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ 7 & 4 & 13 & 2 & 0 \end{bmatrix} \end{array}$$

$A^1[1,3] = 6$

$A^1[1,2] + A^1[2+3] = \infty + 9$

$6 < \infty + 9$

$$A^3 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 3 & 0 & 9 & 6 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ 7 & 4 & 13 & 2 & 0 \end{bmatrix} \end{array}$$

$$A^4 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 4 & 4 & 3 & \infty \\ 3 & 0 & 7 & 6 & \infty \\ \infty & 3 & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ 7 & 3 & 3 & 2 & 0 \end{bmatrix} \end{array}$$

$$A5 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 4 & 4 & 3 & \infty \\ 3 & 0 & 7 & 6 & \infty \\ \infty & 3 & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ 7 & 3 & 3 & 2 & 0 \end{bmatrix} \end{array}$$