

Université d'Ottawa
Faculté de génie

École de science
d'informatique
et de génie électrique



University of Ottawa
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School of Electrical
Engineering
and Computer Science

CSI 4106 Introduction to AI

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Assignment - 3

Logic and Reasoning

Course Coordinator: Caroline Barrière

Group-12

Member1: Meet Mehta (300261159)

Member2: Rakshita Mathur (300215340)

Q1 – Proofs in Propositional Logic

Solution 1:

i) Proof of e:

1. Premise: $S \rightarrow \neg Q$

2. Premise: $P \rightarrow S$

3. Premise: $\neg \neg P$

To Show: $\neg Q$

4. P (double negation)

5. S (Apply Modus Ponens on 1 and 4)

6. $\neg Q$ (Apply Modus Ponens on 2 and 5)

ii) Proof of f:

1. Premise: $T \rightarrow P$

2. Premise: $Q \rightarrow S$

3. Premise: $S \rightarrow T$

4. Premise: $\neg P$

To Show: $\neg Q$

5. $\neg T$ (Modus Tollens on 1 and 4)

6. $\neg S$ (Modus Tollens on 3 and 5)

7. $\neg Q$ (Modus Tollens on 2 and 6)

iii) Proof of g:

1. Premise: P

2. Premise: R

3. Premise: $(P \rightarrow (R \rightarrow Q))$

To Show: Q

4. $R \rightarrow Q$ (Modus Ponens on Premise 1 and 3)

5. Q (Modus Ponens on Premise 2 and 4)

iv) Proof of k:

1. Premise: P

2. Premise: $P \rightarrow R$

3. Premise: $(P \rightarrow (R \rightarrow Q))$

To Show: Q

4. R (Modus Ponens on Premise 1 and 2)

5. $(R \rightarrow Q)$ (Modus Ponens on Premise 1 and 3)

6. Q (Modus Ponens on Step 4 and 5)

Q2 – Predicate Logic from NL – expressive power

Solution 2:

1. All graduate students have an undergraduate degree.
 $\forall (x): (\text{GraduateStudent}(x) \rightarrow \text{HasUndergraduateDegree}(x))$
2. Anyone with an undergraduate degree studied in at least one university.
 $\forall y \exists x: (\text{UndergraduateDegree}(y) \wedge \text{StudiedInUniversity}(x, y))$
3. A university, at any particular year, delivers courses to many students.
 $\forall y \exists x, z: (\text{University}(x) \wedge \text{Year}(y) \wedge \text{DeliversCoursesToStudents}(x, y, z))$
4. For a professor to teach a course, there must be at least a student registered in it.
 $\forall y, x: (\text{Teach}(x, y) \rightarrow \exists z \text{ RegisteredInCourse}(z, y))$
5. A class is called a large class if at least 100 students are part of it.
 $\forall y \exists x: (\text{Class}(x, y) \wedge (\text{NumberOfStudents}(y, x) \geq 100))$
6. Classmates are students taking the same course.
 $\forall y, x: (\text{Classmate}(x, y) \rightarrow \exists z \text{ TakingCourse}(x, z) \wedge \text{TakingCourse}(y, z))$
7. A graduate student takes fewer courses than an undergraduate student.
 $\forall y, x: (\text{GraduateStudent}(x) \wedge \text{UndergraduateStudent}(y) \rightarrow \text{TakesFewerCourses}(x, y))$

Explanation:

- Each statement is expressed using quantifiers \forall for universal, \exists for existential) and predicates representing different relationships and properties.
 - The expressions capture the relationships between entities such as students, degrees, universities, courses, professors, classes, and classmates, as well as certain conditions specified in the statements.
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Q3 – Fuzzy logic world

Solution3:

Linguistic Variable

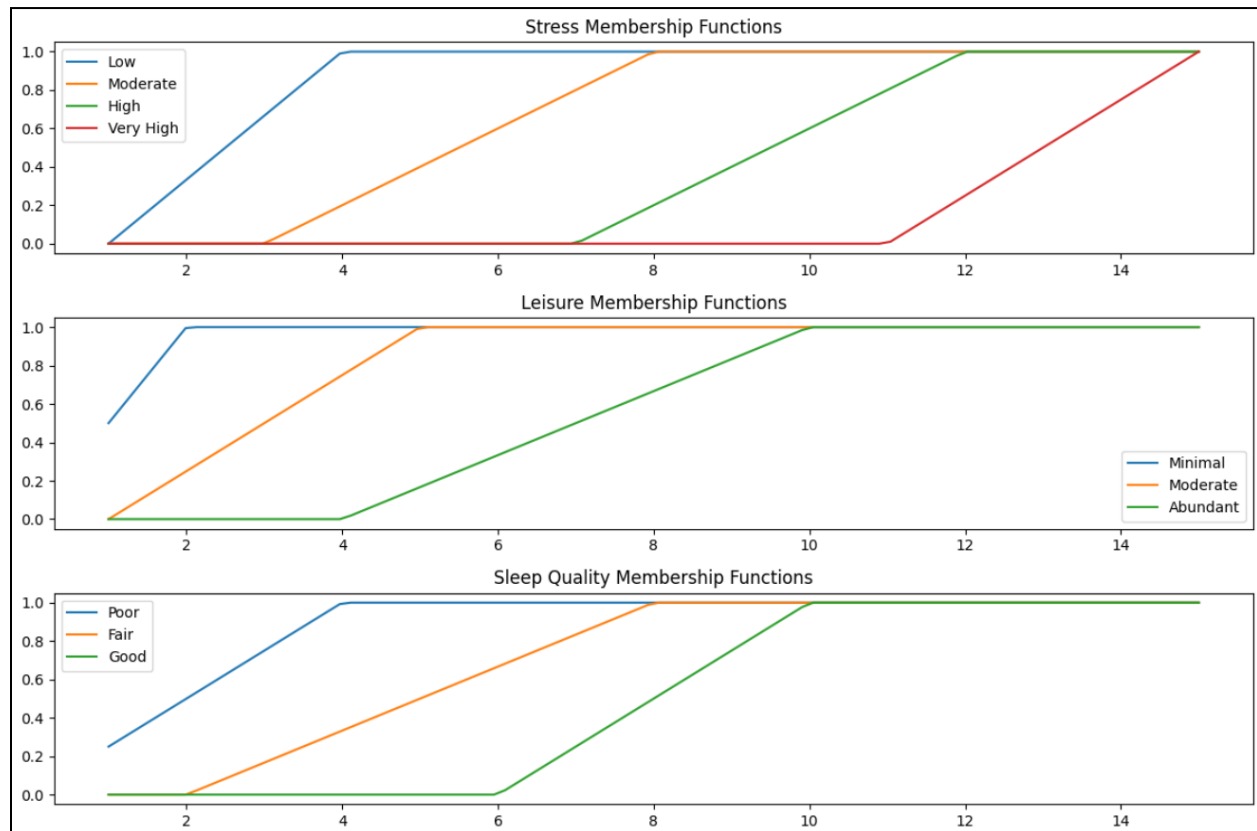
1. Stress:
 - V: Stress
 - X_V : {1, 2, ..., 15} (Range of possible values for stress)
 - T_V : {Low, Moderate, High, Very High} with membership functions:
 - Low: Triangular, (1, 0, 4)
 - Moderate: Triangular, (3, 5, 8)
 - High: Triangular, (7, 10, 12)
 - Very High: Triangular, (11, 15, 15)

2. Leisure:

- V: Leisure
- X_V : {0, 1, 2, ..., 10} (Range of possible values for leisure hours)
- T_V : {Minimal, Moderate, Abundant} with membership functions:
 - Minimal: Triangular, (0, 0, 2)
 - Moderate: Triangular, (1, 3, 5)
 - Abundant: Triangular, (4, 7, 10)

3. Sleep Quality:

- V: Sleep Quality
- X_V : {0, 1, 2, ..., 10} (Range of possible values for sleep quality)
- T_V : {Poor, Fair, Good} with membership functions:
 - Poor: Triangular, (0, 0, 4)
 - Fair: Triangular, (2, 5, 8)
 - Good: Triangular, (6, 10, 10)



[Source](#)

Fuzzy Rules:

1. Rule 1: If Stress is High and Leisure is Minimal, Then Sleep Quality is Poor.
2. Rule 2: If Stress is Moderate and Leisure is Moderate, Then Sleep Quality is Fair.
3. Rule 3: If Stress is Low and Leisure is Abundant, Then Sleep Quality is Good.

Example Fuzzy Inference:

Let's consider an example with Stress = 9 and Leisure = 2 hours.

1. Transform Input to Membership Values:

- Stress:

- High: $\mu_{\text{High}}(9) = 0.2$
- Moderate: $\mu_{\text{MODERATE}}(9) = 0.8$
- Low: $\mu_{\text{LOW}}(9) = 0$
- Leisure:
 - Minimal: $\mu_{\text{MINIMAL}}(2) = 0.5$
 - Moderate: $\mu_{\text{MODERATE}}(2) = 0.5$
 - Abundant: $\mu_{\text{ABUNDANT}}(2) = 0$
- 2. Apply Fuzzy Rules:
 - Rule 1: $\min(0.2, 0.5) = 0.2$ (Poor)
 - Rule 2: $\min(0.8, 0.5) = 0.5$ (Fair)
 - Rule 3: $\min(0, 0) = 0$ (Good)
- 3. Combine Rules:
 - Aggregated Output: $\max(0.2, 0.5, 0) = 0.5$
- 4. Defuzzify:
 - Centroid Method: Sleep Quality = $0.5 * 8 = 4$

So, for the input (Stress = 9, Leisure = 2 hours), the estimated Sleep Quality is 4.

Q4 – Examples of reasoning in human tests

Solution 4:

1. Constraint-Based Reasoning:

- Solving Q.5 - Finding the fares for cities B and C from A.
- Source: [link](#)
- Explanation: * This problem involves constraints on the total cost of tickets for different routes. The constraints are represented by the given equations that describe the cost of tickets from A to B and A to C.
- Generic vs. Specific: The generic aspect is understanding how to set up and solve systems of equations. The specific part involves the constraints related to ticket prices in this scenario.

2. Inductive Reasoning:

- Solving Q.3 - Finding the missing number in the series.
- Source: [link](#)
- Explanation: The series follows a pattern where each number is obtained by doubling the previous number and subtracting one. Inductive reasoning is applied by observing the pattern and predicting the next number in the sequence.
- Generic vs. Specific: The generic aspect is recognizing and applying patterns in a sequence. The specific part is the doubling and subtracting pattern in this particular series.

3. Analogical Reasoning:

- Solving Q.4 - Completing the alphabet series (A, B, N, C, D, O, E, F, P, ?, ?, ?).
- Source: [link](#)

- Explanation: Analogical reasoning can be used to recognize the pattern in the series. It involves understanding that the series alternates between two groups of letters: (A, B, C, ...) and (N, O, P, ...).
- Generic vs. Specific: The generic part is the ability to recognize and apply patterns in alphabet series. The specific part is the alternating pattern of two letter groups.

4. Temporal Reasoning:

- Solving Q.2 - Completing the letter series (OTE, PUF, QVG, RWH, ?).
- Source: [link](#)
- Explanation: Temporal reasoning involves understanding the temporal sequence of letters in the series. In this case, each letter represents a step forward or backward in the alphabet, creating a temporal sequence.
- Generic vs. Specific: The generic aspect is understanding the temporal sequence in letter series. The specific part is the direction and magnitude of the letter changes in this series.

5. Spatial Reasoning:

- Solving Q.6 - Completing the letter series (Z, W, S, P, L, I, E, ?).
- Source: [link](#)
- Explanation: Spatial reasoning can be applied to this problem by recognizing the spatial relationship between letters in the alphabet. In this case, the letters are moving backward through the alphabet, indicating a spatial pattern.
- Generic vs. Specific: The generic part is the ability to interpret spatial relationships in letter series. The specific part is the spatial movement of letters in this particular series.

Q5 – Causal fallacies

Solution 5:

Here are the 5 fallacies we have selected:

- **Glittering generality:**

This is a fallacy that portrays a vague and generalized statement that cannot be proven or disproven.

- Example: The advertisement from Gillet cited below campaigns on the tagline saying ‘Gillet - The Best Man Can Get’. This uses vague and disproportionated language that cannot be proven or disproven.

From the perspective of training an LLM like ChatGPT on sources from the internet, this can create a bias, training the model thinking Gillet in fact is the best that a consumer can buy, without any concrete evidence.

- Source: [The Best Men Can Be - Wikipedia](#)

- **Card Stacking:**

Card stacking is a propaganda technique that involves presenting only one side of an argument, often by selectively omitting information that contradicts the argument.

- Example: The blog post below is heavily biased and only focuses on the positive part (for the most part of the article) about the phone and ignores any negative aspects of the phone.

From the perspective of training an LLM like ChatGPT on sources from the internet, this can create a bias, training the model thinking Samsung in fact could be the best phone for every consumer.

- Source: [Here's Why You Should Buy the Samsung Galaxy S23 Ultra \(howtogeek.com\)](https://howtogeek.com/2022/05/12/why-you-should-buy-the-samsung-galaxy-s23-ultra/)

- **The bandwagon fallacy:**

Also known as the appeal to the masses, is a logical fallacy of claiming that something is true only because everyone believes it to be true.

- Example: The blog post below shows a list of some of the viral TickTock challenges that have proven to be fatal.

This shows that blindly following what is the most popular (viral in this case) is not always the most logical choice to make and is not true (or safe in this case) because everyone simply believes it to be.

- Source: <https://nypost.com/2022/05/12/craziest-tiktok-challenges-so-far/>

- **Unrelated Testimonial:**

An unrelated testimonial fallacy is a type of logical fallacy that occurs when someone uses the endorsement or opinion of a person who is not an expert or relevant to the topic as evidence for their argument.

- Example: In the post below, we can see that A-List celebrities are being sued after endorsing FTX. For context, FTX is (was) a big name in Crypto exchange before the founder and CEO was found guilty on all charges of 'Fraud'. The athletes who are endorsing the brand do not have any expertise in the field of crypto exchange and yet are portrayed as the face of the company.

- Source: <https://www.cbc.ca/news/business/bankruptcy-class-action-ftx-cryptocurrency-bailout-bankman-fried-1.6655836>

- **Name Calling:**

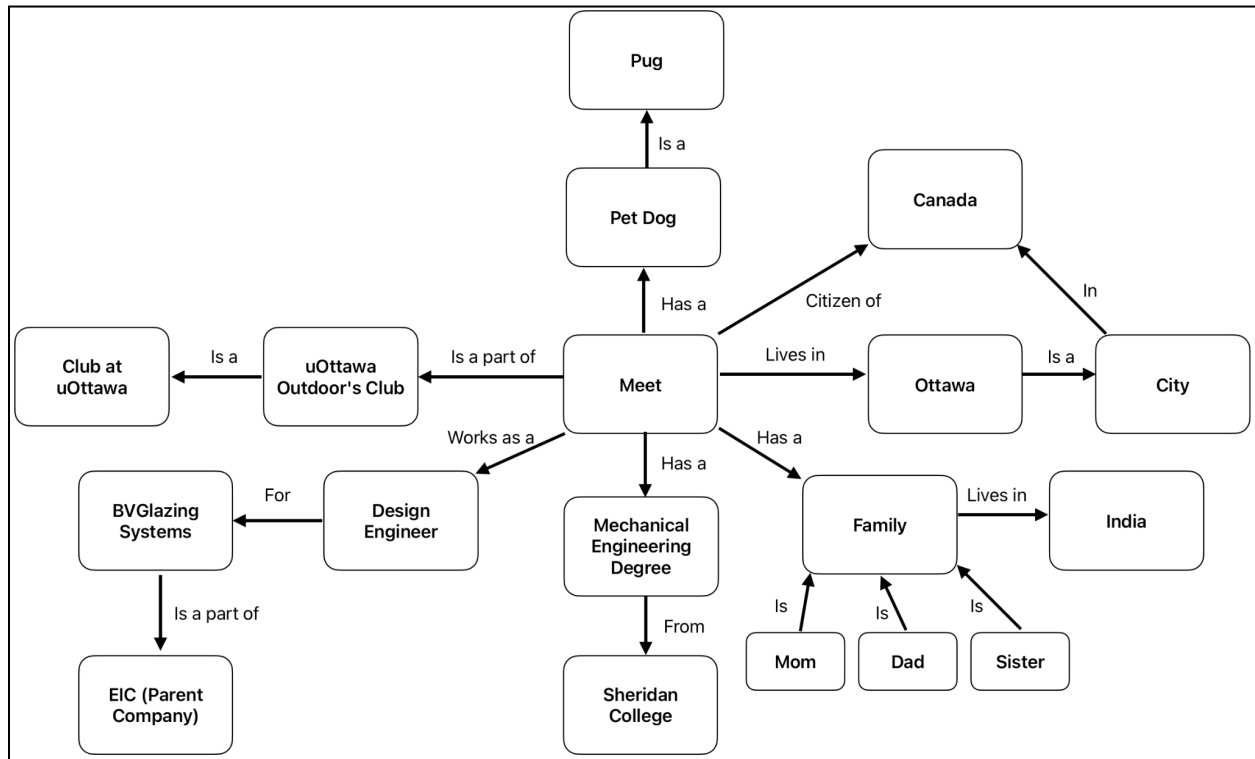
The name-calling fallacy involves attacking the person making an argument rather than addressing the argument itself.

- Example: In the article cited below we can see that Donald Trump accuses and calls out President Joe Biden calling him 'crooked Joe' and blaming him for the border control issues that America is facing instead of addressing the issue itself. This is a very common political tactic to drill opposition and create a 'bad reputation' of the opposition leader in public.

- Source: [Trump campaign calls out 'crooked Joe Biden' after admin stresses 'immediate need' for border wall | Fox News](https://www.foxnews.com/politics/trump-campaign-calls-out-crooked-joe-biden-admin-stresses-immediate-need-border-wall)

Q6 – Knowledge Representation

Solution 6:



Q7 – ChatGPT as “reasoner” (15 points)

Since ChatGPT is a non-deterministic system, we are submitting the same question in 2 different chat boxes to see if we can see any difference in the answers. We have selected the following 5 reasoning types:

- Common Sense Reasoning
- Mathematical Reasoning
- Similarity-Based reasoning
- Spatial Reasoning
- Constraint-Based Reasoning

Common Sense Reasoning

Question 1:

Alex is Charlie's father. Which of them was born later?

Answer 1:

Charlie was born later than Alex.

Answer 2:

Charlie was born later than Alex.

Discussion

We can clearly tell that Charlie was born later using common sense reasoning that parents must be born before the children. Here Chat GPT is able to answer the logical question correctly both times.

Question 2:

A brother and a sister were born in summer and winter. The sister was not born in winter. Who was born in the summer?

Answer 1:

The brother was born in the summer.

Answer 2:

The brother was born in the summer.

Discussion:

The given scenario states that a brother and a sister were born in summer and winter. It is explicitly mentioned that the sister was not born in winter. This implies that the sister must have been born in the summer. Here the model is unable to distinguish between a double negation and provides an incorrect answer.

Question 3:

" 'thing' is a 5-letter word. Name a longer 5-letter word."

Answer 1:

The word "smile" is a longer 5-letter word than "thing."

Answer 2:

The longer 5-letter word is "giant."

Discussion:

Clearly, the question asks to provide any word that is longer than 5 letters in length but the model is unable to identify the basic task and is instead providing a word that is 5 letters long in both cases. We believe the model interprets the question as 'provide any other 5 letter word'.

Mathematical Reasoning**Question 1:**

I left 5 items of clothing out to dry and it took 5 hours.
How long would it take to dry 30 items of clothing?

Answer 1:

The time it takes to dry clothing is not necessarily linear with the number of items. However, for the sake of this simple calculation, if we assume a linear relationship, it might still take 5 hours.

Answer 2:

The time it takes to dry the clothing is likely independent of the number of items. Assuming similar conditions, it should still take around 5 hours to dry 30 items.

Discussion:

Here the model is successfully able to answer the question each time the prompt was provided. Interestingly the previous versions of ChatGPT (I've tested this with version 3) were unable to answer a similar question. It was unable to understand that the attribute can be independent of the parameter in the question.

Question 2:

I have a 12-liter jug and a 6-liter jug and I want to measure 6 liters. How do I do it?

Answer 1:

You can measure 6 liters using the 12 and 6 liter jugs in the following steps:

- Fill the 6-liter jug.
- Pour water from the 6-liter jug into the 12-liter jug until it's full, leaving you with exactly 6 liters in the 6-liter jug.

Answer 2:

You can measure exactly 6 liters using the 12-liter and 6-liter jugs as follows:

- Fill the 6-liter jug.
- Pour the water from the 6-liter jug into the 12-liter jug.
- Fill the 6-liter jug again.
- Carefully pour water from the 6-liter jug into the 12-liter jug until the 12-liter jug is full. Since you already have 6 liters in the 12-liter jug, only 6 liters will fit.
- At this point, you have exactly 6 liters in the 12-liter jug.
- This way, you've measured out 6-liter using the 12-liter and 6-liter jugs.

Discussion:

Here in both cases, the model fails miserably. The question clearly states that we already have a 6-liter jug and all we need is 6 liters of water. Instead of jumping through the hoops to get the 6-liter water, we can just fill the 6-liter jug and get to the result. The model is unable to identify the hidden answer present in the question.

Question 3:

One rabbit saw 6 elephants while going towards River. Every elephant saw 2 monkeys going towards the river. Every monkey holds one tortoise in their hands. How many animals are going towards the river?

Answer 1:

The number of animals going towards the river is 9.

Answer 2:

The number of animals going towards the river is 19: 1 rabbit, 6 elephants, and 12 monkeys (2 monkeys per elephant). Each monkey is holding a tortoise.

Discussion:

This question takes a bit of Mathematical reasoning and Logical Reasoning

This is a tricky question even for humans. Using logical reasoning we can tell that 1 rabbit is going towards the river not the 6 elephants. These 6 elephants see 2 monkeys that are going towards the river and each of them is holding a tortoise.

So the answer should be $1 \text{ Rabbit} + 2 \text{ Monkeys} + 2 \text{ Tortoise} = 5 \text{ Animals}$

This was a tricky question for a Large language model like Chat GPT to answer and shows the limitations in logical and mathematical reasoning.

Similarity-Based reasoning

Question 1:

which does not belong in the group?

- Milk
- Snow
- kale
- Cloud

Answer 1:

The item that does not belong in the group is "Milk." The other items (Snow, Kale, Cloud) are related to weather or nature, while Milk is a food product.

Answer 2:

Kale does not belong in the group, as it is not typically white. The other items (Milk, Snow, and Cloud) are associated with the color white.

Discussion:

While Answer 2 is correct, Answer 1 is incorrect. It should be Kale since all other items are naturally white.

Also, the reasoning in Answer 1 is incorrect, since Kale is also a food item. The model is unable to identify the similarity between the given options.

Question 2:

which does not belong to the group:

- Apple Inc.
- Amazon
- IBM
- Walmart

Answer 1:

Among the given options, "Walmart" does not belong to the group, as it is primarily a retail corporation, while the others (Apple Inc., Amazon, IBM) are technology-related companies.

Answer 2:

The item that does not belong to the group is "Walmart." The other items (Apple Inc., Amazon, IBM) are primarily associated with the technology and e-commerce industry, while Walmart is a multinational retail corporation.

Discussion:

Here the model is successfully able to identify the similarities between the companies and able to find the option that does not belong to the list.

Question 3:

which does not belong in the group:

- Smarties
- Disc
- Pizza
- Tacos
- Coins

Answer 1:

The item that does not belong in the group is "Coins." The other items (Smarties, Disc, Pizza, Tacos) are all types of food or related to food, while Coins are a form of currency.

Answer 2:

Pizza does not belong in the group, as it is a type of food, while the others are objects that can be associated with various activities or categories (Smarties as a candy, Disc for sports or entertainment, Tacos as a food, and Coins as currency).

Discussion:

Incorrect, the answer should be Tacos since all other objects are round in shape. Here the model is unable to identify the similarities between the given options and so is unable to provide a correct answer.

Spatial Reasoning**Question 1:**

Suppose I'm in the middle of South Dakota and I'm looking straight down towards the center of Texas. Is Boston to my left or to my right?

Answer 1:

If you are in the middle of South Dakota, looking straight down towards the center of Texas, Boston would be to your right. The reason is that South Dakota is west of the central part of the United States, and Boston is located to the east of that central point. When you look straight down towards Texas, you are facing south, so locations to the east, including Boston, would be on your right.

Answer 2:

In your current orientation, if you're looking straight down towards the center of Texas from South Dakota, Boston would be to your right.

Discussion:

Incorrect: The correct answer is Boston will be on your left. This requires the model to not only have a general idea about where each state is but also to deduce the answer based on the geographical location of the given state on a map. The reasoning provided by the model is also incorrect, it seems like the model knows the geographical location of each state, however, is unable to 'map' the orientation and provide a definitive answer.

Question 2:

If I am facing north right now, then I move 10 steps forward, take a left turn, move 10 steps forward, and take a right turn, which direction am I facing?

Answer 1:

If you start facing north, move 10 steps forward, take a left turn (west), move 10 steps forward, and then take a right turn (north), you will be facing east at the end of these movements.

Answer 2:

If you start facing north, take 10 steps forward, then take a left turn (90 degrees counterclockwise), move 10 steps forward, and finally take a right turn (90 degrees clockwise), you will be facing west.

Discussion:

Incorrect. If we start facing north and turn left, we are facing west, and based on the relative position, when we turn right (right of west), we face north again. The model is unable to identify the 'relative direction' based on the current position. The confusion for the model was at the second turn (right) where turning right should be relative to the current position which is facing west.

Constraint-Based Reasoning**Question 1:**

Nancy's commute takes 20 minutes to get to work. John's commute takes 10 minutes to get to work. On Friday, John left for work on time and arrived at work at 8:00 am. Nancy left 10 minutes before John did, at what time did Nancy get to work?

Answer 1:

If John arrived at work at 8:00 am and his commute takes 10 minutes, he must have left home at:

- $8:00 - 10 \text{ minutes} = 7:50$
- $8:00 \text{ am} - 10 \text{ minutes} = 7:50 \text{ am}$.
- Nancy left 10 minutes before John, so she left at
- $7:50 - 10 \text{ minutes} = 7:40$
- $7:50 - 10 \text{ minutes} = 7:40 \text{ am}$.

Therefore, Nancy got to work at 7:40 am.

Answer 2:

- If John arrives at work at 8:00 am, and his commute takes 10 minutes, he leaves home at
- $8:00 \text{ am} - 10 \text{ minutes} = 7:50 \text{ am}$
- $8:00 \text{ am} - 10 \text{ minutes} = 7:50 \text{ am}$.
- Nancy left 10 minutes before John, so she left at
- $7:50 \text{ am} - 10 \text{ minutes} = 7:40 \text{ am}$
- $7:50 \text{ am} - 10 \text{ minutes} = 7:40 \text{ am}$. Therefore, Nancy arrived at work at 8:00 am plus her 20-minute commute, which means she got to work at
- $8:00 \text{ am} + 20 \text{ minutes} = 8:20 \text{ am}$
- $8:00 \text{ am} + 20 \text{ minutes} = 8:20 \text{ am}$.

Therefore, Nancy got to work at 8:20 am.

Discussion:

Both answers are incorrect since both would arrive at the same time at 8. The specific constraints were provided in the question about the commute time that Nancy and John take to get to work. The model was expected to perform a 'reverse' calculation to get to the result but was unable to do so.

Question 2:

Furniture arrangement:

A, B, C, D, E are the furniture items that must be arranged. Find a possible combination that satisfies the following constraints.

- A cannot be beside B,
- Nothing can be on the right of E
- Nothing can be on the Left of A
- C cannot be beside D
- D cannot be beside E

Answer 1:

A possible arrangement could be:
CDAEB

Answer 2:

A possible arrangement is:
CEDAB

Discussion:

Incorrect, the possible solution is
A, D, B, C, E

Based on the provided constraints the model was not able to produce a correct result, in each of the answers, one or more constraints were violated.

Overall Feedback

In general, ChatGPT proves to be an extremely powerful tool with plenty of room for improvement. In most of the questions provided above, humans would easily be able to reason and arrive at the correct answer. However, having the ability to reason like a human brain and possess cognitive abilities is not easy for a language model, and there is still a long way to go for a tool like ChatGPT to be able to reason similarly or even better than a human.
