Problems marked with (P) are to be done using a computer program. Problems not so designated can be solved by either hand, computer program, or spreadsheet.

Exercise 1

(P) (40 points) (Problem 6.10 in Chapra and Canale, 8th edition, modified) Determine the lowest positive root of $f(x) = 8sin(x)e^{-x} - 1$:

(A) Use the graphical method and identify the two integers closest to the root on both sides.

Apply the two integers identified from the graphical method as the two initial guesses with tolerance $\varepsilon_s = 10^{-7}$:

(B) Use the bisection method.

(C) Use the false-position method.

(D) Use the Newton-Raphson method with an initial guess of one of the two identified integers.

(E) Use the secant method.

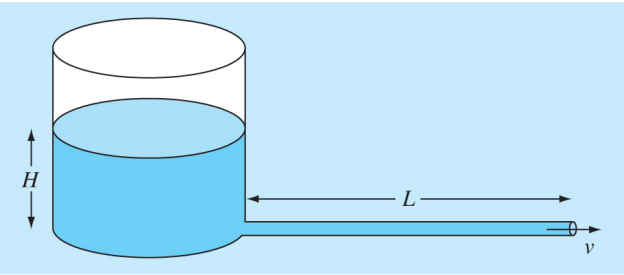
For (B) to (E), **print** iteration number, root estimates, and approximation percent relative errors for each iteration.

Exercise 2

(P) (60 points) (Problem 5.15 in Chapra and Canale, 8th edition, modified) The velocity of water v (m/s) discharged from a cylindrical tank through a long pipe can be computed as:

$$v = \sqrt{2gH}\tanh\frac{\sqrt{2gH}}{2L}t$$

where g = 9.81 (m/s^2), H = initial head (m), L = pipe length (m), and t = elapsed time (s). We would like to determine H needed to achieve v = 5m/s in t = 2.5s for L = 4m.



(A) **Plot** the function with Python between 0 and 2 (use a step of 0.1)

(B) Find H with a stopping criterion of $\varepsilon_a < \varepsilon_s = 10^{-7}$ with the Bisection, False-position, Newton-Raphson, and Secant Methods. For bracketing method, use initial guesses of $H_l = 0$ and $H_u = 2$. For open method, use initial guesses of $H_0 = 0$ and $H_{-1} = 1$. **Show** Python code for each method, and **print** iteration number, root estimates, and approximation percent relative errors for each iteration.

(C) **Plot** root estimate (y axis) vs iteration number (x axis) of the results from the four methods in the same chart for a meaningful visual comparison of the convergence.