## Appendix On Forecast Stability

## Appendix A. Full results

Tables A.1, A.2 and A.3 respectively show the MASE, RMSSE, and sMAPE based accuracy and stability results of vertical stability experiments across the four experimental datasets. In addition to MASE, RMSSE, and their corresponding stability measures, we report here also sMAPE and corresponding vertical and horizontal stability measures, which are defined as follows:

$$sMAPE = \frac{200\%}{h} \sum_{i=1}^{h} \frac{|y_{t+i} - \hat{y}_{t+i|t}|}{|y_{t+i}| + |\hat{y}_{t+i|t}|}$$
(A.1)

$$sMAPC(V) = \frac{200\%}{(h-1)} \sum_{i=1}^{h-1} \frac{|\hat{y}_{t+i|t} - \hat{y}_{t+i|t-1}|}{|\hat{y}_{t+i|t}| + |\hat{y}_{t+i|t-1}|}$$
(A.2)

$$sMAPC(H) = \frac{200\%}{(h-1)} \sum_{i=2}^{h} \frac{|\hat{y}_{t+i|t} - \hat{y}_{t+i-1|t}|}{|\hat{y}_{t+i|t}| + |\hat{y}_{t+i-1|t}|}$$
(A.3)

The error measures are defined across one to h-step ahead forecasts resulting from a specific forecasting origin t, where h is the forecast horizon,  $y_{t+i}$  is the actual series value at time t+i and  $\hat{y}_{t+i|t}$  is the forecast corresponding with time t+i made at time t.

Here, sMAPC(V) measures the change of one to h-step ahead forecasts corresponding with two adjacent forecast origins, t and t-1. Thus, it provides a measurement of up to which extent the forecasts generated at origin t are stable compared to the forecasts generated at origin t-1 for the same time period. Analogously, sMAPC(H) measures the change between two forecasts from the same origin but for adjacent targets.

			M	ASE			M	ASC		MASCJ					
		M4	М3	Favorita	M5	M4	М3	Favorita	M5	M4	М3	Favorita	M5		
N-BEATS	Base	0.6385	0.6431	_	_	0.3073	0.2603	_	_	0.4561	0.3921	_	-		
N-BEATS	Stable	0.6485	0.6391	-	-	0.2532	0.1931	-	-	0.4155	0.3314	-	-		
N-BEATS	PI_0.2	0.6346	0.6423	-	-	0.2762	0.2113	-	-	0.4358	0.3589	-	_		
N-BEATS	PI_0.4	0.6382	0.6450	-	-	0.2239	0.1769	-	-	0.3983	0.3296	-	-		
N-BEATS	PI_0.5	0.6425	0.6478	-	-	0.2099	0.1676	-	-	0.3820	0.3166	-	-		
N-BEATS	PI_0.6	0.6485	0.6515	-	-	0.2056	0.1639	-	-	0.3676	0.3048	-	-		
N-BEATS	PI_0.8	0.6650	0.6615	-	-	0.2201	0.1715	-	-	0.3445	0.2852	-	-		
N-BEATS N-BEATS	PI_1 FI_0.2	0.6871 $0.6343$	0.6748 $0.6419$	-	-	0.2545 $0.2754$	0.1924	-	-	0.3298 $0.4299$	0.2710 $0.3538$	-			
N-BEATS	FI_0.2 FI_0.4	0.6374	0.6419 $0.6439$	-	-	0.2098	0.2098 $0.1632$	-	-	0.4299 $0.3691$	0.3050	_	_		
N-BEATS	FI_0.5	0.6424	0.6466	_	_	0.1785	0.1403	_	_	0.3317	0.2744	_	_		
N-BEATS	FI_0.6	0.6505	0.6512	_	-	0.1473	0.1169	_	_	0.2875	0.2381	_	_		
N-BEATS	FI_0.8	0.6826	0.6700	-	-	0.0815	0.0659	-	_	0.1713	0.1420	-	-		
N-BEATS	FI_1	0.7532	0.7164	-	-	0.0000	0.0000	-	-	0.0000	0.0000	-	-		
PR	Base	0.7911	0.7552	0.7737	1.3569	0.2186	0.1901	0.1666	0.4012	0.3721	0.2875	0.1966	0.6465		
PR	PI_0.2	0.7983	0.7593	0.7715	1.3493	0.1969	0.1574	0.1313	0.3123	0.3546	0.2597	0.1740	0.6063		
PR	PI_0.4	0.8075	0.7650	0.7705	1.3465	0.1695	0.1319	0.1022	0.2512	0.3231	0.2344	0.1540	0.5735		
PR	PI_0.5	0.8128	0.7684	0.7704	1.3469	0.1604	0.1236	0.0932	0.2371	0.3084	0.2230	0.1452	0.5604		
PR	PI_0.6	0.8186	0.7721	0.7707	1.3485	0.1546	0.1188	0.0881	0.2355	0.2946	0.2123	0.1373	0.5496		
PR	PI_0.8	0.8316	0.7807	0.7723	1.3555	0.1520	0.1188	0.0883	0.2653	0.2699	0.1940	0.1250	0.5357		
PR	PI_1	0.8465	0.7908	0.7752	1.3674	0.1596	0.1286	0.1010	0.3261	0.2497	0.1804	0.1187	0.5325		
PR	FI_0.2	0.7989	0.7598	0.7713	1.3485	0.1948	0.1562	0.1312	0.3122	0.3495	0.2559	0.1717	0.6010		
PR	FI0.4	0.8108	0.7676	0.7699	1.3426	0.1569	0.1233	0.0983	0.2352	0.2990	0.2168	0.1432	0.5455		
PR	FI_0.5	0.8189	0.7731	0.7696	1.3404	0.1368	0.1065	0.0824	0.1997	0.2676	0.1931	0.1267	0.5099		
PR	FI_0.6	0.8291	0.7800	0.7699	1.3387	0.1153	0.0889	0.0669	0.1652	0.2309	0.1657	0.1081	0.4650		
PR	FI_0.8	0.8589	0.7999	0.7723	1.3398	0.0657	0.0497	0.0354	0.0945	0.1358	0.0964	0.0624	0.3217		
PR	FI_1	0.9111	0.8332	0.7796	1.3875	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000		
LightGBM	Base	0.8540	0.7664	0.9142	1.3416	0.2815	0.2395	0.1520	0.2728	0.4452	0.3504	0.2274	0.5206		
LightGBM	PI_0.2	0.8580	0.7692	0.9180	1.3399	0.2608	0.1941	0.1251	0.2203	0.4255	0.3180	0.2049	0.4936		
LightGBM	PI_0.4	0.8664	0.7745	0.9226	1.3408	0.2154	0.1603	0.1035	0.1842	0.3869	0.2894	0.1842	0.4711		
LightGBM	PI_0.5	0.8723	0.7782	0.9251	1.3420	0.2016	0.1502	0.0961	0.1752	0.3696	0.2766	0.1747	0.4613		
LightGBM	PI_0.6	0.8792	0.7825	0.9277	1.3436	0.1957	0.1459	0.0917	0.1759	0.3539	0.2649	0.1658	0.4525		
LightGBM	PI_0.8	0.8960	0.7930	0.9334	1.3485	0.2022	0.1514	0.0901	0.1931	0.3272	0.2452	0.1499	0.4383		
LightGBM	PI_1	0.9165	0.8059	0.9397	1.3561	0.2261	0.1698	0.0965	0.2267	0.3084	0.2313	0.1372	0.4304		
LightGBM	FI_0.2	0.8585	0.7696	0.9184	1.3395	0.2594	0.1931	0.1243	0.2186	0.4196	0.3136	0.2022	0.4890		
LightGBM LightGBM	FI_0.4 FI_0.5	0.8693 $0.8777$	0.7767 $0.7823$	0.9249 $0.9291$	1.3392 1.3396	0.2023 $0.1738$	0.1503 $0.1291$	0.0977 $0.0842$	0.1705 $0.1473$	0.3585 $0.3210$	0.2679 $0.2399$	0.1713 $0.1527$	0.4479 $0.4202$		
LightGBM	FI_0.5 FI_0.6	0.8890	0.7823	0.9291 $0.9342$	1.3406	0.1445	0.1291 $0.1074$	0.0842	0.1473	0.3210 $0.2773$	0.2399 $0.2071$	0.1327 $0.1312$	0.4202		
LightGBM	FI_0.8	0.9244	0.8131	0.9482	1.3471	0.0804	0.0599	0.0702	0.1240 $0.0735$	0.1638	0.1222	0.1312	0.2665		
LightGBM	FI_1	0.9904	0.8574	0.9698	1.3903	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000		
EXERC	D	0.0550	0.6160	0.7744	1 0000	0.0001	0.0001	0.0000	0.1791	0.4500	0.9971	0.1410	0.2240		
ETS ETS	Base PL0.2	0.6558 $0.6574$	0.6169 $0.6175$	0.7744 $0.7742$	1.2669 1.2668	0.2631	0.2091	0.0920 $0.0752$	0.1731 $0.1424$	0.4529 $0.4395$	0.3371 $0.3106$	0.1410	0.3349 $0.3172$		
ETS	PI_0.4	0.6644	0.6214	0.7745	1.2678	0.2424 $0.2072$	0.1711 $0.1449$	0.0732	0.1424 $0.1223$	0.4393	0.3100 $0.2866$	0.1291	0.3172		
ETS	PI_0.5	0.6698	0.6214 $0.6245$	0.7749	1.2688	0.2072 $0.1974$	0.1449 $0.1382$	0.0602	0.1223	0.3889	0.2360 $0.2756$	0.1134	0.3020 $0.2955$		
ETS	PI_0.6	0.6764	0.6283	0.7754	1.2701	0.1933	0.1365	0.0587	0.1170	0.3742	0.2654	0.1190	0.2898		
ETS	PI_0.8	0.6927	0.6382	0.7769	1.2736	0.1987	0.1428	0.0604	0.1259	0.3486	0.2476	0.1016	0.2809		
ETS	PI_1	0.7128	0.6505	0.7790	1.2783	0.2180	0.1582	0.0668	0.1462	0.3290	0.2337	0.0960	0.2757		
ETS	FI_0.2	0.6568	0.6173	0.7740	1.2666	0.2397	0.1697	0.0746	0.1410	0.4333	0.3061	0.1273	0.3140		
ETS	FI_0.4	0.6634	0.6212	0.7740	1.2671	0.1895	0.1334	0.0582	0.1115	0.3748	0.2655	0.1096	0.2858		
ETS	FI0.5	0.6697	0.6250	0.7741	1.2677	0.1643	0.1155	0.0501	0.0969	0.3377	0.2396	0.0986	0.2671		
ETS	FI_0.6	0.6792	0.6307	0.7744	1.2689	0.1381	0.0970	0.0419	0.0818	0.2934	0.2085	0.0855	0.2431		
ETS	FI_0.8	0.7145	0.6520	0.7760	1.2746	0.0791	0.0556	0.0237	0.0483	0.1753	0.1250	0.0509	0.1670		
ETS	FI_1	0.7901	0.6997	0.7822	1.3011	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000		
ARIMA	Base	0.6339	0.6180	0.7847	1.2881	0.2752	0.2058	0.0997	0.1528	0.4346	0.3263	0.1261	0.3073		
ARIMA	PI_0.2	0.6349	0.6183	0.7835	1.2883	0.2255	0.1681	0.0793	0.1242	0.3982	0.2992	0.1124	0.2907		
ARIMA	PI_0.4	0.6414	0.6221	0.7832	1.2894	0.1908	0.1422	0.0638	0.1050	0.3653	0.2748	0.1003	0.2762		
ARIMA	PI_0.5	0.6465	0.6251	0.7833	1.2903	0.1809	0.1353	0.0587	0.1000	0.3502	0.2636	0.0949	0.2699		
ARIMA	PI_0.6	0.6527	0.6289	0.7836	1.2914	0.1766	0.1325	0.0560	0.0984	0.3363	0.2532	0.0901	0.2644		
ARIMA	PI_0.8	0.6683	0.6387	0.7848	1.2945	0.1814	0.1368	0.0563	0.1054	0.3124	0.2352	0.0825	0.2556		
ARIMA	PI_1	0.6877	0.6513	0.7868	1.2986	0.2004	0.1512	0.0637	0.1235	0.2945	0.2216	0.0782	0.2503		
ARIMA	FI_0.2	0.6347	0.6182	0.7834	1.2883	0.2235	0.1667	0.0791	0.1234	0.3926	0.2950	0.1109	0.2880		
ARIMA	FI_0.4	0.6416	0.6220	0.7827	1.2894	0.1758	0.1311	0.0601	0.0972	0.3384	0.2547	0.0932	0.2627		
ARIMA	FI_0.5	0.6481	0.6259	0.7826	1.2905	0.1520	0.1134	0.0509	0.0845	0.3044	0.2293	0.0828	0.2458		
ARIMA	FI_0.6	0.6576	0.6316	0.7827	1.2921	0.1274	0.0952	0.0417	0.0716	0.2640	0.1990	0.0709	0.2243		
A D I N I A	$FI_0.8$	0.6916	0.6524	0.7840	1.2985	0.0725	0.0543	0.0224	0.0430	0.1572	0.1187	0.0413	0.1548		
ARIMA ARIMA	$FI_{-}1$	0.7613	0.6981	0.7886	1.3242	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000		

Table A.2: RMSSE, RMSSC and RMSSC\_I results of vertical stability experiments.

			RM	ISSE			RN	ISSC		RMSSCJ					
		M4	М3	Favorita	M5	M4	М3	Favorita	M5	M4	М3	Favorita	M5		
N-BEATS	Base	0.5607	0.5929	_	_	0.2341	0.2112	_	_	0.3781	0.3456	_	_		
N-BEATS	Stable	0.5664	0.5900	-	-	0.1814	0.1573	-	-	0.3469	0.2948	-	-		
N-BEATS	PI_0.2	0.5574	0.5921	-	-	0.2214	0.1723	-	-	0.3742	0.3202	-	-		
N-BEATS	PI_0.4	0.5600	0.5945	-	-	0.1813	0.1464	-	-	0.3467	0.2984	-	-		
N-BEATS	PI_0.5	0.5634	0.5968	-	-	0.1711	0.1399	-	-	0.3354	0.2892	-	-		
N-BEATS	PI_0.6	0.5682	0.5999	-	-	0.1688	0.1381	-	-	0.3261	0.2813	-	-		
N-BEATS	PI_0.8	0.5813	0.6083	-	-	0.1863	0.1481	-	-	0.3139	0.2698	-	-		
N-BEATS N-BEATS	PI_1 FI_0.2	0.5989 $0.5571$	0.6193 $0.5918$	-	-	0.2266 $0.2206$	0.1739 $0.1708$	-	-	0.3115	0.2652 $0.3157$		-		
N-BEATS	FI_0.2 FI_0.4	0.5571 $0.5593$	0.5918 $0.5936$	_	-	0.2200	0.1708	-	-	0.3095 $0.3218$	0.3137 $0.2765$	_	_		
N-BEATS	FI_0.4	0.5631	0.5960	_	_	0.1442	0.1161	_	_	0.3218	0.2703 $0.2512$	_	_		
N-BEATS	FI_0.6	0.5696	0.5999	_	_	0.1197	0.0976	_	_	0.2556	0.2203	_	_		
N-BEATS	FI_0.8	0.5959	0.6165	_	_	0.0674	0.0563	-	_	0.1561	0.1347	-	_		
N-BEATS	FI <sub>-</sub> 1	0.6577	0.6584	-	-	0.0000	0.0000	-	-	0.0000	0.0000	-	-		
PR	Base	0.6896	0.6886	0.5863	1.0086	0.1683	0.1527	0.1062	0.2514	0.3297	0.2543	0.1333	0.4365		
PR	PI_0.2	0.6943	0.6914	0.5848	1.0035	0.1496	0.1327	0.1002	0.1975	0.3004	0.2328	0.1192	0.4116		
PR	PI_0.4	0.7009	0.6955	0.5843	1.0035	0.1430	0.1273	0.0661	0.1613	0.3004	0.2326	0.1132	0.3917		
PR	PI_0.5	0.7047	0.6980	0.5844	1.0018	0.1253	0.1029	0.0606	0.1529	0.2680	0.2050	0.1017	0.3839		
PR	PI_0.6	0.7090	0.7009	0.5847	1.0028	0.1220	0.1000	0.0576	0.1518	0.2586	0.1973	0.0971	0.3778		
PR	PI_0.8	0.7189	0.7076	0.5861	1.0074	0.1231	0.1027	0.0591	0.1702	0.2428	0.1845	0.0905	0.3708		
PR	PI_1	0.7303	0.7157	0.5885	1.0151	0.1338	0.1148	0.0698	0.2105	0.2315	0.1764	0.0884	0.3717		
PR	FI_0.2	0.6947	0.6917	0.5848	1.0029	0.1478	0.1262	0.0838	0.1970	0.2961	0.2295	0.1177	0.4082		
PR	FI0.4	0.7028	0.6972	0.5841	0.9988	0.1206	0.1008	0.0631	0.1501	0.2576	0.1977	0.0995	0.3734		
PR	FI_0.5	0.7086	0.7012	0.5841	0.9972	0.1061	0.0878	0.0532	0.1282	0.2329	0.1778	0.0888	0.3508		
PR	FI_0.6	0.7159	0.7063	0.5845	0.9960	0.0904	0.0740	0.0435	0.1068	0.2031	0.1542	0.0765	0.3221		
PR	FI_0.8	0.7386	0.7218	0.5871	0.9962	0.0530	0.0424	0.0234	0.0620	0.1224	0.0919	0.0452	0.2270		
PR	FI_1	0.7814	0.7497	0.5940	1.0267	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000		
LightGBM	Base	0.7392	0.7003	0.6312	0.9871	0.2243	0.2028	0.0952	0.1796	0.3847	0.3155	0.1461	0.3550		
LightGBM	PI_0.2	0.7412	0.7018	0.6324	0.9853	0.2091	0.1641	0.0778	0.1444	0.3694	0.2895	0.1326	0.3372		
LightGBM	PI_0.4	0.7471	0.7058	0.6340	0.9852	0.1745	0.1366	0.0644	0.1212	0.3404	0.2671	0.1205	0.3227		
LightGBM	PI_0.5	0.7515	0.7087	0.6351	0.9858	0.1645	0.1289	0.0600	0.1158	0.3281	0.2575	0.1151	0.3168		
LightGBM	PI_0.6	0.7567	0.7121	0.6362	0.9869	0.1606	0.1262	0.0576	0.1157	0.3173	0.2492	0.1102	0.3118		
LightGBM	PI_0.8	0.7697	0.7207	0.6389	0.9903	0.1706	0.1350	0.0584	0.1281	0.3013	0.2367	0.1022	0.3050		
LightGBM	$PI_{-}1$	0.7859	0.7314	0.6420	0.9955	0.2001	0.1589	0.0660	0.1536	0.2940	0.2310	0.0971	0.3031		
LightGBM	FI_0.2	0.7415	0.7020	0.6325	0.9850	0.2078	0.1632	0.0774	0.1434	0.3644	0.2856	0.1309	0.3342		
LightGBM	FI_0.4	0.7486	0.7070	0.6349	0.9838	0.1628	0.1275	0.0607	0.1117	0.3159	0.2477	0.1122	0.3075		
LightGBM	FI_0.5	0.7546	0.7112	0.6366	0.9835	0.1405	0.1099	0.0523	0.0966	0.2855	0.2238	0.1008	0.2895		
LightGBM	FI_0.6	0.7629	0.7169	0.6388	0.9836	0.1177	0.0920	0.0437	0.0815	0.2491	0.1953	0.0873	0.2662		
LightGBM	FI_0.8	0.7905	0.7359	0.6453	0.9864	0.0669	0.0523	0.0246	0.0487	0.1507	0.1181	0.0522	0.1879		
LightGBM	FI_1	0.8467	0.7745	0.6570	1.0139	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000		
ETS	Base	0.5713	0.5690	0.5670	0.9332	0.2025	0.1677	0.0526	0.1017	0.3931	0.2994	0.0900	0.2123		
ETS	PI_0.2	0.5724	0.5695	0.5670	0.9331	0.1845	0.1384	0.0435	0.0839	0.3713	0.2788	0.0835	0.2022		
ETS	PI_0.4	0.5778	0.5728	0.5673	0.9337	0.1607	0.1195	0.0374	0.0727	0.3471	0.2609	0.0778	0.1936		
ETS	$PI_{-}0.5$	0.5820	0.5754	0.5676	0.9343	0.1544	0.1149	0.0359	0.0701	0.3366	0.2531	0.0754	0.1901		
ETS	PI_0.6	0.5871	0.5786	0.5679	0.9351	0.1523	0.1140	0.0353	0.0698	0.3272	0.2462	0.0732	0.1870		
ETS	PI_0.8	0.5996	0.5867	0.5690	0.9372	0.1603	0.1220	0.0372	0.0757	0.3126	0.2356	0.0699	0.1826		
ETS	PI_1	0.6150	0.5969	0.5704	0.9400	0.1834	0.1411	0.0428	0.0889	0.3050	0.2301	0.0682	0.1807		
ETS	FI_0.2	0.5721	0.5694	0.5669	0.9330	0.1822	0.1370	0.0430	0.0830	0.3662	0.2750	0.0824	0.2002		
ETS ETS	FI_0.4	0.5775 $0.5827$	0.5729 $0.5762$	0.5671	0.9332	0.1460	0.1092	0.0341	0.0661	0.3220 $0.2931$	0.2421	0.0722	0.1838		
ETS ETS	FI_0.5 FI_0.6	0.5827 $0.5903$	0.5762	0.5673 $0.5677$	0.9335 $0.9341$	0.1277 $0.1085$	0.0954 $0.0810$	0.0297 $0.0251$	0.0577 $0.0490$	0.2931 $0.2575$	0.2205 $0.1939$	0.0655 $0.0575$	0.1727 $0.1583$		
ETS	FI_0.8	0.6190	0.5812 $0.6000$	0.5694	0.9341 $0.9370$	0.1083	0.0810 $0.0477$	0.0231	0.0490 $0.0295$	0.2575 $0.1579$	0.1939 $0.1192$	0.0373	0.1363		
ETS	FI_1	0.6837	0.6434	0.5094 $0.5741$	0.9524	0.0000	0.0000	0.0000	0.0293	0.0000	0.0000	0.0000	0.0000		
ARIMA	Base	0.5547	0.5683	0.5732	0.9510	0.2104	0.1685	0.0644	0.0958	0.3656	0.2921	0.0855	0.2025		
ARIMA	PI_0.2	0.5554	0.5686	0.5724	0.9509	0.1739	0.1387	0.0514	0.0782	0.3389	0.2708	0.0766	0.1924		
ARIMA	PI_0.4	0.5604	0.5717	0.5721	0.9514	0.1498	0.1193	0.0416	0.0666	0.3154	0.2520	0.0689	0.1837		
ARIMA	PI_0.5	0.5643	0.5743	0.5722	0.9519	0.1432	0.1142	0.0384	0.0635	0.3051	0.2438	0.0656	0.1800		
ARIMA ARIMA	PI_0.6 PI_0.8	0.5691 $0.5810$	0.5776 $0.5858$	0.5724 $0.5733$	0.9526 $0.9544$	0.1407 $0.1477$	0.1125 $0.1186$	0.0367 $0.0376$	0.0626 $0.0670$	0.2960 $0.2818$	0.2365 $0.2251$	0.0628 $0.0588$	0.1769 $0.1722$		
ARIMA ARIMA	PI_0.8 PI_1	0.5810 $0.5958$	0.5858 $0.5962$	0.5748	0.9544 $0.9569$	0.1477	0.1186	0.0376	0.0670	0.2818 $0.2745$	0.2251 $0.2191$	0.0588 $0.0576$	0.1722		
ARIMA	FI_0.2	0.5553	0.5685	0.5748	0.9509	0.1090	0.1304	0.0440	0.0790	0.2745	0.2191	0.0576	0.1099		
ARIMA	FI_0.2 FI_0.4	0.5606	0.5085	0.5718	0.9509 $0.9513$	0.1720 $0.1371$	0.1374 $0.1093$	0.0312	0.0776	0.3342 $0.2927$	0.2340	0.0736	0.1907		
ARIMA	FI_0.4 FI_0.5	0.5657	0.5718 $0.5752$	0.5718	0.9518	0.1371	0.1093	0.0333	0.0014 $0.0535$	0.2658	0.2340 $0.2125$	0.0573	0.1731		
ARIMA	FI_0.6	0.5037 $0.5732$	0.5802	0.5719	0.9518 $0.9525$	0.1133	0.0933	0.0333 $0.0274$	0.0355 $0.0455$	0.2330	0.2123	0.0373	0.1512		
ARIMA	FI_0.8	0.6006	0.5988	0.5730	0.9556	0.0590	0.0471	0.0149	0.0435 $0.0276$	0.2330 $0.1423$	0.1138	0.0293	0.1064		
ARIMA	FI_1	0.6601	0.6409	0.5767	0.9699	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000		
1110110111	1 1-1	0.0001	0.0400	0.0101	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000		

Table A.3: sMAPE, sMAPC and sMAPC I results of vertical stability experiments.

			sM	APE		sMAPC			sMAPC_I				
		M4	М3	Favorita	M5	M4	М3	Favorita	M5	M4	М3	Favorita	M5
N-BEATS	Base	9.296	11.485	-	-	4.717	3.932	-	-	6.468	5.786	-	-
N-BEATS	Stable	9.279	11.390			3.336	2.820	-	-	5.270	4.705		
N-BEATS N-BEATS	PI_0.2 PI_0.4	9.265 $9.324$	11.461 11.484	-	-	$3.745 \\ 3.050$	3.181 2.649	-	-	5.886 $5.386$	5.288 $4.850$	-	-
N-BEATS N-BEATS	PI_0.4 PI_0.5	9.324	11.484	_	_	2.865	2.549 $2.508$	_	_	5.169	4.650 $4.657$	_	-
N-BEATS	PI_0.6	9.464	11.553	_	_	2.808	2.451	_	_	4.976	4.485	_	_
N-BEATS	PI_0.8	9.677	11.673	_	_	2.998	2.569	_	_	4.665	4.202	_	_
N-BEATS	PI_1	9.956	11.831	_	_	3.450	2.889	_	_	4.462	4.000	_	-
N-BEATS	FI_0.2	9.261	11.454	-	-	3.732	3.158	-	-	5.806	5.212	-	_
N-BEATS	$FI_0.4$	9.318	11.458	-	-	2.851	2.444	-	-	4.991	4.483	-	-
N-BEATS	FI_0.5	9.390	11.478	-	-	2.429	2.095	-	-	4.489	4.029	-	-
N-BEATS	FI_0.6	9.502	11.518	-	-	2.008	1.743	-	-	3.897	3.492	-	-
N-BEATS	FI_0.8	9.919	11.703	-	-	1.119	0.977	-	-	2.339	2.082	-	-
N-BEATS	FI_1	10.806	12.219	-	-	0.000	0.000		-	0.000	0.000	-	
PR	Base	10.632	12.622	103.351	54.964	3.245	3.518	29.190	17.559	5.282	4.896	36.772	26.876
PR	PI_0.2 PI_0.4	10.713 10.822	12.638	103.255	54.579	2.743	2.880	24.303	13.857	4.817	4.407	33.580 30.665	25.222
PR PR	PI_0.4 PI_0.5	10.822	12.680 $12.712$	103.173 103.196	54.392 54.374	2.350 $2.221$	2.385 $2.222$	20.319 19.011	11.260 $10.650$	4.390 $4.192$	3.971 $3.777$	29.261	23.894 23.366
PR	PI_0.6	10.886	12.712 $12.751$	103.190	54.406	2.142	2.132	18.160	10.565	4.192 $4.007$	3.604	27.930	22.935
PR	PI_0.8	11.116	12.751	103.566	54.629	2.118	2.152 $2.154$	17.694	11.810	3.679	3.317	25.568	22.392
PR	PI_1	11.302	12.981	104.084	55.087	2.244	2.388	18.739	14.375	3.418	3.128	23.677	22.273
PR	FI_0.2	10.720	12.638	103.627	54.532	2.715	2.858	24.670	13.856	4.748	4.341	33.216	24.994
PR	FI_0.4	10.860	12.685	103.530	54.162	2.177	2.217	19.386	10.504	4.064	3.661	28.954	22.720
PR	FI_0.5	10.957	12.726	103.563	53.987	1.893	1.894	16.809	8.925	3.640	3.253	26.342	21.269
PR	FI_0.6	11.080	12.783	103.673	53.812	1.594	1.565	13.911	7.361	3.144	2.786	23.174	19.446
PR	FI_0.8	11.448	12.972	104.248	53.556	0.909	0.856	8.077	4.213	1.860	1.618	14.923	13.662
PR	FI_1	12.103	13.333	105.775	54.936	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
LightGBM	Base	11.125	12.653	101.341	52.207	3.566	3.478	13.216	9.992	5.450	4.887	18.787	17.961
LightGBM	PI_0.2	11.211	12.680	101.229	52.100	2.950	2.812	10.796	8.108	4.939	4.412	16.703	16.996
LightGBM	PI_0.4	11.332	12.738	101.171	52.081	2.466	2.304	8.846	6.792	4.479	3.996	14.821	16.204
LightGBM	PI_0.5	11.405	12.779	101.155	52.092	2.307	2.146	8.146	6.445	4.270	3.813	13.963	15.862
LightGBM	PI_0.6	11.487	12.827	101.149	52.116	2.223	2.072	7.686	6.466	4.077	3.648	13.163	15.554
LightGBM	PI_0.8	11.673	12.944	101.164	52.209 52.398	2.237 $2.440$	2.146	7.359	7.063	3.744	3.375	11.764	15.057
LightGBM LightGBM	PI_1 FI_0.2	11.889 11.221	13.089 12.683	101.222 101.214	52.079	2.440	2.417 2.798	7.697	8.259 8.029	3.496 4.868	3.190 4.349	10.659 16.457	14.796 16.826
LightGBM	FI_0.2 FI_0.4	11.386	12.065 $12.757$	101.214	51.988	2.310	2.198	8.257	6.264	4.000	3.693	13.698	15.369
LightGBM	FI_0.5	11.503	12.815	101.120	51.946	1.993	1.852	7.025	5.398	3.701	3.297	12.087	14.393
LightGBM	FI_0.6	11.652	12.892	101.067	51.907	1.663	1.533	5.768	4.523	3.187	2.838	10.274	13.138
LightGBM	FI_0.8	12.087	13.136	101.107	51.888	0.929	0.845	3.095	2.636	1.868	1.665	5.861	9.062
LightGBM	FI_1	12.827	13.610	101.323	52.690	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
ETS	Base	9.984	11.344	104.820	50.866	3.916	3.210	14.192	6.450	6.995	5.011	20.956	11.689
ETS	PI_0.2	10.007	11.332	104.752	50.823	3.652	2.623	12.224	5.375	6.632	4.615	19.505	11.081
ETS	PI_0.4	10.114	11.369	104.742	50.821	3.153	2.213	10.764	4.682	6.130	4.265	18.117	10.560
ETS	$PI_{-}0.5$	10.195	11.404	104.762	50.834	3.015	2.101	10.332	4.520	5.900	4.106	17.454	10.337
ETS	PI_0.6	10.291	11.449	104.799	50.858	2.960	2.077	10.072	4.488	5.686	3.958	16.817	10.138
ETS	PI_0.8	10.530	11.572	104.928	50.937	3.038	2.186	10.001	4.778	5.306	3.702	15.616	9.825
ETS	PI_1	10.823	11.735	105.120	51.059	3.292	2.432	10.328	5.460	4.996	3.500	14.373	9.646
ETS	FI_0.2	9.995	11.328	104.732	50.811	3.601	2.600	12.106	5.313	6.538	4.552	19.279	10.968
ETS ETS	FI_0.4 FI_0.5	10.086 $10.175$	11.356 $11.392$	$104.653 \\ 104.621$	50.768 50.751	2.862 $2.488$	2.042 $1.764$	9.982	4.248 $3.707$	5.682 $5.137$	3.958 $3.581$	$17.022 \\ 15.560$	10.001 $9.354$
ETS	FI_0.5 FI_0.6	10.175	11.392 $11.451$	104.621	50.737	2.488	1.704	8.833 7.612	3.131	5.157 4.485	3.128	13.820	9.334 8.524
ETS	FI_0.8	10.831	11.695	104.630	50.757	1.229	0.854	4.620	1.848	2.733	1.909	8.957	5.892
ETS	FI_1	11.949	12.303	105.123	51.212	0.000	0.004	0.000	0.000	0.000	0.000	0.000	0.000
ARIMA	Base	9.771	11.748	103.635	51.050	4.148	3.169	12.386	5.457	6.571	4.996	15.665	10.441
ARIMA	PI_0.2	9.781	11.745	103.460	51.030	3.413	2.603	10.135	4.489	6.035	4.579	14.095	9.873
ARIMA	PI_0.4	9.876	11.790	103.366	51.017	2.908	2.211	8.294	3.832	5.550	4.202	12.704	9.380
ARIMA	PI_0.5	9.952	11.829	103.352	51.029	2.766	2.097	7.667	3.657	5.329	4.030	12.072	9.166
ARIMA	PI_0.6	10.044	11.878	103.362	51.048	2.704	2.049	7.354	3.606	5.124	3.869	11.485	8.977
ARIMA	PI_0.8	10.275	12.006	103.441	51.111	2.777	2.098	7.347	3.837	4.765	3.587	10.495	8.681
	PI_1	10.560	12.176	103.624	51.206	3.050	2.296	7.906	4.428	4.484	3.369	9.690	8.503
ARIMA		9.777	11.742	103.443	51.012	3.379	2.579	10.055	4.455	5.951	4.516	13.923	9.780
ARIMA	FI_0.2												
ARIMA ARIMA	FI0.4	9.874	11.783	103.298	50.990	2.671	2.039	7.805	3.535	5.151	3.901	11.887	
ARIMA ARIMA ARIMA	FI_0.4 FI_0.5	9.874 $9.967$	11.783 11.828	$103.298 \\ 103.248$	50.990 50.986	2.671 $2.317$	1.769	6.715	3.084	4.647	3.516	10.689	8.915 8.343
ARIMA ARIMA ARIMA ARIMA	FI_0.4 FI_0.5 FI_0.6	9.874 9.967 10.105	11.783 11.828 11.894	103.298 103.248 103.214	50.990 50.986 50.987	2.671 2.317 1.952	1.769 $1.488$	6.715 $5.590$	3.084 $2.623$	$4.647 \\ 4.048$	$3.516 \\ 3.060$	10.689 $9.323$	8.343 7.615
ARIMA ARIMA ARIMA	FI_0.4 FI_0.5	9.874 $9.967$	11.783 11.828	$103.298 \\ 103.248$	50.990 50.986	2.671 $2.317$	1.769	6.715	3.084	4.647	3.516	10.689	8.343

Tables A.4, A.5 and A.6 respectively show the MASE, RMSSE, and sMAPE based accuracy and stability results of horizontal stability experiments across the four experimental datasets. The accuracy measures are calculated based on final forecasts and stability measures are calculated based on remainders.

## Appendix B. Vertical stability results per horizon

In Figures B.1 to B.9 we show results for some of the models and datasets for sMAPE and sMAPC(V) for vertical stability, separated per horizon. We can see that the forecast for the largest horizon always is a new forecast that is not stabilised, as such, it is the same across all stabilisation methods. Consequently, for the second-largest horizon, as we stabilise against the largest horizon from the previous origin, partial interpolation and full interpolation yields the same result.

Furthermore, the plots show that as is to be expected, accuracy drops in all cases as horizons become larger. For stability, the picture is less clear. In many cases, stability increases as the horizon increases, for some cases it is the opposite, and for some cases, there is no monotonous relationship.

We hypothesise that this depends on characteristics of the datasets and the methods. For example, in our experiments we stabilise forecasts from the same methodology across different origins. Thus, the forecasts will not be independent. If a method is systematically biased this will affect accuracy negatively but not stability. Only variance in the forecasts will be relevant for stability. Furthermore, as horizons become larger, autocorrelations become less important relative to seasonalities and trends. So we expect that there will be datasets where forecasts for longer horizons are more stable than for shorter ones. Also, the relative difference is smaller in larger horizons. For example, the forecast horizon doubles from horizon 1 to horizon 2. This percentage difference is much smaller for larger horizons, e.g., horizon 5 to horizon 6.

		MASE					M	IASC		MASC_I				
			M3	Favorita	M5		M3	Favorita	M5		M3	Favorita	M5	
N DEATS	Daga	0.796	0.675			0.104	0.000			0.212	0.025			
N-BEATS N-BEATS	Base PI_0.2	0.726 $0.727$	$\frac{0.675}{0.676}$	-	-	0.104 $0.091$	0.089 $0.076$	-	-	0.313 $0.297$	0.235 $0.221$			
N-BEATS	PI_0.4	0.729	0.676	_	_	0.084	0.069	_	_	0.280	0.207	_	_	
N-BEATS	PI_0.5	0.731	0.676	-	-	0.082	0.067	-	-	0.273	0.201	-	-	
N-BEATS	PI_0.6	0.732	0.676	-	-	0.082	0.067	-	-	0.265	0.196	-	-	
N-BEATS	PI_0.8	0.737	0.677	-	-	0.084	0.069	-	-	0.250	0.186	-	-	
N-BEATS N-BEATS	PI_1 FI_0.2	0.741 $0.727$	0.678 $0.676$	-	-	0.090	0.075 $0.075$	-	-	0.235 $0.293$	0.179	-		
N-BEATS	FI_0.4	0.729	0.676	_	_	0.077	0.062	_	_	0.261	0.192	-	_	
N-BEATS	FI0.5	0.731	0.676	-	-	0.070	0.055	-	-	0.238	0.175	-	-	
N-BEATS	FI_0.6	0.733	0.676	-	-	0.062	0.047	-	-	0.210	0.153	-	-	
N-BEATS N-BEATS	FI_0.8 FI_1	$0.740 \\ 0.754$	0.677 $0.678$	-	-	0.038 $0.000$	0.023 $0.000$	-	-	0.126 $0.000$	0.089 $0.000$	-	-	
	F1_1	0.754	0.076			0.000				0.000	0.000			
PR PR	Base PL0.2	0.803	0.718	1.012 1.011	1.781 1.782	0.114	0.125 $0.102$	0.252 0.199	0.103 $0.087$	0.323 $0.307$	0.250 $0.236$	0.359	0.369 $0.363$	
PR PR	PI_0.2 PI_0.4	0.802	0.718	1.011	1.782	0.101 $0.094$	0.102 $0.085$	0.199 $0.161$	0.087 $0.078$	0.307 $0.290$	0.230 $0.222$	0.339 $0.322$	0.358	
PR	PI_0.5	0.801	0.718	1.013	1.783	0.092	0.082	0.153	0.077	0.283	0.216	0.315	0.356	
PR	PI_0.6	0.801	0.718	1.016	1.784	0.092	0.082	0.153	0.079	0.275	0.211	0.310	0.354	
PR	PI_0.8	0.800	0.718	1.022	1.785	0.094	0.090	0.172	0.088	0.260	0.201	0.302	0.350	
PR	PI_1	0.800	0.719	1.029	1.787	0.100	0.105	0.204	0.102	0.245	0.194	0.298	0.347	
PR PR	FI_0.2 FI_0.4	0.802 $0.801$	0.718 0.718	1.012 1.015	1.782 1.783	$0.100 \\ 0.087$	0.102 $0.081$	0.197 0.146	0.084 $0.066$	0.303 $0.271$	0.233 $0.207$	0.335 0.301	$0.362 \\ 0.352$	
PR	FI_0.4 FI_0.5	0.800	0.719	1.019	1.785	0.080	0.031 $0.072$	0.140	0.050	0.248	0.207 $0.190$	0.301	0.332 $0.343$	
PR	FI_0.6	0.800	0.720	1.024	1.787	0.072	0.062	0.100	0.047	0.220	0.168	0.248	0.331	
PR	FI_0.8	0.800	0.723	1.044	1.800	0.048	0.038	0.055	0.027	0.136	0.104	0.157	0.276	
PR	FI_1	0.802	0.731	1.088	1.881	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	
LightGBM	Base	0.790	0.709	1.005	1.779	0.119	0.126	0.097	0.092	0.346	0.234	0.164	0.207	
LightGBM	PI_0.2	0.789	0.708	1.004	1.778	0.105	0.101	0.080	0.083	0.328	0.219	0.156	0.203	
LightGBM LightGBM	PI_0.4 PI_0.5	0.787 $0.787$	0.708 $0.708$	1.004 $1.004$	1.778 1.778	0.097 $0.095$	$0.085 \\ 0.082$	0.068 $0.066$	0.078 $0.077$	0.311 $0.303$	0.207 $0.201$	0.149 $0.146$	$0.200 \\ 0.199$	
LightGBM	PI_0.6	0.786	0.708	1.004	1.778	0.096	0.084	0.066	0.077	0.305	0.201 $0.196$	0.140	0.198	
LightGBM	PI_0.8	0.786	0.708	1.005	1.779	0.100	0.094	0.072	0.082	0.279	0.188	0.140	0.196	
LightGBM	PI_1	0.785	0.708	1.006	1.779	0.107	0.110	0.083	0.089	0.264	0.182	0.137	0.196	
LightGBM	FI_0.2	0.788	0.708	1.005	1.778	0.104	0.100	0.078	0.080	0.324	0.216	0.154	0.202	
LightGBM LightGBM	FI_0.4	0.786	0.707 $0.707$	1.005 $1.006$	1.778 1.778	0.090 $0.084$	0.079 $0.069$	$0.061 \\ 0.052$	$0.065 \\ 0.056$	0.291 $0.267$	$0.192 \\ 0.176$	0.139	0.194 $0.189$	
LightGBM	FI_0.6	0.783	0.707	1.007	1.778	0.076	0.059	0.043	0.046	0.236	0.175	0.115	0.182	
LightGBM	FI_0.8	0.780	0.708	1.013	1.779	0.050	0.036	0.025	0.024	0.146	0.095	0.073	0.152	
LightGBM	$FI_{-}1$	0.778	0.712	1.026	1.794	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	
ETS	Base	0.749	0.698	0.984	1.776	0.126	0.013	0.259	0.145	0.198	0.018	0.322	0.155	
ETS	PI_0.2	0.750	0.699	0.985	1.775	0.104	0.010	0.207	0.118	0.184	0.016	0.296	0.148	
ETS	PI_0.4	0.752	0.699	0.989	1.775	0.089	0.008	0.169	0.101	0.172	0.016	0.276	0.143	
ETS ETS	PI_0.5 PI_0.6	0.754 $0.755$	0.699 $0.699$	0.992 $0.996$	1.775 $1.776$	0.085 $0.085$	0.007 $0.008$	0.157 $0.153$	0.098 $0.099$	0.167 $0.163$	0.015 $0.015$	0.269 $0.264$	$0.142 \\ 0.141$	
ETS	PI_0.8	0.760	0.700	1.007	1.778	0.000	0.009	0.133 $0.170$	0.033	0.156	0.013	0.258	0.141	
ETS	PI_1	0.764	0.701	1.021	1.780	0.101	0.010	0.205	0.135	0.152	0.014	0.258	0.146	
ETS	FI_0.2	0.750	0.699	0.984	1.775	0.103	0.010	0.206	0.116	0.181	0.016	0.291	0.146	
ETS	FI_0.4	0.752	0.699	0.987	1.775	0.081	0.008	0.156	0.088	0.159	0.014	0.253	0.135	
ETS ETS	FI_0.5 FI_0.6	0.754 $0.756$	0.699 $0.699$	0.991 $0.995$	1.775 $1.775$	$0.070 \\ 0.059$	$0.006 \\ 0.005$	$0.132 \\ 0.107$	0.074 $0.059$	$0.145 \\ 0.127$	0.013 $0.012$	0.229 $0.200$	$0.128 \\ 0.120$	
ETS	FI_0.8	0.763	0.099 $0.700$	1.010	1.775	0.039	0.003	0.107 $0.056$	0.039 $0.029$	0.127	0.012 $0.007$	0.200 $0.122$	0.120 $0.096$	
ETS	FI <sub>-</sub> 1	0.777	0.701	1.044	1.776	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	
ARIMA	Base	0.777	0.712	1.029	1.790	0.308	0.196	0.221	0.163	0.581	0.320	0.477	0.645	
ARIMA	PI_0.2	0.775	0.710	1.036	1.792	0.256	0.156	0.184	0.135	0.545	0.297	0.454	0.633	
ARIMA	PI_0.4	0.775	0.710	1.044	1.795	0.221	0.128	0.160	0.117	0.514	0.278	0.433	0.623	
ARIMA ARIMA	PI_0.5 PI_0.6	0.775 $0.776$	$0.710 \\ 0.710$	1.049 $1.054$	1.797 1.799	0.213 $0.213$	$0.122 \\ 0.123$	$0.155 \\ 0.155$	0.115 $0.118$	$0.500 \\ 0.487$	$0.271 \\ 0.264$	$0.424 \\ 0.416$	$0.619 \\ 0.615$	
ARIMA	PI_0.6 PI_0.8	0.776	0.710	1.054 $1.065$	1.799	0.213 $0.228$	0.123 $0.138$	0.155 $0.168$	0.118 $0.135$	0.487 $0.466$	0.264 $0.254$	0.416 $0.401$	0.615 $0.609$	
ARIMA	PI_1	0.783	0.714	1.078	1.808	0.257	0.163	0.189	0.160	0.450	0.248	0.388	0.604	
ARIMA	FI_0.2	0.774	0.710	1.038	1.793	0.253	0.156	0.182	0.133	0.538	0.293	0.448	0.631	
ARIMA	FI_0.4	0.772	0.708	1.053	1.798	0.203	0.120	0.145	0.104	0.478	0.260	0.406	0.610	
ARIMA	FI_0.5	0.771	0.707	1.065	1.804	0.178	0.103	0.128	0.090	0.438	0.238	0.375	0.596	
ARIMA ARIMA	FI_0.6 FI_0.8	0.771 $0.771$	0.706 $0.706$	1.081 1.134	1.812 1.856	0.152 $0.091$	0.087 $0.050$	0.110 $0.069$	0.075 $0.049$	0.387 $0.240$	0.210 $0.131$	0.335 $0.212$	0.574 $0.476$	
ARIMA	FI_1	0.779	0.710	1.238	2.100	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	
												_		

 ${\it Table A.5: RMSSE, RMSSC and RMSSC\_I \ results \ of \ horizontal \ stability \ experiments.}$ 

-		RMSSE					Di	MSSC		RMSSC_I				
		- 3.5.4			3.65	3.54			3.65	7.54			3.65	
		M4	М3	Favorita	M5	M4	М3	Favorita	M5	M4	М3	Favorita	M5	
N-BEATS	Base	0.619	0.612	_	_	0.101	0.106	_	_	0.239	0.190	_	_	
N-BEATS	PI_0.2	0.620	0.612	_	-	0.080	0.083	-	-	0.228	0.178	-		
N-BEATS	PI_0.4	0.622	0.613	-	-	0.065	0.067	-	-	0.218	0.169	-	-	
N-BEATS	PI_0.5	0.624	0.613	-	-	0.061	0.064	-	-	0.214	0.165	-	-	
N-BEATS	PI_0.6	0.626	0.614	-	-	0.061	0.065	-	-	0.211	0.163	-	-	
N-BEATS	PI_0.8	0.630	0.614	-	-	0.070	0.079	-	-	0.205	0.161	-	-	
N-BEATS	PI_1	0.636	0.615	-	-	0.087	0.101	-	-	0.202	0.162	-		
N-BEATS N-BEATS	FI_0.2 FI_0.4	$0.620 \\ 0.622$	0.612 $0.613$	-	-	0.079 $0.059$	$0.082 \\ 0.062$	-	-	0.225 $0.203$	$0.176 \\ 0.156$	-	-	
N-BEATS	FI_0.4 FI_0.5	0.624	0.613	_	_	0.039	0.052	_	_	0.203 $0.187$	0.130 $0.143$	_	_	
N-BEATS	FI_0.6	0.626	0.613	_	_	0.039	0.042	_	_	0.166	0.115	_	_	
N-BEATS	FI_0.8	0.633	0.614	_	_	0.017	0.020	-	_	0.102	0.074	-	_	
N-BEATS	$FI_{-}1$	0.645	0.615	-	-	0.000	0.000	-	-	0.000	0.000	-	-	
PR	Base	0.685	0.651	0.723	1.253	0.099	0.112	0.194	0.096	0.249	0.205	0.256	0.237	
PR	PI_0.2	0.684 $0.684$	0.650	0.723 $0.724$	1.253	0.086	0.091	0.152	0.080	0.238 $0.228$	0.193	0.237	0.231	
PR PR	PI_0.4 PI_0.5	0.684 $0.683$	$0.650 \\ 0.650$	0.724 $0.725$	1.253 $1.253$	0.078 $0.076$	$0.075 \\ 0.072$	$0.122 \\ 0.115$	$0.070 \\ 0.069$	0.228 $0.224$	0.184 $0.180$	0.225 $0.221$	$0.228 \\ 0.227$	
PR	PI_0.6	0.683	0.650	0.725 $0.727$	1.253 $1.254$	0.077	0.072	0.116	0.009	0.224 $0.221$	0.178	0.221	0.227	
PR	PI_0.8	0.683	0.650	0.731	1.255	0.083	0.084	0.136	0.080	0.215	0.176	0.224	0.228	
PR	PI_1	0.683	0.650	0.737	1.256	0.094	0.103	0.173	0.095	0.212	0.177	0.236	0.231	
PR	FI_0.2	0.684	0.650	0.723	1.253	0.086	0.091	0.151	0.078	0.235	0.191	0.234	0.230	
PR	FI0.4	0.683	0.651	0.725	1.253	0.072	0.072	0.113	0.061	0.213	0.171	0.209	0.222	
PR	FI0.5	0.683	0.651	0.728	1.254	0.065	0.063	0.095	0.053	0.197	0.158	0.192	0.216	
PR	FI_0.6	0.683	0.651	0.731	1.255	0.057	0.053	0.077	0.044	0.176	0.140	0.172	0.208	
PR	FI_0.8	0.682	0.654	0.743	1.262	0.037	0.032	0.042	0.026	0.112	0.089	0.110	0.176	
PR	FI_1	0.685	0.660	0.771	1.311	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	
LightGBM	Base	0.676	0.643	0.720	1.252	0.106	0.116	0.078	0.077	0.264	0.193	0.115	0.144	
LightGBM	PI_0.2	0.675	0.643	0.720	1.252	0.092	0.093	0.062	0.067	0.253	0.181	0.118	0.140	
LightGBM	PI_0.4	0.674	0.642	0.720	1.251	0.083	0.077	0.052	0.061	0.243	0.172	0.103	0.137	
LightGBM	PI_0.5	0.674	0.642	0.720	1.251	0.081	0.074	0.050	0.060	0.239	0.168	0.102	0.136	
LightGBM	PI_0.6	0.674	0.642	0.720	1.251	0.082	0.075	0.050	0.060	0.235	0.166	0.101	0.136	
LightGBM	PI_0.8	0.673	0.643	0.721	1.252	0.089	0.089	0.059	0.066	0.230	0.166	0.102	0.138	
LightGBM	PI_1	0.673	0.643	0.721	1.252	0.103	0.111	0.073	0.076	0.227	0.169	0.106	0.141	
LightGBM	FI_0.2	0.675	0.642	0.720	1.251	0.091	0.092	0.062	0.065	0.249	0.178	0.106	0.138	
LightGBM	FI_0.4	0.673 $0.672$	0.642	0.720	1.251	0.076	0.072	0.047	0.051	0.227 $0.210$	0.159	0.096	0.130	
LightGBM LightGBM	FI_0.5 FI_0.6	0.672 $0.671$	0.642 $0.641$	0.721 $0.722$	1.251 $1.251$	0.069 $0.060$	0.062 $0.052$	0.040 $0.033$	0.044 $0.036$	0.210 $0.188$	$0.146 \\ 0.130$	0.088 $0.079$	$0.125 \\ 0.119$	
LightGBM	FI_0.8	0.669	0.641	0.722	1.251 $1.251$	0.000	0.032 $0.030$	0.033	0.030	0.133	0.130 $0.081$	0.079	0.119	
LightGBM	FI_1	0.668	0.646	0.734	1.261	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	
		0.000	0.0.0	0.,02		0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	
ETS	Base	0.642	0.635	0.700	1.249	0.111	0.012	0.212	0.097	0.161	0.016	0.250	0.107	
ETS	PI_0.2	0.643	0.635	0.701	1.249	0.090	0.009	0.166	0.078	0.149	0.014	0.227	0.098	
ETS	PI_0.4	0.645	0.636	0.705	1.249	0.075	0.007	0.132	0.065	0.139	0.013	0.211	0.093	
ETS	PI_0.5	0.647	0.636	0.708	1.249	0.071	0.007	0.124	0.063	0.136	0.013	0.206	0.092	
ETS	PI_0.6	0.649	0.637	0.711	1.249	0.071	0.007	0.123	0.064	0.134	0.013	0.204	0.093	
ETS ETS	PI_0.8 PI_1	0.653 $0.659$	0.637 $0.638$	$0.720 \\ 0.732$	1.250 $1.252$	$0.080 \\ 0.097$	0.008 $0.010$	$0.145 \\ 0.184$	0.075 $0.094$	0.133 $0.136$	0.013 $0.013$	$0.208 \\ 0.221$	$0.096 \\ 0.104$	
ETS	FI_0.2	0.643	0.635	0.701	1.249	0.089	0.010	0.164	0.034	0.136	0.013	0.224	$\frac{0.104}{0.097}$	
ETS	FI_0.4	0.645	0.636	0.704	1.248	0.069	0.007	0.124	0.057	0.128	0.011	0.193	0.086	
ETS	FI_0.5	0.647	0.636	0.707	1.248	0.059	0.006	0.104	0.048	0.116	0.011	0.174	0.080	
ETS	FI0.6	0.649	0.636	0.710	1.248	0.049	0.005	0.084	0.038	0.102	0.010	0.152	0.073	
ETS	FI_0.8	0.656	0.637	0.721	1.249	0.027	0.003	0.044	0.019	0.063	0.006	0.093	0.056	
ETS	$FI_{-1}$	0.668	0.638	0.743	1.250	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	
ARIMA	Base	0.664	0.647	0.734	1.258	0.275	0.181	0.178	0.150	0.462	0.268	0.325	0.417	
ARIMA	PI_0.2	0.663	0.645	0.734	1.258	0.275	0.181	0.178	0.130	0.402	0.208	0.323	0.417	
ARIMA	PI_0.4	0.664	0.644	0.744	1.261	0.225 $0.190$	0.144 $0.116$	0.147 $0.125$	0.124 $0.108$	0.432 $0.409$	0.247 $0.232$	0.309 $0.297$	0.403	
ARIMA	PI_0.5	0.665	0.645	0.747	1.262	0.182	0.110	0.120	0.106	0.401	0.227	0.293	0.400	
ARIMA	PI_0.6	0.666	0.645	0.750	1.263	0.183	0.111	0.121	0.108	0.395	0.224	0.291	0.399	
ARIMA	PI_0.8	0.670	0.647	0.759	1.266	0.207	0.131	0.137	0.124	0.392	0.224	0.291	0.400	
ARIMA	PI_1	0.675	0.650	0.768	1.269	0.250	0.165	0.166	0.149	0.398	0.232	0.296	0.405	
ARIMA	FI_0.2	0.663	0.645	0.739	1.259	0.223	0.143	0.145	0.123	0.426	0.244	0.305	0.406	
ARIMA	FI_0.4	0.662	0.643	0.748	1.262	0.175	0.109	0.114	0.097	0.379	0.216	0.277	0.392	
ARIMA	FI_0.5	0.661	0.642	0.756	1.265	0.151	0.093	0.099	0.084	0.348	0.198	0.258	0.382	
ARIMA ARIMA	FI_0.6 FI_0.8	$0.661 \\ 0.662$	0.642 $0.641$	$0.765 \\ 0.796$	1.270 $1.295$	0.127 $0.074$	0.077 $0.044$	0.084 $0.049$	0.071 $0.043$	0.309 $0.195$	$0.175 \\ 0.110$	0.232 $0.150$	0.369 $0.311$	
ARIMA	FI_0.8 FI_1	0.662	0.641 $0.645$	0.796	1.295 $1.445$	0.074 $0.000$	0.044 $0.000$	0.049	0.043 $0.000$	0.195 $0.000$	0.110	0.150	0.311 $0.000$	
1 1 1 (1 1 V I T I	1.1-1	0.010	0.040	0.001	1.770	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	

Table A.6: sMAPE, sMAPC and sMAPC I results of horizontal stability experiments.

		sMAPE sMAPC							sMAPC.I				
		——————————————————————————————————————	M3	Favorita	M5	M4	M3	Favorita	M5	M4	M3	Favorita	
N DEATES	D												
N-BEATS N-BEATS	Base PL0.2	$\frac{12.072}{12.076}$	14.335		-	108.159 100.386	117.581 108.179	-	-	156.298 150.404	157.155 153.321	<u> </u>	
N-BEATS	PI_0.4	12.088	14.339	_	_	94.433	100.549	_	_	143.624	147.908	_	_
N-BEATS	PI_0.5	12.095	14.340	-	-	92.795	98.710	-	-	139.978	144.557	-	-
N-BEATS	PI_0.6	12.103	14.341	-	-	92.398	98.646	-	-	136.218	140.780	-	-
N-BEATS	PI_0.8	12.123	14.344	-	-	92.428	99.887	-	-	128.432	132.068	-	-
N-BEATS N-BEATS	PI_1 FI_0.2	12.145 12.076	14.346	-	-	91.560 100.337	99.001 108.747	-	-	120.178 149.312	121.740 152.661	-	
N-BEATS	FI_0.2 FI_0.4	12.070	14.338	-	-	93.264	100.747	-	-	137.745	144.304	-	-
N-BEATS	FI_0.5	12.097	14.339	-	-	88.640	95.846	-	-	128.731	137.574	-	-
N-BEATS	FI_0.6	12.110	14.340	-	-	80.242	89.790	-	-	115.818	127.704	-	-
N-BEATS	FI_0.8	12.152	14.343	-	-	37.434	56.336	-	-	69.065	87.017	-	-
N-BEATS	FI_1	12.233	14.347	-	_	0.000	0.000	-	-	0.000	0.000	-	-
PR	Base	12.823	14.716	123.639	70.398	112.223	126.150	141.205	127.812	160.959	155.614	162.814	172.046
PR	PI_0.2	12.811	14.712	123.638	70.414	105.292	116.008	134.031	121.223	155.714	152.627	163.914	172.290
PR PR	PI_0.4 PI_0.5	12.803 12.799	14.714 $14.717$	123.737 123.821	70.443 70.461	99.659 97.911	105.596 102.311	123.615 119.676	115.695 114.908	149.297 145.637	147.943 144.779	161.724 158.939	$171.250 \\ 170.165$
PR	PI_0.6	12.796	14.717	123.924	70.481	96.877	102.311	117.799	115.123	141.696	144.779	155.165	168.643
PR	PI_0.8	12.791	14.737	124.185	70.531	95.781	102.232	115.845	116.719	133.153	131.967	144.815	164.614
PR	PI_1	12.789	14.759	124.524	70.592	94.120	101.900	111.655	117.791	123.858	121.658	132.355	159.924
PR	FI_0.2	12.810	14.718	123.669	70.417	105.709	117.247	135.071	121.516	154.853	151.827	163.580	172.810
PR PR	FI_0.4	12.798 12.791	14.741	123.903	70.469	100.622	108.428	127.158	115.529	144.648	143.869	160.859	173.815
PR PR	FI_0.5 FI_0.6	12.791	14.763 $14.795$	124.125 124.444	70.518 70.599	97.735 93.543	103.744 $97.252$	123.593 118.409	113.266 109.228	136.591 124.900	137.069 126.784	156.906 148.979	173.831 172.215
PR	FI_0.8	12.788	14.908	125.578	71.032	39.543	49.569	70.872	55.345	73.186	79.909	101.283	143.614
PR	$FI_{-}1$	12.839	15.148	127.890	73.509	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
LightCDM	Daga	12.659	14 560	124 022	70.406	105 120	114 501	116 159	107 264	15/170	155.055	150.040	149 020
LightGBM LightGBM	Base PI_0.2	12.658 12.634	14.569	124.022 124.009	70.406	105.129 97.356	114.501 105.099	116.152 108.565	107.264 105.942	154.178 148.284	155.055 151.221	150.049 148.601	148.920 149.383
LightGBM	PI_0.4	12.613	14.543	124.017	70.390	91.403	97.469	101.347	104.658	141.504	145.808	144.975	148.655
LightGBM	PI_0.5	12.603	14.543	124.032	70.392	89.765	95.630	98.419	103.933	137.858	142.457	142.107	147.794
LightGBM	PI_0.6	12.594	14.546	124.055	70.396	89.368	95.566	96.401	103.186	134.098	138.680	138.717	146.623
LightGBM LightGBM	PI_0.8 PI_1	12.578 $12.564$	14.558 $14.580$	124.116 124.194	70.411 70.435	89.398 88.530	96.807 95.921	94.034 90.811	101.842 100.106	126.312 118.058	129.968 119.640	$130.685 \\ 121.037$	143.541 139.313
LightGBM	FI_0.2	12.630	14.548	124.134	70.433	97.307	105.667	108.578	106.149	147.192	150.561	148.261	150.178
LightGBM	FI_0.4	12.591	14.534	124.067	70.380	90.234	97.299	99.444	105.687	135.625	142.204	142.656	152.467
LightGBM	FI0.5	12.567	14.529	124.128	70.381	85.610	92.766	93.028	105.429	126.611	135.474	137.074	154.007
LightGBM	FI_0.6	12.539	14.527	124.223	70.391	77.212	86.710	84.062	104.886	113.698	125.604	128.122	155.273
LightGBM LightGBM	FI_0.8 FI_1	12.478 $12.442$	14.555 $14.689$	124.585 125.388	70.485 $71.169$	34.404 0.000	53.256 0.000	50.244 0.000	84.171 0.000	66.945 $0.000$	84.917 0.000	90.047 0.000	142.331 0.000
LightGDM	1.1-1	12.442	14.003	120.300	71.103	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
ETS	Base	12.095	14.358	122.399	70.365	9.500	0.727	81.620	35.587	13.033	0.976	97.368	35.751
ETS ETS	PI_0.2	12.099 12.111	14.359	122.279 122.432	70.331	8.918 8.357	0.690 $0.591$	77.446 71.906	33.836 32.086	12.733 12.335	0.967 $0.937$	96.244 93.788	37.235 37.727
ETS	PI_0.4	12.111	14.363	122.432	70.310 $70.314$	8.131	0.591 $0.561$	68.795	31.523	12.066	0.937	92.092	37.590
ETS		12.126	14.364	122.810	70.318	8.043	0.583	67.050	31.380	11.761	0.904	90.145	37.219
ETS		12.146	14.367	123.341	70.339	7.948	0.595	67.089	32.168	11.052	0.842	85.733	35.867
ETS		12.168	14.369	124.040	70.381	7.662	0.574	65.360	33.274	10.117	0.749	78.119	33.420
ETS ETS	FI_0.2	12.099 12.111	14.359 $14.361$	122.263 122.328	70.326 70.295	8.926 8.280	0.683 $0.607$	77.940 73.617	34.135 33.069	12.681 $12.058$	0.954 $0.917$	95.969 92.697	37.259 38.278
ETS ETS	FI_0.4 FI_0.5	12.111 $12.120$	14.361 $14.362$	122.328	70.295	7.850	0.557	70.492	32.525	12.058	0.917	92.697 89.708	38.278
ETS	FI_0.6	12.133	14.363	122.652	70.271	7.295	0.520	66.764	32.004	10.814	0.831	85.277	39.060
ETS	FI_0.8	12.175	14.366	123.476	70.257	5.038	0.369	48.776	27.871	7.883	0.638	64.328	36.683
ETS	FI_1	12.256	14.370	125.483	70.313	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
ARIMA	Base	12.741	14.729	124.112	70.769	118.195	128.660	129.235	129.085	151.142	152.250	165.513	174.413
ARIMA	PI_0.2	12.672	14.672	124.457	70.871	111.258	125.315	120.771	121.352	146.357	147.546	162.845	174.215
ARIMA		12.630	14.646	124.845	70.993	104.654	119.112	112.656	115.004	141.286	144.042	158.691	173.267
ARIMA	PI_0.5	12.618	14.643	125.037	71.060	102.276	117.045	109.946	113.752	138.271	141.769	155.688	172.185
ARIMA ARIMA	PI_0.6 PI_0.8	12.612 12.616	14.647 $14.675$	$125.250 \\ 125.729$	71.131 71.285	100.933 99.504	116.009 115.189	108.362 106.394	114.833 118.215	134.976 127.851	139.241 133.463	152.034 143.081	170.595 166.463
ARIMA	PI_1	12.643	14.721	126.251	71.450	97.112	113.036	103.625	120.608	119.772	126.663	133.113	161.841
ARIMA	FI_0.2	12.664	14.669	124.560	70.900	109.693	123.646	121.894	122.359	145.347	146.572	162.416	174.364
ARIMA		12.588	14.623	125.363	71.152	97.906	112.762		115.388	135.942	139.429	155.416	173.695
ARIMA	FI_0.5	12.548	14.605	125.960	71.367	90.845	106.256	111.004	111.698	128.414	133.566	148.717	172.532
ARIMA ARIMA	FI_0.6 FI_0.8	12.506 12.430	$14.592 \\ 14.592$	126.752 129.089	71.699 73.296	81.791 49.708	98.183 73.412	104.948 48.383	107.916 49.302	117.690 79.044	125.126 95.329	137.568 84.225	169.705 137.414
ARIMA		12.448	14.708	132.640	80.491	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
			00	3=.310		0.500	5.500	5.500	5.500	0.500	5.500	5.500	0.000

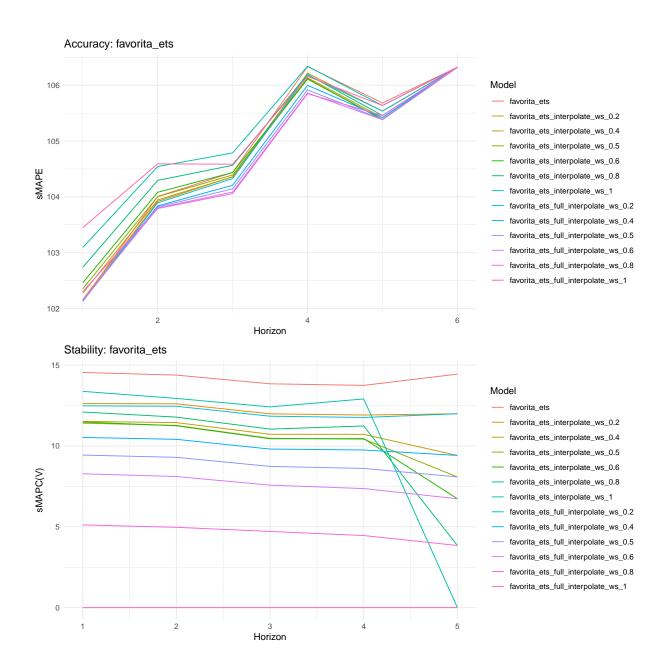


Figure B.1: Accuracy (top) and Vertical Stability (bottom) for ETS for the Favorita dataset. We can see that the forecast for the largest horizon always is a new forecast that is not stabilised, as such, it is the same across all stabilisation methods. Consequently, for the second-largest horizon, as we stabilise against the largest horizon from the previous origin, partial interpolation and full interpolation yields the same result. Furthermore, we can see that accuracy decreases with an increasing horizon, whereas stability stays relatively constant across horizons.

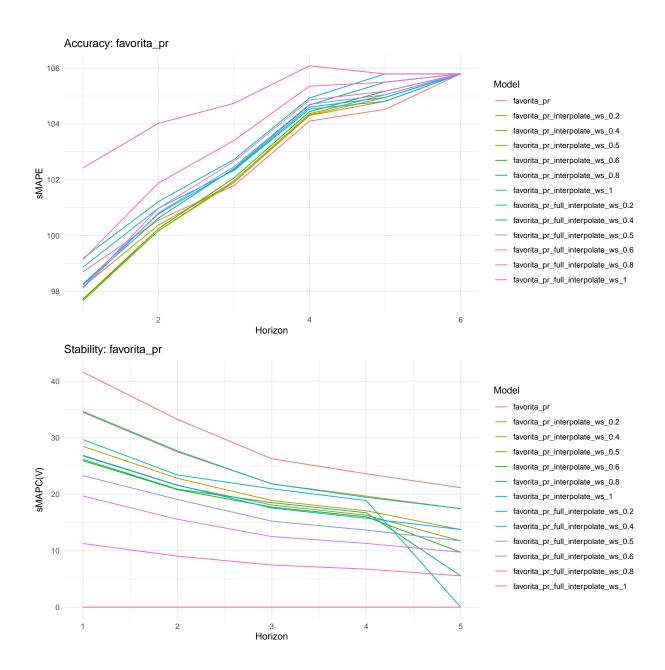


Figure B.2: Accuracy (top) and Vertical Stability (bottom) for PR for the Favorita dataset. We can see that accuracy decreases (as sMAPE increases) with an increasing horizon, whereas stability increases (as sMAPC decreases).

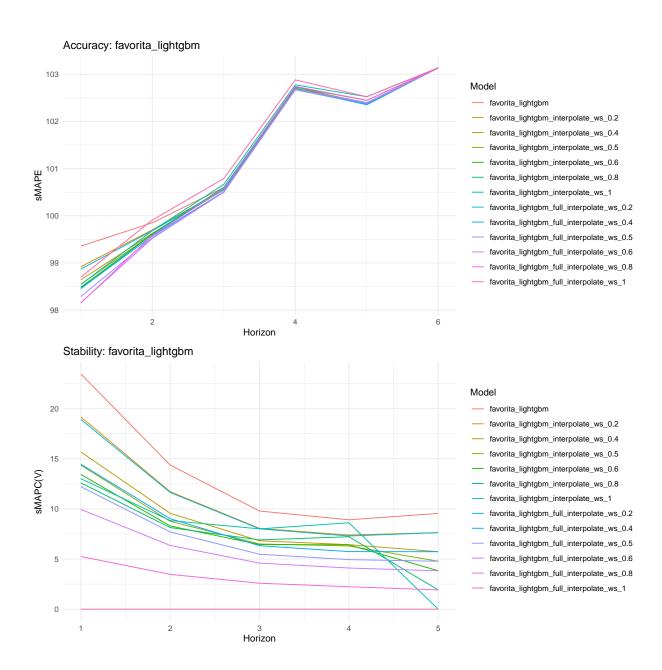


Figure B.3: Accuracy (top) and Vertical Stability (bottom) for LightGBM for the Favorita dataset. We can see that accuracy decreases (as sMAPE increases) with an increasing horizon, whereas stability mostly increases (as sMAPC decreases).

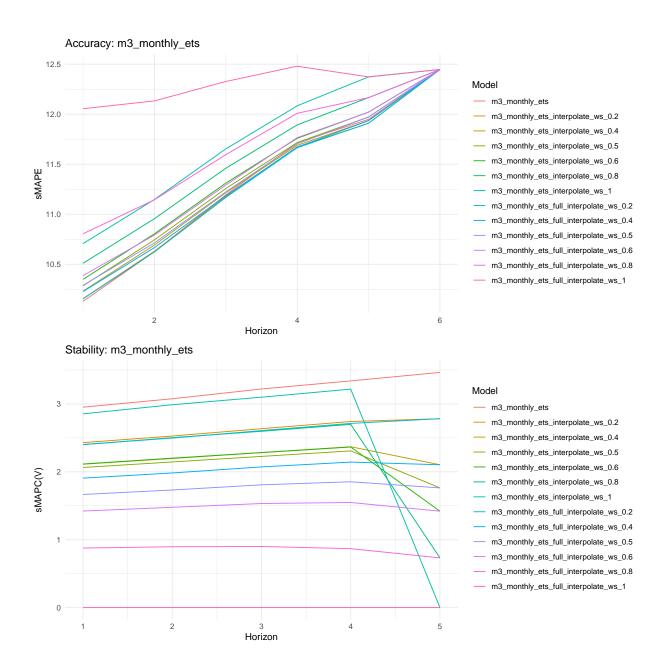


Figure B.4: Accuracy (top) and Vertical Stability (bottom) for ETS for the Monthly M3 dataset. We can see that accuracy decreases (as sMAPE increases) with an increasing horizon, and stability also decreases (as sMAPC increases), except for the artefacts that stability for the second-largest horizon of partial interpolation is identical with full interpolation, and therefore is more stable.

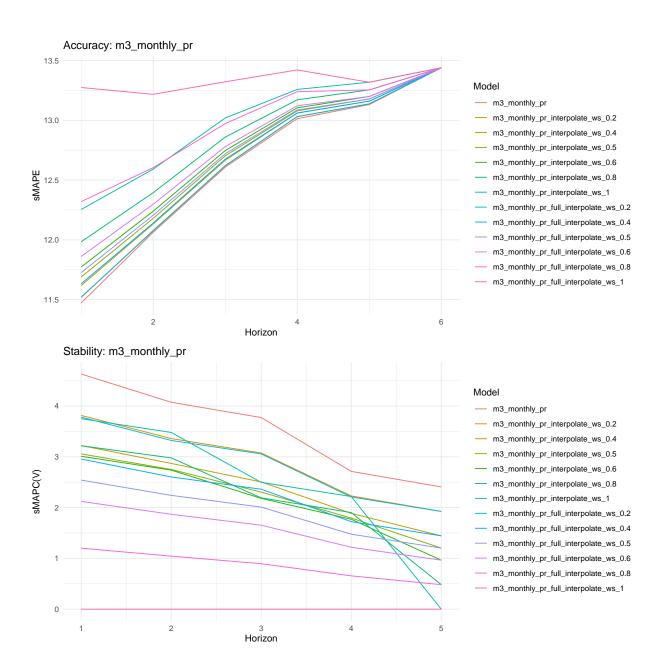


Figure B.5: Accuracy (top) and Vertical Stability (bottom) for PR for the Monthly M3 dataset. We can see that accuracy decreases (as sMAPE increases) with an increasing horizon, whereas stability mostly increases (as sMAPC decreases).

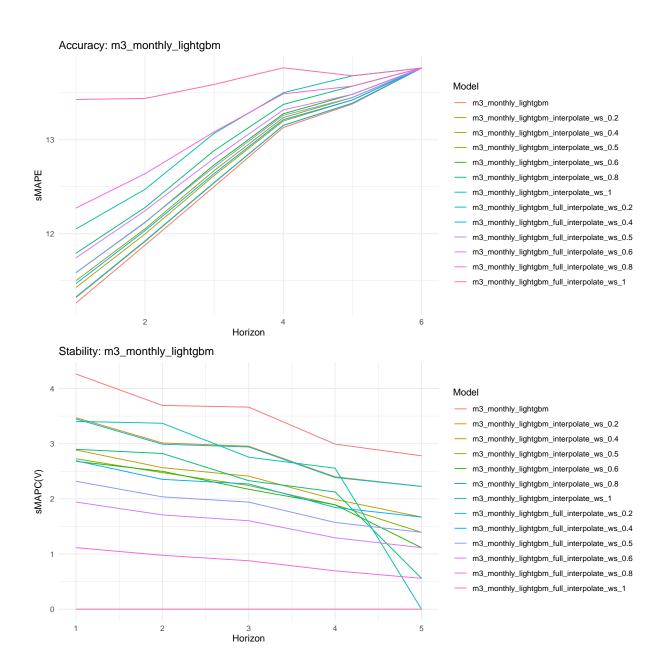


Figure B.6: Accuracy (top) and Vertical Stability (bottom) for LightGBM for the Monthly M3 dataset. We can see that accuracy decreases (as sMAPE increases) with an increasing horizon, whereas stability mostly increases (as sMAPC decreases).

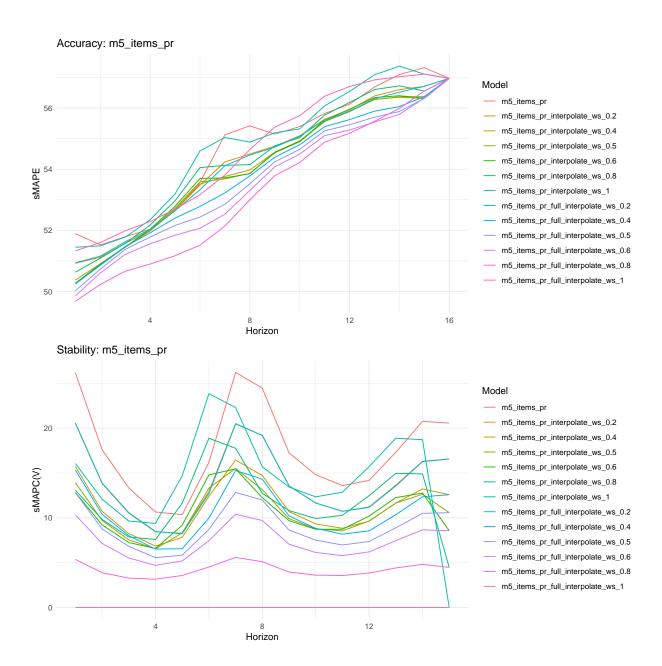


Figure B.7: Accuracy (top) and Vertical Stability (bottom) for PR for the M5 items dataset. We can see that accuracy decreases (as sMAPE increases) with an increasing horizon, whereas stability has no monotonous relationship with the horizon.

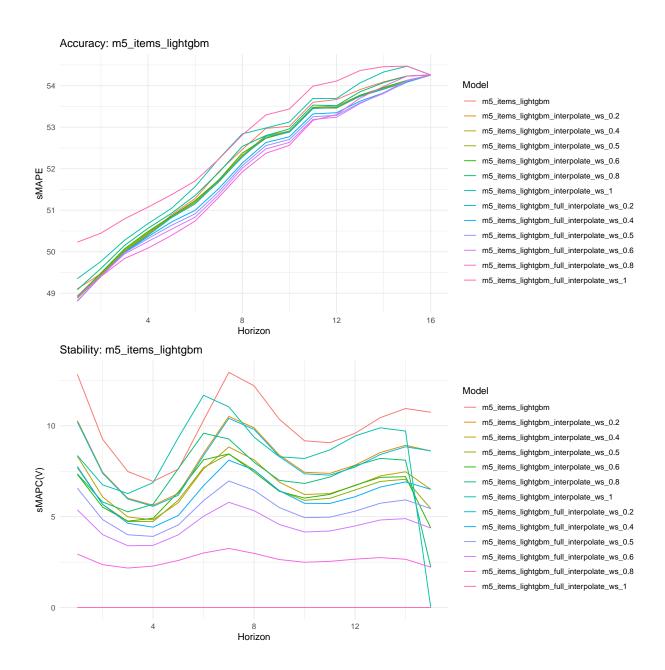


Figure B.8: Accuracy (top) and Vertical Stability (bottom) for LightGBM for the M5 items dataset. We can see that accuracy decreases (as sMAPE increases) with an increasing horizon, whereas stability has no monotonous relationship with the horizon.

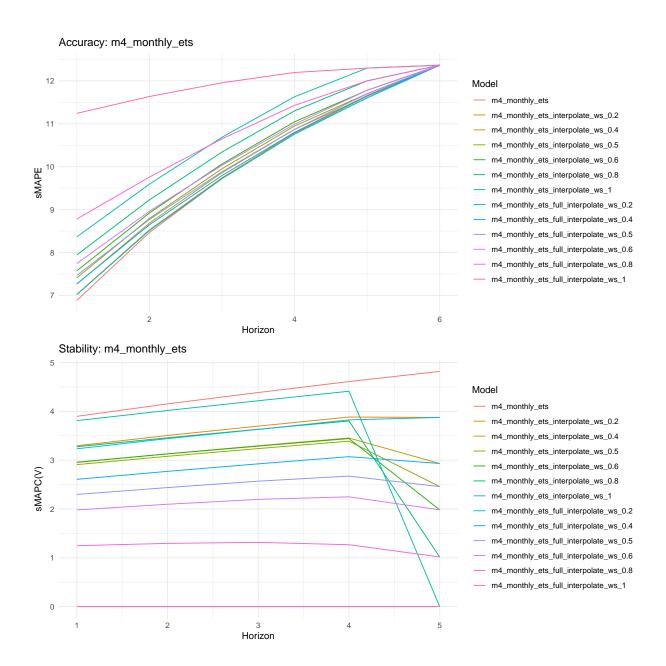


Figure B.9: Accuracy (top) and Vertical Stability (bottom) for ETS for the Monthly M4 dataset. We can see that accuracy decreases (as sMAPE increases) with an increasing horizon, and stability also decreases (as sMAPC increases), except for the artefacts that stability for the second-largest horizon of partial interpolation is identical with full interpolation, and therefore is more stable.