

## Appendix

### On Forecast Stability

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#### Appendix A. Full results

Tables A.1, A.2 and A.3 respectively show the MASE, RMSSE, and sMAPE based accuracy and stability results of vertical stability experiments across the four experimental datasets. In addition to MASE, RMSSE, and their corresponding stability measures, we report here also sMAPE and corresponding vertical and horizontal stability measures, which are defined as follows:

$$sMAPE = \frac{200\%}{h} \sum_{i=1}^h \frac{|y_{t+i} - \hat{y}_{t+i|t}|}{|y_{t+i}| + |\hat{y}_{t+i|t}|} \quad (\text{A.1})$$

$$sMAPC(V) = \frac{200\%}{(h-1)} \sum_{i=1}^{h-1} \frac{|\hat{y}_{t+i|t} - \hat{y}_{t+i|t-1}|}{|\hat{y}_{t+i|t}| + |\hat{y}_{t+i|t-1}|} \quad (\text{A.2})$$

$$sMAPC(H) = \frac{200\%}{(h-1)} \sum_{i=2}^h \frac{|\hat{y}_{t+i|t} - \hat{y}_{t+i-1|t}|}{|\hat{y}_{t+i|t}| + |\hat{y}_{t+i-1|t}|} \quad (\text{A.3})$$

The error measures are defined across one to  $h$ -step ahead forecasts resulting from a specific forecasting origin  $t$ , where  $h$  is the forecast horizon,  $y_{t+i}$  is the actual series value at time  $t+i$  and  $\hat{y}_{t+i|t}$  is the forecast corresponding with time  $t+i$  made at time  $t$ .

Here, sMAPC(V) measures the change of one to  $h$ -step ahead forecasts corresponding with two adjacent forecast origins,  $t$  and  $t-1$ . Thus, it provides a measurement of up to which extent the forecasts generated at origin  $t$  are stable compared to the forecasts generated at origin  $t-1$  for the same time period. Analogously, sMAPC(H) measures the change between two forecasts from the same origin but for adjacent targets.

Table A.1: MASE, MASC and MASC-I results of vertical stability experiments.

		MASE				MASC				MASC.I			
		M4	M3	Favorita	M5	M4	M3	Favorita	M5	M4	M3	Favorita	M5
N-BEATS	Base	0.6385	0.6431	-	-	0.3073	0.2603	-	-	0.4561	0.3921	-	-
N-BEATS	Stable	0.6485	0.6391	-	-	0.2532	0.1931	-	-	0.4155	0.3314	-	-
N-BEATS	PI.0.2	0.6346	0.6423	-	-	0.2762	0.2113	-	-	0.4358	0.3589	-	-
N-BEATS	PI.0.4	0.6382	0.6450	-	-	0.2239	0.1769	-	-	0.3983	0.3296	-	-
N-BEATS	PI.0.5	0.6425	0.6478	-	-	0.2099	0.1676	-	-	0.3820	0.3166	-	-
N-BEATS	PI.0.6	0.6485	0.6515	-	-	0.2056	0.1639	-	-	0.3676	0.3048	-	-
N-BEATS	PI.0.8	0.6650	0.6615	-	-	0.2201	0.1715	-	-	0.3445	0.2852	-	-
N-BEATS	PI.1	0.6871	0.6748	-	-	0.2545	0.1924	-	-	0.3298	0.2710	-	-
N-BEATS	FI.0.2	0.6343	0.6419	-	-	0.2754	0.2098	-	-	0.4299	0.3538	-	-
N-BEATS	FI.0.4	0.6374	0.6439	-	-	0.2098	0.1632	-	-	0.3691	0.3050	-	-
N-BEATS	FI.0.5	0.6424	0.6466	-	-	0.1785	0.1403	-	-	0.3317	0.2744	-	-
N-BEATS	FI.0.6	0.6505	0.6512	-	-	0.1473	0.1169	-	-	0.2875	0.2381	-	-
N-BEATS	FI.0.8	0.6826	0.6700	-	-	0.0815	0.0659	-	-	0.1713	0.1420	-	-
N-BEATS	FI.1	0.7532	0.7164	-	-	0.0000	0.0000	-	-	0.0000	0.0000	-	-
PR	Base	0.7911	0.7552	0.7737	1.3569	0.2186	0.1901	0.1666	0.4012	0.3721	0.2875	0.1966	0.6465
PR	PI.0.2	0.7983	0.7593	0.7715	1.3493	0.1969	0.1574	0.1313	0.3123	0.3546	0.2597	0.1740	0.6063
PR	PI.0.4	0.8075	0.7650	0.7705	1.3465	0.1695	0.1319	0.1022	0.2512	0.3231	0.2344	0.1540	0.5735
PR	PI.0.5	0.8128	0.7684	0.7704	1.3469	0.1604	0.1236	0.0932	0.2371	0.3084	0.2230	0.1452	0.5604
PR	PI.0.6	0.8186	0.7721	0.7707	1.3485	0.1546	0.1188	0.0881	0.2355	0.2946	0.2123	0.1373	0.5496
PR	PI.0.8	0.8316	0.7807	0.7723	1.3555	0.1520	0.1188	0.0883	0.2653	0.2699	0.1940	0.1250	0.5357
PR	PI.1	0.8465	0.7908	0.7752	1.3674	0.1596	0.1286	0.1010	0.3261	0.2497	0.1804	0.1187	0.5325
PR	FI.0.2	0.7989	0.7598	0.7713	1.3485	0.1948	0.1562	0.1312	0.3122	0.3495	0.2559	0.1717	0.6010
PR	FI.0.4	0.8108	0.7676	0.7699	1.3426	0.1569	0.1233	0.0983	0.2352	0.2990	0.2168	0.1432	0.5455
PR	FI.0.5	0.8189	0.7731	0.7696	1.3404	0.1368	0.1065	0.0824	0.1997	0.2676	0.1931	0.1267	0.5099
PR	FI.0.6	0.8291	0.7800	0.7699	1.3387	0.1153	0.0889	0.0669	0.1652	0.2309	0.1657	0.1081	0.4650
PR	FI.0.8	0.8589	0.7999	0.7723	1.3398	0.0657	0.0497	0.0354	0.0945	0.1358	0.0964	0.0624	0.3217
PR	FI.1	0.9111	0.8332	0.7796	1.3875	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
LightGBM	Base	0.8540	0.7664	0.9142	1.3416	0.2815	0.2395	0.1520	0.2728	0.4452	0.3504	0.2274	0.5206
LightGBM	PI.0.2	0.8580	0.7692	0.9180	1.3399	0.2608	0.1941	0.1251	0.2203	0.4255	0.3180	0.2049	0.4936
LightGBM	PI.0.4	0.8664	0.7745	0.9226	1.3408	0.2154	0.1603	0.1035	0.1842	0.3869	0.2894	0.1842	0.4711
LightGBM	PI.0.5	0.8723	0.7782	0.9251	1.3420	0.2016	0.1502	0.0961	0.1752	0.3696	0.2766	0.1747	0.4613
LightGBM	PI.0.6	0.8792	0.7825	0.9277	1.3436	0.1957	0.1459	0.0917	0.1759	0.3539	0.2649	0.1658	0.4525
LightGBM	PI.0.8	0.8960	0.7930	0.9334	1.3485	0.2022	0.1514	0.0901	0.1931	0.3272	0.2452	0.1499	0.4383
LightGBM	PI.1	0.9165	0.8059	0.9397	1.3561	0.2261	0.1698	0.0965	0.2267	0.3084	0.2313	0.1372	0.4304
LightGBM	FI.0.2	0.8585	0.7696	0.9184	1.3395	0.2594	0.1931	0.1243	0.2186	0.4196	0.3136	0.2022	0.4890
LightGBM	FI.0.4	0.8693	0.7767	0.9249	1.3392	0.2023	0.1503	0.0977	0.1705	0.3585	0.2679	0.1713	0.4479
LightGBM	FI.0.5	0.8777	0.7823	0.9291	1.3396	0.1738	0.1291	0.0842	0.1473	0.3210	0.2399	0.1527	0.4202
LightGBM	FI.0.6	0.8890	0.7897	0.9342	1.3406	0.1445	0.1074	0.0702	0.1240	0.2773	0.2071	0.1312	0.3843
LightGBM	FI.0.8	0.9244	0.8131	0.9482	1.3471	0.0804	0.0599	0.0391	0.0735	0.1638	0.1222	0.0766	0.2665
LightGBM	FI.1	0.9904	0.8574	0.9698	1.3903	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
ETS	Base	0.6558	0.6169	0.7744	1.2669	0.2631	0.2091	0.0920	0.1731	0.4529	0.3371	0.1410	0.3349
ETS	PI.0.2	0.6574	0.6175	0.7742	1.2668	0.2424	0.1711	0.0752	0.1424	0.4395	0.3106	0.1291	0.3172
ETS	PI.0.4	0.6644	0.6214	0.7745	1.2678	0.2072	0.1449	0.0634	0.1223	0.4048	0.2866	0.1184	0.3020
ETS	PI.0.5	0.6698	0.6245	0.7749	1.2688	0.1974	0.1382	0.0602	0.1176	0.3889	0.2756	0.1136	0.2955
ETS	PI.0.6	0.6764	0.6283	0.7754	1.2701	0.1933	0.1365	0.0587	0.1167	0.3742	0.2654	0.1091	0.2898
ETS	PI.0.8	0.6927	0.6382	0.7769	1.2736	0.1987	0.1428	0.0604	0.1259	0.3486	0.2476	0.1016	0.2809
ETS	PI.1	0.7128	0.6505	0.7790	1.2783	0.2180	0.1582	0.0668	0.1462	0.3290	0.2337	0.0960	0.2757
ETS	FI.0.2	0.6568	0.6173	0.7740	1.2666	0.2397	0.1697	0.0746	0.1410	0.4333	0.3061	0.1273	0.3140
ETS	FI.0.4	0.6634	0.6212	0.7740	1.2671	0.1895	0.1334	0.0582	0.1115	0.3748	0.2655	0.1096	0.2858
ETS	FI.0.5	0.6697	0.6250	0.7741	1.2677	0.1643	0.1155	0.0501	0.0969	0.3377	0.2396	0.0986	0.2671
ETS	FI.0.6	0.6792	0.6307	0.7744	1.2689	0.1381	0.0970	0.0419	0.0818	0.2934	0.2085	0.0855	0.2431
ETS	FI.0.8	0.7145	0.6520	0.7760	1.2746	0.0791	0.0556	0.0237	0.0483	0.1753	0.1250	0.0509	0.1670
ETS	FI.1	0.7901	0.6997	0.7822	1.3011	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
ARIMA	Base	0.6339	0.6180	0.7847	1.2881	0.2752	0.2058	0.0997	0.1528	0.4346	0.3263	0.1261	0.3073
ARIMA	PI.0.2	0.6349	0.6183	0.7835	1.2883	0.2255	0.1681	0.0793	0.1242	0.3982	0.2992	0.1124	0.2907
ARIMA	PI.0.4	0.6414	0.6221	0.7832	1.2894	0.1908	0.1422	0.0638	0.1050	0.3653	0.2748	0.1003	0.2762
ARIMA	PI.0.5	0.6465	0.6251	0.7833	1.2903	0.1809	0.1353	0.0587	0.1000	0.3502	0.2636	0.0949	0.2699
ARIMA	PI.0.6	0.6527	0.6289	0.7836	1.2914	0.1766	0.1325	0.0560	0.0984	0.3363	0.2532	0.0901	0.2644
ARIMA	PI.0.8	0.6683	0.6387	0.7848	1.2945	0.1814	0.1368	0.0563	0.1054	0.3124	0.2352	0.0825	0.2556
ARIMA	PI.1	0.6877	0.6513	0.7868	1.2986	0.2004	0.1512	0.0637	0.1235	0.2945	0.2216	0.0782	0.2503
ARIMA	FI.0.2	0.6347	0.6182	0.7834	1.2883	0.2235	0.1667	0.0791	0.1234	0.3926	0.2950	0.1109	0.2880
ARIMA	FI.0.4	0.6416	0.6220	0.7827	1.2894	0.1758	0.1311	0.0601	0.0972	0.3384	0.2547	0.0932	0.2627
ARIMA	FI.0.5	0.6481	0.6259	0.7826	1.2905	0.1520	0.1134	0.0509	0.0845	0.3044	0.2293	0.0828	0.2458
ARIMA	FI.0.6	0.6576	0.6316	0.7827	1.2921	0.1274	0.0952	0.0417	0.0716	0.2640	0.1990	0.0709	0.2243
ARIMA	FI.0.8	0.6916	0.6524	0.7840	1.2985	0.0725	0.0543	0.0224	0.0430	0.1572	0.1187	0.0413	0.1548
ARIMA	FI.1	0.7613	0.6981	0.7886	1.3242	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table A.2: RMSSE, RMSSC and RMSSC-I results of vertical stability experiments.

		RMSSSE				RMSSC				RMSSC.I			
		M4	M3	Favorita	M5	M4	M3	Favorita	M5	M4	M3	Favorita	M5
N-BEATS	Base	0.5607	0.5929	-	-	0.2341	0.2112	-	-	0.3781	0.3456	-	-
N-BEATS	Stable	0.5664	0.5900	-	-	0.1814	0.1573	-	-	0.3469	0.2948	-	-
N-BEATS	PI.0.2	0.5574	0.5921	-	-	0.2214	0.1723	-	-	0.3742	0.3202	-	-
N-BEATS	PI.0.4	0.5600	0.5945	-	-	0.1813	0.1464	-	-	0.3467	0.2984	-	-
N-BEATS	PI.0.5	0.5634	0.5968	-	-	0.1711	0.1399	-	-	0.3354	0.2892	-	-
N-BEATS	PI.0.6	0.5682	0.5999	-	-	0.1688	0.1381	-	-	0.3261	0.2813	-	-
N-BEATS	PI.0.8	0.5813	0.6083	-	-	0.1863	0.1481	-	-	0.3139	0.2698	-	-
N-BEATS	PI.1	0.5989	0.6193	-	-	0.2266	0.1739	-	-	0.3115	0.2652	-	-
N-BEATS	FI.0.2	0.5571	0.5918	-	-	0.2206	0.1708	-	-	0.3693	0.3157	-	-
N-BEATS	FI.0.4	0.5593	0.5936	-	-	0.1688	0.1341	-	-	0.3218	0.2765	-	-
N-BEATS	FI.0.5	0.5631	0.5960	-	-	0.1442	0.1161	-	-	0.2918	0.2512	-	-
N-BEATS	FI.0.6	0.5696	0.5999	-	-	0.1197	0.0976	-	-	0.2556	0.2203	-	-
N-BEATS	FI.0.8	0.5959	0.6165	-	-	0.0674	0.0563	-	-	0.1561	0.1347	-	-
N-BEATS	FI.1	0.6577	0.6584	-	-	0.0000	0.0000	-	-	0.0000	0.0000	-	-
PR	Base	0.6896	0.6886	0.5863	1.0086	0.1683	0.1527	0.1062	0.2514	0.3297	0.2543	0.1333	0.4365
PR	PI.0.2	0.6943	0.6914	0.5848	1.0035	0.1496	0.1273	0.0839	0.1975	0.3004	0.2328	0.1192	0.4116
PR	PI.0.4	0.7009	0.6955	0.5843	1.0015	0.1311	0.1086	0.0661	0.1613	0.2781	0.2136	0.1070	0.3917
PR	PI.0.5	0.7047	0.6980	0.5844	1.0018	0.1253	0.1029	0.0606	0.1529	0.2680	0.2050	0.1017	0.3839
PR	PI.0.6	0.7090	0.7009	0.5847	1.0028	0.1220	0.1000	0.0576	0.1518	0.2586	0.1973	0.0971	0.3778
PR	PI.0.8	0.7189	0.7076	0.5861	1.0074	0.1231	0.1027	0.0591	0.1702	0.2428	0.1845	0.0905	0.3708
PR	PI.1	0.7303	0.7157	0.5885	1.0151	0.1338	0.1148	0.0698	0.2105	0.2315	0.1764	0.0884	0.3717
PR	FI.0.2	0.6947	0.6917	0.5848	1.0029	0.1478	0.1262	0.0838	0.1970	0.2961	0.2295	0.1177	0.4082
PR	FI.0.4	0.7028	0.6972	0.5841	0.9988	0.1206	0.1008	0.0631	0.1501	0.2576	0.1977	0.0995	0.3734
PR	FI.0.5	0.7086	0.7012	0.5841	0.9972	0.1061	0.0878	0.0532	0.1282	0.2329	0.1778	0.0888	0.3508
PR	FI.0.6	0.7159	0.7063	0.5845	0.9960	0.0904	0.0740	0.0435	0.1068	0.2031	0.1542	0.0765	0.3221
PR	FI.0.8	0.7386	0.7218	0.5871	0.9962	0.0530	0.0424	0.0234	0.0620	0.1224	0.0919	0.0452	0.2270
PR	FI.1	0.7814	0.7497	0.5940	1.0267	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
LightGBM	Base	0.7392	0.7003	0.6312	0.9871	0.2243	0.2028	0.0952	0.1796	0.3847	0.3155	0.1461	0.3550
LightGBM	PI.0.2	0.7412	0.7018	0.6324	0.9853	0.2091	0.1641	0.0778	0.1444	0.3694	0.2895	0.1326	0.3372
LightGBM	PI.0.4	0.7471	0.7058	0.6340	0.9852	0.1745	0.1366	0.0644	0.1212	0.3404	0.2671	0.1205	0.3227
LightGBM	PI.0.5	0.7515	0.7087	0.6351	0.9858	0.1645	0.1289	0.0600	0.1158	0.3281	0.2575	0.1151	0.3168
LightGBM	PI.0.6	0.7567	0.7121	0.6362	0.9869	0.1606	0.1262	0.0576	0.1157	0.3173	0.2492	0.1102	0.3118
LightGBM	PI.0.8	0.7697	0.7207	0.6389	0.9903	0.1706	0.1350	0.0584	0.1281	0.3013	0.2367	0.1022	0.3050
LightGBM	PI.1	0.7859	0.7314	0.6420	0.9955	0.2001	0.1589	0.0660	0.1536	0.2940	0.2310	0.0971	0.3031
LightGBM	FI.0.2	0.7415	0.7020	0.6325	0.9850	0.2078	0.1632	0.0774	0.1434	0.3644	0.2856	0.1309	0.3342
LightGBM	FI.0.4	0.7486	0.7070	0.6349	0.9838	0.1628	0.1275	0.0607	0.1117	0.3159	0.2477	0.1122	0.3075
LightGBM	FI.0.5	0.7546	0.7112	0.6366	0.9835	0.1405	0.1099	0.0523	0.0966	0.2855	0.2238	0.1008	0.2895
LightGBM	FI.0.6	0.7629	0.7169	0.6388	0.9836	0.1177	0.0920	0.0437	0.0815	0.2491	0.1953	0.0873	0.2662
LightGBM	FI.0.8	0.7905	0.7359	0.6453	0.9864	0.0669	0.0523	0.0246	0.0487	0.1507	0.1181	0.0522	0.1879
LightGBM	FI.1	0.8467	0.7745	0.6570	1.0139	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
ETS	Base	0.5713	0.5690	0.5670	0.9332	0.2025	0.1677	0.0526	0.1017	0.3931	0.2994	0.0900	0.2123
ETS	PI.0.2	0.5724	0.5695	0.5670	0.9331	0.1845	0.1384	0.0435	0.0839	0.3713	0.2788	0.0835	0.2022
ETS	PI.0.4	0.5778	0.5728	0.5673	0.9337	0.1607	0.1195	0.0374	0.0727	0.3471	0.2609	0.0778	0.1936
ETS	PI.0.5	0.5820	0.5754	0.5676	0.9343	0.1544	0.1149	0.0359	0.0701	0.3366	0.2531	0.0754	0.1901
ETS	PI.0.6	0.5871	0.5786	0.5679	0.9351	0.1523	0.1140	0.0353	0.0698	0.3272	0.2462	0.0732	0.1870
ETS	PI.0.8	0.5996	0.5867	0.5690	0.9372	0.1603	0.1220	0.0372	0.0757	0.3126	0.2356	0.0699	0.1826
ETS	PI.1	0.6150	0.5969	0.5704	0.9400	0.1834	0.1411	0.0428	0.0889	0.3050	0.2301	0.0682	0.1807
ETS	FI.0.2	0.5721	0.5694	0.5669	0.9330	0.1822	0.1370	0.0430	0.0830	0.3662	0.2750	0.0824	0.2002
ETS	FI.0.4	0.5775	0.5729	0.5671	0.9332	0.1460	0.1092	0.0341	0.0661	0.3220	0.2421	0.0722	0.1838
ETS	FI.0.5	0.5827	0.5762	0.5673	0.9335	0.1277	0.0954	0.0297	0.0577	0.2931	0.2205	0.0655	0.1727
ETS	FI.0.6	0.5903	0.5812	0.5677	0.9341	0.1085	0.0810	0.0251	0.0490	0.2575	0.1939	0.0575	0.1583
ETS	FI.0.8	0.6190	0.6000	0.5694	0.9370	0.0638	0.0477	0.0146	0.0295	0.1579	0.1192	0.0351	0.1110
ETS	FI.1	0.6837	0.6434	0.5741	0.9524	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
ARIMA	Base	0.5547	0.5683	0.5732	0.9510	0.2104	0.1685	0.0644	0.0958	0.3656	0.2921	0.0855	0.2025
ARIMA	PI.0.2	0.5554	0.5686	0.5724	0.9509	0.1739	0.1387	0.0514	0.0782	0.3389	0.2708	0.0766	0.1924
ARIMA	PI.0.4	0.5604	0.5717	0.5721	0.9514	0.1498	0.1193	0.0416	0.0666	0.3154	0.2520	0.0689	0.1837
ARIMA	PI.0.5	0.5643	0.5743	0.5722	0.9519	0.1432	0.1142	0.0384	0.0635	0.3051	0.2438	0.0656	0.1800
ARIMA	PI.0.6	0.5691	0.5776	0.5724	0.9526	0.1407	0.1125	0.0367	0.0626	0.2960	0.2365	0.0628	0.1769
ARIMA	PI.0.8	0.5810	0.5858	0.5733	0.9544	0.1477	0.1186	0.0376	0.0670	0.2818	0.2251	0.0588	0.1722
ARIMA	PI.1	0.5958	0.5962	0.5748	0.9569	0.1696	0.1364	0.0440	0.0790	0.2745	0.2191	0.0576	0.1699
ARIMA	FI.0.2	0.5553	0.5685	0.5723	0.9509	0.1720	0.1374	0.0512	0.0776	0.3342	0.2671	0.0756	0.1907
ARIMA	FI.0.4	0.5606	0.5718	0.5718	0.9513	0.1371	0.1093	0.0392	0.0614	0.2927	0.2340	0.0641	0.1751
ARIMA	FI.0.5	0.5657	0.5752	0.5718	0.9518	0.1195	0.0953	0.0333	0.0535	0.2658	0.2125	0.0573	0.1647
ARIMA	FI.0.6	0.5732	0.5802	0.5719	0.9525	0.1012	0.0807	0.0274	0.0455	0.2330	0.1863	0.0494	0.1512
ARIMA	FI.0.8	0.6006	0.5988	0.5730	0.9556	0.0590	0.0471	0.0149	0.0276	0.1423	0.1138	0.0293	0.1064
ARIMA	FI.1	0.6601	0.6409	0.5767	0.9699	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table A.3: sMAPE, sMAPC and sMAPC-I results of vertical stability experiments.

		sMAPE				sMAPC				sMAPC.I			
		M4	M3	Favorita	M5	M4	M3	Favorita	M5	M4	M3	Favorita	M5
N-BEATS	Base	9.296	11.485	-	-	4.717	3.932	-	-	6.468	5.786	-	-
N-BEATS	Stable	9.279	11.390	-	-	3.336	2.820	-	-	5.270	4.705	-	-
N-BEATS	PI.0.2	9.265	11.461	-	-	3.745	3.181	-	-	5.886	5.288	-	-
N-BEATS	PI.0.4	9.324	11.484	-	-	3.050	2.649	-	-	5.386	4.850	-	-
N-BEATS	PI.0.5	9.384	11.511	-	-	2.865	2.508	-	-	5.169	4.657	-	-
N-BEATS	PI.0.6	9.464	11.553	-	-	2.808	2.451	-	-	4.976	4.485	-	-
N-BEATS	PI.0.8	9.677	11.673	-	-	2.998	2.569	-	-	4.665	4.202	-	-
N-BEATS	PI.1	9.956	11.831	-	-	3.450	2.889	-	-	4.462	4.000	-	-
N-BEATS	FI.0.2	9.261	11.454	-	-	3.732	3.158	-	-	5.806	5.212	-	-
N-BEATS	FI.0.4	9.318	11.458	-	-	2.851	2.444	-	-	4.991	4.483	-	-
N-BEATS	FI.0.5	9.390	11.478	-	-	2.429	2.095	-	-	4.489	4.029	-	-
N-BEATS	FI.0.6	9.502	11.518	-	-	2.008	1.743	-	-	3.897	3.492	-	-
N-BEATS	FI.0.8	9.919	11.703	-	-	1.119	0.977	-	-	2.339	2.082	-	-
N-BEATS	FI.1	10.806	12.219	-	-	0.000	0.000	-	-	0.000	0.000	-	-
PR	Base	10.632	12.622	103.351	54.964	3.245	3.518	29.190	17.559	5.282	4.896	36.772	26.876
PR	PI.0.2	10.713	12.638	103.255	54.579	2.743	2.880	24.303	13.857	4.817	4.407	33.580	25.222
PR	PI.0.4	10.822	12.680	103.173	54.392	2.350	2.385	20.319	11.260	4.390	3.971	30.665	23.894
PR	PI.0.5	10.886	12.712	103.196	54.374	2.221	2.222	19.011	10.650	4.192	3.777	29.261	23.366
PR	PI.0.6	10.956	12.751	103.269	54.406	2.142	2.132	18.160	10.565	4.007	3.604	27.930	22.935
PR	PI.0.8	11.116	12.851	103.566	54.629	2.118	2.154	17.694	11.810	3.679	3.317	25.568	22.392
PR	PI.1	11.302	12.981	104.084	55.087	2.244	2.388	18.739	14.375	3.418	3.128	23.677	22.273
PR	FI.0.2	10.720	12.638	103.627	54.532	2.715	2.858	24.670	13.856	4.748	4.341	33.216	24.994
PR	FI.0.4	10.860	12.685	103.530	54.162	2.177	2.217	19.386	10.504	4.064	3.661	28.954	22.720
PR	FI.0.5	10.957	12.726	103.563	53.987	1.893	1.894	16.809	8.925	3.640	3.253	26.342	21.269
PR	FI.0.6	11.080	12.783	103.673	53.812	1.594	1.565	13.911	7.361	3.144	2.786	23.174	19.446
PR	FI.0.8	11.448	12.972	104.248	53.556	0.909	0.856	8.077	4.213	1.860	1.618	14.923	13.662
PR	FI.1	12.103	13.333	105.775	54.936	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
LightGBM	Base	11.125	12.653	101.341	52.207	3.566	3.478	13.216	9.992	5.450	4.887	18.787	17.961
LightGBM	PI.0.2	11.211	12.680	101.229	52.100	2.950	2.812	10.796	8.108	4.939	4.412	16.703	16.996
LightGBM	PI.0.4	11.332	12.738	101.171	52.081	2.466	2.304	8.846	6.792	4.479	3.996	14.821	16.204
LightGBM	PI.0.5	11.405	12.779	101.155	52.092	2.307	2.146	8.146	6.445	4.270	3.813	13.963	15.862
LightGBM	PI.0.6	11.487	12.827	101.149	52.116	2.223	2.072	7.686	6.466	4.077	3.648	13.163	15.554
LightGBM	PI.0.8	11.673	12.944	101.164	52.209	2.237	2.146	7.359	7.063	3.744	3.375	11.764	15.057
LightGBM	PI.1	11.889	13.089	101.222	52.398	2.440	2.417	7.697	8.259	3.496	3.190	10.659	14.796
LightGBM	FI.0.2	11.221	12.683	101.214	52.079	2.929	2.798	10.701	8.029	4.868	4.349	16.457	16.826
LightGBM	FI.0.4	11.386	12.757	101.120	51.988	2.310	2.165	8.257	6.264	4.144	3.693	13.698	15.369
LightGBM	FI.0.5	11.503	12.815	101.086	51.946	1.993	1.852	7.025	5.398	3.701	3.297	12.087	14.393
LightGBM	FI.0.6	11.652	12.892	101.067	51.907	1.663	1.533	5.768	4.523	3.187	2.838	10.274	13.138
LightGBM	FI.0.8	12.087	13.136	101.107	51.888	0.929	0.845	3.095	2.636	1.868	1.665	5.861	9.062
LightGBM	FI.1	12.827	13.610	101.323	52.690	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
ETS	Base	9.984	11.344	104.820	50.866	3.916	3.210	14.192	6.450	6.995	5.011	20.956	11.689
ETS	PI.0.2	10.007	11.332	104.752	50.823	3.652	2.623	12.224	5.375	6.632	4.615	19.505	11.081
ETS	PI.0.4	10.114	11.369	104.742	50.821	3.153	2.213	10.764	4.682	6.130	4.265	18.117	10.560
ETS	PI.0.5	10.195	11.404	104.762	50.834	3.015	2.101	10.332	4.520	5.900	4.106	17.454	10.337
ETS	PI.0.6	10.291	11.449	104.799	50.858	2.960	2.077	10.072	4.488	5.686	3.958	16.817	10.138
ETS	PI.0.8	10.530	11.572	104.928	50.937	3.038	2.186	10.001	4.778	5.306	3.702	15.616	9.825
ETS	PI.1	10.823	11.735	105.120	51.059	3.292	2.432	10.328	5.460	4.996	3.500	14.373	9.646
ETS	FI.0.2	9.995	11.328	104.732	50.811	3.601	2.600	12.106	5.313	6.538	4.552	19.279	10.968
ETS	FI.0.4	10.086	11.356	104.653	50.768	2.862	2.042	9.982	4.248	5.682	3.958	17.022	10.001
ETS	FI.0.5	10.175	11.392	104.621	50.751	2.488	1.764	8.833	3.707	5.137	3.581	15.560	9.354
ETS	FI.0.6	10.312	11.451	104.600	50.737	2.101	1.480	7.612	3.131	4.485	3.128	13.820	8.524
ETS	FI.0.8	10.831	11.695	104.630	50.750	1.229	0.854	4.620	1.848	2.733	1.909	8.957	5.892
ETS	FI.1	11.949	12.303	105.123	51.212	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
ARIMA	Base	9.771	11.748	103.635	51.050	4.148	3.169	12.386	5.457	6.571	4.996	15.665	10.441
ARIMA	PI.0.2	9.781	11.745	103.460	51.018	3.413	2.603	10.135	4.489	6.035	4.579	14.095	9.873
ARIMA	PI.0.4	9.876	11.790	103.366	51.017	2.908	2.211	8.294	3.832	5.550	4.202	12.704	9.380
ARIMA	PI.0.5	9.952	11.829	103.352	51.029	2.766	2.097	7.667	3.657	5.329	4.030	12.072	9.166
ARIMA	PI.0.6	10.044	11.878	103.362	51.048	2.704	2.049	7.354	3.606	5.124	3.869	11.485	8.977
ARIMA	PI.0.8	10.275	12.006	103.441	51.111	2.777	2.098	7.347	3.837	4.765	3.587	10.495	8.681
ARIMA	PI.1	10.560	12.176	103.624	51.206	3.050	2.296	7.906	4.428	4.484	3.369	9.690	8.503
ARIMA	FI.0.2	9.777	11.742	103.443	51.012	3.379	2.579	10.055	4.455	5.951	4.516	13.923	9.780
ARIMA	FI.0.4	9.874	11.783	103.298	50.990	2.671	2.039	7.805	3.535	5.151	3.901	11.887	8.915
ARIMA	FI.0.5	9.967	11.828	103.248	50.986	2.317	1.769	6.715	3.084	4.647	3.516	10.689	8.343
ARIMA	FI.0.6	10.105	11.894	103.214	50.987	1.952	1.488	5.590	2.623	4.048	3.060	9.323	7.615
ARIMA	FI.0.8	10.598	12.151	103.245	51.038	1.130	0.859	3.203	1.593	2.452	1.846	5.865	5.286
ARIMA	FI.1	11.618	12.760	103.636	51.487	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Tables A.4, A.5 and A.6 respectively show the MASE, RMSSE, and sMAPE based accuracy and stability results of horizontal stability experiments across the four experimental datasets. The accuracy measures are calculated based on final forecasts and stability measures are calculated based on remainders.

## Appendix B. Vertical stability results per horizon

In Figures B.1 to B.9 we show results for some of the models and datasets for sMAPE and sMAPC(V) for vertical stability, separated per horizon. We can see that the forecast for the largest horizon always is a new forecast that is not stabilised, as such, it is the same across all stabilisation methods. Consequently, for the second-largest horizon, as we stabilise against the largest horizon from the previous origin, partial interpolation and full interpolation yields the same result.

Furthermore, the plots show that as is to be expected, accuracy drops in all cases as horizons become larger. For stability, the picture is less clear. In many cases, stability increases as the horizon increases, for some cases it is the opposite, and for some cases, there is no monotonous relationship.

We hypothesise that this depends on characteristics of the datasets and the methods. For example, in our experiments we stabilise forecasts from the same methodology across different origins. Thus, the forecasts will not be independent. If a method is systematically biased this will affect accuracy negatively but not stability. Only variance in the forecasts will be relevant for stability. Furthermore, as horizons become larger, autocorrelations become less important relative to seasonalities and trends. So we expect that there will be datasets where forecasts for longer horizons are more stable than for shorter ones. Also, the relative difference is smaller in larger horizons. For example, the forecast horizon doubles from horizon 1 to horizon 2. This percentage difference is much smaller for larger horizons, e.g., horizon 5 to horizon 6.









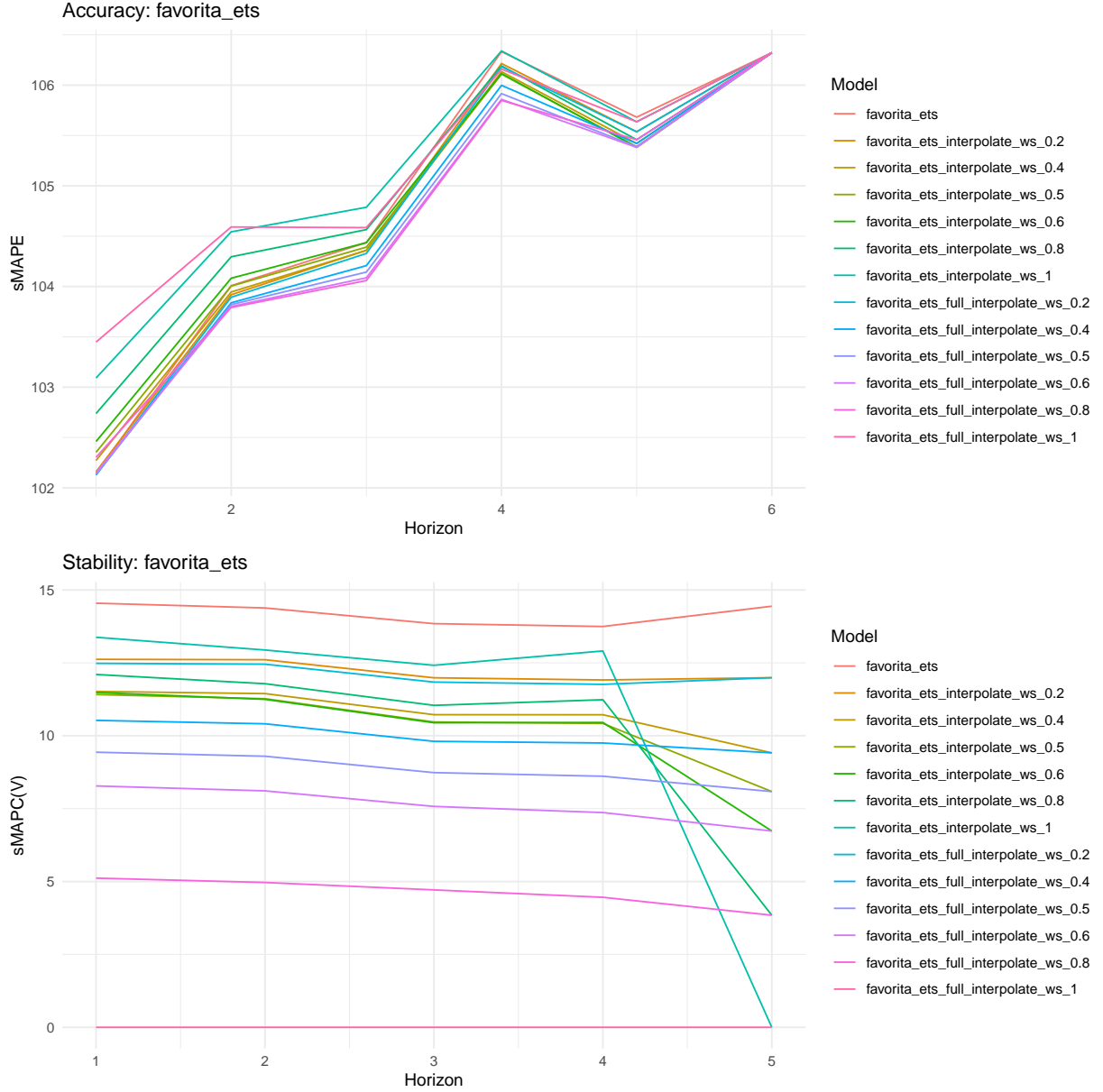


Figure B.1: Accuracy (top) and Vertical Stability (bottom) for ETS for the Favorita dataset. We can see that the forecast for the largest horizon always is a new forecast that is not stabilised, as such, it is the same across all stabilisation methods. Consequently, for the second-largest horizon, as we stabilise against the largest horizon from the previous origin, partial interpolation and full interpolation yields the same result. Furthermore, we can see that accuracy decreases with an increasing horizon, whereas stability stays relatively constant across horizons.

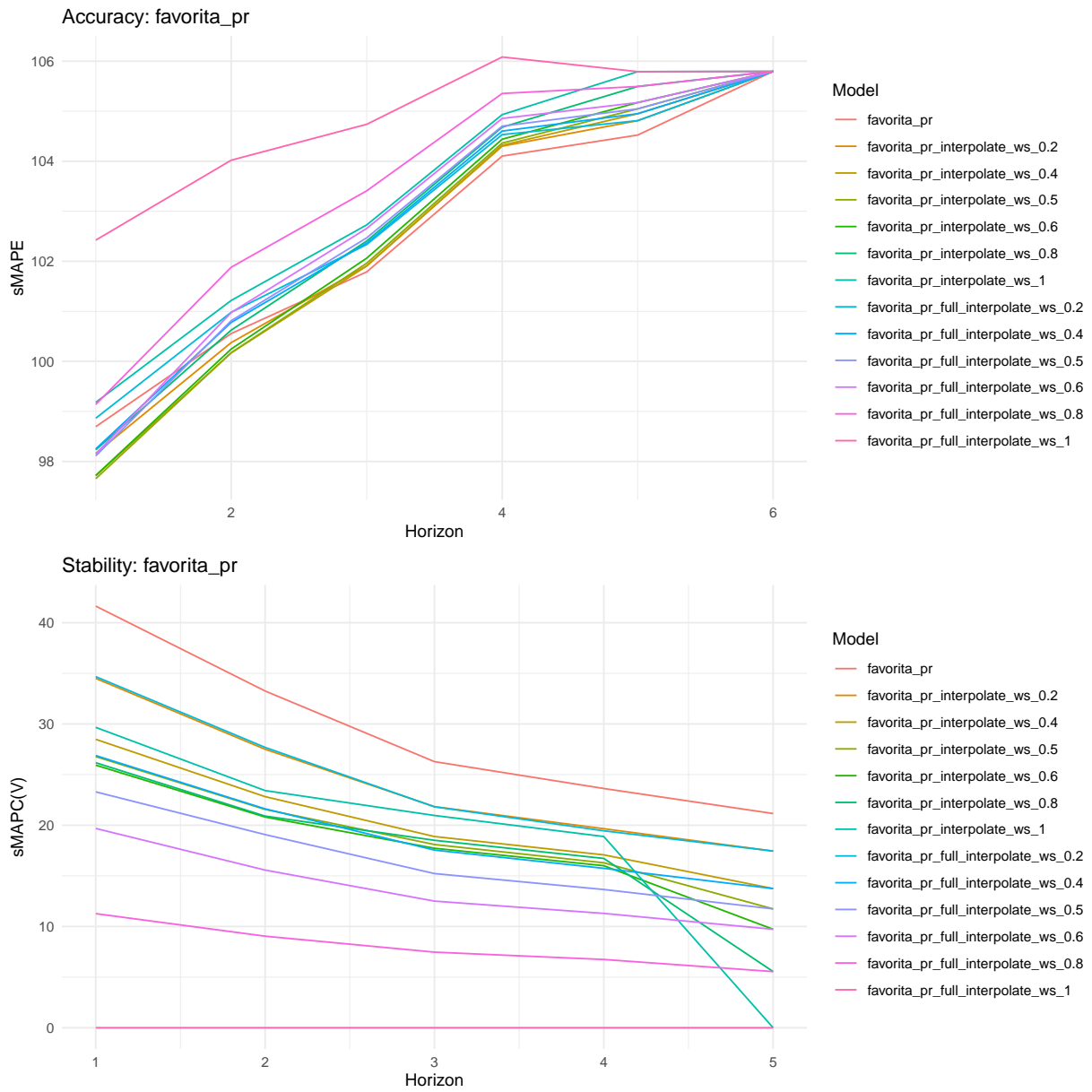


Figure B.2: Accuracy (top) and Vertical Stability (bottom) for PR for the Favorita dataset. We can see that accuracy decreases (as sMAPE increases) with an increasing horizon, whereas stability increases (as sMAPC decreases).

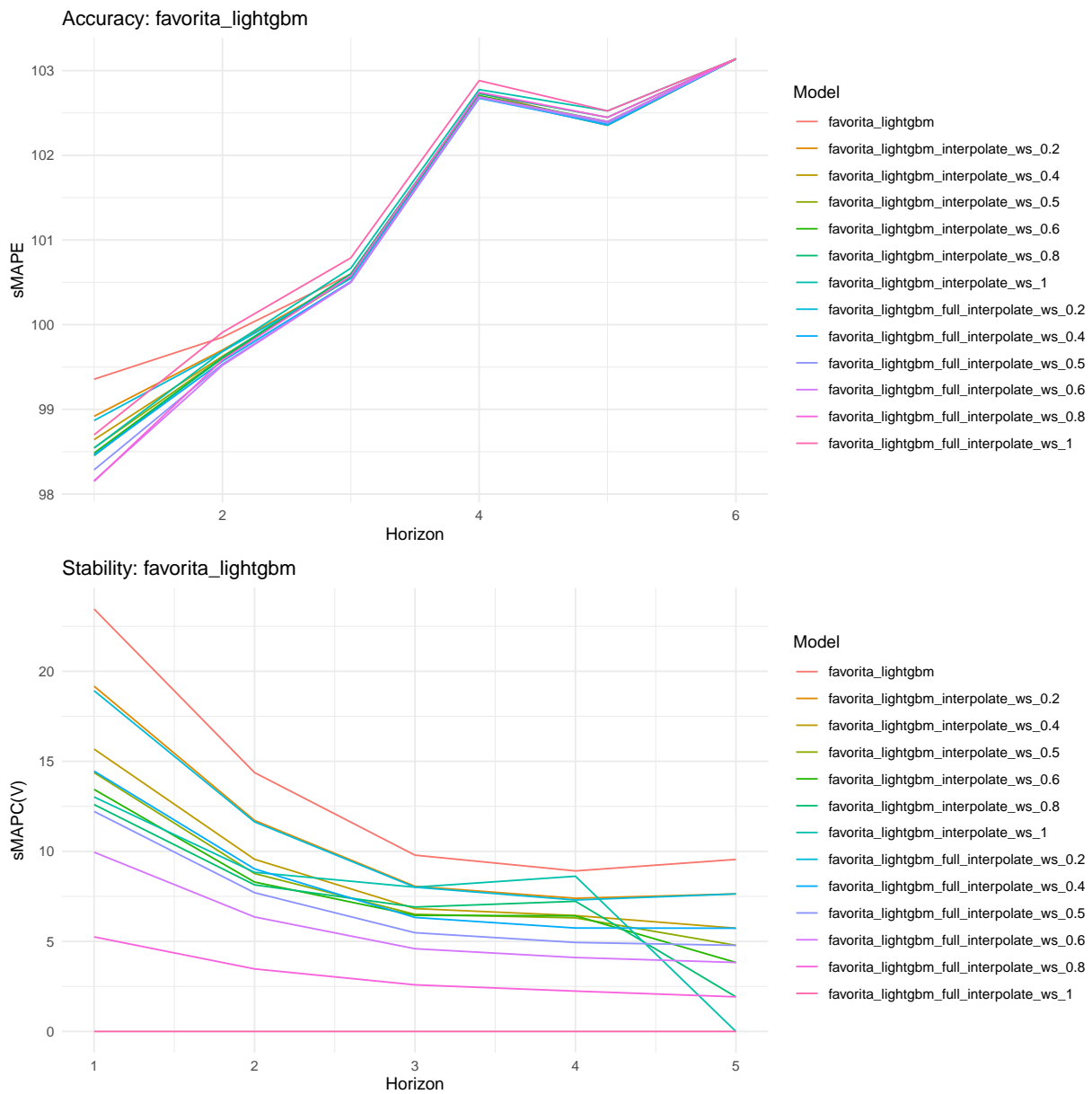


Figure B.3: Accuracy (top) and Vertical Stability (bottom) for LightGBM for the Favorita dataset. We can see that accuracy decreases (as sMAPE increases) with an increasing horizon, whereas stability mostly increases (as sMAPC decreases).

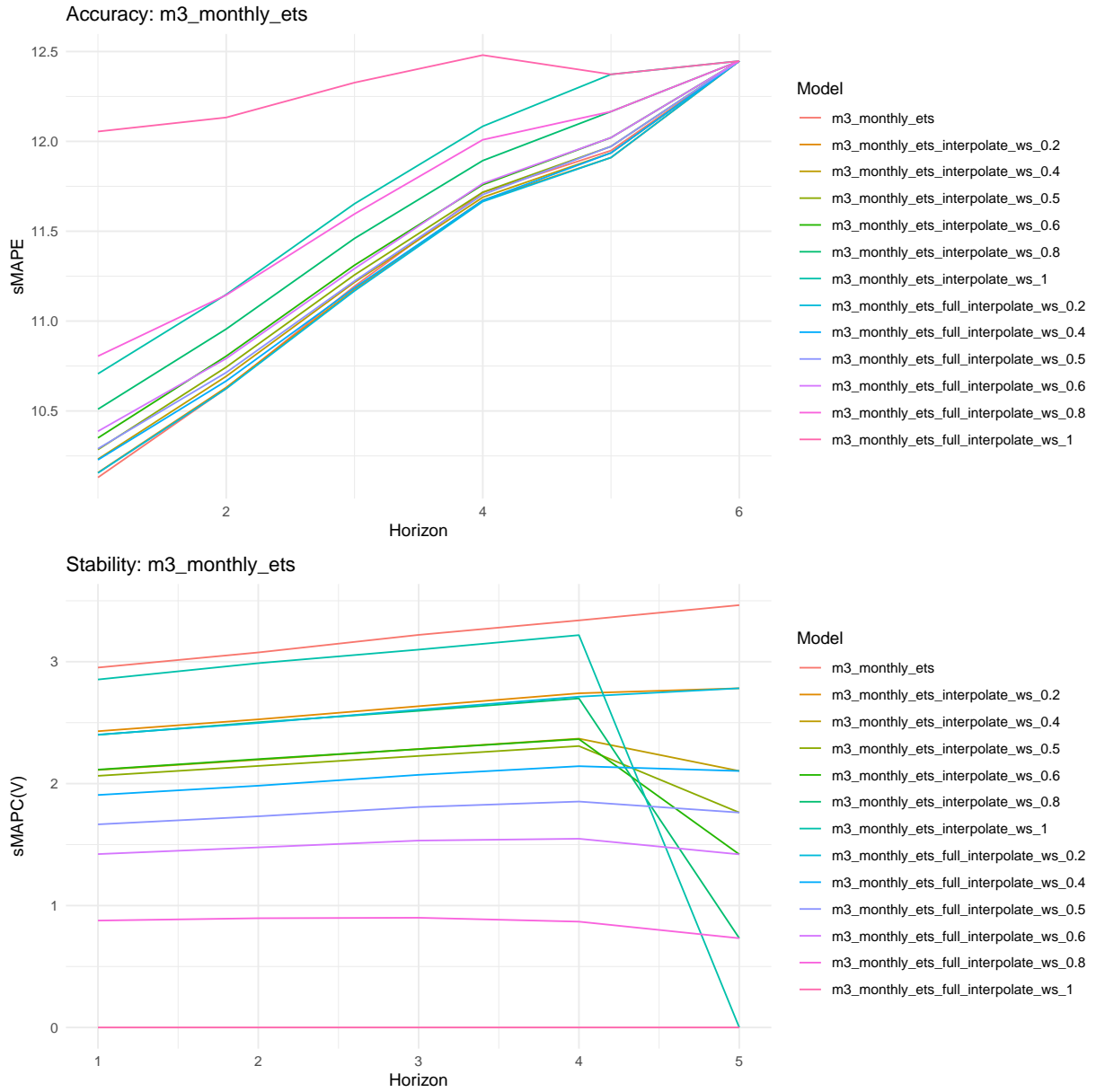


Figure B.4: Accuracy (top) and Vertical Stability (bottom) for ETS for the Monthly M3 dataset. We can see that accuracy decreases (as sMAPE increases) with an increasing horizon, and stability also decreases (as sMAPC increases), except for the artefacts that stability for the second-largest horizon of partial interpolation is identical with full interpolation, and therefore is more stable.

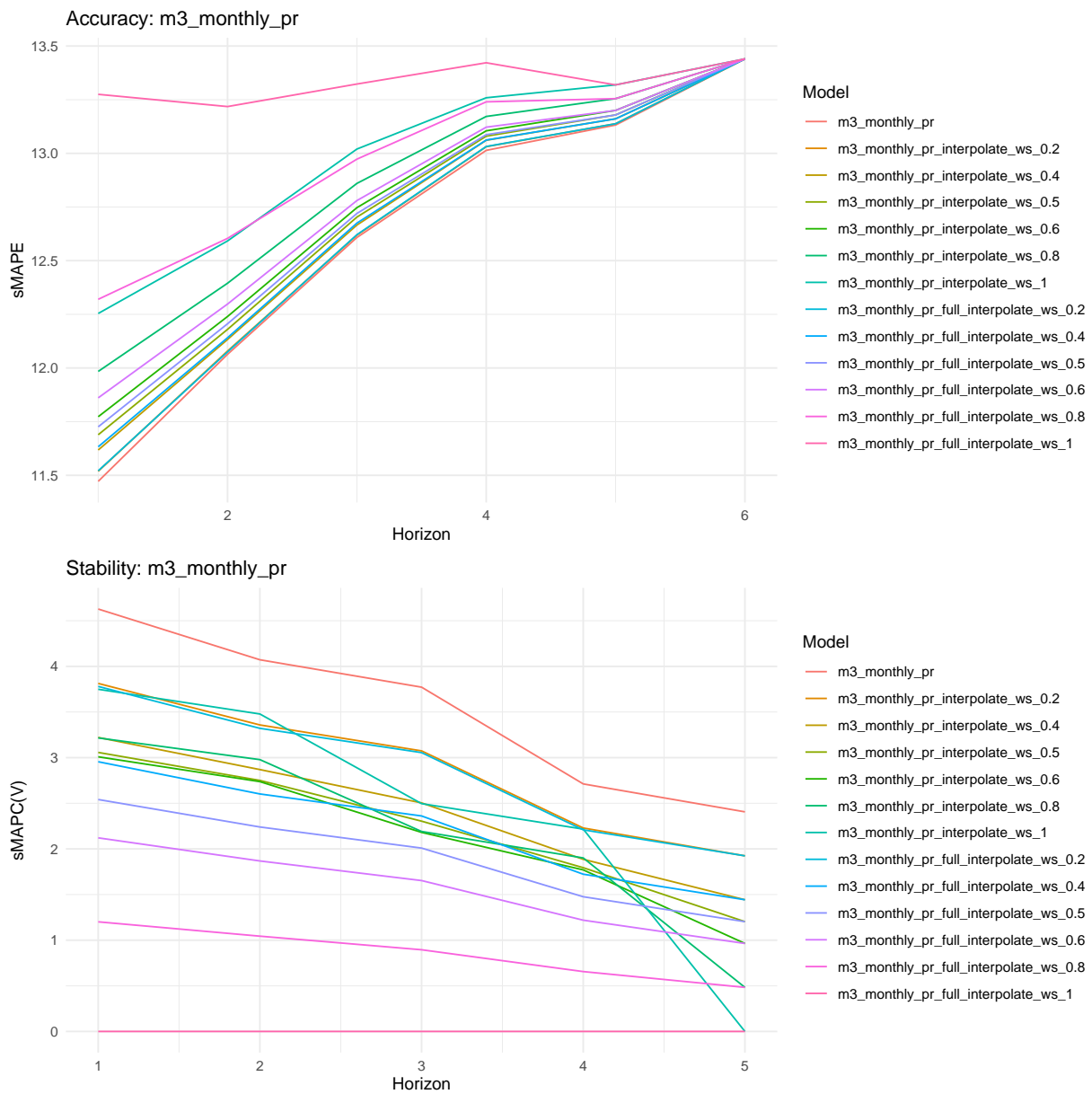


Figure B.5: Accuracy (top) and Vertical Stability (bottom) for PR for the Monthly M3 dataset. We can see that accuracy decreases (as sMAPE increases) with an increasing horizon, whereas stability mostly increases (as sMAPC decreases).

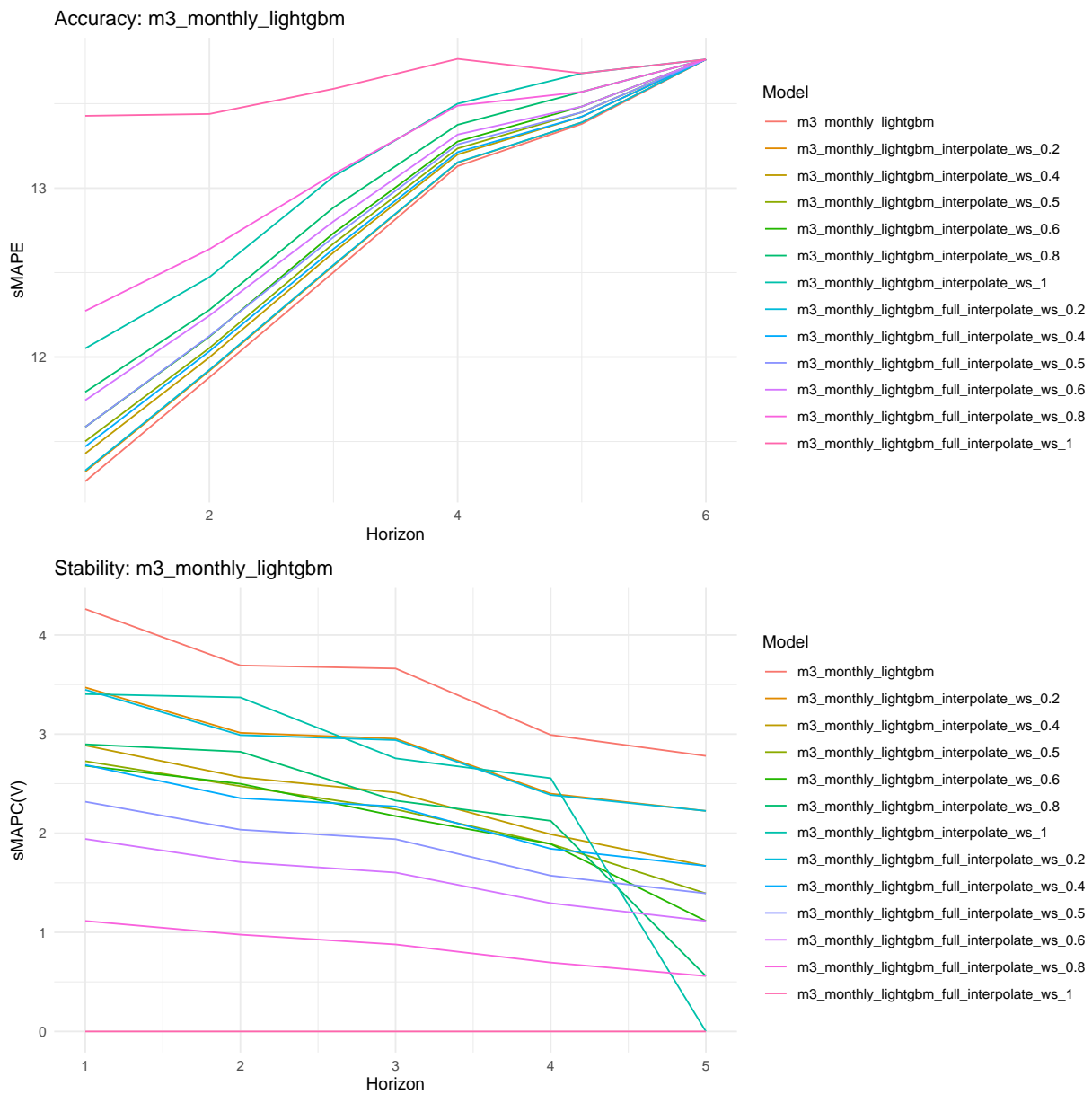


Figure B.6: Accuracy (top) and Vertical Stability (bottom) for LightGBM for the Monthly M3 dataset. We can see that accuracy decreases (as sMAPE increases) with an increasing horizon, whereas stability mostly increases (as sMAPC decreases).

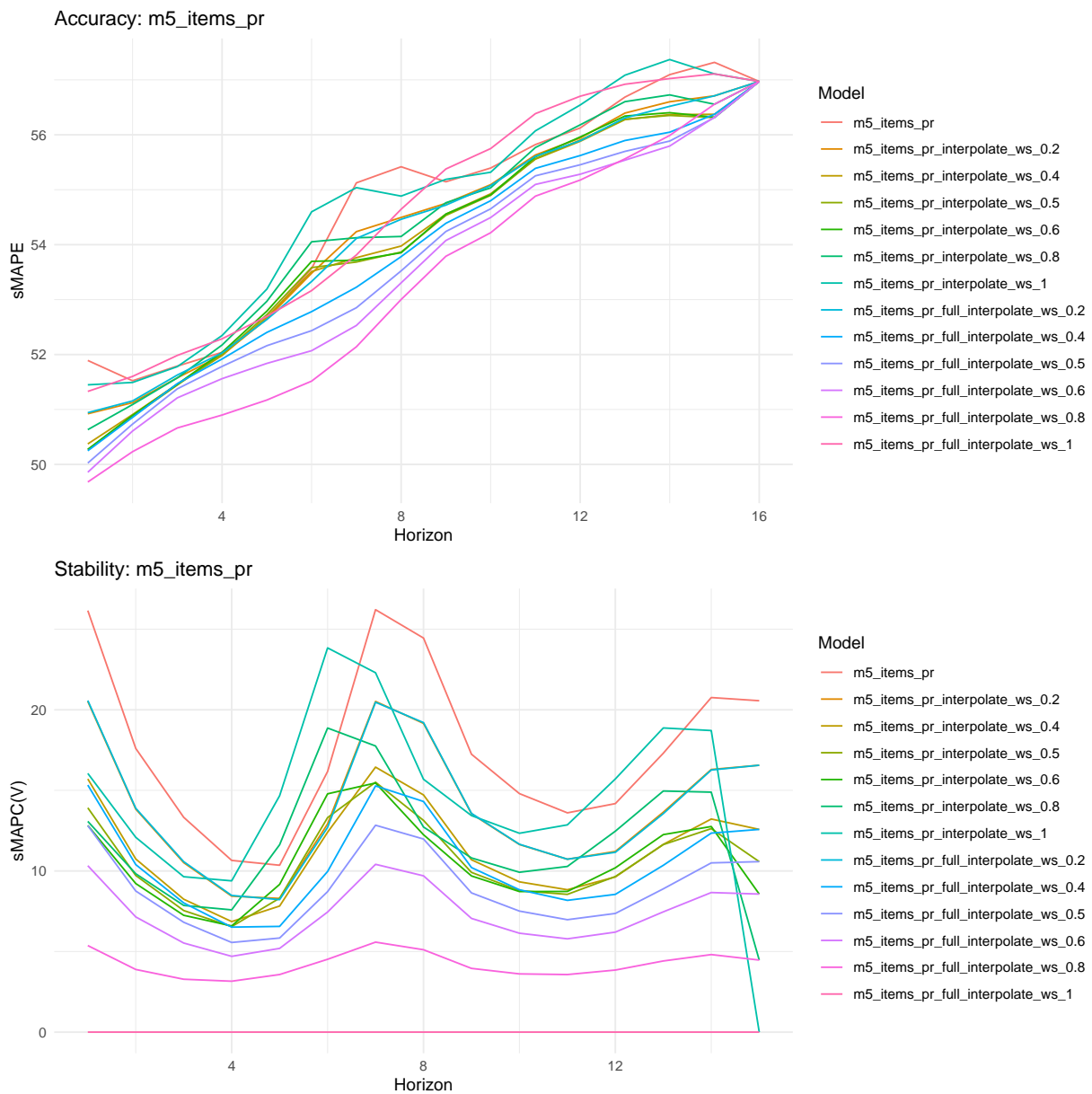


Figure B.7: Accuracy (top) and Vertical Stability (bottom) for PR for the M5 items dataset. We can see that accuracy decreases (as sMAPE increases) with an increasing horizon, whereas stability has no monotonous relationship with the horizon.

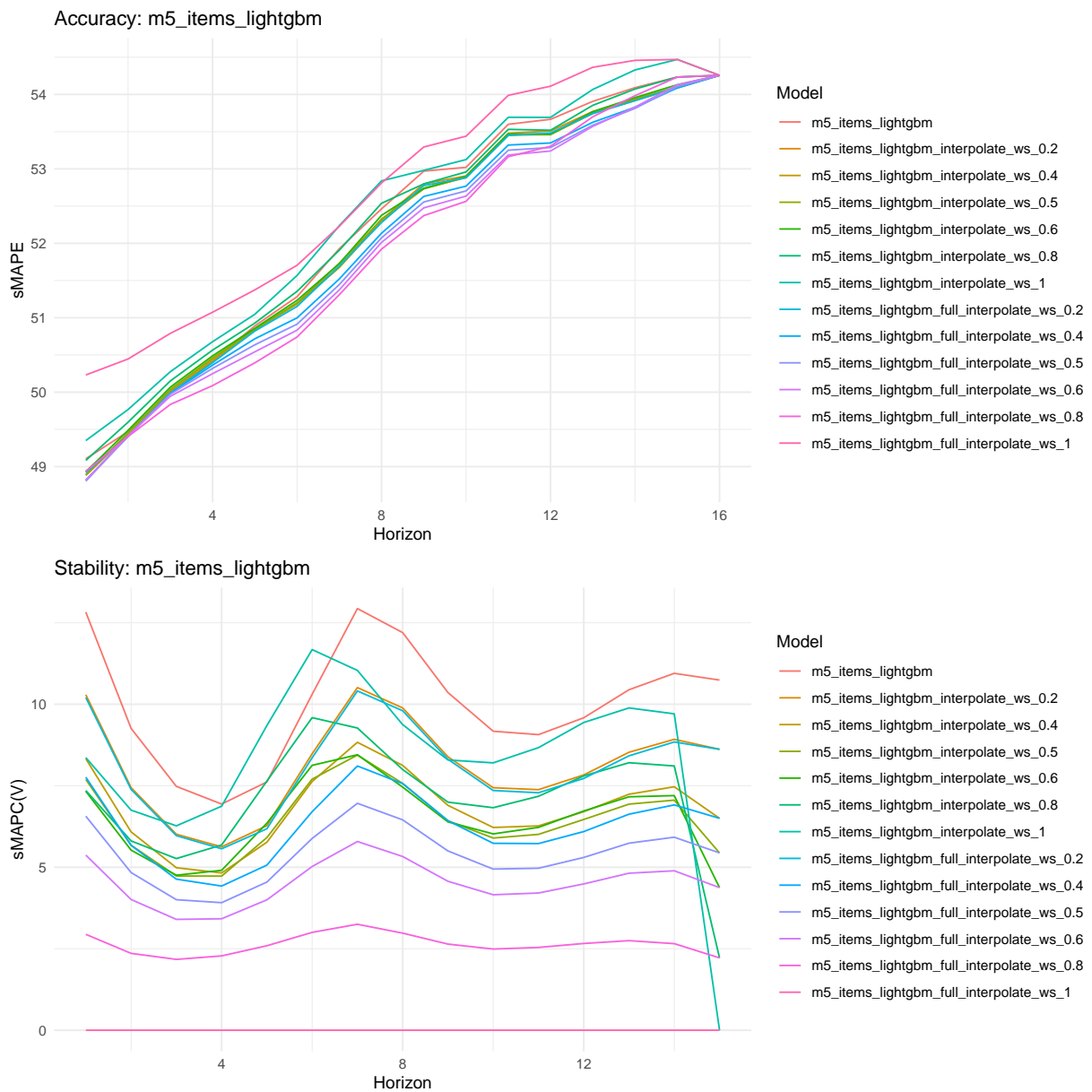


Figure B.8: Accuracy (top) and Vertical Stability (bottom) for LightGBM for the M5 items dataset. We can see that accuracy decreases (as sMAPE increases) with an increasing horizon, whereas stability has no monotonous relationship with the horizon.



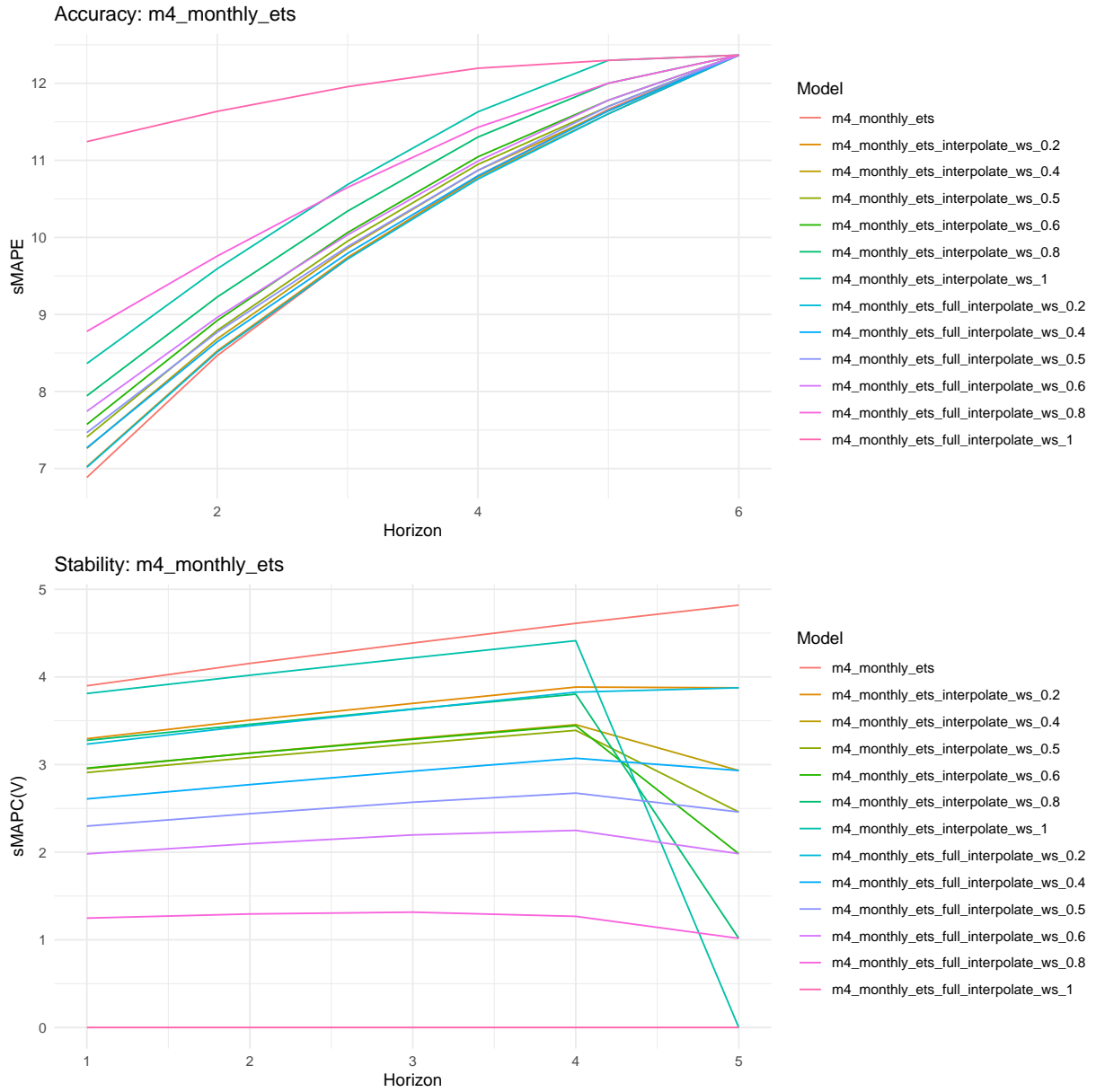


Figure B.9: Accuracy (top) and Vertical Stability (bottom) for ETS for the Monthly M4 dataset. We can see that accuracy decreases (as sMAPE increases) with an increasing horizon, and stability also decreases (as sMAPC increases), except for the artefacts that stability for the second-largest horizon of partial interpolation is identical with full interpolation, and therefore is more stable.