COS-310

Approximation Algorithms for Facility Location Problems

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Uncapacitated Facility Location Problem

Instance:

C = set of clients

F = set of locations where facilities could be opened

f_i = cost of opening a facility at $i \in F$

d is a metric, where $d(i, j)$ is the cost of servicing client j with facility i.

Objective:

Find $S \subseteq F$ and an assignment $A: C \rightarrow S$ s.t $\sum f_i + \sum d(i, j) = c_{\square}(S) + c_{\square}(S) = c(S)$ is minimised.

Observation: Once S is fixed, A becomes automatically fixed - assign each client to nearest open facility!

Uncapacitated Facility Location Problem

01	Deterministic Rounding	4-approximation
02	Randomised Rounding	1.736-approximation (expected cost)
03	Primal-Dual Algorithm	3-approximation
04	Local Search	2.414-approximation

Theorem: There is no α-approx where α < 1.463 unless each problem in NP has O(n^(log log n)) algorithm!

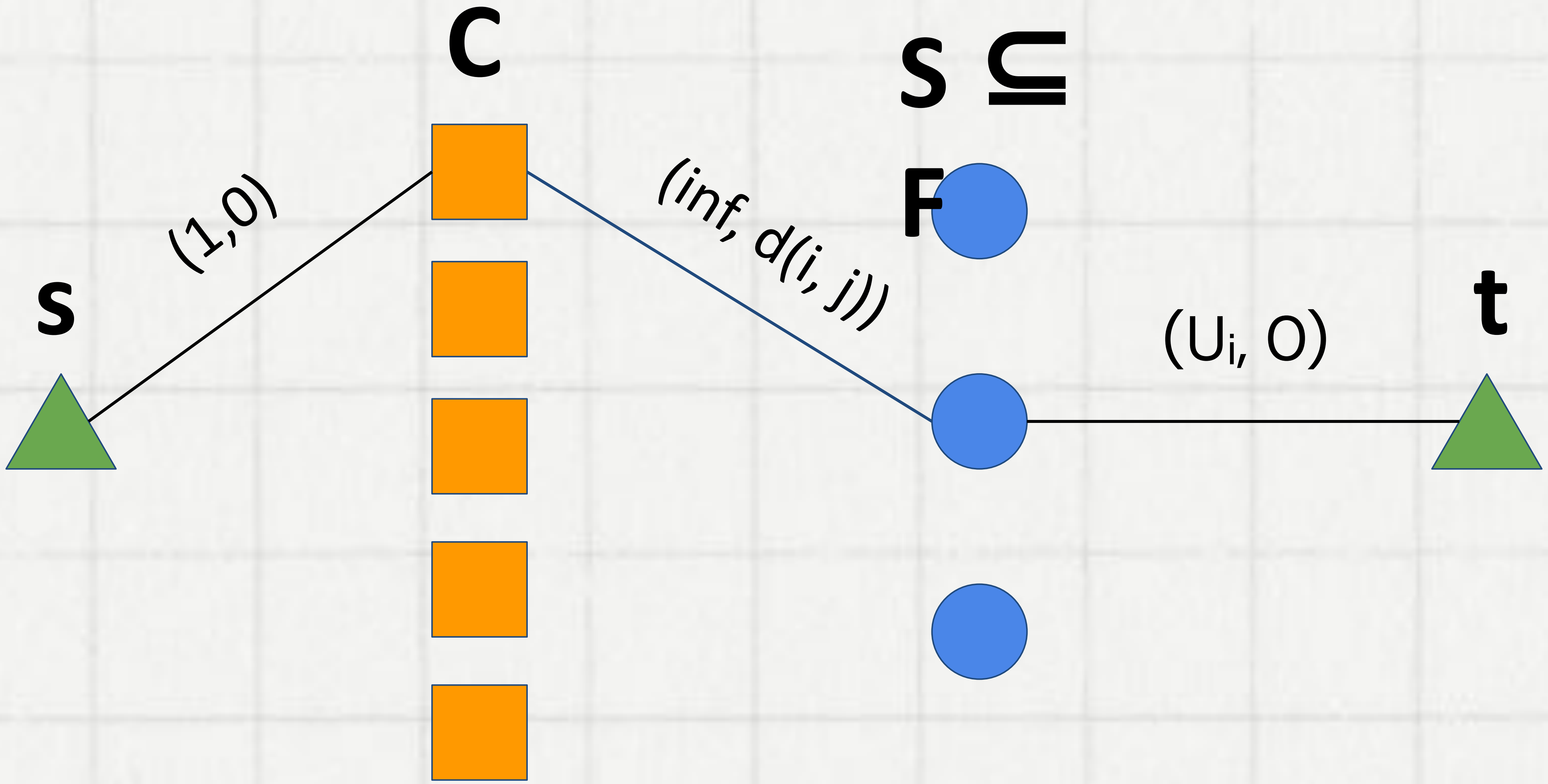
* Reference: Williamson DP, Shmoys DB. *The Design of Approximation Algorithms*. Cambridge University Press; 2011.

Capacitated Facility Location Problem

To the uncapacitated problem, we add an additional constraint – each facility can serve a maximum of U_i clients.

Observation: Once S is fixed, A becomes automatically fixed.

But note that we cannot necessarily send each client to its nearest open facility now!



Local Search Algorithm

Permissible Moves:

1. **Add:** $S \leftarrow S \cup \{i\}$ for some $i \notin S$

2. **Delete:** $S \leftarrow S \setminus \{i\}$ for some $i \in S$

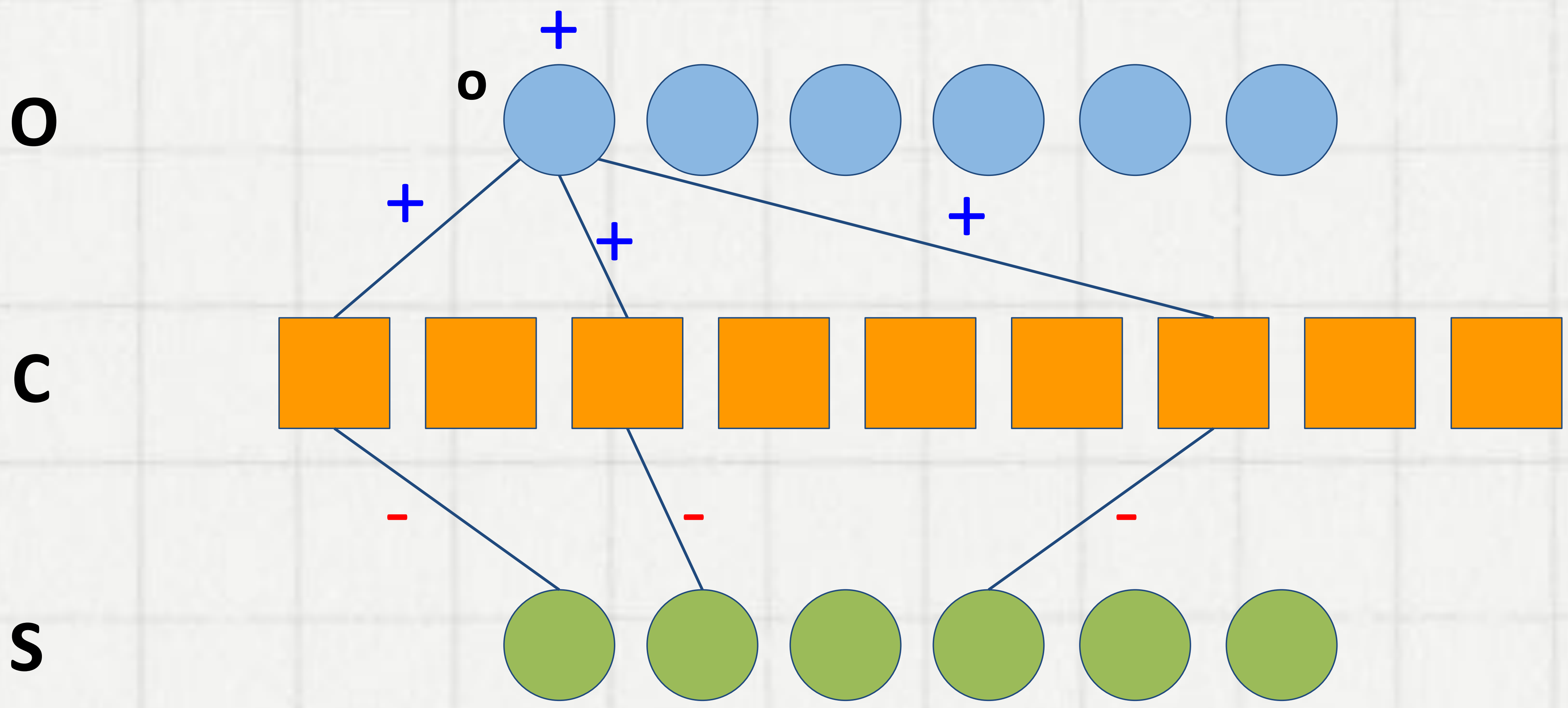
3. **Swap:** $S \leftarrow S \cup \{i\} \setminus \{i'\}$ for some $i \notin S$, $i' \in S$

Local Search Algorithm

Let O be an optimal solution and let S be a locally optimum solution.

<u>Lemma:</u> C□(S) ≤ C(O)

<u>Proof:</u> Using add operations, we can deduce the result.



Local Search Algorithm

6-approximation

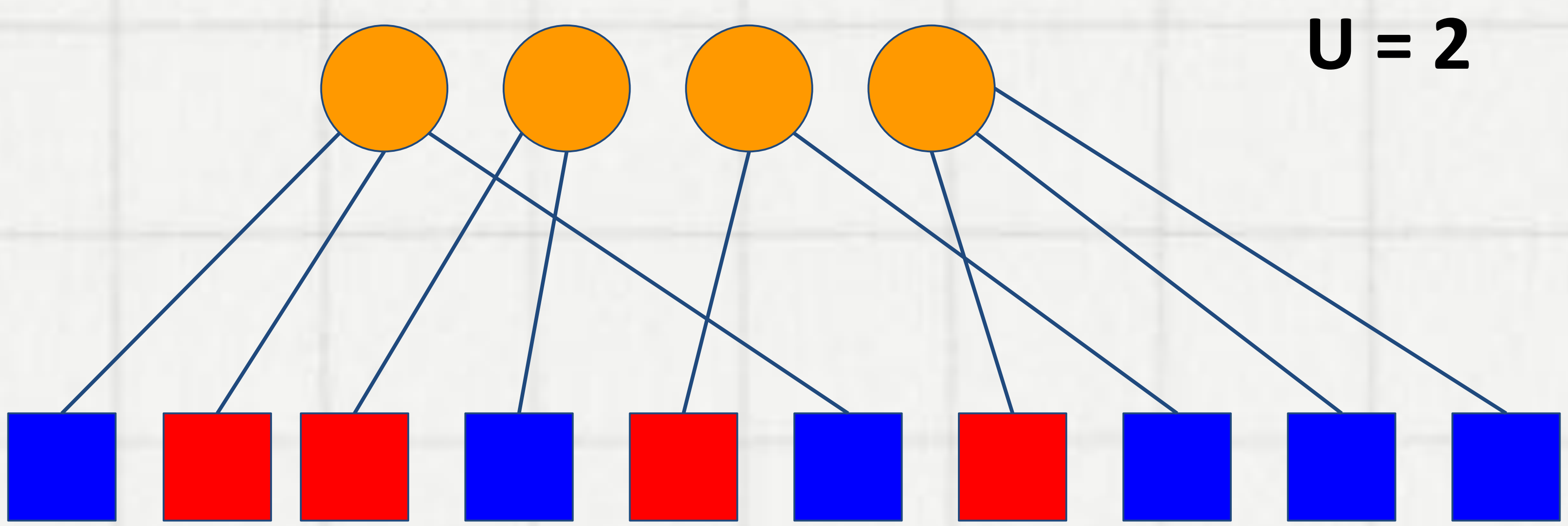
- For uniform capacities.
- Use the light/heavy facility approach.
- *Integral mapping* from clients to clients s.t mapping is one-one, no facility in S is overburdened.
- Clients who do not have a mapping are swapped, the ones who have a mapping are redirected to the facility serving their map.
- Add up all obtained inequalities.

Best Guarantee: **3-approximation**

- For uniform capacities.
- *Fractional mapping* from clients to clients .
- Use a convex combination of all obtained inequalities.

A New Variant of Capacitated Facility Location!

- Each client now demands for **one out of k services** - can be visualised as a k-coloured variant of the previous problem.
- Each facility now has a capacity w.r.t each service.



First we consider the case where capacities are uniform across clients and across services.

Easy Guarantee

We are working on finding an approximation guarantee for the new variant (Special Case: Uniform Capacities across colours and facilities) using the same local search algorithm.

<u>Observation:</u> A 3k approximation can be obtained by solving CFL w.r.t each service (colour) separately and then considering the union.

Note that this guarantee is independent of the # facility locations or # clients, just dependent on k.

<u>Question:</u>
Can we get a better approximation algorithm for the problem?

Better Guarantee?

Have tried out the natural extensions of integral mapping analysis, and the fractional mapping approach – $O(k)$ guarantees!

Also haven't been able to find an example where a local search gives an $O(k)$ approximation.

Scope

Will continue to work on the problem through the next semester:

- (possibly) establish a constant (independent of k) guarantee for the local search algorithm.
- relax the constraint that capacities are uniform across colours.
- relax the constraint that capacities are uniform across facilities.

References

1. Williamson DP, Shmoys DB. *The Design of Approximation Algorithms*. Cambridge University Press; 2011.
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4. An, Hyung-Chan & Singh, Mohit & Svensson, Ola. (2014). LP-Based Algorithms for Capacitated Facility Location. Proceedings - Annual IEEE Symposium on Foundations of Computer Science, FOCS. 10.1109/FOCS.2014.35.
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