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Developing a novel inverse data envelopment analysis (DEA) model for evaluating after-sales units

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Abstract

This paper proposes a novel model of inverse data envelopment analysis (IDEA) based on the slack-based measure (SBM) approach. The developed inverse SBM model can maintain relative efficiency of decision making units (DMUs) with new input and output. This model can also measure the input and output volumes when a decision maker (DM) increases efficiency score. The inverse SBM model is a kind of multi-objective non-linear programming (MONLP) problem, which is not easy to solve. Therefore, we suggest a linear programming model for solving inverse SBM model. In this model efficiency score of DMU under evaluation remains unchanged. Furthermore, we suggest an optimal combination of inputs and outputs in the production possibility set (PPS). A case study is presented to demonstrate the efficacy of our proposed model.

KEYWORDS

data envelopment analysis, DEA, after-sales service, DEA, inverse DEA, SBM, slacks-based measure

1 | INTRODUCTION

After-sales services can be a source of revenue, profit and competitive advantage (Farzipoor Saen & Seyedi Hosseini Nia, 2019; Gaiardelli, Saccani, & Songini, 2007). However, there are only a few papers on the performance measurement of the after-sales services (Gaiardelli et al., 2007). Every organization needs to evaluate the performance of its after-sales services (Gaiardelli et al., 2007). Several methods have been introduced to measure the performance of after-sales services. For instance, Poureisa, Bolouki Asli, Ahmadgourabi, and Efteghar (2013) discussed that balanced scorecard (BSC) could be used for assessing the performance of after-sales services. Izadbakhsh, Hour Ali, Amirkhani, and Monta (2009) presented an integrated methodology for measuring after-sales network performance by data envelopment analysis (DEA) technique. In this paper, DEA is used for assessing and optimizing after-sales services of automotive industries.

DEA is a non-parametric technique for assessing the relative efficiency of decision-making units (DMUs). DEA has been applied for analyzing the performance of various public and private organizations which was first introduced by Charnes, Cooper and Rhodes (CCR) (Charnes, Cooper, & Rhodes, 1978). Slack-based measure (SBM) model is one of the DEA models. SBM makes direct use of slacks and focuses on decreasing inputs and increasing outputs, simultaneously (Tone, 2001).

Inverse DEA is applied for solving two types of problems: (a) Solving resource allocation problems that determines the most appropriate inputs for expected outputs and keeps the efficiency of DMU under evaluation constant compared to other DMUs; (b) Solving investment analysis problems that determines the most appropriate outputs for intended inputs and keeps the efficiency of the DMU_o (DMU under evaluation) constant compared to other DMUs (Lertworasirikul, Charnsethikul, & Fang, 2011). To solve both problems, we need to develop a new inverse DEA model. The SBM model can determine the best possible inputs (outputs) for the outputs (inputs) such that the current relative efficiency of DMU will remain unchanged. For this reason, a new inverse SBM model is developed such that it maintains the efficiency score of the DMU_o and suggests an optimum combination of inputs and outputs in the current production possibility set (PPS).

For the first time, Wei, Zhang, and Zhang (2000) proposed the inverse DEA model for estimating inputs and outputs. Given the work of Yan, Wei, and Hao (2002), Jahanshahloo, Hosseinzadeh Lotfi, and Shoja (2004a) developed an inverse DEA model. Their model estimates outputs of the DMU_o when some or all inputs are increased, and the efficiency score of DMU_o needs to be improved. Besides, Jahanshahloo, Hosseinzadeh Lotfi, and Shoja (2004b) demonstrated that inverse DEA models could estimate inputs when some or all outputs, as well as relative efficiency, are increased or remain constant. Jahanshahloo, Hosseinzadeh Lotfi, and Shoja (2005) presented a revised inverse DEA model. They obtained lower and upper limits of input and output variations.

Lertworasirikul et al. (2011) developed a model based on inverse Banker-Charnes-Cooper (BCC) model, which not only maintains the efficiency score of all DMUs in new PPS but also proposed new inputs and outputs for DMU_o. Hosseinzadeh Lotfi, Ebrahimkhani Ghazi, Ebrahimkhani Ghazi, and Ahadzadeh Namin (2012) estimated DMU's outputs when some or all inputs are increased, and its current efficiency score is increased. Hadi-Vencheh, Foroughi, and Soleimani-Daman (2008) used inverse DEA model for estimating inputs and outputs of DMU_o. Their model was developed based on multiple objective functions to handle DMUs with low efficiency scores. Lin (2010) used inverse DEA and imprecise DEA models to recommend the required efficiency level and desirable revenue. Mahmoodi Rad, Dehghan, and Hosseinzadeh Lotfi (2010) showed that inverse DEA models could be used to estimate outputs with fuzzy data when some or all inputs are increased, and performance is not changed. Hassanzadeh, Yousefi, Farzipoor Saen, and Seyedi Hosseininia (2018) developed two input-oriented and output-oriented inverse semi-oriented radial measures)SORM). They determined resource allocation and investment strategies for assessing the sustainability of countries. Kalantary, Farzipoor Saen, and Toloie Eshlaghy (2018) developed inverse network range adjusted measure (RAM) model in a dynamic context. Yousefi, Farzipoor Saen, and Seyedi Hosseinininia (2019) proposed an inverse range directional measure (RDM) model to deal with positive and negative values. Kalantary and Farzipoor Saen (2019) evaluated the sustainability of supply chains by inverse network dynamic DEA model. Wegener and Amin (2019) illustrated an inverse DEA model in the oil and gas industry. Emrouznejad, Yang, and Amin (2019) proposed an inverse DEA model to allocate undesirable outputs. Table 1 summarizes inverse DEA models.

However, the inputs and outputs in previous inverse DEA papers are not changed simultaneously, and none of them can evaluate variations of both inputs and outputs. Also, in some of the previous papers, the efficiency score is changed that is unacceptable. In this paper, variations of inputs and outputs are analyzed simultaneously while keeping efficiency scores of DMUs unchanged. Furthermore, our proposed model can calculate input and output variations when the decision maker (DM) wants to increase the efficiency score of inefficient DMU. Finally, in classical inverse DEA models, the inputs might be increased. However, in the real world, DM may be unhappy with this event. Our model can produce solutions that the inputs are not increased.

This paper has the following contributions:

- For the first time, the inverse SBM DEA model is introduced. This model determines the minimum input variations and maximum output variations for DMU_o so that the efficiency scores of other DMUs are unchanged.

TABLE 1 Classification of inverse DEA models

Category	Techniques	Descriptions
Inverse general DEA	Input-oriented	Jahanshahloo et al. (2004b) proposed inverse DEA models for estimating inputs of DMU_o when outputs and efficiency are increased.
		Yan et al. (2002) and Hosseinzadeh Lotfi et al. (2012) introduced inverse DEA models for estimating inputs/outputs of DMU_o when efficiency score is unchanged.
		Jahanshahloo et al. (2005) modified inverse DEA models for running sensitivity analysis.
	Output-oriented	Wei et al. (2000) proposed the inverse DEA model for estimating inputs or outputs.
	Input and output-oriented	Jahanshahloo, Soleimani-Damaneh, and Ghobadi (2015) introduced inverse DEA models for estimating input/output under inter-temporal context.
Inverse CCR model	Input-oriented	Hadi-Vencheh et al. (2008) estimated the input/output of DMU if some or all inputs/outputs are changed.
Inverse BCC model	Input-oriented	Lertworasirikul et al. (2011) developed inverse BCC model to keep efficiency score unchanged given new inputs and outputs.
	Output-oriented	Alinezhad, Makui, and Kiani Mavi (2007) estimated inverse BCC model for inputs/outputs given the decision maker's preferences.
Inverse imprecise DEA model	CCR model	Lin (2010) used inverse CCR model to obtain efficiency score for detecting the performance of resource utilization.
Inverse directional SBM model	SBM model	Mirsalehy, Abu Bakar, Lee, Jaafar, and Heydar (2014) proposed the inverse directional SBM model for estimate input/output when the efficiency score is not reduced.

- The inverse SBM model is proposed so that relative efficiency of DMU_o remains unchanged.
- Another feature of our proposed model is that relative efficiencies of other DMUs when inputs and outputs of DMU_o are changed remain unchanged.
- The inverse SBM model is proposed for target setting by changing efficiency score in line with corporate strategies.
- Our inverse SBM model can determine corporate strategies by adding managerial constraints while efficiency score is constant.
- Our inverse SBM model can set targets via changing in level of efficiency in line with corporate strategy.
- Our model determines minimum input variations and maximum output variations of inefficient DMU_o.

The rest of this paper is organized as follows: Proposed models are given in Section 2. In Section 3, we present numerical examples. In Section 4, a case study is presented. Conclusions are given in Section 5.

2 | PROPOSED MODELS

DEA evaluates the relative efficiency of DMUs using the weighted sum of outputs to the weighted sum of inputs. Original models of DEA are CCR and BCC which are known as radial models. Non-radial DEA models include multiplicative model, range adjusted measure (RAM), additive model and SBM model (Tavassoli, Faramarzi, & Farzipoor Saen, 2014). The SBM model as a non-radial model can cope with inputs/outputs individually, but radial models suppose proportional changes in inputs/outputs (Tone & Tsutsui, 2010). As a matter of fact, SBM model deals directly with surplus of inputs and outputs shortage of the DMU which are known as slacks (Chang, 2013). This model projects the inefficient DMU to the farthest point on efficient frontier and finds maximum slacks to minimize the objective function (Chang, 2013). As illustrated by Zhou, Ang, and Poh (2006), the SBM model has better discriminatory power than other traditional DEA models. Therefore, in this paper, we develop inverse DEA of SBM model.

Here, new inverse SBM models are developed in four categories:

- General inverse SBM model
- Inverse SBM model if DM changes efficiency
- Inverse SBM model with the managerial constraint imposed by DM
- Inverse SBM model with managerial constraint and efficiency change by DM

2.1 | General inverse SBM model

SBM is an additive DEA model which not only deals with excess and shortfall slacks but also focuses on reducing inputs and increasing outputs, simultaneously. SBM was first developed by Tone (2001)). In previous DEA models like CCR and BCC, inefficient DMUs can be converted into efficient DMUs by reducing inputs or increasing outputs (Farzipoor Saen, 2005). However, SBM is a non-radial DEA model that decreases inputs and increases outputs simultaneously. The main advantage of the SBM model is to measure the weak efficiency of DMUs. SBM model is applicable in both constant and variable returns to scale.

Now, suppose there are n DMUs (DMU_1,…,DMU_n), and each DMU_j ($j = 1$,…,n) consumes m inputs (x_{1j},…,x_{mj}) to produce s outputs (y_{1j},…,y_{sj}). Also, suppose all the inputs and outputs are non-negative. Then, PPS is given as follows:

$$\text{PPS} = \left\{(x,y) : x \geq \sum_{j=1}^{n} \lambda_j x_{ij}, y \leq \sum_{j=1}^{n} \lambda_j y_{rj}, \lambda_j \geq 0, i = 1,\ldots,m, r = 1,\ldots,s, j = 1,\ldots,n\right\} \tag{1}$$

To assess the relative efficiency of DMU_o, this paper uses the following linear program of SBM model (Cooper, Seiford, & Tone, 2002):

$$\min \Gamma = t - \frac{1}{m}\sum_{i=1}^{m} s_i^- / x_{io}$$

s.t.

$$t + \frac{1}{s}\sum_{r=1}^{s} s_r^+ / y_{ro} = 1 \tag{2}$$

$$X\Lambda + S^- = tx_o$$

$$Y\Lambda - S^+ = ty_o$$

$$\Lambda, S^-, S^+ \geq 0,\ t > 0$$

where x_{io} is the ith input of DMU$_o$ and y_{ro} is the rth output of the DMU$_o$. S_i^- is slack of ith input and S_r^+ is slack of rth output of DMU$_o$. $\Lambda = \lambda t$ and λ_j (j = 1, …, n) is nonnegative vector of variables. t is a positive scalar variable (Charnes & Cooper, 1962). $\Gamma = \rho_o$ displays relative efficiency score. ρ_o is always between 0 and 1.

Definition 1 The DMU$_o$ in SBM model is efficient if and only if its objective function value is 1. This condition is equal to s^{-*}=0, s^{+*} = 0 (Cooper et al., 2002).

2.1.1 | Non-linear inverse SBM model

The results obtained from Model (2) show that by decreasing the inputs by s_i^- and increasing the outputs by s_r^+, inefficient DMUs can be converted into efficient DMUs. Consequently, for the first time, an inverse SBM model is developed to keep relative efficiency of inefficient DMU unchanged. Furthermore, we suggest a new combination of inputs and outputs. Now, suppose ρ_o^* is optimal value in Model (2) and input and output variations are represented by $\Delta x_o$and Δy_o, respectively. To determine feasible variations of inputs and outputs, we should model the objective function of Model (7) in a way that produces variations. Since the objective function of Model (2) is minimized, the objective function of Model (3) should be minimized. Therefore, the objective function of Model (7) is obtained from the following expression:

$$\min \frac{\Delta x_o}{\Delta y_o} \tag{3}$$

Moreover, to obtain variations of inputs and outputs, it is essential to formulate x_{io} and y_{ro} in Model (2) as follows:

$$\begin{pmatrix} x_{io} + \Delta x_{io} \\ y_{ro} + \Delta y_{ro} \end{pmatrix} \tag{4}$$

Accordingly, to estimate variations of both inputs and outputs of DMU$_o$, while its efficiency score remains fixed at ρ_o^*, our proposed model, which is called inverse SBM model, is presented as follows:

$$min \frac{\Delta x_o}{\Delta y_o}$$

s.t.

$$\sum_{j=1}^{n} \lambda_j x_{ij} = \rho_o^*(x_{io} + \Delta x_{io}) \qquad i = 1, .., m \tag{5}$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} = y_{ro} + \Delta y_{ro} \qquad r = 1, \ldots, s$$

$$\lambda_j, \Delta x_{io}, \Delta y_{ro} \geq 0 \qquad j = 1, \ldots, n$$

where Δx_o is the input change and Δy_o is the output change of DMU$_o$. Δ can be a positive or a negative value. ρ_o^* is an efficiency score obtained from Model (2). The model minimizes and maximizes input and output variations, respectively.

2.1.2 | Linear inverse SBM model

Note that the objective function of Model (5) is non-linear. To convert it to a linear one, the following modification is necessary.

$$\Delta y_o = 1 \tag{6}$$

Such an expression converts the objective function to a linear objective function. Thus, linear inverse SBM model is as follows:

$$\min \quad \Delta x_o$$

s.t.

$$\Delta y_0 = 1 \tag{7}$$

$$\sum_{j=1}^{n} \lambda_j x_{ij} = \rho_o^* (x_{io} + \Delta x_{io}) \qquad i = 1, \ldots, m$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} = y_{ro} + \Delta y_{ro} \qquad r = 1, \ldots, s$$

$$\lambda j, \Delta x_{io}, \Delta y_{ro} \geq 0 \qquad j = 1, \ldots, n$$

In Model (7), input variations Δx_o = (x_{1O}, x_{2O}, …, x_{mO}) and output variations Δy_o = (y_{1O}, y_{2O}, …, y_{sO}) are obtained while relative efficiency obtained from Model (2) remains unchanged.

2.2 | Inverse SBM model if DM increases efficiency

In some cases, DM wishes to obtain input and output variations for a certain amount of relative efficiency. In these situations, the efficiency score obtained from Model (7) is replaced by the new efficiency score. Then, Model (8) is used on the condition that we aim at increasing the relative efficiency of DMU_o by α. DM sets the α.

$$\min \quad \Delta x_o$$

s.t.

$$\Delta y_0 = 1 \tag{8}$$

$$\sum_{j=1}^{n} \lambda_j x_{ij} = \rho_n^* (x_{io} + \Delta x_{io}) \qquad i = 1, .., m$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} = y_{ro} + \Delta y_{ro} \qquad r = 1, \ldots, s$$

$$\lambda j, \Delta x_{io}, \Delta y_{ro} \geq 0 \qquad j = 1, \ldots, n$$

where ρ_n^* is new efficiency score, which is calculated as follows:

$$\rho_n^* = \rho_o^* + \alpha \tag{9}$$

2.3 | Inverse SBM model with the managerial constraint imposed by DM

There might be some managerial constraints such as lack of inputs' augmentation. Then, DM should be able to incorporate such constraint into the model. For instance, an organization may not be able to increase the number of staffs. In this case, constraint (10) can be added to Model (7).

In other words, after running Model (7), there might be an increase in inputs of DMU_o, which is unacceptable for DM. To prevent this issue, managerial constraint (10) is added to Model (7).

$$\Delta x_{io} \le 0, \qquad i = 1,\ldots,m \tag{10}$$

By adding constraint (10) to Model (7), the input variations might be out of the PPS. In this case, the efficiency score will be changed. To solve this problem, it is necessary to add new variable $\varPhi$ to both objective function and the first constraint. Hence, Model (11) is obtained as follows:

$$\min \quad \Delta x_o + \varPhi$$

s.t.

$$\begin{aligned}
&\Delta y_{r0} + \varPhi = 1 && r = 1,\ldots,s \\
&\Delta x_{io} \le 0 && i = 1,\ldots,m \\
&\sum_{j=1}^{n} \lambda_j x_{ij} = \rho_o^*(x_{io} + \Delta x_{io}) && i = 1,\ldots,m \\
&\sum_{j=1}^{n} \lambda_j y_{rj} = y_{ro} + \Delta y_{ro} && r = 1,\ldots,s \\
&\lambda_j, \Delta x_{io}, \Delta y_{ro}, \varPhi \ge 0, && j = 1,\ldots,n.
\end{aligned} \tag{11}$$

2.4 | Inverse SBM model with managerial constraint and efficiency change by DM

In Model (11), the relative efficiency of DMU_o can be enhanced by α. The following model can determine variations of inputs and outputs:

$$min \quad \Delta x_0 + \varPhi$$

s.t.

$$\begin{aligned}
&\Delta y_{r0} + \varPhi = 1 && r = 1,\ldots,s \\
&\Delta x_{io} \le 0 && i = 1,\ldots,m \\
&\sum_{j=1}^{n} \lambda_j x_{ij} = \rho_n^*(x_{io} + \Delta x_{io}) && i = 1,\ldots,m \\
&\sum_{j=1}^{n} \lambda_j y_{rj} = y_{ro} + \Delta y_{ro} && r = 1,\ldots,s \\
&\lambda_j, \Delta x_{io}, \Delta y_{ro}, \varPhi \ge 0, && j = 1,\ldots,n.
\end{aligned} \tag{12}$$

Model (12) can calculate changes in inputs and outputs of DMU_o such that efficiency score is increased or decreased by α.

3 | NUMERICAL EXAMPLES

Here, to validate the proposed models, three numerical examples are presented. Also, different number of DMUs (8, 12 and 9 DMUs) are presented to analyze the sensitivity of the proposed models.

Example 1 Consider 8 DMUs with two inputs and one output as reported in Table 2.

By evaluating DMU_1 by Model (2), ρ^*=0.333. Variations of inputs and output are shown in Table 3.

Example 2 Assume that there are 12 DMUs with two inputs and two outputs. Dataset is given in Table 4. Efficiency scores are obtained by Model (2).

Variations of inputs and outputs by Model (7) for DMU_1 are reported in Table 5.

By incorporating the viewpoint of DM (DM wishes to increase efficiency score to 0.65), the efficiency score of DMU_1 is obtained by the following expression:

$$\rho_n^* = 0.43 + 0.22 \rightarrow \rho_n^* = 0.65$$

Furthermore, variations in inputs and outputs of DMU_1 obtained by Model (8) are depicted in Table 6.

TABLE 2 Dataset and efficiency scores

DMUs	x_1	x_2	y_1	Efficiency scores obtained by model (2)
1	4	3	2	0.333
2	7	3	1	0.105
3	8	1	7	1
4	4	2	3	0.53
5	2	4	3	1
6	10	1	2	0.25
7	12	1	3	0.35
8	10	1.5	5	0.52

TABLE 3 Variations of inputs and output by Model (7)

Variations of inputs and output of DMU #1	
x_1	+6.29601
x_2	−1.712999
y_1	+1

TABLE 4 Dataset and efficiency scores

DMUs	x_1	x_2	y_1	y_2	Efficiency scores by model (2)
1	1	0.8	0.6	0.9	0.43
2	0.5	1	0.6	0.7	0.53
3	0.7	0.4	1	0.7	1
4	0.4	1	0.7	0.9	0.84
5	0.5	0.4	0.7	1	1
6	1	0.7	0.4	0.8	0.35
7	0.7	0.3	0.5	0.7	0.72
8	0.3	0.9	0.8	0.7	1
9	0.4	0.9	0.8	0.6	0.74
10	0.4	1	0.8	0.6	0.71
11	0.8	0.7	1	1	0.7
12	1	0.6	0.7	0.8	0.51

Example 3 Consider 9 DMUs with a single input and two outputs as reported in Table 7.

Variations of input and outputs of DMU_4 obtained by Model (11) are shown in Table 8. As is seen in Table 8, there is no change in the input of DMU_4.

If DM wishes to increase efficiency score of DMU_4 to 0.8, then we have

$$\rho_n^* = 0.61 + 0.19 \rightarrow \rho_n^* = 0.8$$

Variations in input and outputs of DMU_4 obtained by Model (12) are depicted in Table 9.

TABLE 5 Variations of inputs and outputs by Model (7)

Variations of inputs and outputs for DMU #1	
x_1	+0.7099863
x_2	+0.5679891
y_1	+0.4294118
y_2	+0.5705882

TABLE 6 Variations in inputs and outputs obtained by Model (8)

Variations in inputs and outputs of DMU_1	
x_1	+0.1312217
x_2	+0.1049774
y_1	+0.4294118
y_2	+0.5705882

TABLE 7 Dataset and efficiency scores

DMUs	x_1	y_1	y_2	Efficiency scores by model (2)
1	0.83	0.66	0.59	0.47
2	0.66	0.49	0.7	0.57
3	0.72	0.85	0.82	0.73
4	0.9	0.73	0.99	0.61
5	1	0.49	0.78	0.39
6	0.55	0.72	1	1
7	0.87	1	0.64	0.55
8	0.92	0.84	0.93	0.61
9	0.67	0.55	1	0.71

TABLE 8 Variations of input and outputs obtained by Model (11)

Variations of input and outputs of DMU_4	
x_1	0
y_1	−0.011
y_2	+0.008

TABLE 9 Variations of input and outputs obtained by Model (12)

Variations of input and outputs of DMU_4	
x_1	0
y_1	+0.2125455
y_2	+0.3190909

4 | CASE STUDY

After-sales services comprise several activities which start after purchasing of the product or service (Onar, Oztaysi, & Kahraman, 2017). After-sales services can lead to a high income for the companies and create competitive advantage (Burström, Wilson, & Wincent, 2020; Onar et al., 2017). Each company should evaluate the performance of its after-sales services (Gaiardelli et al., 2007). Inverse DEA models can be an appropriate method to evaluate efficiency of each after-sales services unit. Inverse DEA can present the optimal strategies given two types of resource allocation and investment analysis problems. Here, we present data set related to 64 after-sales service units (DMUs) of a car manufacturing company in Iran.1 Decision maker needs optimization strategies based on corporate macro policies. These macro policies include keeping efficiency of each after-sales services unit unchanged and level of some indicators are not allowed to be increased. Also, top managers wish to promote the inefficient after-sales services units and move them towards efficient frontier by increasing efficiency score and meeting managerial constraints, simultaneously. Therefore, utilized model should be based on these macro policies and provide us with new combination of resources so that efficiency does not change and managerial constraints can be satisfied. In this regard, due to the limitations in increasing number of staff and amount of debts, managerial constraint (10) is added to the model. Each DMU is evaluated given the following inputs and outputs:

Number of staff (x_1).

Amount of debts (x_2).

Number of standard equipment (x_3).

Number of auto relief services (y_1).

Average of customers' satisfaction (y_2).

Revenue (y_3).

Dataset is given in Table 10. Dataset dates back to 2014. The inputs and outputs are normalized as follows:

$$\begin{pmatrix} \tilde{X} = \left(x_{ij} / \max(x_{ij}) \right) 100; \quad i = 1,\dots,m; \quad j = 1,\dots,n \\ \tilde{Y} = \left(y_{rj} / \max(y_{rj}) \right) 100; \quad r = 1,\dots,s; \quad j = 1,\dots,n \end{pmatrix} \tag{13}$$

Also, results obtained from Model (2) are given in Table 10.

As is shown in Table 10, 22 DMUs are efficient, and other DMUs are inefficient. We wish to get a new combination of inputs and outputs provided that inefficient DMUs remain unchanged. To this end, we use Model (7). For example, the efficiency score of DMU_{60} is ρ^*=0.503644. Assuming the efficiency score of DMU_{60} is unchanged; variations of inputs and outputs are shown in Table 11.

After running Model (7), it is seen that in other combinations, number of staff and amount of debts, in most DMUs, are increased. Since in DMU_{60} it is impossible to increase the number of staff and amount of debts, managerial constraint (6) is added to Model (11). As is shown in Table 12, inputs are not increased.

At this juncture, DM wishes to increase efficiency score of DMUs. To this end, in addition to incorporating $\Delta x_{io} \leq 0$ into Model (11), DM wants to increase the efficiency of all DMUs by 0.15. Results of analysis for DMU_{60} are shown in Table 13.

5 | CONCLUSIONS

5.1 | Concluding remarks

After-sales services are one of the sources of revenue, profit and competitive advantage for companies. Therefore, every organization needs to evaluate the performance of its after-sales services (Gaiardelli et al., 2007). After-sales service is one of the main factors in improving performance, and it leads to customers' satisfaction. Using DEA models and inverse DEA is a proper method to evaluate efficiency. Inverse DEA provides optimal strategies for resource allocation and investment analysis for after-sales service units. The inverse DEA model is used to determine the most appropriate inputs (outputs) of the DMU_o while it keeps the relative efficiency score unchanged. To determine proper policies, organizations try to identify their strengths and weaknesses. Since inverse DEA models can calculate optimal changes, we proposed new inverse DEA models to determine the best possible strategies. In this paper, we proposed an inverse SBM DEA model which can present particular strategies in terms of four different scenarios. In the first scenario, assuming a constant efficiency score, one of the uses of our proposed inverse SBM model (Model (7)) is to generate a new combination of inputs and outputs. Therefore, DM can select his feasible inputs and outputs. The inverse SBM problem is a kind of multi-objective non-linear programming (MONLP) problem which is not easy to solve. In the second scenario, the inverse SBM model (Model (8)) was developed to estimate variations of inputs and outputs of DMU_o when its current efficiency score in PPS remains unchanged. Furthermore, if DM wishes to increase efficiency score and wants to determine its impact on inputs and outputs, he/she can use

TABLE 10 Dataset related to 64 after-sales service units and efficiency scores

DMUs	x_1	x_2	x_3	y_1	y_2	y_3	Efficiency scores obtained from model (2)
1	78	23	76	88	71	89	1
2	40	44	38	71	71	44	1
3	33	37	55	53	86	45	0.721439
4	56	68	69	56	100	41	0.419132
5	51	30	95	53	71	72	0.697744
6	27	47	60	27	71	34	0.436547
7	24	52	84	53	71	99	1
8	38	43	67	64	71	86	0.763012
9	47	60	89	64	86	88	0.599695
10	67	48	95	38	71	67	0.42292
11	62	34	76	68	86	56	0.71411
12	31	25	71	53	71	57	1
13	44	99	78	89	57	26	0.281664
14	42	64	85	86	100	51	0.547039
15	51	90	73	59	71	55	0.415522
16	33	40	51	53	71	47	0.638554
17	36	69	69	97	71	46	0.62819
18	27	46	67	91	57	75	1
19	47	26	55	64	71	43	1
20	31	92	33	69	71	57	1
21	62	73	80	60	57	77	0.42563
22	38	97	78	53	71	29	0.31182
23	36	41	62	83	71	46	1
24	36	58	60	81	71	21	0.39879
25	27	39	45	53	86	93	1
26	38	54	56	53	43	94	0.56057
27	42	44	49	82	86	74	1
28	33	74	71	86	71	75	0.68753
29	29	100	78	57	71	91	0.60008
30	49	54	49	88	86	77	1
31	16	71	35	53	57	50	1
32	22	57	55	57	86	52	0.795294
33	22	23	84	43	71	33	1
34	58	45	100	83	71	66	0.61367
35	36	55	69	94	57	67	0.73793
36	31	32	56	39	71	98	1
37	36	46	75	62	86	26	0.40462
38	29	41	78	31	71	100	0.66355
39	36	39	67	90	43	83	1
40	36	56	73	55	43	99	0.52853
41	73	89	75	60	71	74	0.42015
42	100	57	49	47	71	41	0.40644
43	24	64	62	83	86	89	1
44	49	28	78	64	57	36	0.59083
45	33	44	45	53	86	79	0.85123
46	36	53	53	59	71	32	0.46551
47	47	82	80	91	71	34	0.35606

TABLE 10 (Continued)

DMUs	x_1	x_2	x_3	y_1	y_2	y_3	Efficiency scores obtained from model (2)
48	31	43	58	65	86	22	0.44585
49	16	31	49	38	71	65	1
50	38	49	82	61	71	46	0.50105
51	29	57	65	64	86	29	0.47401
52	76	64	38	57	71	85	1
53	33	71	93	41	71	25	0.29746
54	31	45	78	69	71	57	0.67905
55	67	37	69	48	57	44	0.44625
56	20	68	45	71	86	40	1
57	56	74	71	62	86	97	0.58394
58	22	95	60	98	100	27	1
59	44	24	75	39	71	72	1
60	31	54	85	54	71	46	0.503644
61	20	44	67	73	57	73	1
62	33	38	56	40	57	56	0.589182
63	100	97	100	100	100	100	0.483647
64	16	34	33	27	43	21	0.500

TABLE 11 Variations of inputs and outputs obtained by Model (7)

ΔX and ΔY of DMU_{60}	
x_1	+8.744822
x_2	+3.409187
x_3	−18.75863
y_1	−14.7069
y_2	−7.24137
y_3	+22.94828

TABLE 12 Variations of inputs and outputs obtained by Model (11)

ΔX and ΔY of DMU_{60}	
x_1	0
x_2	−9.2222
x_3	−33.333
y_1	−23.3523
y_2	−21.2698
y_3	+7.777988

TABLE 13 Variations of inputs and outputs obtained by Model (12)

ΔX and ΔY of DMU_{60}	
x_1	−0.2042
x_2	−9.5172
x_3	−33.673
y_1	−14.7069
y_2	−7.24138
y_3	+22.94828

Model (8). Nevertheless, in the third scenario, DM, in some cases, may incorporate some managerial constraints while efficiency does not change. Then, Model (11) can produce an optimal combination of inputs and outputs. Finally, in the fourth scenario, if DM wishes to increase efficiency score and also to incorporate managerial constraints, Model (12) can generate an optimal combination of inputs and outputs. We illustrated the proposed approach via a case study in the after-sales service units of a car manufacturing company in Iran.

5.2 | Further discussions

We suggest that prospective researchers apply our proposed model in other fields, such as marketing and supply chain management. In marketing and sales fields, we can measure the efficiency of sales offices in target markets and determine optimal strategies to improve sales. In the supply chain management field, the suppliers can utilize the proposed models and determine proper strategies in terms of supply chains' goals. Furthermore, given our proposed model, we suggest future researchers to develop fuzzy inverse DEA model. Also, scholars may develop similar models in the presence of dual-role factors. The dual-role factors are the factors that play role of inputs and outputs, simultaneously.

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ENDNOTE

[1]For the sake of confidentiality, we have dropped the name of company and name of after-sales service units.

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