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Symmetric Error Structure in Stochastic DEA

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Abstract
Stochastic programming is an approach for modeling and solving optimization problem that include uncertain data. Chance constrained programming is one of the most important methods of stochastic programming. In many real world data envelopment analysis (DEA) models, exact amount of data can not be determined. Therefore several researchers proposed methods to evaluate stochastic efficiency of units with random inputs and/or outputs. Most of these methods are nonlinear. In this paper by introducing symmetric error structure for random variables, a linear from of stochastic CCR is provided. Finally, the proposed model is applied on an example.
Keywords : Data envelopment analysis; Stochastic programming; Symmetric error structure

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1 Introduction

Data Envelopment Analysis (DEA) is a technique based on mathematical programming to assess the efficiency of a set of Decision Making Units (DMUs). Charnes et al. [2] were pioneers in DEA by introducing CCR model. In various fields, many DEA models have been presented to evaluate DMUs with different kinds of data such as deterministic, fuzzy and interval. However, in many practical problems managers deal with units with imprecise data. Thus they need methods to assess their units. In these situations analysts may consider imprecise data as random variables. While working by random variables with considering the possibility for occurrence of unforeseen events, different aspects of the information can be detected. The main advantage of working with random data in DEA is the prediction of efficiencies in future. Given the need to use random data in practical models, Several researchers initiated stochastic DEA models (see [3,4,5,10,11,15]). Subsequently, Li [12], Sengupta [14], Huang and Li [6], Olesen [13], Khodabakhshi [8,9], Behzadi et al.[1] and Jahanshahloo et al. [7] present more stochastic DEA models. Almost

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all of these models are nonlinear. In this paper using symmetric error structure for inputs and outputs, deterministic equivalent of CCR model is presented which is a linear model.

The paper organized as follows: First the preliminaries on stochastic models and stochastic efficiency is provided in section 2 and then by introducing symmetric error structure the deterministic equivalent of CCR model is obtained in section 3. In section 4 Using numerical example, we will demonstrate how to use the result. Section 5 conclude the paper.

2 Preliminaries

Assume there are n homogeneous DMUs ($DMU_j,\ j = 1, ..., n$) such that all the DMUs use m inputs x_{ij} $(i = 1, ..., m)$ to produce s outputs y_{rj} $(r = 1, ..., s)$. Also assume that $x_j = (x_{1j}, ..., x_{mj})$ and $y_j = (y_{1j}, ..., y_{sj})$ are nonnegative and nonzero vectors. The set of production possibility set (PPS) is defined as $T = \{(X, Y)|Y \geq 0\, can\, be\, produced\, by\, X \geq 0\}$. Here we assume that $T = T_{CCR}$ in which:

$$T_{CCR} = \{(X, Y)| \sum_{j=1}^{n} \lambda_j X_j \leq X, \sum_{j=1}^{n} \lambda_j Y_j \geq Y, \lambda_j \geq 0\}.$$

The input oriented CCR model [2] to estimate the efficiency of DMU_o, $o \in \{1, ..., n\}$ is

$$\begin{array}{lll} \min & \theta & \\ s.t. & \sum_{j=1}^{n} x_{ij}\lambda_j \leq \theta x_{io}, & i = 1, ..., m, \\ & \sum_{j=1}^{n} y_{rj}\lambda_j \geq y_{ro}, & r = 1, ..., s, \\ & \lambda_j \geq 0, & j = 1, ..., n. \end{array} \tag{2.1}$$

In model (2.1) it is assumed that inputs and outputs are deterministic values. Now, let us assume that these data are random variables i.e. $\tilde{X}_j = (\tilde{x}_{1j}, ..., \tilde{x}_{mj})$ and $\tilde{Y}_j = (\tilde{y}_{1j}, ..., \tilde{y}_{sj})$ are random input and output vectors of DMU_j, $j = 1 \ldots, n$ and $X_j = (x_{1j}, ..., x_{mj}) \in R^{m+}$ and $Y_j = (y_{1j}, ..., y_{sj}) \in R^{s+}$ stand for corresponding vectors of expected values of input and output for it. All input and output components have been considered to be normally distributed i.e.

$$\tilde{x}_{ij} \sim N(x_{ij}, \sigma_{ij}^2), \qquad i = 1, ..., m,$$
$$\tilde{y}_{rj} \sim N(y_{rj}, \sigma_{rj}^{'2}), \qquad r = 1, ..., s.$$

Therefore the chance constrained model related to input oriented stochastic CCR model for evaluating DMU_o, $o \in \{1, ..., n\}$ is follows:

$$\begin{array}{lll} \min & \theta & \\ s.t. & p\{\sum_{j=1}^{n} \tilde{x}_{ij}\lambda_j \leq \theta \tilde{x}_{io}\} \geq 1 - \alpha, & i = 1, ..., m, \\ & p\{\sum_{j=1}^{n} \tilde{y}_{rj}\lambda_j \geq \tilde{y}_{ro}\} \geq 1 - \alpha, & r = 1, ..., s, \\ & \lambda_j \geq 0, & j = 1, ..., n. \end{array} \tag{2.2}$$

where in the above model, p means "probability" and α is a level of error between 0 and 1, which is a predetermined number. The deterministic equivalent of model (2.2) which is obtained by Cooper et al.[4] is follows:

$$
\begin{array}{lll}
\min & \theta & \\
s.t. & \sum_{j=1}^{n} x_{ij}\lambda_j + s_i^- - \Phi^{-1}(\alpha)v_i = \theta x_{io}, & i = 1, ..., m, \\
& \sum_{j=1}^{n} y_{rj}\lambda_j - s_r^+ + \Phi^{-1}(\alpha)u_r = y_{ro}, & r = 1, ..., s, \\
& v_i^2 = \sum_{j\neq o}\sum_{k\neq o} \lambda_j\lambda_k cov(\tilde{x}_{ij}, \tilde{x}_{ik}) + 2(\lambda_o - \theta)\sum_{j\neq o} \lambda_j cov(\tilde{x}_{ij}, \tilde{x}_{io}) + & \\
& (\lambda_o - \theta)^2 var(\tilde{x}_{io}), & i = 1, ..., m, \\
& u_r^2 = \sum_{j\neq o}\sum_{k\neq o} \lambda_j\lambda_k cov(\tilde{y}_{rj}, \tilde{y}_{rk}) + 2(\lambda_o - 1)\sum_{j\neq o} \lambda_j cov(\tilde{y}_{rj}, \tilde{y}_{ro}) + & \\
& (\lambda_o - 1)^2 var(\tilde{y}_{ro}), & r = 1, ..., s, \\
& s_i^- \geq 0, \quad s_r^+ \geq 0, & i = 1, ..., m, \quad r = 1, ..., s, \\
& \lambda_j \geq 0, \ u_r \geq 0, \ v_i \geq 0 & j = 1, ..., n, \ r = 1, ..., s, \ i = 1, ..., m.
\end{array}
\tag{2.3}
$$

Here, Φ is the cumulative distribution function of the standard normal distribution and $\Phi^{-1}(\alpha)$, is its inverse in level of α. model (2.3) is an nonlinear and quadratic programming model. Also, DMU_o is defined a stochastic efficient DMU in level of α if and only if $\theta^* = 1$ in the optimal solution of model (2.3).

3 Stochastic Efficiency Based on Symmetric Error Structure

In this section symmetric error structure for random inputs and outputs is introduced. Then using this structure the stochastic CCR model convert to a deterministic linear model.

Assume related inputs and outputs of DMU_j, $j = 1, ..., n$ are as following structure:

$$
\begin{array}{l}
\tilde{x}_{ij} = x_{ij} + a_{ij}\tilde{\varepsilon}_{ij}, \ \ i = 1, ..., m, \\
\tilde{y}_{rj} = y_{rj} + b_{rj}\tilde{\xi}_{rj}, \ \ r = 1, ..., s.
\end{array}
\tag{3.1}
$$

where a_{ij} and b_{rj} are nonnegative real values. Also, $\tilde{\varepsilon}_{ij}$ and $\tilde{\xi}_{rj}$ are random variables with normal distributions, $\tilde{\varepsilon}_{ij} \sim N(0, \bar{\sigma}^2)$ and $\tilde{\xi}_{rj} \sim N(0, \bar{\sigma}^2)$. Therefore $\tilde{\varepsilon}_{ij}$ and $\tilde{\xi}_{rj}$ are errors of inputs and outputs in contrast to the mean values respectively. Since normal distribution is symmetric then the structure in expression (3.1) is named symmetric error structure. Also, the following relations are resulted from expression (3.1):

$$
\begin{array}{l}
\tilde{x}_{ij} \sim N(x_{ij}, \bar{\sigma}^2 a_{ij}^2), \\
\tilde{y}_{rj} \sim N(y_{rj}, \bar{\sigma}^2 b_{rj}^2).
\end{array}
$$

It must be noted that every random variable with normal distribution can be stated as symmetric error structure. Assume that ith input of every DMUs are uncorrelated.

Similarly assume rth output of every DMUs are uncorrelated, too. i.e. for every $j \neq k$,

$$\begin{array}{l} Cov(\tilde{\varepsilon}_{ij}, \tilde{\varepsilon}_{ik}) = 0, \quad i = 1, ..., m, \\ Cov(\tilde{\xi}_{rj}, \tilde{\xi}_{rk}) = 0, \quad r = 1, ..., s. \end{array} \tag{3.2}$$

Therefore according to relations (3.1) and (3.2), it can be considered a same error for all DMUs, i.e. $\tilde{\varepsilon}_i = \tilde{\varepsilon}_{ij}$ and $\tilde{\xi}_r = \tilde{\xi}_{rj}$, for every $j = 1, ..., n$, $i = 1, ..., m$ and $r = 1, ..., s$.

Now, consider ith input constraint of model (2.2),

$$p\{\sum_{j=1}^{n} \tilde{x}_{ij}\lambda_j \leq \theta \tilde{x}_{io}\} \geq 1 - \alpha \tag{3.3}$$

Let $\tilde{h}_i = \sum_{j=1}^{n} \lambda_j \tilde{x}_{ij} - \theta \tilde{x}_{io}$. So (3.1) and (3.2) result:

$$\tilde{h}_i = (\sum_{j=1}^{n} \lambda_j x_{ij} - \theta x_{io}) + \tilde{\varepsilon}_i (\sum_{j=1}^{n} \lambda_j a_{ij} - \theta a_{io})$$

Therefore,

$$\tilde{h}_i \sim N\left((\sum_{j=1}^{n} \lambda_j x_{ij} - \theta x_{io}), \bar{\sigma}^2 (\sum_{j=1}^{n} \lambda_j a_{ij} - \theta a_{io})^2 \right)$$

From the above expression and normal distribution properties, the stochastic constraint (3.3) can be converted to the following deterministic equivalent:

$$\sum_{j=1}^{n} \lambda_j x_{ij} - \Phi^{-1}(\alpha)\bar{\sigma} \left| \sum_{j=1}^{n} \lambda_j a_{ij} - \theta a_{io} \right| \leq \theta x_{io}. \tag{3.4}$$

Similarly, rth output constraint of model (2.2) can be converted to

$$\sum_{j=1}^{n} \lambda_j y_{rj} + \Phi^{-1}(\alpha)\bar{\sigma} \left| \sum_{j=1}^{n} \lambda_j b_{rj} - b_{ro} \right| \geq y_{ro} \tag{3.5}$$

Therefore from (3.4) and (3.5), deterministic equivalent of model (2.2) is

$$\begin{array}{ll} \min & \theta \\ s.t. & \sum_{j=1}^{n} \lambda_j x_{ij} - \Phi^{-1}(\alpha)\bar{\sigma} \left| \sum_{j=1}^{n} \lambda_j a_{ij} - \theta a_{io} \right| \leq \theta x_{io}, i = 1, ..., m, \\ & \sum_{j=1}^{n} \lambda_j y_{rj} + \Phi^{-1}(\alpha)\bar{\sigma} \left| \sum_{j=1}^{n} \lambda_j b_{rj} - b_{ro} \right| \geq y_{ro}, r = 1, ..., s, \\ & \lambda_j \geq 0, \quad j = 1, ..., n. \end{array} \tag{3.6}$$

Model (3.6) is a nonlinear programming but it can be converted to linear using the following transformations:

$$\begin{array}{l} \left| \sum_{j=1}^{n} \lambda_j a_{ij} - \theta a_{io} \right| = (p_i^+ + p_i^-), i = 1, ..., m, \\ \sum_{j=1}^{n} \lambda_j a_{ij} - \theta a_{io} = (p_i^+ - p_i^-), i = 1, ..., m, \\ p_i^+ p_i^- = 0, i = 1, ..., m, \\ \left| \sum_{j=1}^{n} \lambda_j b_{rj} - b_{ro} \right| = (q_r^+ + q_r^-), r = 1, ..., s, \\ \sum_{j=1}^{n} \lambda_j b_{rj} - b_{ro} = (q_r^+ - q_r^-), r = 1, ..., s, \\ q_r^+ q_r^- = 0, r = 1, ..., s, \\ p_i^+ \geq 0, p_i^- \geq 0, q_r^+ \geq 0, q_r^- \geq 0. \end{array}$$

By substituting the above relation in model (3.6), we have:

$$\begin{array}{ll} \min & \theta \\ s.t. & \sum_{j=1}^{n} \lambda_j x_{ij} - \Phi^{-1}(\alpha)\bar{\sigma}(p_i^+ + p_i^-) \leq \theta x_{io}, i = 1, ..., m, \\ & \sum_{j=1}^{n} \lambda_j y_{rj} + \Phi^{-1}(\alpha)\bar{\sigma}(q_r^+ + q_r^-) \geq y_{ro}, r = 1, ..., s, \\ & \sum_{j=1}^{n} \lambda_j a_{ij} - \theta a_{io} = p_i^+ - p_i^-, i = 1, ..., m, \\ & \sum_{j=1}^{n} \lambda_j b_{rj} - b_{ro} = q_r^+ - q_r^-, r = 1, ..., s, \\ & p_i^+ p_i^- = 0,\ i = 1, ..., m, \quad q_r^+ q_r^- = 0,\ r = 1, ..., s, \\ & \lambda_j, p_i^+, p_i^-, q_r^+, q_r^- \geq 0, j = 1, ..., n, i = 1, ..., m, r = 1, ..., s. \end{array} \tag{3.7}$$

Model (3.7) is nonlinear because of the existence of constraints $p_i^+ p_i^- = 0$ and $q_r^+ q_r^- = 0$. It must be noted that for every linear problem which has an optimal solution, there is at least a basic optimal solution. Model (3.7) without constraints $p_i^+ p_i^- = 0$ and $q_r^+ q_r^- = 0$ is a linear model while in every its optimal basic solutions at least one of p_i^+ and p_i^- for every $i = 1, ..., m$ and at least one of q_r^+ and q_r^- for every $r = 1, ..., s$ is zero. Therefore constraints $p_i^+ p_i^- = 0$ and $q_r^+ q_r^- = 0$ will be satisfied in optimal basic solutions of linear model. Thus, assuming the use of optimal basic solution detector algorithms (for example Simplex), constraints $p_i^+ p_i^- = 0$ and $q_r^+ q_r^- = 0$ can be removed. So, the linear deterministic equivalent of model (2.2) is

$$
\begin{aligned}
\theta^*(\alpha) = \min \quad & \theta \\
s.t. \qquad & \sum_{j=1}^{n} \lambda_j x_{ij} - \Phi^{-1}(\alpha)\bar{\sigma}(p_i^+ + p_i^-) \leq \theta x_{io}, i = 1, ..., m, \\
& \sum_{j=1}^{n} \lambda_j y_{rj} + \Phi^{-1}(\alpha)\bar{\sigma}(q_r^+ + q_r^-) \geq y_{ro}, r = 1, ..., s, \\
& \sum_{j=1}^{n} \lambda_j a_{ij} - \theta a_{io} = p_i^+ - p_i^-, i = 1, ..., m, \\
& \sum_{j=1}^{n} \lambda_j b_{rj} - b_{ro} = q_r^+ - q_r^-, r = 1, ..., s, \\
& \lambda_j, p_i^+, p_i^-, q_r^+, q_r^- \geq 0, j = 1, ..., n, i = 1, ..., m, r = 1, ..., s.
\end{aligned}
\tag{3.8}
$$

Theorem 3.1. $0 < \theta^*(\alpha) \leq 1$ *for every* $\alpha < 0.5$ *in model (3.8).*

Proof. Let $\theta = 1$, $\lambda_o = 1$, $\lambda_j = 0$; $j \neq o$, $p_i^+ = 0$, $p_i^- = 0$; $i = 1, ..., m$, $q_r^+ = 0$, $q_r^- = 0$; $r = 1, ..., s$. This solution is feasible for model (3.8) in every α levels of error. Since this model is minimizing then $\theta^*(\alpha) \leq 1$. If $\alpha < 0.5$, then $\Phi^{-1}(\alpha) < 0$. Therefore inputs and output constraint of model (3.8) results

$$\sum_{j=1}^{n} \lambda_j^* x_{ij} \leq \theta^* x_{io}, i = 1, ..., m,$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro}, r = 1, ..., s.$$

Now suppose $\theta^*(\alpha) \leq 0$ (by contradiction hypothesis). Since $x_{ij} \geq 0$, $x_j \neq 0$ and $\lambda_j^* \geq 0$ by definition then the first and the last of above inequalities results $\lambda_j^* = 0$; $j = 1, ..., n$ and $y_{rj} \leq 0$ respectively. which is a contradiction by definitions $y_{rj} \geq 0$, $r = 1, ..., s$ and $y_j \neq 0$. Thus $\theta^*(\alpha) \geq 0$. □

Theorem 3.2. $\theta^*(\alpha') \leq \theta^*(\alpha)$ *for every* $\alpha \leq \alpha'$ *in model (3.8).*

Proof. Let $(\theta^*, \lambda^*, p^{+*}, p^{-*}, q^{+*}, q^{-*})$ be an optimal solution of model (3.8) in α level of error. Since $\Phi^{-1}(\alpha)$ is nondecreasing function, so $\Phi^{-1}(\alpha) \leq \Phi^{-1}(\alpha')$ and

$$0 \geq \sum_{j=1}^{n} \lambda_j^* x_{ij} - \Phi^{-1}(\alpha)\bar{\sigma}(p_i^{+*} + p_i^{-*}) - \theta^* x_{io} \geq \sum_{j=1}^{n} \lambda_j^* x_{ij} - \Phi^{-1}(\alpha')\bar{\sigma}(p_i^{+*} + p_i^{-*}) - \theta^* x_{io},$$

$$0 \leq \sum_{j=1}^{n} \lambda_j^* y_{rj} + \Phi^{-1}(\alpha)\bar{\sigma}(q_r^{+*} + q_r^{-*}) - y_{ro} \leq \sum_{j=1}^{n} \lambda_j^* y_{rj} + \Phi^{-1}(\alpha')\bar{\sigma}(q_r^{+*} + q_r^{-*}) - y_{ro}.$$

The above relations show that $(\theta^*, \lambda^*, p^{+*}, p^{-*}, q^{+*}, q^{-*})$ is a feasible solution of model (3.8) in α' level of error. Since model (3.8) is minimizing, the assertion will be proved. □

Corollary 3.1. *Theorem 3.1 shows if DMU_o is efficient in α' level of error, then it would be efficient for every $\alpha < \alpha'$. Also, if DMU_o is inefficient in α' level of error, then it would be inefficient for every $\alpha' < \alpha$. It is noteworthy to say that for 0.5 level of error model (3.8) is CCR model with mean of data. Thus if a DMU is efficient in CCR mean model, then it would be efficient for every $\alpha < 0.5$. It means permanent efficient DMUs can be detected by assessing them in $\alpha = 0.5$. Also, if a DMU is inefficient in $\alpha = 0$, then it remain inefficient in all levels of error. Since $\Phi^{-1}(0) = -\infty$ but $\Phi^{-1}(0,001) \simeq -3$, so by assessing DMUs in $\alpha = 0,001$ almost all permanent inefficient DMUs can be detected.*

4 An application

In this section, we consider 20 branches of an Iranian bank with three stochastic inputs and five stochastic outputs which are mentioned in Table 4.1.

Table 4.1. inputs and outputs

input 1	personal rate (weighted combination of personal qualifications, quantity, education and others)
input 2	payable benefits (of all deposits)
input 3	delayed requisitions (delay in returning ceded loans and other facilities)
output 1	facilities (sum of business and individual loans)
output 2	amount of deposits (of current, short duration and long duration accounts)
output 3	received benefits (of all ceded loans and facilities)
output 4	received commission (on banking operations, issuance guaranty, transferring money and others)
output 5	other resources of deposits

These data based on consideration ten successive months have normal distribution and their scaled parameters are presented in Tables 4.2 and 4.3. We want to assess the total performance of these units. We suppose that $\bar{\sigma} = 1$ in symmetric error structure, thus $a_{ij} = \sqrt{Var(\tilde{x}_{ij})}$ and $b_{rj} = \sqrt{Var(\tilde{y}_{rj})}$. Here by running model (3.8) stochastic efficiencies of all branches are evaluated and results are gathered in Table 4.4.

Table 4.2 Estimated parameters of inputs normal distributions.

	Input1		Input2		Input3	
DMU	Mean	Variance	Mean	Variance	Mean	Variance
DMU1	9131	0.05	18.79	8.81	7228	0.58
DMU2	10.59	0.53	44.32	24.1	1121	0.02
DMU3	6712	0.86	19.73	27.7	19.21	0.47
DMU4	11.91	0.31	17.43	12.2	59.47	0.85
DMU5	7012	0.02	10.38	2.12	12.23	59.9
DMU6	18.99	0.88	16.67	10.8	568.6	28.1
DMU7	11.16	0.01	25.46	18.6	552.8	43.2
DMU8	15.05	0.48	123.1	42.6	14.78	0.06
DMU9	8787	0.38	36.16	38.4	361.8	23.2
DMU10	19.88	0.25	46.41	53.1	12.81	0.38
DMU11	18.92	0.17	36.88	54.5	24.43	0.01
DMU12	20.45	0.42	100.8	31.8	115.2	19.4
DMU13	12.41	0.12	20.19	10.6	78.02	24.1
DMU14	8051	0.79	33.21	24.3	115.3	15.6
DMU15	18.48	0.92	45.36	92.6	57.52	12.8
DMU16	10.35	0.27	11.16	3.32	43.32	36.1
DMU17	9511	0.01	31.49	38.5	173.3	3.13
DMU18	13.71	0.18	40.32	51.4	10.88	0.12
DMU19	11.69	0.26	26.44	26.2	31.22	0.05
DMU20	7823	0.58	17.74	10.1	13.06	8.88

Table 4.3 Estimated parameters of outputs normal distributions.

	output1		output2		output3		output4		output5	
DMU	Mean	Variance	Mean	Variance	Mean	Variance	Mean	Variance	Mean	Variance
DMU1	149.85	48.51	49621	48.01	4701	0.41	4748	0.41	30.09	24.34
DMU2	50772	8011	73132	13.11	1815	0.02	3035	0.68	5823	0.689
DMU3	259.91	295.9	108.04	225.6	6016	0.01	10.06	2.98	2721	4392
DMU4	137.51	21.65	44972	13.78	4923	1.65	4212	3.84	63.61	33.73
DMU5	95901	2521	31633	38.94	2718	0.44	9024	7.23	64.55	31.25
DMU6	112.58	3562	71958	70.05	13.19	9.15	41.89	13.2	291.3	92.52
DMU7	192.97	145.6	78015	195.6	7791	3.56	15.89	2.34	7499	12.68
DMU8	724.38	660.2	219.69	375.6	35.32	15.3	23.98	10.4	361.6	48.35
DMU9	548.15	418.6	86225	48.31	17.64	3.67	86.23	17.2	565.2	175.1
DMU10	1229.1	69.02	194.58	17.35	25.91	12.3	86.76	22.1	600.6	86.34
DMU11	11557	718.1	155.32	49.13	166.6	14.1	8142	31.1	119.9	14.89
DMU12	1132.1	353.5	248.16	238.9	46.88	26.3	31.85	45.1	96.21	44.47
DMU13	438.39	174.1	104.41	257.1	10.68	10.4	30.22	37.7	331.9	171.6
DMU14	260.82	15.32	87369	234.4	8415	3.52	6101	2.86	36.93	5777
DMU15	11190	1214	166.44	106.9	65.12	16.1	132.7	13.9	919.1	133.1
DMU16	709.85	42.19	159.48	327.8	36.89	6.28	12.15	25.1	79.66	105.2
DMU17	308.11	43.72	107.03	99.39	11.87	1.76	13.63	15.7	342.3	134.9
DMU18	259.21	342.1	81779	51.65	5212	19.4	8021	3.52	107.9	130.7
DMU19	381.33	573.8	72993	69.26	5165	1.79	50.32	34.7	577.2	113.4
DMU20	399.25	10.41	40985	25.93	11.51	0.99	6432	4.25	82.13	98.91

Table 4.4 Computational results of model (3.8).

DMU	$\alpha = 0.999$	$\alpha = 0.5$	$\alpha = 0.1$	$\alpha = 0.05$	$\alpha = 0.001$
DMU1	0.03	0.59	0.67	0.7	0.77
DMU2	0	1	1	1	1
DMU3	0.02	1	1	1	1
DMU4	0.1	0.28	0.3	0.3	0.34
DMU5	-1.81	0.49	0.98	1	1
DMU6	0.03	0.92	0.97	0.99	1
DMU7	0.02	0.48	0.56	0.58	0.66
DMU8	0.12	1	1	1	1
DMU9	0.08	1	1	1	1
DMU10	0.51	1	1	1	1
DMU11	1	1	1	1	1
DMU12	0.1	0.79	0.89	0.91	0.99
DMU13	1	0.91	0.95	0.96	0.99
DMU14	0.01	0.69	0.73	0.74	0.77
DMU15	1	1	1	1	1
DMU16	0.06	1	1	1	1
DMU17	0.08	0.93	0.99	1	1
DMU18	0.08	0.56	0.61	0.62	0.67
DMU19	0.16	1	1	1	1
DMU20	0.04	0.46	0.54	0.57	0.73

Results in Table 4.4 show the efficiency may be negative while the level of error is less than half (note efficiency of DMU_5 in 0.999 level of error). Also, efficiency of each DMU increases during decreasing level of error. DMUs 2, 3, 8, 9, 10, 11, 15, 16 and 19, which are efficient in 0.5 level of error, are permanent efficient DMUs. DMUs 1, 4, 7, 12, 13, 14, 18 and 20, which are inefficient in 0.001 level of error, are permanent inefficient DMUs.

5 Conclusion

In this paper considering symmetric error structure, equivalent nonlinear deterministic model of stochastic CCR model is converted to linear model. It is showed that stochastic efficiency may negative while level of error is greater than fifty percent. The efficiency is also nonincreasing function of level of error, so a DMU will be permanent inefficient DMU, if it is not efficient in the lowest level of error. However, efficient units at a level of error may be inefficient in other levels of error. These results emphasis that it should pay more attention to the level of error in order to evaluate stochastic efficiency. Applying symmetric error structure in other DEA models is an interesting area for future researches.

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