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Computer Vision

Assignment -1

① Geometric image formation:-

(a) let $f=10$ be the focal length of a camera, let $P=(3,2,1)$ be a world point. Find the co-ordinate of the point p when projecting it onto the image. Assume that the projection is done in camera co-ordinate so there is no need for a transformation between co-ordinate system.

Solution:

given data, $f=10$

Point $P=[3,2,1]$, $x=3, y=2, z=1$

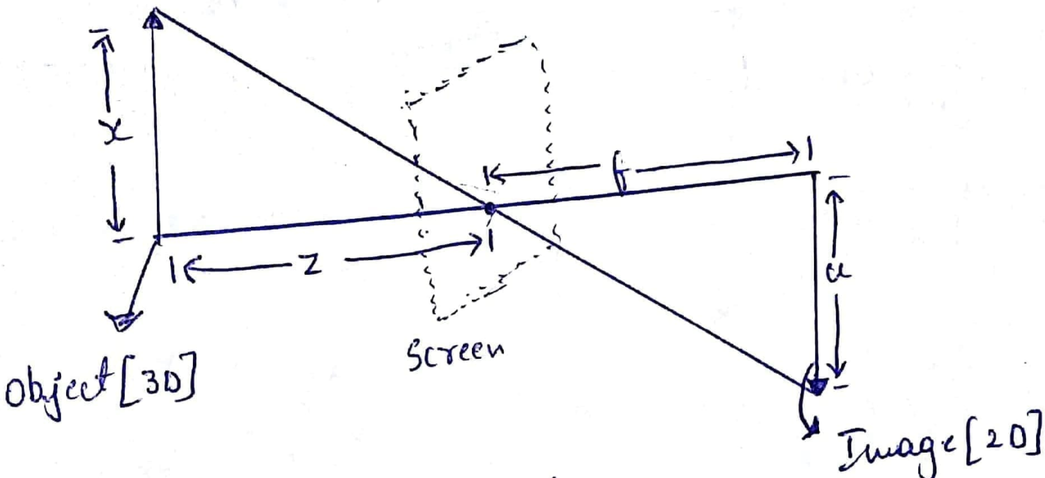
$$\mu = f \cdot \frac{x}{z} \Rightarrow \frac{-10(3)}{1} = -30$$

$$V = -f \cdot \frac{y}{z} = \frac{-10(2)}{1} = -20$$

(b) Explain the difference between the pinhole camera model where the image plane is behind the COP and the pinhole camera model where the image plane is in front of the centre of projection. which model corresponds better to a physical pinhole camera model? How is the other model Justified?

Solution:-

Pinhole camera model:-



From the above figure, the projection eqn

$$\frac{-u}{f} = \frac{x}{Z} \Rightarrow -u = f \cdot \frac{x}{Z}$$

$$\Rightarrow -f \cdot \frac{x}{Z}$$

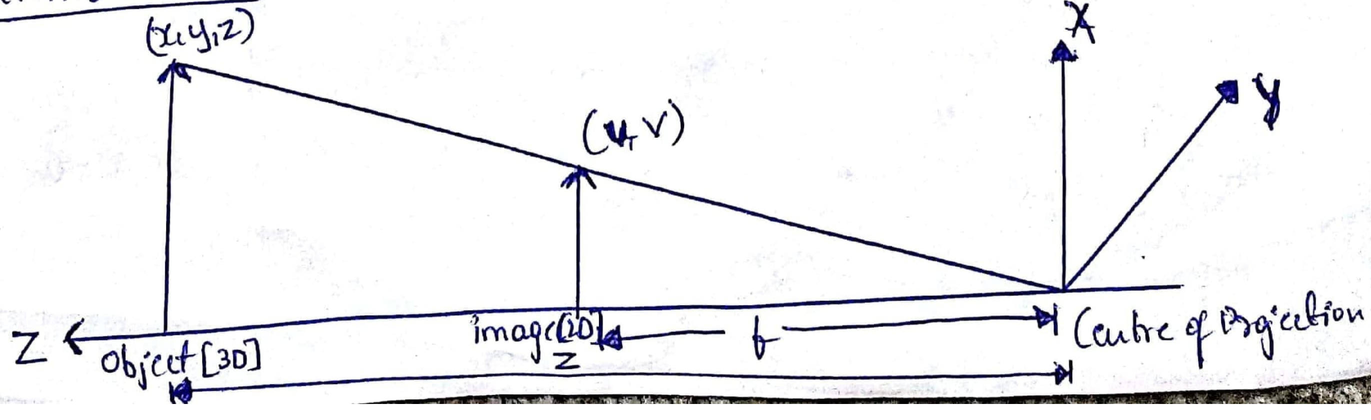
Illy, Projection Equation, $\frac{-v}{f} = \frac{y}{Z} \Rightarrow -v = f \cdot y/Z$

$$v = -f \cdot \frac{y}{Z}$$

$$\begin{bmatrix} u \\ v \end{bmatrix} = \frac{1}{Z} \begin{bmatrix} -f & 0 \\ 0 & -f \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \quad \text{—(1)}$$

[Non linear projection Equation]

(2) Alternate Model:-



From the figure in alternate model, Projection Equation will be

$$\frac{X}{Z} = \frac{u}{f} \Rightarrow u = f \cdot \frac{X}{Z}$$

Similarly,

$$\frac{Y}{Z} = \frac{v}{f} = v = f \cdot \frac{Y}{Z}$$

∴ from above Equations, we can form a non-linear Equation.

$$\begin{bmatrix} u \\ v \end{bmatrix} = \frac{1}{Z} \begin{bmatrix} f & 0 \\ 0 & f \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} \quad \text{——— (3)}$$

From the Equations (1) and (3), we can conclude that the alternative model is better because the projection equation of pinhole camera gives a negative value.

(C) Explain what happens to the projection of an object when the focal length gets bigger and what happens to the projection when the distance to the object gets bigger.

Refering to the pinhole camera model, projection equation is

$$u = -f \cdot \frac{X}{Z}, \quad f: \text{focal length}$$

Z: distance of the object.

(i) As the focal length and Projection of the object are ~~inversely~~ directly Proportional to each other, the projection of the object increases when the focal length gets bigger.
i.e [$f \propto u$]

(ii) As the projection of object and distance are inversely proportional to each other, the projection of the object gets smaller when distance gets bigger.
i.e $\left[\propto 1/z \right]$

ⓓ Given the 2D point (1,1). find the co-ordinates in homogeneous co-ordinates (2DH). Find another 2DH point that corresponds to the same 2D point.

The given 2D point (1,1) to convert to 2DH, we're adding 1 ~~poi~~ at the end, i.e (1,1,1) is 1 2DH point
$[x, y]_{2D} \longrightarrow [x, y, 1]_{2DH}$

To get another 2DH point multiply 2DH point (1,1,1) by 2
=> (2,2,2) is another point.

(e) Give the 2DH point (1,1,2) find the 2D point corresponding to it.

given 2DH point is (1,1,2)
Now to get 2D point, we ~~will~~ have to make the w-co-ordinate as 1, so dividing the co-ordinates by 2.

$(1,1,2)/2 \Rightarrow (1/2, 1/2, 1/2) \Rightarrow (1/2, 1/2, 1)$

So 2D point is $\underline{\underline{(1/2, 1/2)}}$

(f) Explain the meaning of the 2DH point (1,1,0)

Solution:

The meaning of 2DH point (1, 1, 0) is that the value of w-co-ordindate is zero. This point is called the point at infinity direction, which represents the vector's direction.

(g) Explain what makes it possible to write the non-linear projection equation as a linear equation in homogenous coordinates.

Solution:

Homogenity which ~~self~~ refers to the intersecting with plane w=1 in 2DH point, makes it possible to write the non-linear projection equation as a linear equation in homogeneous co-ordinates.

(h) Given the projection matrix, $M = K[I][0]$, write the dimensions of M and the submatrix K, I, O.

Solution:

$$M = K[I][0] = K$$

Here, $K = 3 \times 3 \Rightarrow \begin{bmatrix} + & & \\ & + & 1 \end{bmatrix}$

M & K are 3 x 3 dimension matrix.

$$I = \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix}$$ is a identity matrix of 3x3.

$$O = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$ is a 3 x 1 Matrix.

(i) Given a projection matrix M whose rows are [1, 2, 3, 4], [5, 6, 7, 8], [1, 2, 1, 2] and a 3D point P = [1, 2, 3]. find the co-ordinate of the 2D point P obtained by Projecting P using M.

Solution:

$$M = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 1 & 2 & 1 & 2 \end{bmatrix} \qquad P = [1, 2, 3]$$

2D projection of matrix = M.P.

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 1 & 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times1 + 2\times2 + 3\times3 + 4\times1 \\ 5\times1 + 6\times2 + 7\times3 + 8\times1 \\ 1\times1 + 2\times2 + 1\times3 + 2\times1 \end{bmatrix} = \begin{bmatrix} \cancel{1+4+9+4} \\ \cancel{5} \\ \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+9+4 \\ 5+12+21+8 \\ 1+4+3+2 \end{bmatrix} = \begin{bmatrix} 18 \\ 46 \\ 10 \end{bmatrix}$$

∴ Projection in 2D

$$\left(\frac{18}{10}, \frac{46}{10}, \frac{10}{10}\right) = \left(\frac{9}{5}, \frac{23}{5}\right)$$

(2) Modeling transformations

(a) Given the point (1,1). find its coordinates after translating it by (2,3). Perform the computation using a transformation matrix.

Solution:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

$$y' = x + t_x$$

$$y' = y + t_y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

(b) Given the point (1,1). find its coordinates after scaling it by (2, 2). Perform the computation using a transformation matrix.

Given:

$X = 1, \quad Y = 1$

$S_x = 2 \qquad x' = S_x \cdot X = 2 \cdot 1 = 2$

$S_y = 2 \qquad y' = S_y \cdot Y = \cancel{2I}\, 2 \cdot 1 = 2 \quad \Rightarrow \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

(c) Given the point (1,1) find its coordinates after rotating it by 45°.

Solution: given $x=1$, $y=1$, $\theta=45°$

$x' = x\cos\theta - y\sin\theta$

$1.\cos 45 - 1\sin 45'$

$\left(1 \cdot 1/\sqrt{2}\right) - \left(1 \cdot 1/\sqrt{2}\right)$

$= \underline{\underline{0}}$

$y' = x\sin\theta + y\cos\theta$

$1\sin 45 + 1.\cos 45$

$\left(1 \cdot 1/\sqrt{2}\right) + \left(1 \cdot 1/\sqrt{2}\right)$

$2 \cdot 1/\sqrt{2} = 2 \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$\underline{\underline{\sqrt{2}}}$

(d) given that point (1,1) find its coordinates after rotating it by 45° about point (2,2)

Solution: given $x=1$, $y=1$, $\theta=45$

$R(\theta) = T(P)\, R_u(\theta)\, T(-P)$

$T(-P) = T(-2,-2) = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$

$R_u(\theta) = R_{(-1,-1)}(45)$, $x' = x\cos\theta - y\sin\theta$

$-1.\cos 45 - (-1).\sin 45'$

$-1/\sqrt{2} + 1/\sqrt{2} = \underline{\underline{0}}$

$y' = x\sin\theta + y\cos\theta$

$(-1)\sin 45' + (-1).\cos 45'$

$= -1/\sqrt{2} - 1/\sqrt{2} = -2/\sqrt{2} = -\sqrt{2}$

$(x', y') = (0, -\sqrt{2})$

$$T(P) = T(2,2) = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -\sqrt{2} \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2-\sqrt{2} \\ 1 \end{bmatrix}$$

$\therefore (2, 2-\sqrt{2})$ are the co-ordinates

$2\cos 45 - 2\sin 45$

$2 \cdot 1/\sqrt{2} - 2 \cdot 1/\sqrt{2}$

$\sqrt{2} - \sqrt{2} = \underline{\underline{0}}$

$(x', y') = (0, 2\sqrt{2})$

$y' = x_1 \sin\theta + y_1 \cos\theta$

$= 2\sin 45 + 2\cos 45$

$= 2 \cdot 1/\sqrt{2} + 2 \cdot 1/\sqrt{2}$

$= 4 \cdot 1/\sqrt{2}$

$= \underline{\underline{2\sqrt{2}}}$

(C) Given that I want first to Rotate an object using a matrix R and then translate it using a a matrix T. what should be the combined matrix (expressed in terms of R and T) that needs to be applied to the object.

Given $P' = TRP$ wher T → Translation
R → Rotation.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \begin{bmatrix} x\cos\theta - y\sin\theta \\ x\sin\theta + y\cos\theta \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x\cos\theta & -y\sin\theta \\ x\sin\theta & +y\cos\theta \end{bmatrix}$$

Hence, After rotation, we get

$$\begin{bmatrix} x\cos\theta & -y\sin\theta \\ x\sin\theta & +y\cos\theta \end{bmatrix}$$

Now, applying translation,

$$\begin{bmatrix} x'' \\ y'' \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix} + \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x\cos\theta & -y\sin\theta \\ x\sin\theta & + y\cos\theta \end{bmatrix} + \begin{bmatrix} x \\ y \end{bmatrix}$$

(F) Let M be a 2D transformation matrix in homogeneous co-ordinates whose rows are [3,0,0], [0,2,0], [0,0,1]. What is the effect of applying this matrix to transform a point P.

Solution:-

The given transformation matrix M whose rows are [3,0,0] [0,2,0] [0,0,1]

$$M = \begin{bmatrix} 3 & 0 & 0 & t_x \\ 0 & 2 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow \text{Translating the Matrix}$$

Now, on scaling the matrix,

$$M = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ This is the output of the matrix on point P in translation or scaling.

(g) Let M be 2D transformation matrix is homogeneous co-ordinates where rows are [1,0,1], [0,1,2], [0,0,1]. what is the effect of applying that matrix to transform a point P.

Solution:

$$M = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$ (This is 2D transformation matrix)

The translation matrix which is similar to is, $\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$

$t_x = 1, \; t_y = 2$

M translates point P by (1, 2)

(h) Let M be a 2D transformation matrix in homogeneous co-ordinates where rows [3,0,0], [0,2,0], [0,0,1]. what is the transformation matrix will reverse the effects of this transformation.

Solution: Given $M = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} S_x & & \\ & S_y & \\ & & 1 \end{bmatrix}$

Inverse of M is $M^{-1} = \begin{bmatrix} 1/3 & & \\ & 1/3 & \\ & & 1 \end{bmatrix}$

(i) let $M = R(45)\,T(1,2)$ be a transformation matrix in homogeneous coordinates composed of rotation by 45° and a translate by (1,2). Express the inverse of this transformation in terms of a rotation and translation matrix?

Solution:- $M = R(45) \cdot T(1,2)$

$$M^{-1} = \left[R(45)\,T(1,2)\right]^{-1}$$

From special cases, $R^{-1}(\theta) = R(-\theta)$

$$R^{-1}(45) = R(-45)$$

$$T^{-1}(t_x, t_y) = T^{-1}(1,2)$$

$$T(-1,-2)$$

$$M^{-1} = \left[T(-1,-2),\ R(-45)\right] m$$

(j) find the vector which is $\perp^r$ to vector (1,3).

Solution:- let vector be $V = (v_1, v_2)$, $u = (1,3)$

$\therefore u \cdot v = (1,3)(v_1, v_2)$

$$v_1 + 3v_2 \Rightarrow v_1 = -3v_2$$

The dot product is 0 for $\perp^r$ vectors, let $v_2 = 1$ $\therefore v_1 = -3(1) = -3$ $\therefore$ $v_1, v_2 (-3,1)$ vector $\perp$ to (1,3) is (-3,1)

(4) find the projection of vector (1,3) onto the direction defined by vector (2,5).

Solution:- let $\vec{u} = (1,3)$

$\vec{v} = (2,5)$

$$|\vec{v}| = \sqrt{2^2+5^2} = \sqrt{4+25} = \sqrt{29}$$

$$P = \left(\frac{\vec{u}\cdot\vec{v}}{|v|^2}\right)\vec{v} = \left[\frac{(1,3)\cdot(2,5)}{(\sqrt{29})^2}\right](2,5)$$

$$= \left(\frac{2+15}{29}\right)(2,5)$$

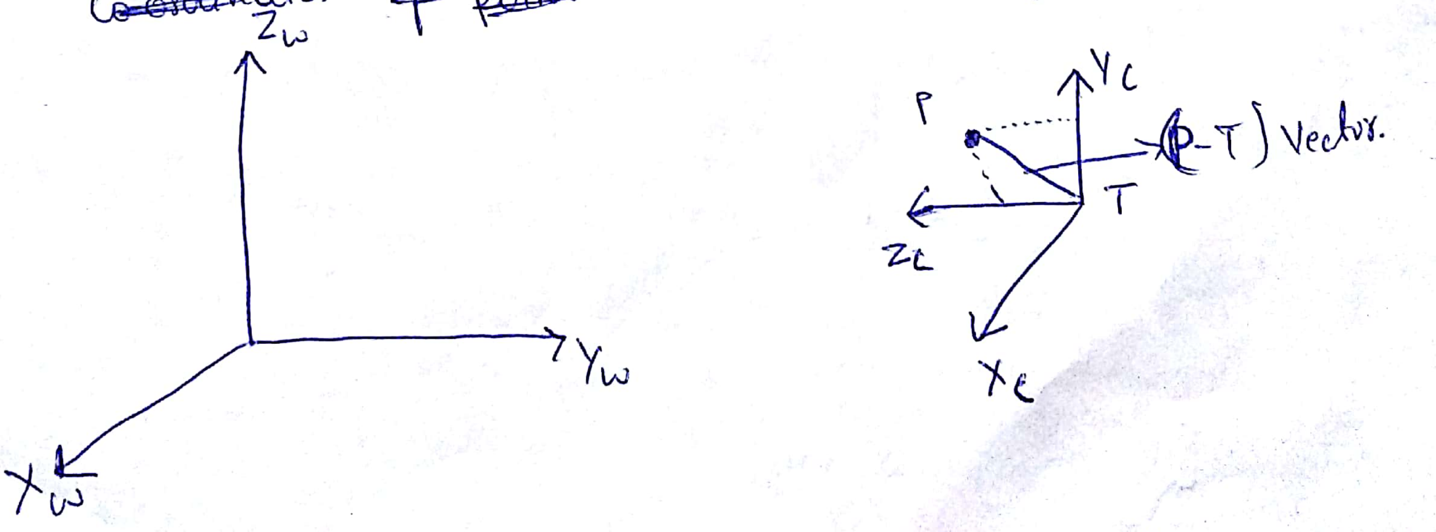
$$\left(\frac{34}{29}, \frac{85}{29}\right) = (1.17, 2.93)$$

③ General camera Model:-

(a) Explain the need for a general projection matrix that uses different co-ordinate system for camera and image.

As it ~~can~~ have a sequence of transformation matrices that will turn world co-ordinates to camera co-ordinates and camera co-ordinates to image co-ordinates, also to move back and forth between 2D and 3D co-ordinate systems.

(b) Given that the camera is rotated by R and translated by T w.r.t the world, write the transformation matrix that will convert world to camera co-ordinate. ~~Find co-ordinates of point P in camera system~~



Solution:-

Project $(P-T)$ onto $\underbrace{X_c, Y_c, Z_c}$ → unit vectors.

$(P-T)$ → vector from T to P

$$X' = (P-T) \cdot X_c = \hat{X}_c^T (P-T)$$
$$Y' = (P-T) \cdot Y_c = \hat{Y}_c^T (P-T)$$
$$Z' = (P-T) \cdot Z_c = \hat{Z}_c^T (P-T)$$

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} \hat{X}_c^T \\ \hat{Y}_c^T \\ \hat{Z}_c^T \end{bmatrix} (P-T)$$

(P is point in world)

$(P-T)$ ← Translate to origin

$\begin{bmatrix} \hat{X}_c^T \\ \hat{Y}_c^T \\ \hat{Z}_c^T \end{bmatrix}$ ← Rotate

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(c) Given three unit vectors $\hat{X}, \hat{Y}, \hat{Z}$. write the rotation matrix describing the rotation of the camera w.r.t the world.

Define → $R^T = \begin{bmatrix} X_c^T \\ Y_c^T \\ Z_c^T \end{bmatrix}$

$\therefore P' = R^T (P-T)$

$\therefore R^T$ aligns camera with model.

example:- $\begin{bmatrix} \hat{X}_c^T & X_c \\ \hat{Y}_c^T & Y_c \\ \hat{Z}_c^T & Z_c \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = X_W$

The product will be zero as both are orthogonal.

Therefore, the aligning of the camera with the world helps us to get the transformation between world and camera.

(c) Given three unit vectors, $\hat{X}, \hat{Y}, \hat{Z}$, write the rotation matrix describing the rotation of camera with respect to the world.

Solution:- $R = \begin{bmatrix} \hat{X} \\ \hat{Y} \\ \hat{Z} \end{bmatrix}$ → Rotation Matrix

$P' = R^T (P - T)$, R^T → inverse rotation

T → inverse translation.

R^T aligns camera with world.

example: $R^T \hat{Z}' = \begin{bmatrix} \hat{X}^T & \hat{Z} \\ \hat{Y}^T & \hat{Z} \\ \hat{Z}^T & \hat{Z} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = Z_w$ (Point in world)

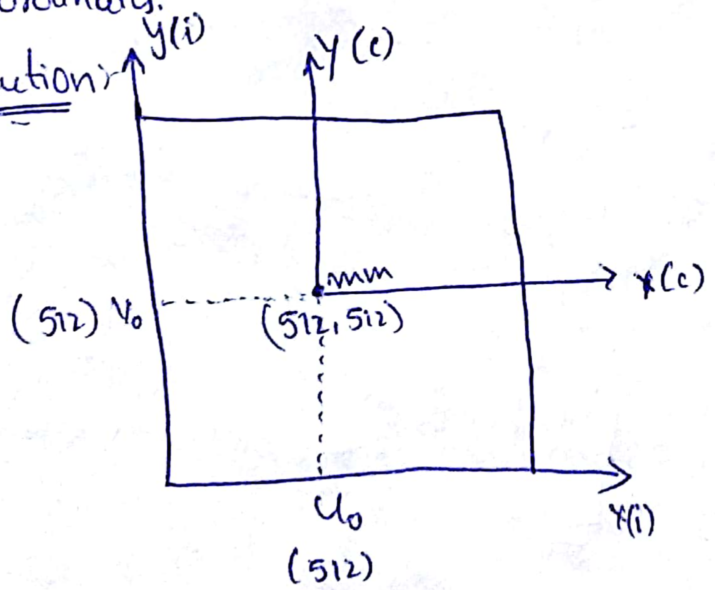
(d) given the transformation matrix between world and camera co-ordinates $M = \begin{bmatrix} R^* & T^* \\ 0 & 1 \end{bmatrix}$. Explain the meaning of R^* and T^*

Solution:

R^* is the rotation and T^* is the translation of world with respect to the camera. R is the rotation, T is the translation of camera with respect to the world.

(e) Given that there are K_u pixels per mm in the x direction, K_v pixels per mm in the y direction, and that the optical center of the camera is translated by $(u_0, v_0) = (512, 512)$ pixels, write the transformation matrix that will convert camera co-ordinates to image co-ordinates.

Solution:



where u_0, v_0: translation of optical center [pixels]

K_u, K_v = scale (pixel/mm)

Align camera with image,

$$M_{c \leftarrow i} = \begin{bmatrix} 1/K_u & 0 & 0 \\ 0 & 1/K_v & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -u_0 \\ 0 & 1 & +v_0 \\ 0 & 0 & 1 \end{bmatrix}$$

inverse scale   inverse translation.

$$M_{i \leftarrow c} = M^{-1}_{c \leftarrow i} = \begin{bmatrix} 1 & 0 & -u_0 \\ 0 & 1 & -v_0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1/K_u & 0 & 0 \\ 0 & 1/K_v & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 1 & 0 & u_0 \\ 0 & 1 & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} K_u & 0 & 0 \\ 0 & K_v & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} K_u & 0 & u_0 \\ 0 & K_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} K_u & 0 & 512 \\ 0 & K_v & 512 \\ 0 & 0 & 1 \end{bmatrix}$$

(f) let the projection matrix M of a general camera be given by $K^*[R^* | T^*]$. explain which parts contain the intrinsic and extrinsic parameters of the camera

Solution:

Projection matrix is given by,

$$p^{(i)} = M_{i \leftarrow c} \quad K[I|0] \, M_{c \leftarrow w} \, P^{(w)}$$

$$= \begin{bmatrix} k_u & 0 & u_0 \\ 0 & k_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} [I \,|\, 0] \left[\begin{array}{c|c} R^* & T^* \\ \hline 0 & 1 \end{array}\right] p^{(w)}$$

$$= \begin{bmatrix} fk_u & 0 & u_0 \\ 0 & fk_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} [R^* \,|\, T^*] \; p^{(w)}$$

$$M = p^{(i)} = K^* \, [R^* \,|\, T^*] \; p^{(w)}$$

where $K^* \rightarrow$ intrinsic parameter

$[R^* \,|\, T^*] \Rightarrow$ extrinsic parameter.

(g) Explain the reason for including a 2D skew parameter in the camera model.

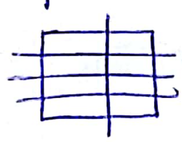
To figure out whether the detector array has a non-orthogonal structure or when the array is not orthogonal to optical axis, this is the reason for including a 2D skew parameter in the camera model

(h) Explain what happens to the camera model when taking into account radial lens distortion. What is the complication introduced by the radial lens distortion?

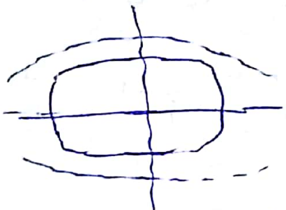
Solution:-

Radial distortion can be found in wide-angle lenses which is ~~clear~~ displays as a visible uncurvature while projecting on straight lens

Ex:-



Normal cabinet



Distorted radially cabinet

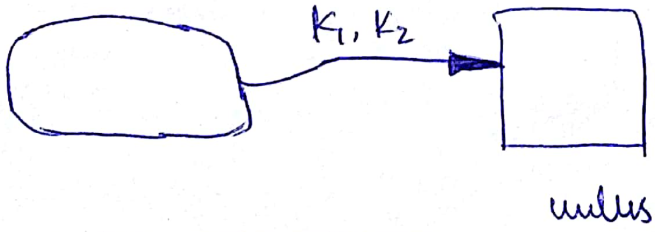
Equation:-

$$p^{(i)} = \begin{bmatrix} 1/\lambda & 0 & 0 \\ 0 & 1/\lambda & 0 \\ 0 & 0 & 1 \end{bmatrix} K^{*} \left[R^{*} \mid T^{*} \right] p^{(w)}$$

$\lambda = 1 + k_1 d + k_2 d^2$ where k_1 = linear distortion co-efficient

k_2 = quadratic distortion co-efficient

d = distance from centre.



The complication introduced by radial distortion is that its impossible to create accurate photo-realistic reconstructions unless its taken into the account.

(3)(i) Explain the meaning of a weak- perspective camera and of an affine camera.

Solution:- A weak perspective camera is correct when depth variation on screen ~~[illegible]~~ and the distance from camera are both compared.

$$e = \left| M_{\infty} P - MP \right|$$

$$= \frac{\Delta}{d_0} \left(M_p - P_0 \right) \rightarrow \text{distance from center.}$$

where, M_{∞} = weak perspective camera.

M = perspective camera.

Δ = depth variation.

d_0 = distance from the camera

weak perspective camera:- its a linear approximation of full perspective projection

Affine cameras:- the linear mathematical model ~~to~~ of which is used to approximate the perspective projection ~~and~~ followed by an ideal pinhole camera.

(4) color and photometric image formation:-

(a) Explain the difference b/w surface radiance and image irradiance.

Surface radiance	image irradiance
* the energy going out of a source will be radiance	* the energy incoming to a surface is called irradiance
* its the measure of flux density per unit, like solid, angle of view.	* its the measure of radiometric flux per unit area, flux density.
* Represented by watt per steradian per square meter i.e, $(W/sr/m^2)$	* Represented by watt per square meter, i.e (W/m^2)
* the brightness of scene related to energy radiated from a surface.	* Related to energy flux falling on the image plane at an angle.

(b) write the radiosity equations relating surface radiance and image irradiance.

Solutions

$$E(P) = L(P) \cdot \frac{\pi}{4}\left(\frac{d}{f}\right)^2 (\cos \alpha)^4$$

where $E(p)$ = image irradiance. (light at the image)
$L(p)$ = Surface radiance. (light at the surface)
d: diameter of the lens
f = focal length.
$\cos\alpha$ = angle between principle axis and surface normal

* $d \propto E(p)$ [directly proportional]
* $f \frac{1}{\alpha} E(p)$ [inversely proportional]
* $\cos\alpha \; 1/_\alpha \; E(p)$ [inversely proportional]

(c) Define the albedo of a surface.

~~Albedo is the ratio of of irradiance reflected~~

Its the measure of spreading of solar light out from the surface received of any body. Its the ratio of irradiance reflected to the irradiance received by surface.

$$I_{ref} = I . S \cos\theta$$

where S is the surface albedo $\in [0,1]$.

(d) Explain what's the reason for using the RGB color model to represent colors.

Solution:-

The main reason is that the Human Vision is sensitive to red, green, blue and it recognizes quickly for the RGB wavelengths. Its even more sensitive to green color. Hence ~~its~~ ~~the~~ RGB color model is used to represent colors.

(e) Given the RGB color cube, what are the colors along the line that connects (0,0,0) with (1,1,1)

Solution:-

The colors along the line that connects (0,0,0) with (1,1,1) are Black, grey and white.

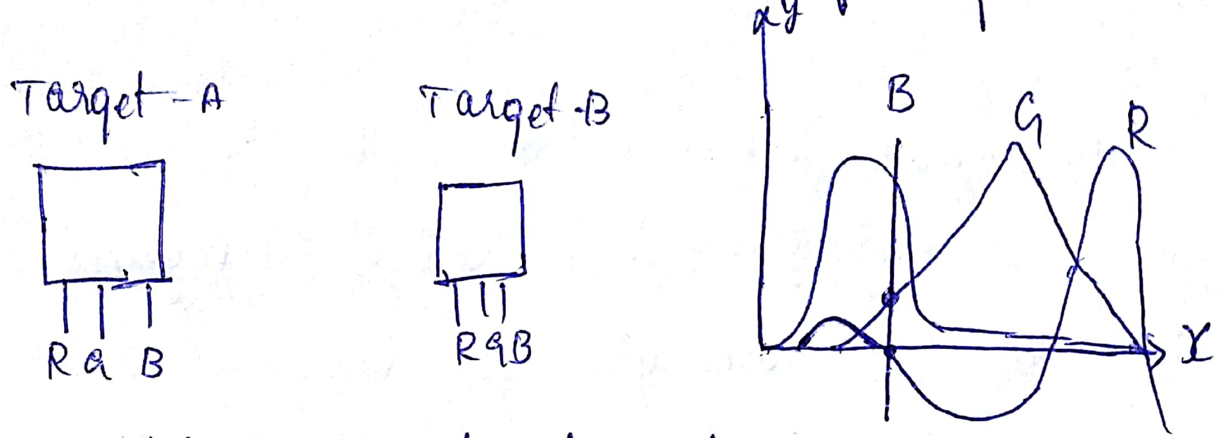
(f) Explain the way by which RGB colors are mapped to real world colors.

Solution:-

The RGB colors are mapped by CIE table, by using ~~mapping~~ wavelengths to RGB intensities.

(645 nm) R	(526 nm) G	(444 nm) B

using CIE table, select a color ~~box~~ targets -A & B and target -B is used to match the color of target A.



To match the color of real world. -ve values are added to target-B.

(g) Given the CIE RGB color model & its conversion to the XYZ model. explain is the use for the luminance component Y?

Solution: The green parts of the spectrum is used to perceive light, ~~are~~ brighter than red, blue light. The luminance component Y represents the perceived brightness of different wavelength. is comparable to spectral sensitivity of M-cones. Therefore the CIE model takes on this fact by defining Y as luminance.

(h) Explain the advantage of LAB color space.

Solution:

* LAB color is designed in such a way that its indefinite to our human vision.

* We can split the image into independent brightness which is referred as L layer (luminous) and 2 independent information.

* Color replacement can also be implemented without masking repeatedly.