COMPUTER VISION RAKSHITH CHURCHAGUNDI AMARNATH

CWID:-

CS 512 HW0

(A)

$A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $B = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$, $C = \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}$, find:-

(1) $2A - B$

$$= 2\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} - \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} - \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \\ 0 \end{bmatrix}$$

(2) $\|A\|$ and the angle of A relative to the positive x-axis

$$|A| = \sqrt{(1)^2 + (2)^2 + (3)^2} = \sqrt{1+4+9} = \sqrt{14}$$

$$\vec{A} = [\hat{a} + 2\hat{b} + 3\hat{c}]$$

$$\vec{A} \cdot a = |A|\,|a| \cos\theta$$

$$1 = \sqrt{14} \cdot \cos\theta$$

$$\therefore \theta = \cos^{-1}\left(1/\sqrt{14}\right) = 74.49^\circ$$

(3) $\hat{A}$, a unit vector in the direction of A

$$\vec{A} = \frac{\hat{a} + 2\hat{b} + 3\hat{c}}{\sqrt{14}}$$

$$\hat{A} = \frac{\vec{A}}{|A|} = \left(1/\sqrt{14}, 2/\sqrt{14}, 3/\sqrt{14}\right)$$

$$= (0.2672, 0.5345, 0.8017)$$

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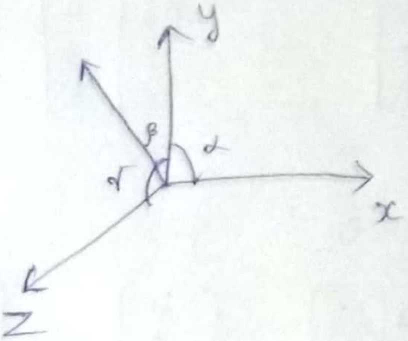
④ the direction cosine of A.

$\cos\alpha = \frac{1}{\sqrt{14}}$, $\cos\beta = \frac{2}{\sqrt{14}}$, $\cos\gamma = \frac{3}{\sqrt{14}}$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$



⑤ A.B and B.A

$A.B = \begin{bmatrix}1\\2\\3\end{bmatrix} \times \begin{bmatrix}4\\5\\6\end{bmatrix} = \begin{bmatrix}1\times4 + 2\times5 + 3\times6\end{bmatrix} = \begin{bmatrix}4 + 10 + 18\end{bmatrix} = \underline{\underline{32}}$

$B.A = \begin{bmatrix}4\\5\\6\end{bmatrix} \times \begin{bmatrix}1\\2\\3\end{bmatrix} = \begin{bmatrix}4\times1 + 5\times2 + 6\times3\end{bmatrix} = \begin{bmatrix}4 + 10 + 18\end{bmatrix} = \underline{\underline{32}}$

⑥ the angle b/w A & B.

$A.B = |A|.|B|.\cos\theta = (a + 2b + 3c).(4a + 5b + 6c)$

$= |A||B|\cos\theta = \frac{4 + 10 + 18}{\sqrt{14}.\sqrt{77}} = \cos\theta \Rightarrow \theta = \cos^{-1}\left(\frac{32}{\sqrt{14}\sqrt{77}}\right)$

$= \underline{\underline{12.93^\circ}}$

(7) A vector which is $\perp^r$ to A.

$$\vec{A} = \vec{x}\cdot\vec{A} = 0 \Rightarrow \begin{bmatrix} a \\ b \\ c \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = 0$$

$$a + 2b + 3c = 0$$

Now, let $a = 2, \quad b = 3$

$$\therefore 2 + 6 + 3c = 0$$

$$c = -8/3$$

$\therefore$ Vector $\perp$ to A is $[2, 3, -8/3]$

(8) A x B and B x A

$$\times\ \vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix} = \hat{i}\begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix} - \hat{j}\begin{vmatrix} 1 & 3 \\ 4 & 0 \end{vmatrix} + \hat{k}\begin{vmatrix} 1 & 2 \\ 4 & 5 \end{vmatrix}$$

$$= \hat{i}(12-15) - \hat{j}(6-12) + \hat{k}(5-8)$$

$$-3\hat{i} + 6\hat{j} - 3\hat{k}$$

$$\times\ \vec{B} \times \vec{A} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 5 & 6 \\ 1 & 2 & 3 \end{vmatrix} = \hat{i}\begin{vmatrix} 5 & 6 \\ 2 & 3 \end{vmatrix} - \hat{j}\begin{vmatrix} 4 & 6 \\ 1 & 3 \end{vmatrix} + \hat{k}\begin{vmatrix} 4 & 3 \\ 1 & 2 \end{vmatrix}$$

$$= \hat{i}(15-12) - \hat{j}(12-6) + \hat{k}(8-5) = 3\hat{i} - 6\hat{j} + 3\hat{k}$$

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(9)

A Vector which is ⊥ to both A and B.

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix} = i\begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix} - j\begin{vmatrix} 1 & 3 \\ 4 & 6 \end{vmatrix} + k\begin{vmatrix} 1 & 2 \\ 4 & 5 \end{vmatrix}$$

$$= i(12-15) - j(6-12) + k(5-8) = -3\hat{i} + 6\hat{j} - 3\hat{k}.$$

(10) Linear dependency b/w A, B, C

$$x_1 A + x_2 B + x_3 C = 0$$

(x_1, x_2, x_3 are scalars)

$$x_1\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + x_2\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} + x_3\begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Converting into augmented matrix $\begin{bmatrix} 1 & 4 & -1 \\ 2 & 5 & 1 \\ 3 & 6 & 3 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Reducing to get echleon form

$$\begin{bmatrix} 1 & 4 & -1 \\ 2 & 5 & 1 \\ -3 & -9 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 4 & -1 \\ 3 & 9 & 0 \\ -3 & -9 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad R_2 \to R_2 + R_1$$

$R_3 \to R_3 + R_2$

$$\left[\begin{array}{ccc|c} 1 & 4 & -1 & 0 \\ 3 & 9 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \overset{R_2 \to R_2 - 3R_1}{=} \left[\begin{array}{ccc|c} 1 & 4 & -1 & 0 \\ 0 & -3 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow$$

Eq^n: $x_1 + 4x_2 - x_3 = 0$

$-3x_2 + 3x_3 = 0$

$x_2 = x_3$

lets take $x_3 = x$ ∴ non trivial sol^n $(-3n, n, n)$

there's comon factor 'n', the vectors are linearly dependent.

(ii) $A^T B$ and $A B^T$

$$A^T B = \begin{bmatrix}1 & 2 & 3\end{bmatrix}\begin{bmatrix}4\\5\\6\end{bmatrix} = \begin{bmatrix}1\times4 + 2\times5 + 3\times6\end{bmatrix} = \underline{32}$$

$$AB^T = \begin{bmatrix}1\\2\\3\end{bmatrix}\begin{bmatrix}4 & 5 & 6\end{bmatrix} = \begin{bmatrix}\cancel{1\times4 + 2\times5 + 3\times6}\end{bmatrix}$$

$$\begin{bmatrix}1\times4 & 1\times5 & 1\times6\\ 2\times4 & 2\times5 & 2\times6\\ 3\times4 & 3\times5 & 3\times6\end{bmatrix} = \begin{bmatrix}4 & 5 & 6\\ 8 & 10 & 12\\ 12 & 15 & 18\end{bmatrix}$$

B.

Let $A = \begin{bmatrix}1 & 2 & 3\\ 4 & -2 & 3\\ 0 & 5 & -1\end{bmatrix}$ $B = \begin{bmatrix}1 & 2 & 1\\ 2 & 1 & -4\\ 3 & -2 & 1\end{bmatrix}$

$C = \begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ -1 & 1 & 3\end{bmatrix}$.

(1) $2A - B = 2\begin{bmatrix}1 & 2 & 3\\ 4 & -2 & 3\\ 0 & 5 & -1\end{bmatrix} - \begin{bmatrix}1 & 2 & 1\\ 2 & 1 & -4\\ 3 & -2 & 1\end{bmatrix}$

$$= \begin{bmatrix}2 & 4 & 6\\ 8 & -4 & 6\\ 0 & 10 & 2\end{bmatrix} - \begin{bmatrix}1 & 2 & 1\\ 2 & 1 & -4\\ 3 & -2 & 1\end{bmatrix} = \begin{bmatrix}1 & 2 & 5\\ 6 & -5 & 10\\ -3 & 12 & 1\end{bmatrix}$$

② AB and BA.

$$AB = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -5 & 3 \\ 0 & 5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1+4+9 & 2+2-6 & 1-8+3 \\ 4-4+9 & 8-2-6 & 4+8+3 \\ 0+10-3 & 0+5-2 & 0-20-1 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & -2 & -4 \\ 9 & 0 & 15 \\ 7 & 7 & -21 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & -2 & 3 \\ 0 & 5 & -1 \end{bmatrix} = \begin{bmatrix} 9 & 3 & 8 \\ 6 & -18 & 13 \\ -5 & 15 & 2 \end{bmatrix}$$

③ $(AB)^T$ and $B^T A^T$

$$(AB)^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & -2 \\ 1 & -4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 & 0 \\ 2 & -2 & 5 \\ 3 & 3 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+9 & 4-4+9 & 0+10-3 \\ 2+2-6 & 8-2-6 & 0+5+2 \\ 1-8+3 & 4+8+3 & 0-20-1 \end{bmatrix} = \begin{bmatrix} 14 & 9 & 7 \\ -2 & 0 & 7 \\ -4 & 15 & -21 \end{bmatrix}$$

④ $|A|$ and $|C|$.

$$|A| = 1\begin{vmatrix} -2 & 3 \\ 5 & -1 \end{vmatrix} - 2\begin{vmatrix} 4 & 3 \\ 0 & -1 \end{vmatrix} + 3\begin{vmatrix} 4 & -2 \\ 0 & 5 \end{vmatrix} = -13 + 8 + 60$$

$$= \underline{\underline{55}}$$

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$|c| = 1\begin{vmatrix} 5 & 6 \\ 1 & 3 \end{vmatrix} - 2\begin{vmatrix} 4 & 6 \\ -1 & 3 \end{vmatrix} + 3\begin{vmatrix} 4 & 5 \\ -1 & 1 \end{vmatrix}$

$= 9 - 36 + 27 = \underline{0}$

(5) The matrix (A, B or C) in which the row vectors form an orthogonal set.

A: $x_1 = (1,2,3) \quad x_2 = (4,-2,3) \quad x_3 = (0,5,-1)$

$x_1 \cdot x_2 = (1,2,3) \cdot (4,-2,3) = 4-4+9$

$x_2 \cdot x_3 = (4,-2,3) \cdot (0,5,-1) = 0-10-3 = -13$

$x_1 \cdot x_3 = (1,2,3) \cdot (0,5,-1) = 0+10-3 = \underline{7}$

B: $x_1 = (1,2,1) \quad x_2 = (2,1,-4) \quad x_3 = (-1,1,3)$

$x_1 \cdot x_2 = (1,2,1) \cdot (2,1,4) = 2+2-4 = 0$

$x_2 \cdot x_3 = \overset{2,1,-4}{(\cancel{4,5,6})} \cdot (3,-2,1) = 6-2-4 = 0$

$x_1 \cdot x_3 = (1,2,3) \cdot (3,-2,1) = 3-6+3 = \underline{0}$

C: $x_1 = (1,2,3) \quad x_2 = (4,5,6), x_3 = (-1,1,3)$

$x_1 \cdot x_2 = (1,2,3) \cdot (4,5,6) = 4+10+18$

$x_2 \cdot x_3 = (4,5,6) \cdot (-1,1,3) \neq 0$

$x_3 \cdot x_1 = (-1,1,3) \cdot (1,2,3) \neq 0$

$\therefore$ orthogonal set $[(1,2,1), (2,1,-4), (3,-2,1)]$

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(6) A^{-1} and B^{-1}

Matrix of Minors $= \begin{bmatrix} 2+15 & -4-0 & 20-0 \\ -2-15 & -1-0 & 5-0 \\ 6+6 & 3-12 & -2-8 \end{bmatrix} = \begin{bmatrix} -13 & -4 & 20 \\ -17 & -1 & 5 \\ 12 & .9 & -10 \end{bmatrix}$

Co-factor of $A = \begin{bmatrix} -13 & 4 & 20 \\ -17 & -1 & -5 \\ 12 & 9 & 10 \end{bmatrix}$

$\text{Adj}(A) = |C(A)|^T = \begin{bmatrix} -13 & 17 & 12 \\ 4 & -1 & 9 \\ 20 & -5 & -10 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{Adj } A = \frac{1}{55} \begin{bmatrix} -13 & 17 & 12 \\ 4 & -1 & 9 \\ 20 & -5 & -10 \end{bmatrix} = \begin{bmatrix} -0.236 & 0.309 & 0.218 \\ 0.072 & -0.018 & 0.0163 \\ 0.0363 & -0.909 & -0.181 \end{bmatrix}$

B^{-1} Matrix of Minors $= \begin{bmatrix} 1-8 & 2+12 & -4-3 \\ 2+2 & 1-3 & -2-6 \\ 1+8 & -4-2 & 1-4 \end{bmatrix} = \begin{bmatrix} -7 & -14 & -7 \\ 4 & -2 & 8 \\ 9 & 6 & -3 \end{bmatrix}$

$\text{adj}(B) = \begin{bmatrix} -7 & 4 & -9 \\ -14 & -2 & 6 \\ -7 & 8 & -3 \end{bmatrix}$

$|B| = 1.(1-8) - 2(2+12) + 1(-4-3) = -42$

$B^{-1} = \frac{1}{|B|} \cdot \text{adj } B$

$= -\frac{1}{42} \begin{bmatrix} -7 & -4 & -9 \\ -14 & -2 & 6 \\ -7 & 8 & -3 \end{bmatrix} = \begin{bmatrix} 0.166 & 0.095 & 0.214 \\ 0.333 & 0.047 & -0.142 \\ 0.166 & -0.190 & 0.071 \end{bmatrix}$

C.

let $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix}$, find:-

① the eigen values and corresponding eigen vectors of

$|A - I\lambda| = 0$

$\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = 0 \Rightarrow \begin{bmatrix} 1-\lambda & 2 \\ 3 & 2-\lambda \end{bmatrix} = 0$

$(1-\lambda)(2-\lambda) - 6 = 0$

$2 - \lambda - 2\lambda + \lambda^2 - 6 = 0$

$\lambda^2 - 3\lambda - 4 = 0$

$\lambda^2 - 4\lambda + \lambda = 0$

$\lambda(\lambda - 4) + 1(\lambda - 4) = 0$

$\underline{\lambda = -1, 4}$

The eigen values corresponding to A are $\underline{-1, 4}$.

eigen vectors

$|A - I\lambda| |x| = 0$

$\begin{bmatrix} 1-\lambda & 2 \\ 3 & 2-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

Substituting $\lambda = 4$

$\begin{bmatrix} -3 & 2 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

$-3x + 2x_2 = 0$

$\Rightarrow 3x - 2x_2 = 0$

$x_1 = \frac{2}{3} x_2$

$$x_2 = \begin{bmatrix} 3/2 \\ x_1 \end{bmatrix} = \begin{bmatrix} 3/2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.66 \\ 1 \end{bmatrix}$$

Substituting $\lambda = -1$

$$\begin{bmatrix} 2 & 2 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$2x_1 + 2x_2 = 0$

$3x_1 + 3x_2 = 0 \Rightarrow x_1 + x_2 = 0$

$x_1 = -x_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

③ Dot Product b/w eigen vectors of A

$$\begin{bmatrix} 0.66 \\ 1 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -0.66 \\ 1 \end{bmatrix}$$

④ Eigen values of B

$|B - I\lambda| = 0$

$$\begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = 0 \quad \begin{bmatrix} 2-\lambda & -2 \\ -2 & 5-\lambda \end{bmatrix} = 0$$

$(2-\lambda)(5-\lambda) - 4 = 0$

$10 - 2\lambda - 5\lambda + \lambda^2 - 4 = 0$

$\lambda^2 - 7\lambda + 6 = 0$

$\lambda^2 - 6\lambda - \lambda + 6 = 0$

$\lambda(\lambda - 6) - 1(\lambda - 6) = 0$

$\underline{\lambda = 1, 6}$ Eigen values of $B = \underline{1, 6}$

Eigen values of B

$$\begin{bmatrix} 2-\lambda & -2 \\ -2 & 5-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

Substitute $\lambda = 1$

$$\begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$x_1 - 2x_2 = 0$

$-2x_1 + 4x_2 = 0$

$x_1 = 2x_2$

$$x_1 = \begin{bmatrix} 2x \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

Substitute $\lambda = 6$

$$\begin{bmatrix} -4 & -2 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \quad \Rightarrow -2x_1 = x_2$$

$$x_2 = \begin{bmatrix} -2x_1 \\ x_1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

Dot Product b/w eigen vectors of B

$$\Rightarrow \begin{bmatrix} -2 \\ 1 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

D.

$f(x) = x^2+3$, $g(x,y) = x^2+y^2$

① $f'(x) = 2x$

$f''(x) = 2$

② $g(x,y) = x^2+y^2 \Rightarrow \frac{\partial g}{\partial x} = 2x$

$\frac{\partial g}{\partial y} = 2y$

③ $\nabla g(x,y) = \nabla (x^2+y^2) = 2x\hat{i} + 2y\hat{j}$

④ The probability density function of a univariate gaussian distribution

$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\left(\frac{-(x-\mu)^2}{2\sigma^2}\right)}$ $| \sigma > 0$

$\boxed{\sigma = S.D}$