In [ ]:
```
import numpy as np
import matplotlib.pyplot as plt
```

Problem-1: A network consists of n stations, labeled $1, \ldots, n$. A path through the network is a subset of the stations. This data can be represented as an $N \times n$-matrix P, where

$$P_{ij} = \begin{cases} 1, & \text{if station } j \text{ is on path } i, \\ 0, & \text{otherwise.} \end{cases}$$

The code snippet below simulates the matrix P for 10 stations and 100 paths.

In [ ]:
```
# Simulating a network tomography matrix
np.random.seed(1)
npaths = 100
nstations = 10
P = np.random.choice(np.arange(0,2), (npaths, nstations))
```

Problem-1.1: Busy paths have *at least* 7 stations in them? What are the busy paths and how many of them do we have?

In [ ]:
```
result = np.sum(np.dot(P, np.ones((nstations, 1))) >= 7, axis=1)
print(result)
print(result.shape[0])
```

```
[1 0 0 0 1 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 1 0 1 0 0 0
 0 0 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0
 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 1 1 0 0]
100
```

Problem-1.2: Busy stations show up in *at least* 50 paths? What are the busy stations and how many of them do we have?

In [ ]:
```
result = np.sum(np.dot(P.T, np.ones((npaths, 1))) >= 50, axis=0)
print(result)
print(result.shape[0])
```

```
[3]
1
```

Problem-1.3: Station-1 is *most similar* to which other station?

In [ ]:
```
result = np.linalg.norm(P.T - P.T[0], axis = 0)
print(result)
np.argmin(result)
```

```
[1.73205081 2.         1.73205081 2.23606798 1.41421356 2.23606798
 2.44948974 2.23606798 2.         2.44948974 2.23606798 2.23606798
 2.64575131 2.23606798 2.23606798 2.44948974 2.44948974 2.
 1.41421356 2.23606798 2.23606798 2.23606798 2.         1.73205081
 1.73205081 2.23606798 2.23606798 2.23606798 2.         2.
 2.         1.73205081 2.         1.41421356 1.         2.
 2.64575131 2.         2.44948974 1.73205081 2.44948974 1.73205081
 2.44948974 2.23606798 2.64575131 1.         2.23606798 2.44948974
 2.         1.73205081 2.64575131 2.23606798 1.73205081 2.23606798
 1.73205081 2.64575131 1.73205081 1.73205081 1.73205081 2.23606798
 1.73205081 2.64575131 2.         2.         1.41421356 1.41421356
 2.         2.         2.82842712 2.         2.44948974 1.41421356
 2.44948974 1.41421356 2.23606798 2.         2.44948974 2.
 2.44948974 1.41421356 1.73205081 2.44948974 2.23606798 1.73205081
 2.64575131 2.64575131 2.         2.23606798 2.44948974 1.41421356
 2.23606798 2.         2.23606798 1.73205081 2.23606798 2.23606798
 2.64575131 1.73205081 1.41421356 1.41421356]
```

Out[ ]:

```
34
```

Problem-1.4: Express the number of paths common to each pair of stations as a product of two matrices. How many paths are common to station-4 and station-10?

In [ ]:

```
result = np.dot(P.T, P)
print(result[3, 9])
```

```
23
```

Problem-1.5: Express the number of stations common to each pair of paths as a product of two matrices. How many stations are common to the 1st path and the 100th path? How many stations do we have in the 100th path?

In [ ]:

```
result = np.dot(P, P.T)
# Stations common to the 1st path and the 100th path
print(result[0, 99])
# Number of stations in the 100th path
print(result[99,99])
```

```
1
2
```

Problem-2: A *compartmental system* is a model used to describe the movement of some material over time among a set of n compartments of a system and the outside world. It is widely used in pharmaco-kinetics, the study of how the concentration of a drug varies over time in the body. In this application, the material is a drug, and the compartments are the bloodstream, lungs, heart, liver, kidneys, and so on. Compartmental systems are special cases of linear dynamical systems. In this problem we will consider a very simple compartmental system with 3 compartments. We let $(x_t)_i$ denote the amount of the material (say, a drug) in compartment i at time stamp t. Between time stamps t and $t+1$, the material moves as follows:

- 20% of the material in compartment 1 moves to compartment 2. (This decreases the amount in compartment 1 and increases the amount in compartment 2.)
- 5% of the material in compartment 2 moves to compartment 1.
- 5% of the material in compartment 2 moves to compartment 3.
- 10% of the material in compartment 2 is eliminated.
- 5% of the material in compartment 3 moves to compartment 1

- 5% of the material in compartment 3 moves to compartment 2.

This compartmental system can be modeled as a linear dynamical system, $x_{t+1} = Ax_t$, where A is the linear dynamics matrix.

Problem-2.1: Construct the linear dynamics matrix A.

In [ ]:
```
# Linear dynamical system matrix
A = np.array([[0.8, 0.05, 0.05],
              [0.20, 0.85, 0.05],
              [0.05, 0.10, 0.9]])
```

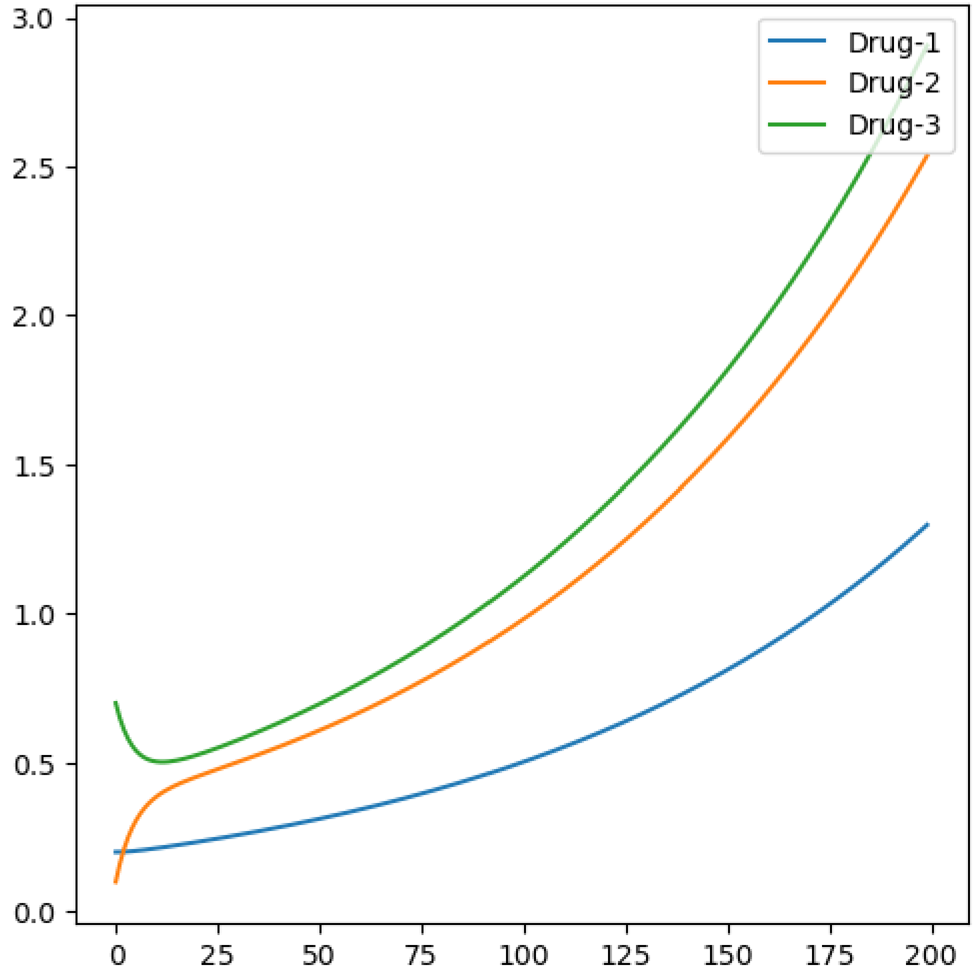
Problem-2.2: Suppose that the initial concentration of the three drugs is 20%, 10%, and 70%, respectively. We want to simulate the linear dynamical system for 200 time stamps.

In [ ]:
```
# Time period
T = 200

# Initialize the three drug concentration values to zeros for all time stamps
X = np.zeros((3, T))

# Initial state vector
X[:, 0] = np.array([0.20, 0.10, 0.70])

# Simulate the linear dynamical system for all time stamps
for j in np.arange(1, T):
    X[:, j] = np.dot(A, X[:, j-1])
```

Problem-2.3: Plot the concentrations of the three drugs for all time stamps. Which drug component shows an initial increase and then a decrease in its concentration? Approximately, at what time stamp does that drug concentration peak? Which drug component shows the least rapid change in the initial time stamps?

In [ ]:
```
fig, ax = plt.subplots(1, 1, figsize = (6,6))
fig.tight_layout(pad = 4.0)
ax.plot(X[0, :])
ax.plot(X[1, :])
ax.plot(X[2, :])
plt.legend(["Drug-1", "Drug-2", "Drug-3"], loc ="upper right")
```

Out[ ]:
```
<matplotlib.legend.Legend at 0x1fe89debcd0>
```


Drug-1
Drug-2
Drug-3
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
25
50
75
100
125
150
175
200