

# COMP9414 Artificial Intelligence

## Assignment 1: Constraint Satisfaction Search

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**Due Date:** Week 5, Friday, October 17, 5.00pm

### Objective

This assignment concerns developing optimal solutions to a scheduling problem inspired by the scenario of a manufacturing plant that has to fulfil multiple customer orders with varying deadlines, but where there may be constraints on tasks and on relationships between tasks. Any number of tasks can be scheduled at the same time, but it is possible that some tasks cannot be finished before their deadline. A task finishing late is acceptable, however incurs a cost, which for this assignment is a simple (dollar) amount per hour that the task is late.

A *fuzzy scheduling* problem in this scenario is simplified by ignoring customer orders and having just one machine and a number of *tasks*, each with a fixed duration in hours. Each task must start and finish on the same day, within working hours (9am to 5pm). In addition, there can be *constraints*, both on single tasks and between two tasks. One type of constraint is that a task can have a deadline, which can be “hard” (the deadline must be met in any valid schedule) or “soft” (the task may be finished late – though still at or before 5pm – but with a “cost” per hour for missing the deadline). The aim is to develop an overall schedule for all the tasks (in a single week) that minimizes the total cost of all the tasks that finish late, provided that all the hard constraints on tasks are satisfied.

More technically, this assignment is an example of a *constraint optimization problem* (or *constrained optimization problem*), a problem that has constraints like a standard Constraint Satisfaction Problem (CSP), but also a *cost* associated with each solution. For this assignment, we will use a *greedy* algorithm to find optimal solutions to fuzzy scheduling problems that are specified as text strings. However, unlike the greedy search algorithm described in the lectures on search, this greedy algorithm has the property that it is guaranteed to find an optimal solution for any problem (if a solution exists).

The assignment will use the AI Python code of Poole & Mackworth. You are given code to translate fuzzy scheduling problems specified as text strings into CSPs with a cost, and you are given code for several constraint solving algorithms – based on domain splitting and arc consistency, and based on depth-first search. The assignment will be to implement some missing procedures and to analyse the performance of the constraint solving methods, both analytically and experimentally.

### Submission Instructions

- This is an individual assignment.
- Write your answers in **this** notebook and submit **this** notebook on Moodle under **Assignment 1**.
- Name your submission `<zid>-<firstname>-<lastname>.ipynb` where `<firstname>-<lastname>` is your **real** (not Moodle) name.
- Make sure you set up AIPython (as done below) so the code can be run on either CSE machines or a marker's own machine.
- Do not submit any AIPython code. Hence do not change any AIPython code to make your code run.
- Make sure your notebook runs cleanly (restart the kernel, clear all outputs and run each cell to check).
- After checking that your notebook runs cleanly, run all cells and submit the notebook **with** the outputs included (do not submit the empty version).
- Make sure images (for plots/graphs) are **included** in the notebook you submit (sometimes images are saved on your machine but are not in the notebook).
- Do not modify the existing code in this notebook except to answer the questions. Marks will be given as and where indicated.
- If you want to submit additional code (e.g. for generating plots), add that at the end of the notebook.
- **Important: Do not distribute any of this code on the Internet. This includes ChatGPT. Do not put this assignment into any LLM.**

## Late Penalties

Standard UNSW late penalties apply (5% of the value of the assignment per day or part day late).

**Note:** Unlike the CSE systems, there is no grace period on Moodle. The due date and time is 5pm **precisely** on Friday October 17.

**Important: You can submit as many times as you want before the due date, but if you do submit before the due date, you cannot submit on Moodle after the due date. If you do not submit before the due date, you can submit on Moodle after the due date.**

## Plagiarism

Remember that ALL work submitted for this assignment must be your own work and no sharing or copying of code or answers is allowed. You may discuss the assignment with other students but must not collaborate on developing answers to the questions. You

may use code from the Internet only with suitable attribution of the source. You may not use ChatGPT or any similar software to generate any part of your explanations, evaluations or code. Do not use public code repositories on sites such as github or file sharing sites such as Google Drive to save any part of your work – make sure your code repository or cloud storage is private and do not share any links. This also applies after you have finished the course, as we do not want next year's students accessing your solution, and plagiarism penalties can still apply after the course has finished.

All submitted assignments will be run through plagiarism detection software to detect similarities to other submissions, including from past years. You should **carefully** read the UNSW policy on academic integrity and plagiarism (linked from the course web page), noting, in particular, that collusion (working together on an assignment, or sharing parts of assignment solutions) is a form of plagiarism.

Finally, do not use any contract cheating "academies" or online "tutoring" services. This counts as serious misconduct with heavy penalties up to automatic failure of the course with 0 marks, and expulsion from the university for repeat offenders.

## Fuzzy Scheduling

A CSP for this assignment is a set of variables representing tasks, binary constraints on pairs of tasks, and unary constraints (hard or soft) on tasks. The domains are all the working hours in one week, and a task duration is in hours. Days are represented (in the input and output) as strings 'mon', 'tue', 'wed', 'thu' and 'fri', and times are represented as strings '9am', '10am', '11am', '12pm', '1pm', '2pm', '3pm', '4pm' and '5pm'. The only possible values for the start and end times of a task are combinations of a day and times, e.g. 'mon 9am'. Each task name is a string (with no spaces), and the only soft constraints are the soft deadline constraints.

There are three types of constraint:

- **Binary Constraints:** These specify a hard requirement for the relationship between two tasks.
- **Hard Domain Constraints:** These specify hard requirements for the tasks themselves.
- **Soft Deadline Constraints:** These constraints specify that a task may finish late, but with a given cost.

Each soft constraint has a function defining the *cost* associated with violating the preference, that the constraint solver must minimize, while respecting all the hard constraints. The *cost* of a solution is simply the sum of the costs for the soft constraints that the solution violates (and is always a non-negative integer).

This is the list of possible constraints for a fuzzy scheduling problem (comments below are for explanation and do **not** appear in the input specification; however, the code we supply *should* work with comments that take up a full line):

```

# binary constraints
constraint, <t1> before <t2>          # t1 ends when or before
t2 starts
constraint, <t1> after <t2>          # t1 starts after or when
t2 ends
constraint, <t1> same-day <t2>      # t1 and t2 are scheduled
on the same day
constraint, <t1> starts-at <t2>      # t1 starts exactly when
t2 ends

# hard domain constraints
domain, <t>, <day>, hard              # t
starts on given day at any time
domain, <t>, <time>, hard             # t
starts at given time on any day
domain, <t>, starts-before <day> <time>, hard # t
starts at or before day, time
domain, <t>, starts-after <day> <time>, hard # t
starts at or after day, time
domain, <t>, ends-before <day> <time>, hard # t
ends at or before day, time
domain, <t>, ends-after <day> <time>, hard # t
starts at or after day, time
domain, <t>, starts-in <day1> <time1>-<day2> <time2>, hard # day-
time range for start time; includes day1, time1 and day2, time2
domain, <t>, ends-in <day1> <time1>-<day2> <time2>, hard # day-
time range for end time; includes day1, time1 and day2, time2
domain, <t>, starts-before <time>, hard # t
starts at or before time on any day
domain, <t>, ends-before <time>, hard # t
ends at or before time on any day
domain, <t>, starts-after <time>, hard # t
starts at or after time on any day
domain, <t>, ends-after <time>, hard # t
ends at or after time on any day

# soft deadline constraint
domain, <t>, ends-by <day> <time> <cost>, soft # cost per
hour of missing deadline

```

The input specification will consist of several “blocks”, listing the tasks, binary constraints, hard unary constraints and soft deadline constraints for the given problem. A “declaration” of each task will be included before it is used in a constraint. A sample input specification is as follows. Comments are for explanation and do **not** have to be included in the input.

```

# two tasks with two binary constraints and soft deadlines
task, t1 3
task, t2 4
# two binary constraints
constraint, t1 before t2
constraint, t1 same-day t2
# domain constraint
domain, t2 mon

```

```
# soft deadline constraints
domain, t1 ends-by mon 3pm 10
domain, t2 ends-by mon 3pm 10
```

## Preparation

### 1. Set up AIPython

You will need AIPython for this assignment. To find the aipython files, the aipython directory has to be added to the Python path.

Do this temporarily, as done here, so we can find AIPython and run your code (you will not submit any AI Python code).

You can add either the full path (using `os.path.abspath`), or as in the code below, the relative path.

```
In [1]: import sys
sys.path.append('aipython') # change to your directory
sys.path # check that aipython is now on the path
```

```
Out[1]: ['C:\\Users\\32883\\Desktop\\Jupyter Notebook\\COMP9414\\Ass1',
'D:\\Anaconda3\\python312.zip',
'D:\\Anaconda3\\DLLs',
'D:\\Anaconda3\\Lib',
'D:\\Anaconda3',
'',
'D:\\Anaconda3\\Lib\\site-packages',
'D:\\Anaconda3\\Lib\\site-packages\\win32',
'D:\\Anaconda3\\Lib\\site-packages\\win32\\lib',
'D:\\Anaconda3\\Lib\\site-packages\\Pythonwin',
'aipython']
```

### 2. Representation of Day Times

Input and output are day time strings such as 'mon 10am' or a range of day time strings such as 'mon 10am-mon 4pm'.

The CSP will represent these as integer hour numbers in the week, ranging from 0 to 39.

The following code handles the conversion between day time strings and hour numbers.

```
In [2]: # -*- coding: utf-8 -*-

""" day_time string format is a day plus time, e.g. Mon 10am, Tue 4pm, or just T
    if only day or time, returns day number or hour number only
    day_time strings are converted to and from integer hours in the week from 0
    """

class Day_Time():
    num_hours_in_day = 8
    num_days_in_week = 5

    def __init__(self):
        self.day_names = ['mon', 'tue', 'wed', 'thu', 'fri']
```

```

self.time_names = ['9am', '10am', '11am', '12pm', '1pm', '2pm', '3pm', '4pm']

def string_to_week_hour_number(self, day_time_str):
    """ convert a single day_time into an integer hour in the week """
    value = None
    value_type = None
    day_time_list = day_time_str.split()
    if len(day_time_list) == 1:
        str1 = day_time_list[0].strip()
        if str1 in self.time_names: # this is a time
            value = self.time_names.index(str1)
            value_type = 'hour_number'
        else:
            value = self.day_names.index(str1) # this is a day
            value_type = 'day_number'
        # if not day or time, throw an exception
    else:
        value = self.day_names.index(day_time_list[0].strip())*self.num_hours_in_day + self.time_names.index(day_time_list[1].strip())
        value_type = 'week_hour_number'
    return (value_type, value)

def string_to_number_set(self, day_time_list_str):
    """ convert a list of day-times or ranges 'Mon 9am, Tue 9am-Tue 4pm' into a set of integers
    e.g. 'mon 9am-1pm, mon 4pm' -> [0,1,2,3,4,7] """
    number_set = set()
    type1 = None
    for str1 in day_time_list_str.lower().split(','):
        if str1.find('-') > 0:
            # day time range
            type1, v1 = self.string_to_week_hour_number(str1.split('-')[0].strip())
            type2, v2 = self.string_to_week_hour_number(str1.split('-')[1].strip())
            if type1 != type2: return None # error, types in range spec are different
            number_set.update({n for n in range(v1, v2+1)})
        else:
            # single day time
            type2, value2 = self.string_to_week_hour_number(str1)
            if type1 != None and type1 != type2: return None # error: type mismatch
            type1 = type2
            number_set.update({value2})
    return (type1, number_set)

# convert integer hour in week to day time string
def week_hour_number_to_day_time(self, week_hour_number):
    hour = self.day_hour_number(week_hour_number)
    day = self.day_number(week_hour_number)
    return self.day_names[day]+' '+self.time_names[hour]

# convert integer hour in week to integer day and integer time in day
def hour_day_split(self, week_hour_number):
    return (self.day_hour_number(week_hour_number), self.day_number(week_hour_number))

# convert integer hour in week to integer day in week
def day_number(self, week_hour_number):
    return int(week_hour_number / self.num_hours_in_day)

# convert integer hour in week to integer time in day
def day_hour_number(self, week_hour_number):
    return week_hour_number % self.num_hours_in_day

```

```
def __repr__(self):
    day_hour_number = self.week_hour_number % self.num_hours_in_day
    day_number = int(self.week_hour_number / self.num_hours_in_day)
    return self.day_names[day_number]+' '+self.time_names[day_hour_number]
```

### 3. Constraint Satisfaction Problems with Costs over Tasks with Durations

Since AI Python does not provide the CSP class with an explicit cost, we implement our own class that extends `CSP`.

We also store the cost functions and the durations of all tasks explicitly in the CSP.

The durations of the tasks are used in the `hold` function to evaluate constraints.

```
In [3]: from cspProblem import CSP, Constraint

# We need to override Constraint, because tasks have durations
class Task_Constraint(Constraint):
    """A Task_Constraint consists of
    * scope: a tuple of variables
    * spec: text description of the constraint used in debugging
    * condition: a function that can applied to a tuple of values for the variab
    * durations: durations of all tasks
    * func_key: index to the function used to evaluate the constraint
    """
    def __init__(self, scope, spec, condition, durations, func_key):
        super().__init__(scope, condition, spec)
        self.scope = scope
        self.condition = condition
        self.durations = durations
        self.func_key = func_key

    def holds(self, assignment):
        """returns the value of Constraint con evaluated in assignment.

        precondition: all variables are assigned in assignment

        CSP has only binary constraints
        condition is in the form week_hour_number1, week_hour_number2
        add task durations as appropriate to evaluate condition
        """
        if self.func_key == 'before':
            # t1 ends before t2 starts, so we need add duration to t1 assignment
            ass0 = assignment[self.scope[0]] + self.durations[self.scope[0]]
            ass1 = assignment[self.scope[1]]
        elif self.func_key == 'after':
            # t2 ends before t1 starts so we need add duration to t2 assignment
            ass0 = assignment[self.scope[0]]
            ass1 = assignment[self.scope[1]] + self.durations[self.scope[1]]
        elif self.func_key == 'starts-at':
            # t1 starts exactly when t2 ends, so we need add duration to t2 assi
            ass0 = assignment[self.scope[0]]
            ass1 = assignment[self.scope[1]] + self.durations[self.scope[1]]
        else:
            return self.condition(*tuple(assignment[v] for v in self.scope))
        # condition here comes from get_binary_constraint
```

```

        return self.condition(*tuple([ass0, ass1]))

# implement nodes as CSP problems with cost functions
class CSP_with_Cost(CSP):
    """ cost_functions maps a CSP var, here a task name, to a list of functions
    def __init__(self, domains, durations, constraints, cost_functions, soft_day
        self.domains = domains
        self.variables = self.domains.keys()
        super().__init__("title of csp", self.variables, constraints)
        self.durations = durations
        self.cost_functions = cost_functions
        self.soft_day_time = soft_day_time
        self.soft_costs = soft_costs
        self.cost = self.calculate_cost()

# specific to fuzzy scheduling CSP problems
    def calculate_cost(self):
        """ this is really a function f = path cost + heuristic to be used by th
        cost = 0
        #TODO
        for task, domain in self.domains.items():
            dur = self.durations[task]
            soft_dt = self.soft_day_time[task]
            soft_cost = int(self.soft_costs[task])
            cost_func = self.cost_functions[task][0] # ends-by / no-cost

            if len(domain) == 1:
                # assigned task cost
                assigned_time = list(domain)[0]
                task_cost = cost_func(assigned_time, soft_dt, dur, soft_cost)
                if task_cost is not None:
                    cost += task_cost
            elif len(domain) > 1:
                # estimate unassigned cost
                earliest_time = min(domain)
                task_cost = cost_func(earliest_time, soft_dt, dur, soft_cost)
                if task_cost is not None:
                    cost += task_cost
        return cost

    def __repr__(self):
        """ string representation of an arc"""
        return "CSP_with_Cost("+str(list(self.domains.keys()))+":'+str(self.cost

```

This formulates a solver for a CSP with cost as a search problem, using domain splitting with arc consistency to define the successors of a node.

In [4]:

```

from cspConsistency import Con_solver, select, partition_domain
from searchProblem import Arc, Search_problem
from operator import eq, le, ge

# rewrites rather than extends Search_with_AC_from_CSP
class Search_with_AC_from_Cost_CSP(Search_problem):
    """ A search problem with domain splitting and arc consistency """
    def __init__(self, csp):
        self.cons = Con_solver(csp) # copy of the CSP with access to arc consist
        self.domains = self.cons.make_arc_consistent(csp.domains)
        self.constraints = csp.constraints

```



```

self.cost_functions = csp.cost_functions
self.durations = csp.durations
self.soft_day_time = csp.soft_day_time
self.soft_costs = csp.soft_costs
csp.domains = self.domains # after arc consistency
self.csp = csp

def is_goal(self, node):
    """ node is a goal if all domains have exactly 1 element """
    return all(len(node.domains[var]) == 1 for var in node.domains)

def start_node(self):
    return CSP_with_Cost(self.domains, self.durations, self.constraints,
                        self.cost_functions, self.soft_day_time, self.soft_

def neighbors(self, node):
    """returns the neighboring nodes of node.
    """
    neighs = []
    var = select(x for x in node.domains if len(node.domains[x]) > 1) # chos
    if var:
        dom1, dom2 = partition_domain(node.domains[var])
        self.display(2, "Splitting", var, "into", dom1, "and", dom2)
        to_do = self.cons.new_to_do(var, None)
        for dom in [dom1, dom2]:
            newdoms = node.domains | {var: dom} # overwrite domain of var wi
            cons_doms = self.cons.make_arc_consistent(newdoms, to_do)
            if all(len(cons_doms[v]) > 0 for v in cons_doms):
                # all domains are non-empty
                # make new CSP_with_Cost node to continue the search
                csp_node = CSP_with_Cost(cons_doms, self.durations, self.con
                    self.cost_functions, self.soft_day_time, self.soft_
                neighs.append(Arc(node, csp_node))
            else:
                self.display(2, "...", var, "in", dom, "has no solution")
    return neighs

def neighbors_new(self, node, k):
    """returns the neighboring nodes of node.
    """
    neighs = []
    var = select(x for x in node.domains if len(node.domains[x]) > 1) # chos
    if var:
        partitions = partition_domain_new(node.domains[var], k)
        self.display(2, "Splitting", var, "into", partitions)
        to_do = self.cons.new_to_do(var, None)
        for dom in partitions:
            newdoms = node.domains | {var: dom} # overwrite domain of var wi
            cons_doms = self.cons.make_arc_consistent(newdoms, to_do)
            if all(len(cons_doms[v]) > 0 for v in cons_doms):
                # all domains are non-empty
                # make new CSP_with_Cost node to continue the search
                csp_node = CSP_with_Cost(cons_doms, self.durations, self.con
                    self.cost_functions, self.soft_day_time, self.soft_
                neighs.append(Arc(node, csp_node))
            else:
                self.display(2, "...", var, "in", dom, "has no solution")
    return neighs

```

```
def heuristic(self, n):
    return n.cost
```

## 4. Fuzzy Scheduling Constraint Satisfaction Problems

The following code sets up a CSP problem from a given specification.

Hard (unary) domain constraints are applied to reduce the domains of the variables before the constraint solver runs.

```
In [5]: # domain specific CSP builder for week schedule
class CSP_builder():
    # list of text lines without comments and empty lines
    _, default_domain = Day_Time().string_to_number_set('mon 9am-fri 4pm') # sho

    # hard unary constraints: domain is a list of values, params is a single val
    # starts-before, ends-before (for starts-before duration should be 0)
    # vals in domain are actual task start/end date/time, so must be val <= what
    def apply_before(self, param_type, params, duration, domain):
        domain_orig = domain.copy()
        param_val = params.pop()
        for val in domain_orig: # val is week_hour_number
            val1 = val + duration
            h, d = Day_Time().hour_day_split(val1)
            if param_type == 'hour_number' and h > param_val:
                if val in domain: domain.remove(val)
            if param_type == 'day_number' and d > param_val:
                if val in domain: domain.remove(val)
            if param_type == 'week_hour_number' and val1 > param_val:
                if val in domain: domain.remove(val)
        return domain

    def apply_after(self, param_type, params, duration, domain):
        domain_orig = domain.copy()
        param_val = params.pop()
        for val in domain_orig: # val is week_hour_number
            val1 = val + duration
            h, d = Day_Time().hour_day_split(val1)
            if param_type == 'hour_number' and h < param_val:
                if val in domain: domain.remove(val)
            if param_type == 'day_number' and d < param_val:
                if val in domain: domain.remove(val)
            if param_type == 'week_hour_number' and val1 < param_val:
                if val in domain: domain.remove(val)
        return domain

    # day time range only
    # includes starts-in, ends-in
    # duration is 0 for starts-in, task duration for ends-in
    def apply_in(self, params, duration, domain):
        domain_orig = domain.copy()
        for val in domain_orig: # val is week_hour_number
            # task must be within range
            if val in domain and val+duration not in params:
                domain.remove(val)
        return domain

    # task must start at day/time
```

```

def apply_at(self, param_type, param, domain):
    domain_orig = domain.copy()
    for val in domain_orig:
        h, d = Day_Time().hour_day_split(val)
        if param_type == 'hour_number' and param != h:
            if val in domain: domain.remove(val)
        if param_type == 'day_number' and param != d:
            if val in domain: domain.remove(val)
        if param_type == 'week_hour_number' and param != val:
            if val in domain: domain.remove(val)
    return domain

# soft deadline constraints: return cost to break constraint
# ends-by implementation: domain_dt is the day, hour from the domain
# constr_dt is the soft const spec, dur is the duration of task
# soft_cost is the unit cost of completion delay
# so if the tasks starts on domain_dt, it ends on domain_dt+dur
"""

<t> ends-by <day> <time>, both must be specified
delay = day_hour(T2) - day_hour(T1) + 24*(D2 - D1),
where day_hour(9am) = 0, day_hour(5pm) = 7
"""

def ends_by(self, domain_dt, constr_dt_str, dur, soft_cost):
    param_type, params = Day_Time().string_to_number_set(constr_dt_str)
    param_val = params.pop()
    dom_h, dom_d = Day_Time().hour_day_split(domain_dt+dur)
    if param_type == 'week_hour_number':
        con_h, con_d = Day_Time().hour_day_split(param_val)
        return 0 if domain_dt + dur <= param_val else soft_cost*(dom_h - con
    else:
        return None # not good, must be day and time

def no_cost(self, day ,hour):
    return 0

# hard binary constraint, the rest are implemented as gt, lt, eq
def same_day(self, week_hour1, week_hour2):
    h1, d1 = Day_Time().hour_day_split(week_hour1)
    h2, d2 = Day_Time().hour_day_split(week_hour2)
    return d1 == d2

# domain is a list of values
def apply_hard_constraint(self, domain, duration, spec):
    tokens = func_key = spec.split(' ')
    if len(tokens) > 1:
        func_key = spec.split(' ')[0].strip()
        param_type, params = Day_Time().string_to_number_set(spec[len(func_key)
    if func_key == 'starts-before':
        # duration is 0 for starts before, since we do not modify the time
        return self.apply_before(param_type, params, 0, domain)
    if func_key == 'ends-before':
        return self.apply_before(param_type, params, duration, domain)
    if func_key == 'starts-after':
        return self.apply_after(param_type, params, 0, domain)
    if func_key == 'ends-after':
        return self.apply_after(param_type, params, duration, domain)
    if func_key == 'starts-in':
        return self.apply_in(params, 0, domain)
    if func_key == 'ends-in':
        return self.apply_in(params, duration, domain)

```

```

else:
    # here we have task day or time, it has no func key so we need to par
    param_type, params = Day_Time().string_to_week_hour_number(spec)
    return self.apply_at(param_type, params, domain)

def get_cost_function(self, spec):
    func_dict = {'ends-by':self.ends_by, 'no-cost':self.no_cost}
    return [func_dict[spec]]

# spec is the text of a constraint, e.g. 't1 before t2'
# durations are durations of all tasks
def get_binary_constraint(self, spec, durations):
    tokens = spec.strip().split(' ')
    if len(tokens) != 3: return None # error in spec
    # task1 relation task2
    fun_dict = {'before':le, 'after':ge, 'starts-at':eq, 'same-day':self.sam
    return Task_Constraint((tokens[0].strip(), tokens[2].strip()), spec, fun

def get_CSP_with_Cost(self, input_lines):
    # Note: It would be more elegant to make task a class but AIPython is no
    # CSP_with_Cost inherits from CSP, which takes domains and constraints f
    domains = dict()
    constraints = []
    cost_functions = dict()
    durations = dict() # durations of tasks
    soft_day_time = dict() # day time specs of soft constraints
    soft_costs = dict() # costs of soft constraints

    for input_line in input_lines:
        func_spec = None
        input_line_tokens = input_line.strip().split(',')
        if len(input_line_tokens) != 2:
            return None # must have number of tokens = 2
        line_token1 = input_line_tokens[0].strip()
        line_token2 = input_line_tokens[1].strip()
        if line_token1 == 'task':
            tokens = line_token2.split(' ')
            if len(tokens) != 2:
                return None # must have number of tokens = 3
            key = tokens[0].strip()
            # check the duration and save it
            duration = int(tokens[1].strip())
            if duration > Day_Time().num_hours_in_day:
                return None
            durations[key] = duration
            # set zero cost function for this task as default, may add real
            cost_functions[key] = self.get_cost_function('no-cost')
            soft_costs[key] = '0'
            soft_day_time[key] = 'fri 5pm'
            # restrict domain to times that are within allowed range
            # that is start 9-5, start+duration in 9-5
            domains[key] = {x for x in self.default_domain \
                            if Day_Time().day_number(x+duration) \
                            == Day_Time().day_number(x)}
        elif line_token1 == 'domain':
            tokens = line_token2.split(' ')
            if len(tokens) < 2:
                return None # must have number of tokens >= 2
            key = tokens[0].strip()
            # if soft constraint, it is handled differently from hard constr

```

```

        if tokens[1].strip() == 'ends-by':
            # need to retain day time and cost from the line
            # must have task, 'end-by', day, time, cost
            # or task, 'end-by', day, cost
            # or task, 'end-by', time, cost
            if len(tokens) != 5:
                return None
            # get the rest of the line after 'ends-by'
            soft_costs[key] = int(tokens[len(tokens)-1].strip()) # Last
            # pass the day time string to avoid passing param_type
            day_time_str = tokens[2] + ' ' + tokens[3]
            soft_day_time[key] = day_time_str
            cost_functions[key] = self.get_cost_function(tokens[1].strip()
        else:
            # the rest of domain spec, after key, are hard unary domain
            # func spec has day time, we also need duration
            dur = durations[key]
            func_spec = line_token2[len(key):].strip()
            domains[key] = self.apply_hard_constraint(domains[key], dur,
    elif line_token1 == 'constraint': # all binary constraints
        constraints.append(self.get_binary_constraint(line_token2, durat
    else:
        return None

    return CSP_with_Cost(domains, durations, constraints, cost_functions, so

def create_CSP_from_spec(spec: str):
    input_lines = list()
    spec = spec.split('\n')
    # strip comments
    for input_line in spec:
        input_line = input_line.split('#')
        if len(input_line[0]) > 0:
            input_lines.append(input_line[0])
            print(input_line[0])
    # construct initial CSP problem
    csp = CSP_builder()
    csp_problem = csp.get_CSP_with_Cost(input_lines)
    return csp_problem

```

## 5. Greedy Search Constraint Solver using Domain Splitting and Arc Consistency

Create a GreedySearcher to search over the CSP.

The cost function for CSP nodes is used as the heuristic, but is actually a direct estimate of the total path cost function  $f$  used in A\* Search.

```

In [6]: from searchGeneric import AStarSearcher

class GreedySearcher(AStarSearcher):
    """ returns a searcher for a problem.
    Paths can be found by repeatedly calling search().
    """
    def add_to_frontier(self, path):
        """ add path to the frontier with the appropriate cost """
        # value = path.cost + self.problem.heuristic(path.end()) -- A* definitio

```

```

value = path.end().cost
self.frontier.add(path, value)    ### f = path + heuristic

```

Run the GreedySearcher on the CSP derived from the sample input.

**Note: The solution cost will always be 0 (which is wrong for the sample input) until you write the cost function in the cell above.**

In [7]: *# Sample problem specification*

```

sample_spec = """
# two tasks with two binary constraints and soft deadlines
task, t1 3
task, t2 4
# two binary constraints
constraint, t1 before t2
constraint, t1 same-day t2
# domain constraint
domain, t2 mon
# soft deadlines
domain, t1 ends-by mon 3pm 10
domain, t2 ends-by mon 3pm 10
"""

```

In [8]: *# display details (0 turns off)*  
*# Con\_solver.max\_display\_level = 0*  
*# Search\_with\_AC\_from\_Cost\_CSP.max\_display\_level = 2*  
*# GreedySearcher.max\_display\_level = 0*

```

def test_csp_solver(searcher):
    final_path = searcher.search()
    if final_path == None:
        print('No solution')
    else:
        domains = final_path.end().domains
        result_str = ''
        for name, domain in domains.items():
            for n in domain:
                result_str += '\n'+str(name)+': '+Day_Time().week_hour_number_to
        print(result_str[1:]+\nncost: '+str(final_path.end().cost))

csp_problem = create_CSP_from_spec(sample_spec)
solver = GreedySearcher(Search_with_AC_from_Cost_CSP(csp_problem))
test_csp_solver(solver)

```

```

task, t1 3
task, t2 4
constraint, t1 before t2
constraint, t1 same-day t2
domain, t2 mon
domain, t1 ends-by mon 3pm 10
domain, t2 ends-by mon 3pm 10
Solution: title of csp (cost: 0)
  1 paths have been expanded and 0 paths remain in the frontier
t1: mon 9am
t2: mon 12pm
cost: 10

```

## 6. Depth-First Search Constraint Solver

The Depth-First Constraint Solver in AI Python by default uses a random ordering of the variables in the CSP.

We need to modify this code to make it compatible with the arc consistency solver.

Run the solver by calling `dfs_solve1` (first solution) or `dfs_solve_all` (all solutions).

```
In [9]: num_expanded = 0
display = False

def dfs_solver(constraints, domains, context, var_order):
    """ generator for all solutions to csp
        context is an assignment of values to some of the variables
        var_order is a list of the variables in csp that are not in context
    """
    global num_expanded, display
    to_eval = {c for c in constraints if c.can_evaluate(context)}
    if all(c.holds(context) for c in to_eval):
        if var_order == []:
            print("Nodes expanded to reach solution:", num_expanded)
            yield context
        else:
            rem_cons = [c for c in constraints if c not in to_eval]
            var = var_order[0]
            for val in domains[var]:
                if display:
                    print("Setting", var, "to", val)
                num_expanded += 1
                yield from dfs_solver(rem_cons, domains, context|{var:val}, var_order[1:])

def dfs_solve_all(csp, var_order=None):
    """ depth-first CSP solver to return a list of all solutions to csp """
    global num_expanded
    num_expanded = 0
    if var_order == None: # use an arbitrary variable order
        var_order = list(csp.domains)
    return list(dfs_solver(csp.constraints, csp.domains, {}, var_order))

def dfs_solve1(csp, var_order=None):
    """ depth-first CSP solver """
    global num_expanded
    num_expanded = 0
    if var_order == None: # use an arbitrary variable order
        var_order = list(csp.domains)
    for sol in dfs_solver(csp.constraints, csp.domains, {}, var_order):
        return sol # return first one
```

Run the Depth-First Solver on the sample problem.

**Note: Again there are no costs calculated.**

```
In [10]: def test_dfs_solver(csp_problem):
solution = dfs_solve1(csp_problem)
if solution == None:
    print('No solution')
```

```

else:
    result_str = ''
    for name in solution.keys():
        result_str += '\n'+str(name)+': '+Day_Time().week_hour_number_to_day
    print(result_str[1:])

# call the Depth-First Search solver
csp_problem = create_CSP_from_spec(sample_spec)
test_dfs_solver(csp_problem) # set display to True to see nodes expanded

```

```

task, t1 3
task, t2 4
constraint, t1 before t2
constraint, t1 same-day t2
domain, t2 mon
domain, t1 ends-by mon 3pm 10
domain, t2 ends-by mon 3pm 10
Nodes expanded to reach solution: 5
t1: mon 9am
t2: mon 12pm

```

## 7. Depth-First Search Constraint Solver using Forward Checking with MRV Heuristic

The Depth-First Constraint Solver in AI Python by default uses a random ordering of the variables in the CSP.

We redefine the `dfs_solver` methods to implement the MRV (Minimum Remaining Values) heuristic using forward checking.

Because the AI Python code is designed to manipulate domain sets, we also need to redefine `can_evaluate` to handle partial assignments.

```

In [11]: num_expanded = 0
display = False

def can_evaluate(c, assignment):
    """ assignment is a variable:value dictionary
    returns True if the constraint can be evaluated given assignment
    """
    return assignment != {} and all(v in assignment.keys() and type(assignment[v]

def mrv_dfs_solver(constraints, domains, context, var_order):
    """ generator for all solutions to csp.
    context is an assignment of values to some of the variables.
    var_order is a list of the variables in csp that are not in context.
    """
    global num_expanded, display
    if display:
        print("Context", context)
    to_eval = {c for c in constraints if can_evaluate(c, context)}
    if all(c.holds(context) for c in to_eval):
        if var_order == []:
            print("Nodes expanded to reach solution:", num_expanded)
            yield context
        else:
            rem_cons = [c for c in constraints if c not in to_eval] # constraint
            var = var_order[0]

```



```

rem_vars = var_order[1:]
for val in domains[var]:
    if display:
        print("Setting", var, "to", val)
    num_expanded += 1
    rem_context = context|{var:val}
    # apply forward checking on remaining variables
    if len(var_order) > 1:
        rem_vars_original = list((v, list(domains[v].copy())) for v
        if display:
            print("Original domains:", rem_vars_original)
        # constraints that can't already be evaluated in rem_cons
        rem_cons_ff = [c for c in constraints if c in rem_cons and n
        for rem_var in rem_vars:
            # constraints that can be evaluated by adding a value of
            any_value = list(domains[rem_var])[0]
            rem_to_eval = {c for c in rem_cons_ff if can_evaluate(c,
            # new domain for rem_var are the values for which all ne
            rem_vals = domains[rem_var].copy()
            for rem_val in domains[rem_var]:
                # no constraint with rem_var in the existing context
                for c in rem_to_eval:
                    if not c.holds(rem_context|{rem_var: rem_val}):
                        if rem_val in rem_vals:
                            rem_vals.remove(rem_val)
            domains[rem_var] = rem_vals
            # order remaining variables by MRV
            rem_vars.sort(key=lambda v: len(domains[v]))
        if display:
            print("After forward checking:", list((v, domains[v]) fo
        if rem_vars == [] or all(len(domains[rem_var]) > 0 for rem_var i
            yield from mrv_dfs_solver(rem_cons, domains, context|{var:va
        # restore original domains if changed through forward checking
        if len(var_order) > 1:
            if display:
                print("Restoring original domain", rem_vars_original)
            for (v, domain) in rem_vars_original:
                domains[v] = domain
    if display:
        print("Nodes expanded so far:", num_expanded)

def mrv_dfs_solve_all(csp, var_order=None):
    """ depth-first CSP solver to return a list of all solutions to csp """
    global num_expanded
    num_expanded = 0
    if var_order == None: # order variables by MRV
        var_order = list(csp.domains)
        var_order.sort(key=lambda var: len(csp.domains[var]))
    return list(mrv_dfs_solver(csp.constraints, csp.domains, {}, var_order))

def mrv_dfs_solve1(csp, var_order=None):
    """ depth-first CSP solver """
    global num_expanded
    num_expanded = 0
    if var_order == None: # order variables by MRV
        var_order = list(csp.domains)
        var_order.sort(key=lambda var: len(csp.domains[var]))
    for sol in mrv_dfs_solver(csp.constraints, csp.domains, {}, var_order):
        return sol # return first one

```

Run this solver on the sample problem.

**Note: Again there are no costs calculated.**

```
In [12]: def test_mrv_dfs_solver(csp_problem):
          solution = mrv_dfs_solve1(csp_problem)
          if solution == None:
              print('No solution')
          else:
              result_str = ''
              for name in solution.keys():
                  result_str += '\n'+str(name)+' : '+Day_Time().week_hour_number_to_day
              print(result_str[1:])

          # call the Depth-First MRV Search solver
          csp_problem = create_CSP_from_spec(sample_spec)
          test_mrv_dfs_solver(csp_problem) # set display to True to see nodes expanded
```

```
task, t1 3
task, t2 4
constraint, t1 before t2
constraint, t1 same-day t2
domain, t2 mon
domain, t1 ends-by mon 3pm 10
domain, t2 ends-by mon 3pm 10
Nodes expanded to reach solution: 5
t2: mon 12pm
t1: mon 9am
```

## Assignment

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### Question 1 (4 marks)

Consider the search spaces for the fuzzy scheduling CSP solvers – domain splitting with arc consistency and the DFS solver (without forward checking).

- Describe the search spaces in terms of start state, successor functions and goal state(s) (1 mark)
- What is the branching factor and maximum depth to find any solution for the two algorithms (ignoring costs)? (1 mark)
- What is the worst case time and space complexity of the two search algorithms? (1 mark)
- Give one example of a fuzzy scheduling problem that is *easier* for the domain splitting with arc consistency solver than it is for the DFS solver, and explain why (1 mark)

For the second and third part-questions, give the answer in a general form in terms of fuzzy scheduling CSP size parameters.

## Answers for Question 1

(1)

1. Start state: DS\_ARC: None of the variables is assigned, and domains are the values that satisfy the constraints.
2. Start state: DFS: None of the variables is assigned and the domain of each variable is complete.
3. Successor functions: DS\_ARC: First, choose the task domain with more than 1 values and split it into two sub-domains. Second, use the arc consistency to filter possible values and exclude impossible values. And then repeat the processes.
4. Successor functions: DFS: Choose a empty task and assign a value. Then check the value whether satisfy the constraints. and repeat it. If one branch is check, back to nearest father node to check another branch. And repeat.
5. Goal states: DS\_ARC: All of the task has a possible value and do not violate the constraints.
6. Goal states: DFS: All of the task is assigned by a possible value and do not violate the constraints.

(2)

1. Branch factor of DS\_ARC: Split the domain into two sub-domains. So, factor = 2.
2. Branch factor of DFS: Number of values in the domain of the chosen variable.
3. Maximum depth of DS\_ARC: The number of variables (tasks), each layer is a variable.
4. Maximum depth of DFS: The Domain of The variable is splitted into two domains, so for each is  $\log(\text{Domain})$ , and we have  $n$  tasks, so the depth is  $n \cdot \log(\text{Domain})$ .

(3)

1. DS\_ARC: Time complexity:  $O(\text{domain}^n \cdot e \cdot \text{domain}^3)$ , Space : complexity:  $O(\text{domain} \cdot n)$ , where  $n$  is the number of variables.
2. DFS: Time complexity:  $O(\text{domain}^n)$ , Space complexity:  $O(\text{domain} \cdot n)$

(4)

1. For example, whrn we have a domain for taks 1 start time such that {1,2,3,4}, so , it will not search in these value branches. So ,is is easier for the DS\_ARC to get the solution for CSP.
2. For the Domain split with arc consistency, when check the solution, the first step for it is checking the values that violate the constraints, so it reduce the difficulty of finding the solutions instead of finding the solution by searching in all the values.

## Question 2 (5 marks)

Define the *cost* function for a fuzzy scheduling CSP (i.e. a node in the search space for domain splitting and arc consistency) as the total cost of the soft deadline constraints violated for all of the variables, assuming that each variable is assigned one of the best

possible values from its domain, where a “best” value for a variable  $v$  is one that has the lowest cost to violate the soft deadline constraint (if any) for that variable  $v$ .

- Implement the cost function in the indicated cell and place a copy of the code below (3 marks)
- What is its computational complexity (give a general form in terms of fuzzy scheduling CSP size parameters)? (1 mark)
- Show that the cost function  $f$  never decreases along a path, and explain why this means the search algorithm is optimal (1 mark)

```
In [13]: # Code for Question 2
# Place a copy of your code here and run it in the relevant cell
# specific to fuzzy scheduling CSP problems
def calculate_cost(self):
    """ this is really a function f = path cost + heuristic to be used by the co
    cost = 0
    #TODO
    for task, domain in self.domains.items():
        dur = self.durations[task]
        soft_dt = self.soft_day_time[task]
        soft_cost = int(self.soft_costs[task])
        cost_func = self.cost_functions[task][0] # ends-by / no-cost

        if len(domain) == 1:
            # assigned task cost
            assigned_time = list(domain)[0]
            task_cost = cost_func(assigned_time, soft_dt, dur, soft_cost)
            if task_cost is not None:
                cost += task_cost
        elif len(domain) > 1:
            # estimate unassigned cost
            earliest_time = min(domain)
            task_cost = cost_func(earliest_time, soft_dt, dur, soft_cost)
            if task_cost is not None:
                cost += task_cost
    return cost
```

### Answers for Question 2

(2) The computation complexity of the cost function is  $O(\text{domain} \cdot n)$ , the  $n$  is the number of variables.

(3) According to the A\* algorithm, it can know that the  $f = g + h$ . In the cost calculate method, the  $g$  is calculated by the past cost as the assigned values and variables. The other part of the un-assigned tasks belong to the  $h$ , it is calculated by the minimum value in its domain. So, as the  $g$  increase after each epoch or assignment, the value of the  $h$  decrease with a smaller value than the  $g$  because the real cost in the  $g$  is always bigger than the estimated value in the  $h$ .

### Question 3 (4 marks)

Conduct an empirical evaluation of the domain splitting CSP solver using the cost function defined as above compared to using no cost function (i.e. the zero cost function,

as originally defined in the above cell). Use the *average number of nodes expanded* as a metric to compare the two algorithms.

- Write a function `generate_problem(n)` that takes an integer `n` and generates a problem specification with `n` tasks and a random set of hard constraints and soft deadline constraints in the correct format for the constraint solvers (2 marks)

Run the CSP solver (with and without the cost function) over a number of problems of size `n` for a range of values of `n`.

- Plot the performance of the two constraint solving algorithms on the above metric against `n` (1 mark)
- Quantify the performance gain (if any) achieved by the use of this cost function (1 mark)

```
In [14]: # Code for Question 3
# Place your code here
import random

def generate_problem(n):
    days = ["Mon", "Tue", "Wed", "Thu", "Fri"]
    times = ["9am", "10am", "11am", "12pm", "1pm", "2pm", "3pm", "4pm"]
    lines = []

    # task and durations
    for i in range(n):
        task = f"t{i+1}"
        duration = random.randint(1, 5)
        lines.append(f"task, {task} {duration}")

    # soft
    for i in range(n):
        task = f"t{i+1}"
        deadline_day = random.choice(days)
        deadline_time = random.choice(times)
        penalty = 10
        lines.append(f"domain, {task} ends-by {deadline_day} {deadline_time} {penalty}")

    #hard
    task_names = [f"t{i+1}" for i in range(n)]
    for _ in range(random.randint(1, n)):
        t1, t2 = random.sample(task_names, 2)
        spec = random.choice(["before", "same-day"])
        lines.append(f"constraint, {t1} {spec} {t2}")

    return "\n".join(lines)

# n = 4
# spec_text = generate_problem(n)
# print(spec_text)
```

### Answers for Question 3

Write the other answers here.

(1)

```

In [15]: import matplotlib.pyplot as plt

ns = [2,3,4,5,6,7,8]
num_epochs = 5
avg_nodes_with_cost = []
avg_nodes_no_cost = []

for n in ns:
    nodes_with_cost = []
    nodes_no_cost = []

    for _ in range(num_epochs):
        spec_text = generate_problem(n)

        # Has cost function
        csp_problem = create_CSP_from_spec(spec_text)
        solver = GreedySearcher(Search_with_AC_from_Cost_CSP(csp_problem))
        solver.search()          ## test_csp_solver rewriter part
        nodes_with_cost.append(solver.num_expanded) # get nodes number

        # No cost function
        csp_problem_nocost = create_CSP_from_spec(spec_text)
        for task in csp_problem_nocost.cost_functions:
            csp_problem_nocost.cost_functions[task] = [lambda *args, **kwargs: 0
            solver = GreedySearcher(Search_with_AC_from_Cost_CSP(csp_problem_nocost)
            solver.search()          ## test_csp_solver rewrite
            nodes_no_cost.append(solver.num_expanded)

    avg_nodes_with_cost.append(sum(nodes_with_cost)/num_epochs)
    avg_nodes_no_cost.append(sum(nodes_no_cost)/num_epochs)

```

```

task, t1 3
task, t2 1
domain, t1 ends-by Mon 10am 10
domain, t2 ends-by Tue 1pm 10
constraint, t1 before t2
constraint, t1 before t2
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp (cost: 9)
  10 paths have been expanded and 9 paths remain in the frontier
task, t1 3
task, t2 1
domain, t1 ends-by Mon 10am 10
domain, t2 ends-by Tue 1pm 10
constraint, t1 before t2
constraint, t1 before t2
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp (cost: 9)
  10 paths have been expanded and 9 paths remain in the frontier
task, t1 5
task, t2 3
domain, t1 ends-by Thu 11am 10
domain, t2 ends-by Wed 11am 10
constraint, t2 same-day t1
constraint, t2 same-day t1
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp (cost: 5)
  6 paths have been expanded and 5 paths remain in the frontier
task, t1 5
task, t2 3
domain, t1 ends-by Thu 11am 10
domain, t2 ends-by Wed 11am 10
constraint, t2 same-day t1
constraint, t2 same-day t1
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp (cost: 5)
  6 paths have been expanded and 5 paths remain in the frontier
task, t1 1
task, t2 3
domain, t1 ends-by Mon 3pm 10
domain, t2 ends-by Mon 1pm 10
constraint, t2 same-day t1
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp (cost: 7)
  8 paths have been expanded and 7 paths remain in the frontier
task, t1 1
task, t2 3
domain, t1 ends-by Mon 3pm 10
domain, t2 ends-by Mon 1pm 10
constraint, t2 same-day t1
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp (cost: 7)
  8 paths have been expanded and 7 paths remain in the frontier
task, t1 4
task, t2 5
domain, t1 ends-by Fri 10am 10
domain, t2 ends-by Mon 2pm 10
constraint, t2 same-day t1
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit

```

le of csp --> title of csp (cost: 5)  
 6 paths have been expanded and 5 paths remain in the frontier  
 task, t1 4  
 task, t2 5  
 domain, t1 ends-by Fri 10am 10  
 domain, t2 ends-by Mon 2pm 10  
 constraint, t2 same-day t1  
 Solution: title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp (cost: 5)  
 6 paths have been expanded and 5 paths remain in the frontier  
 task, t1 4  
 task, t2 1  
 domain, t1 ends-by Mon 3pm 10  
 domain, t2 ends-by Mon 9am 10  
 constraint, t1 same-day t2  
 constraint, t2 same-day t1  
 Solution: title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp (cost: 6)  
 7 paths have been expanded and 6 paths remain in the frontier  
 task, t1 4  
 task, t2 1  
 domain, t1 ends-by Mon 3pm 10  
 domain, t2 ends-by Mon 9am 10  
 constraint, t1 same-day t2  
 constraint, t2 same-day t1  
 Solution: title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp (cost: 6)  
 7 paths have been expanded and 6 paths remain in the frontier  
 task, t1 3  
 task, t2 3  
 task, t3 1  
 domain, t1 ends-by Fri 1pm 10  
 domain, t2 ends-by Thu 1pm 10  
 domain, t3 ends-by Tue 11am 10  
 constraint, t1 before t3  
 constraint, t2 before t1  
 constraint, t3 before t1  
 No (more) solutions. Total of 1 paths expanded.  
 task, t1 3  
 task, t2 3  
 task, t3 1  
 domain, t1 ends-by Fri 1pm 10  
 domain, t2 ends-by Thu 1pm 10  
 domain, t3 ends-by Tue 11am 10  
 constraint, t1 before t3  
 constraint, t2 before t1  
 constraint, t3 before t1  
 No (more) solutions. Total of 1 paths expanded.  
 task, t1 3  
 task, t2 1  
 task, t3 2  
 domain, t1 ends-by Wed 1pm 10  
 domain, t2 ends-by Wed 4pm 10  
 domain, t3 ends-by Thu 3pm 10  
 constraint, t3 before t1  
 Solution: title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp (cost: 9)  
 10 paths have been expanded and 9 paths remain in the frontier  
 task, t1 3



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task, t2 1
task, t3 2
domain, t1 ends-by Wed 1pm 10
domain, t2 ends-by Wed 4pm 10
domain, t3 ends-by Thu 3pm 10
constraint, t3 before t1
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp (cost: 9)
  10 paths have been expanded and 9 paths remain in the frontier
task, t1 1
task, t2 2
task, t3 4
domain, t1 ends-by Thu 1pm 10
domain, t2 ends-by Mon 4pm 10
domain, t3 ends-by Mon 2pm 10
constraint, t1 same-day t2
constraint, t1 same-day t2
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp --> title of csp --> title of csp (cost: 11)
  12 paths have been expanded and 11 paths remain in the frontier
task, t1 1
task, t2 2
task, t3 4
domain, t1 ends-by Thu 1pm 10
domain, t2 ends-by Mon 4pm 10
domain, t3 ends-by Mon 2pm 10
constraint, t1 same-day t2
constraint, t1 same-day t2
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp --> title of csp --> title of csp (cost: 11)
  12 paths have been expanded and 11 paths remain in the frontier
task, t1 3
task, t2 4
task, t3 1
domain, t1 ends-by Fri 11am 10
domain, t2 ends-by Fri 11am 10
domain, t3 ends-by Wed 1pm 10
constraint, t2 before t1
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp --> title of csp --> title of csp (cost: 11)
  12 paths have been expanded and 11 paths remain in the frontier
task, t1 3
task, t2 4
task, t3 1
domain, t1 ends-by Fri 11am 10
domain, t2 ends-by Fri 11am 10
domain, t3 ends-by Wed 1pm 10
constraint, t2 before t1
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp --> title of csp --> title of csp (cost: 11)
  12 paths have been expanded and 11 paths remain in the frontier
task, t1 3
task, t2 1
task, t3 3
domain, t1 ends-by Wed 4pm 10

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domain, t2 ends-by Thu 4pm 10
domain, t3 ends-by Tue 1pm 10
constraint, t2 same-day t3
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp (cost: 11)
12 paths have been expanded and 11 paths remain in the frontier
task, t1 3
task, t2 1
task, t3 3
domain, t1 ends-by Wed 4pm 10
domain, t2 ends-by Thu 4pm 10
domain, t3 ends-by Tue 1pm 10
constraint, t2 same-day t3
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp (cost: 11)
12 paths have been expanded and 11 paths remain in the frontier
task, t1 1
task, t2 5
task, t3 3
task, t4 2
domain, t1 ends-by Wed 9am 10
domain, t2 ends-by Tue 9am 10
domain, t3 ends-by Wed 2pm 10
domain, t4 ends-by Tue 9am 10
constraint, t3 before t1
constraint, t2 before t3
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp (cost: 9)
10 paths have been expanded and 9 paths remain in the frontier
task, t1 1
task, t2 5
task, t3 3
task, t4 2
domain, t1 ends-by Wed 9am 10
domain, t2 ends-by Tue 9am 10
domain, t3 ends-by Wed 2pm 10
domain, t4 ends-by Tue 9am 10
constraint, t3 before t1
constraint, t2 before t3
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp (cost: 9)
10 paths have been expanded and 9 paths remain in the frontier
task, t1 3
task, t2 2
task, t3 1
task, t4 2
domain, t1 ends-by Thu 3pm 10
domain, t2 ends-by Tue 2pm 10
domain, t3 ends-by Mon 1pm 10
domain, t4 ends-by Mon 2pm 10
constraint, t4 same-day t1
constraint, t1 same-day t3
constraint, t2 before t1
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp (co
st: 8)

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9 paths have been expanded and 8 paths remain in the frontier

task, t1 3  
task, t2 2  
task, t3 1  
task, t4 2  
domain, t1 ends-by Thu 3pm 10  
domain, t2 ends-by Tue 2pm 10  
domain, t3 ends-by Mon 1pm 10  
domain, t4 ends-by Mon 2pm 10  
constraint, t4 same-day t1  
constraint, t1 same-day t3  
constraint, t2 before t1  
Solution: title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp (cost: 8)

9 paths have been expanded and 8 paths remain in the frontier

task, t1 5  
task, t2 1  
task, t3 1  
task, t4 3  
domain, t1 ends-by Tue 9am 10  
domain, t2 ends-by Fri 1pm 10  
domain, t3 ends-by Tue 10am 10  
domain, t4 ends-by Tue 2pm 10  
constraint, t2 same-day t3  
constraint, t4 before t2  
Solution: title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp (cost: 10)

11 paths have been expanded and 10 paths remain in the frontier

task, t1 5  
task, t2 1  
task, t3 1  
task, t4 3  
domain, t1 ends-by Tue 9am 10  
domain, t2 ends-by Fri 1pm 10  
domain, t3 ends-by Tue 10am 10  
domain, t4 ends-by Tue 2pm 10  
constraint, t2 same-day t3  
constraint, t4 before t2  
Solution: title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp (cost: 10)

11 paths have been expanded and 10 paths remain in the frontier

task, t1 2  
task, t2 4  
task, t3 2  
task, t4 4  
domain, t1 ends-by Fri 3pm 10  
domain, t2 ends-by Tue 10am 10  
domain, t3 ends-by Thu 4pm 10  
domain, t4 ends-by Thu 10am 10  
constraint, t3 before t2  
constraint, t3 before t1  
constraint, t4 same-day t3  
constraint, t1 before t3  
No (more) solutions. Total of 1 paths expanded.

task, t1 2  
task, t2 4  
task, t3 2

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task, t4 4
domain, t1 ends-by Fri 3pm 10
domain, t2 ends-by Tue 10am 10
domain, t3 ends-by Thu 4pm 10
domain, t4 ends-by Thu 10am 10
constraint, t3 before t2
constraint, t3 before t1
constraint, t4 same-day t3
constraint, t1 before t3
No (more) solutions. Total of 1 paths expanded.
task, t1 4
task, t2 4
task, t3 5
task, t4 1
domain, t1 ends-by Wed 9am 10
domain, t2 ends-by Mon 1pm 10
domain, t3 ends-by Mon 12pm 10
domain, t4 ends-by Thu 2pm 10
constraint, t1 before t3
constraint, t1 same-day t2
constraint, t3 same-day t4
constraint, t3 before t2
No (more) solutions. Total of 1 paths expanded.
task, t1 4
task, t2 4
task, t3 5
task, t4 1
domain, t1 ends-by Wed 9am 10
domain, t2 ends-by Mon 1pm 10
domain, t3 ends-by Mon 12pm 10
domain, t4 ends-by Thu 2pm 10
constraint, t1 before t3
constraint, t1 same-day t2
constraint, t3 same-day t4
constraint, t3 before t2
No (more) solutions. Total of 1 paths expanded.
task, t1 1
task, t2 2
task, t3 5
task, t4 3
task, t5 5
domain, t1 ends-by Tue 1pm 10
domain, t2 ends-by Fri 1pm 10
domain, t3 ends-by Fri 4pm 10
domain, t4 ends-by Thu 1pm 10
domain, t5 ends-by Mon 3pm 10
constraint, t4 same-day t3
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp --> title of csp --> title of csp --> title of csp --> title of csp
--> title of csp --> title of csp --> title of csp --> title of csp (cost: 17)
18 paths have been expanded and 17 paths remain in the frontier
task, t1 1
task, t2 2
task, t3 5
task, t4 3
task, t5 5
domain, t1 ends-by Tue 1pm 10
domain, t2 ends-by Fri 1pm 10
domain, t3 ends-by Fri 4pm 10

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domain, t4 ends-by Thu 1pm 10
domain, t5 ends-by Mon 3pm 10
constraint, t4 same-day t3
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp --> title of csp --> title of csp --> title of csp --> title of csp
--> title of csp --> title of csp --> title of csp --> title of csp (cost: 17)
18 paths have been expanded and 17 paths remain in the frontier
task, t1 5
task, t2 5
task, t3 5
task, t4 4
task, t5 4
domain, t1 ends-by Thu 9am 10
domain, t2 ends-by Thu 3pm 10
domain, t3 ends-by Tue 1pm 10
domain, t4 ends-by Wed 12pm 10
domain, t5 ends-by Tue 9am 10
constraint, t3 before t2
constraint, t5 before t2
constraint, t3 same-day t1
constraint, t5 before t1
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp --> title of csp --> title of csp --> title of csp --> title of csp
(cost: 13)
14 paths have been expanded and 13 paths remain in the frontier
task, t1 5
task, t2 5
task, t3 5
task, t4 4
task, t5 4
domain, t1 ends-by Thu 9am 10
domain, t2 ends-by Thu 3pm 10
domain, t3 ends-by Tue 1pm 10
domain, t4 ends-by Wed 12pm 10
domain, t5 ends-by Tue 9am 10
constraint, t3 before t2
constraint, t5 before t2
constraint, t3 same-day t1
constraint, t5 before t1
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp --> title of csp --> title of csp --> title of csp --> title of csp
(cost: 13)
14 paths have been expanded and 13 paths remain in the frontier
task, t1 4
task, t2 2
task, t3 1
task, t4 1
task, t5 4
domain, t1 ends-by Fri 12pm 10
domain, t2 ends-by Wed 9am 10
domain, t3 ends-by Tue 4pm 10
domain, t4 ends-by Thu 9am 10
domain, t5 ends-by Tue 10am 10
constraint, t1 same-day t3
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp --> title of csp --> title of csp --> title of csp --> title of csp

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--> title of csp --> title of csp --> title of csp --> title of csp --> title of
csp --> title of csp (cost: 19)
  20 paths have been expanded and 19 paths remain in the frontier
task, t1 4
task, t2 2
task, t3 1
task, t4 1
task, t5 4
domain, t1 ends-by Fri 12pm 10
domain, t2 ends-by Wed 9am 10
domain, t3 ends-by Tue 4pm 10
domain, t4 ends-by Thu 9am 10
domain, t5 ends-by Tue 10am 10
constraint, t1 same-day t3
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp --> title of csp --> title of csp --> title of csp --> title of csp
--> title of csp --> title of csp --> title of csp --> title of csp --> title of
csp --> title of csp (cost: 19)
  20 paths have been expanded and 19 paths remain in the frontier
task, t1 3
task, t2 3
task, t3 5
task, t4 1
task, t5 1
domain, t1 ends-by Wed 9am 10
domain, t2 ends-by Wed 2pm 10
domain, t3 ends-by Wed 10am 10
domain, t4 ends-by Mon 12pm 10
domain, t5 ends-by Tue 11am 10
constraint, t4 same-day t1
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp --> title of csp --> title of csp --> title of csp --> title of csp
--> title of csp --> title of csp --> title of csp --> title of csp --> title of
csp (cost: 18)
  19 paths have been expanded and 18 paths remain in the frontier
task, t1 3
task, t2 3
task, t3 5
task, t4 1
task, t5 1
domain, t1 ends-by Wed 9am 10
domain, t2 ends-by Wed 2pm 10
domain, t3 ends-by Wed 10am 10
domain, t4 ends-by Mon 12pm 10
domain, t5 ends-by Tue 11am 10
constraint, t4 same-day t1
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp --> title of csp --> title of csp --> title of csp --> title of csp
--> title of csp --> title of csp --> title of csp --> title of csp --> title of
csp (cost: 18)
  19 paths have been expanded and 18 paths remain in the frontier
task, t1 1
task, t2 1
task, t3 4
task, t4 5
task, t5 3
domain, t1 ends-by Thu 3pm 10

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domain, t2 ends-by Wed 1pm 10
domain, t3 ends-by Tue 9am 10
domain, t4 ends-by Wed 3pm 10
domain, t5 ends-by Fri 1pm 10
constraint, t5 before t1
constraint, t1 same-day t5
constraint, t2 same-day t4
constraint, t1 same-day t2
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp --> title of csp --> title of csp --> title of csp (cost: 12)
  13 paths have been expanded and 12 paths remain in the frontier
task, t1 1
task, t2 1
task, t3 4
task, t4 5
task, t5 3
domain, t1 ends-by Thu 3pm 10
domain, t2 ends-by Wed 1pm 10
domain, t3 ends-by Tue 9am 10
domain, t4 ends-by Wed 3pm 10
domain, t5 ends-by Fri 1pm 10
constraint, t5 before t1
constraint, t1 same-day t5
constraint, t2 same-day t4
constraint, t1 same-day t2
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp --> title of csp --> title of csp --> title of csp (cost: 12)
  13 paths have been expanded and 12 paths remain in the frontier
task, t1 3
task, t2 3
task, t3 4
task, t4 2
task, t5 1
task, t6 3
domain, t1 ends-by Wed 12pm 10
domain, t2 ends-by Tue 12pm 10
domain, t3 ends-by Wed 10am 10
domain, t4 ends-by Fri 3pm 10
domain, t5 ends-by Mon 12pm 10
domain, t6 ends-by Tue 4pm 10
constraint, t1 same-day t5
constraint, t3 same-day t4
constraint, t6 before t3
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp --> title of csp --> title of csp --> title of csp --> title of csp
--> title of csp --> title of csp --> title of csp (cost: 16)
  17 paths have been expanded and 16 paths remain in the frontier
task, t1 3
task, t2 3
task, t3 4
task, t4 2
task, t5 1
task, t6 3
domain, t1 ends-by Wed 12pm 10
domain, t2 ends-by Tue 12pm 10
domain, t3 ends-by Wed 10am 10
domain, t4 ends-by Fri 3pm 10

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domain, t5 ends-by Mon 12pm 10  
 domain, t6 ends-by Tue 4pm 10  
 constraint, t1 same-day t5  
 constraint, t3 same-day t4  
 constraint, t6 before t3  
 Solution: title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp (cost: 20)  
 21 paths have been expanded and 20 paths remain in the frontier  
 task, t1 2  
 task, t2 1  
 task, t3 5  
 task, t4 1  
 task, t5 3  
 task, t6 3  
 domain, t1 ends-by Wed 4pm 10  
 domain, t2 ends-by Mon 2pm 10  
 domain, t3 ends-by Mon 1pm 10  
 domain, t4 ends-by Tue 4pm 10  
 domain, t5 ends-by Mon 12pm 10  
 domain, t6 ends-by Mon 11am 10  
 constraint, t1 same-day t3  
 constraint, t5 same-day t3  
 Solution: title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp (cost: 21)  
 22 paths have been expanded and 21 paths remain in the frontier  
 task, t1 2  
 task, t2 1  
 task, t3 5  
 task, t4 1  
 task, t5 3  
 task, t6 3  
 domain, t1 ends-by Wed 4pm 10  
 domain, t2 ends-by Mon 2pm 10  
 domain, t3 ends-by Mon 1pm 10  
 domain, t4 ends-by Tue 4pm 10  
 domain, t5 ends-by Mon 12pm 10  
 domain, t6 ends-by Mon 11am 10  
 constraint, t1 same-day t3  
 constraint, t5 same-day t3  
 Solution: title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp (cost: 21)  
 22 paths have been expanded and 21 paths remain in the frontier  
 task, t1 5  
 task, t2 4  
 task, t3 1  
 task, t4 4  
 task, t5 2  
 task, t6 5  
 domain, t1 ends-by Fri 11am 10  
 domain, t2 ends-by Wed 2pm 10  
 domain, t3 ends-by Wed 3pm 10



domain, t4 ends-by Fri 12pm 10  
 domain, t5 ends-by Thu 11am 10  
 domain, t6 ends-by Wed 9am 10  
 constraint, t5 before t3  
 constraint, t4 before t1  
 constraint, t2 same-day t6  
 constraint, t1 before t5  
 constraint, t2 same-day t6  
 Solution: title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp (cost: 14)  
 15 paths have been expanded and 14 paths remain in the frontier  
 task, t1 5  
 task, t2 4  
 task, t3 1  
 task, t4 4  
 task, t5 2  
 task, t6 5  
 domain, t1 ends-by Fri 11am 10  
 domain, t2 ends-by Wed 2pm 10  
 domain, t3 ends-by Wed 3pm 10  
 domain, t4 ends-by Fri 12pm 10  
 domain, t5 ends-by Thu 11am 10  
 domain, t6 ends-by Wed 9am 10  
 constraint, t5 before t3  
 constraint, t4 before t1  
 constraint, t2 same-day t6  
 constraint, t1 before t5  
 constraint, t2 same-day t6  
 Solution: title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp (cost: 14)  
 15 paths have been expanded and 14 paths remain in the frontier  
 task, t1 4  
 task, t2 1  
 task, t3 3  
 task, t4 2  
 task, t5 2  
 task, t6 4  
 domain, t1 ends-by Wed 9am 10  
 domain, t2 ends-by Tue 9am 10  
 domain, t3 ends-by Wed 9am 10  
 domain, t4 ends-by Thu 11am 10  
 domain, t5 ends-by Thu 1pm 10  
 domain, t6 ends-by Mon 2pm 10  
 constraint, t5 same-day t6  
 constraint, t3 same-day t4  
 constraint, t3 same-day t1  
 constraint, t4 same-day t5  
 constraint, t1 same-day t6  
 Solution: title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp (cost: 17)  
 18 paths have been expanded and 17 paths remain in the frontier  
 task, t1 4  
 task, t2 1  
 task, t3 3

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task, t4 2
task, t5 2
task, t6 4
domain, t1 ends-by Wed 9am 10
domain, t2 ends-by Tue 9am 10
domain, t3 ends-by Wed 9am 10
domain, t4 ends-by Thu 11am 10
domain, t5 ends-by Thu 1pm 10
domain, t6 ends-by Mon 2pm 10
constraint, t5 same-day t6
constraint, t3 same-day t4
constraint, t3 same-day t1
constraint, t4 same-day t5
constraint, t1 same-day t6
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp --> title of csp --> title of csp --> title of csp --> title of csp
--> title of csp --> title of csp --> title of csp --> title of csp (cost: 17)
  18 paths have been expanded and 17 paths remain in the frontier
task, t1 1
task, t2 4
task, t3 1
task, t4 3
task, t5 3
task, t6 2
domain, t1 ends-by Tue 3pm 10
domain, t2 ends-by Thu 9am 10
domain, t3 ends-by Mon 3pm 10
domain, t4 ends-by Wed 9am 10
domain, t5 ends-by Mon 9am 10
domain, t6 ends-by Mon 1pm 10
constraint, t6 before t5
constraint, t4 before t6
constraint, t6 before t2
constraint, t1 before t5
constraint, t1 before t5
constraint, t5 same-day t4
No (more) solutions. Total of 1 paths expanded.
task, t1 1
task, t2 4
task, t3 1
task, t4 3
task, t5 3
task, t6 2
domain, t1 ends-by Tue 3pm 10
domain, t2 ends-by Thu 9am 10
domain, t3 ends-by Mon 3pm 10
domain, t4 ends-by Wed 9am 10
domain, t5 ends-by Mon 9am 10
domain, t6 ends-by Mon 1pm 10
constraint, t6 before t5
constraint, t4 before t6
constraint, t6 before t2
constraint, t1 before t5
constraint, t1 before t5
constraint, t5 same-day t4
No (more) solutions. Total of 1 paths expanded.
task, t1 1
task, t2 4
task, t3 2

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```

task, t4 1
task, t5 4
task, t6 2
task, t7 1
domain, t1 ends-by Tue 10am 10
domain, t2 ends-by Fri 2pm 10
domain, t3 ends-by Thu 11am 10
domain, t4 ends-by Fri 1pm 10
domain, t5 ends-by Thu 10am 10
domain, t6 ends-by Mon 4pm 10
domain, t7 ends-by Tue 4pm 10
constraint, t4 before t7
constraint, t2 before t7
constraint, t4 before t1
constraint, t5 same-day t3
constraint, t2 same-day t1
constraint, t3 same-day t5
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp --> title of csp --> title of csp --> title of csp --> title of csp
--> title of csp --> title of csp --> title of csp --> title of csp --> title of
csp --> title of csp --> title of csp --> title of csp (cost: 21)
  22 paths have been expanded and 21 paths remain in the frontier
task, t1 1
task, t2 4
task, t3 2
task, t4 1
task, t5 4
task, t6 2
task, t7 1
domain, t1 ends-by Tue 10am 10
domain, t2 ends-by Fri 2pm 10
domain, t3 ends-by Thu 11am 10
domain, t4 ends-by Fri 1pm 10
domain, t5 ends-by Thu 10am 10
domain, t6 ends-by Mon 4pm 10
domain, t7 ends-by Tue 4pm 10
constraint, t4 before t7
constraint, t2 before t7
constraint, t4 before t1
constraint, t5 same-day t3
constraint, t2 same-day t1
constraint, t3 same-day t5
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp --> title of csp --> title of csp --> title of csp --> title of csp
--> title of csp --> title of csp --> title of csp --> title of csp --> title of
csp --> title of csp --> title of csp --> title of csp (cost: 21)
  22 paths have been expanded and 21 paths remain in the frontier
task, t1 2
task, t2 5
task, t3 4
task, t4 4
task, t5 1
task, t6 4
task, t7 5
domain, t1 ends-by Thu 2pm 10
domain, t2 ends-by Tue 4pm 10
domain, t3 ends-by Thu 9am 10
domain, t4 ends-by Fri 12pm 10

```

```

domain, t5 ends-by Fri 1pm 10
domain, t6 ends-by Mon 12pm 10
domain, t7 ends-by Thu 11am 10
constraint, t7 before t6
constraint, t5 before t2
constraint, t7 before t2
constraint, t1 same-day t7
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp --> title of csp --> title of csp --> title of csp --> title of csp
--> title of csp --> title of csp --> title of csp --> title of csp --> title of
csp --> title of csp --> title of csp --> title of csp --> title of csp (cost: 2
2)
23 paths have been expanded and 22 paths remain in the frontier
task, t1 2
task, t2 5
task, t3 4
task, t4 4
task, t5 1
task, t6 4
task, t7 5
domain, t1 ends-by Thu 2pm 10
domain, t2 ends-by Tue 4pm 10
domain, t3 ends-by Thu 9am 10
domain, t4 ends-by Fri 12pm 10
domain, t5 ends-by Fri 1pm 10
domain, t6 ends-by Mon 12pm 10
domain, t7 ends-by Thu 11am 10
constraint, t7 before t6
constraint, t5 before t2
constraint, t7 before t2
constraint, t1 same-day t7
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp --> title of csp --> title of csp --> title of csp --> title of csp
--> title of csp --> title of csp --> title of csp --> title of csp --> title of
csp --> title of csp --> title of csp --> title of csp --> title of csp --> title
of csp --> title of csp (cost: 24)
25 paths have been expanded and 24 paths remain in the frontier
task, t1 5
task, t2 1
task, t3 1
task, t4 5
task, t5 1
task, t6 4
task, t7 1
domain, t1 ends-by Tue 2pm 10
domain, t2 ends-by Fri 1pm 10
domain, t3 ends-by Tue 11am 10
domain, t4 ends-by Tue 1pm 10
domain, t5 ends-by Mon 11am 10
domain, t6 ends-by Thu 1pm 10
domain, t7 ends-by Wed 3pm 10
constraint, t4 before t2
constraint, t1 same-day t2
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp --> title of csp --> title of csp --> title of csp --> title of csp
--> title of csp --> title of csp --> title of csp --> title of csp --> title of
csp --> title of csp --> title of csp --> title of csp --> title of csp --> title
of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title
of csp --> title of csp (cost: 24)

```



task, t5 3

```

task, t6 2
task, t7 4
task, t8 4
domain, t1 ends-by Thu 11am 10
domain, t2 ends-by Tue 4pm 10
domain, t3 ends-by Fri 4pm 10
domain, t4 ends-by Fri 12pm 10
domain, t5 ends-by Thu 3pm 10
domain, t6 ends-by Mon 9am 10
domain, t7 ends-by Mon 9am 10
domain, t8 ends-by Tue 10am 10
constraint, t8 same-day t6
constraint, t6 same-day t5
constraint, t2 before t6
constraint, t4 before t3
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp --> title of csp --> title of csp --> title of csp --> title of csp
--> title of csp --> title of csp --> title of csp --> title of csp --> title of
csp --> title of csp --> title of csp --> title of csp --> title of csp --> title
of csp --> title of csp --> title of csp (cost: 25)
  26 paths have been expanded and 25 paths remain in the frontier
task, t1 4
task, t2 2
task, t3 2
task, t4 5
task, t5 3
task, t6 2
task, t7 4
task, t8 4
domain, t1 ends-by Thu 11am 10
domain, t2 ends-by Tue 4pm 10
domain, t3 ends-by Fri 4pm 10
domain, t4 ends-by Fri 12pm 10
domain, t5 ends-by Thu 3pm 10
domain, t6 ends-by Mon 9am 10
domain, t7 ends-by Mon 9am 10
domain, t8 ends-by Tue 10am 10
constraint, t8 same-day t6
constraint, t6 same-day t5
constraint, t2 before t6
constraint, t4 before t3
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp --> title of csp --> title of csp --> title of csp --> title of csp
--> title of csp --> title of csp --> title of csp --> title of csp --> title of
csp --> title of csp --> title of csp --> title of csp --> title of csp --> title
of csp --> title of csp --> title of csp (cost: 25)
  26 paths have been expanded and 25 paths remain in the frontier
task, t1 1
task, t2 5
task, t3 1
task, t4 4
task, t5 1
task, t6 5
task, t7 1
task, t8 5
domain, t1 ends-by Fri 9am 10
domain, t2 ends-by Mon 9am 10
domain, t3 ends-by Fri 1pm 10

```





```

domain, t7 ends-by Thu 1pm 10
domain, t8 ends-by Fri 2pm 10
constraint, t5 before t2
constraint, t1 before t2
constraint, t6 before t3
constraint, t8 before t7
constraint, t8 before t6
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp --> title of csp --> title of csp --> title of csp --> title of csp
--> title of csp --> title of csp --> title of csp --> title of csp --> title of
csp --> title of csp --> title of csp --> title of csp (cost: 21)
  22 paths have been expanded and 21 paths remain in the frontier
task, t1 2
task, t2 1
task, t3 2
task, t4 4
task, t5 3
task, t6 2
task, t7 5
task, t8 1
domain, t1 ends-by Fri 4pm 10
domain, t2 ends-by Fri 4pm 10
domain, t3 ends-by Thu 11am 10
domain, t4 ends-by Mon 11am 10
domain, t5 ends-by Tue 3pm 10
domain, t6 ends-by Tue 3pm 10
domain, t7 ends-by Thu 1pm 10
domain, t8 ends-by Fri 2pm 10
constraint, t5 before t2
constraint, t1 before t2
constraint, t6 before t3
constraint, t8 before t7
constraint, t8 before t6
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp --> title of csp --> title of csp --> title of csp --> title of csp
--> title of csp --> title of csp --> title of csp --> title of csp --> title of
csp --> title of csp --> title of csp (cost: 20)
  21 paths have been expanded and 20 paths remain in the frontier
task, t1 3
task, t2 4
task, t3 2
task, t4 3
task, t5 2
task, t6 1
task, t7 1
task, t8 5
domain, t1 ends-by Wed 4pm 10
domain, t2 ends-by Thu 10am 10
domain, t3 ends-by Thu 12pm 10
domain, t4 ends-by Thu 3pm 10
domain, t5 ends-by Mon 3pm 10
domain, t6 ends-by Fri 4pm 10
domain, t7 ends-by Thu 11am 10
domain, t8 ends-by Mon 4pm 10
constraint, t4 before t1
constraint, t1 same-day t4
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->

```

title of csp --> title of csp --> title of csp --> title of csp --> title of csp  
--> title of csp --> title of csp --> title of csp --> title of csp --> title of  
csp --> title of csp --> title of csp --> title of csp --> title of csp --> title  
of csp --> title of csp --> title of csp --> title of csp --> title of csp --> ti  
tle of csp (cost: 28)

29 paths have been expanded and 28 paths remain in the frontier

task, t1 3  
task, t2 4  
task, t3 2  
task, t4 3  
task, t5 2  
task, t6 1  
task, t7 1  
task, t8 5

domain, t1 ends-by Wed 4pm 10  
domain, t2 ends-by Thu 10am 10  
domain, t3 ends-by Thu 12pm 10  
domain, t4 ends-by Thu 3pm 10  
domain, t5 ends-by Mon 3pm 10  
domain, t6 ends-by Fri 4pm 10  
domain, t7 ends-by Thu 11am 10  
domain, t8 ends-by Mon 4pm 10  
constraint, t4 before t1  
constraint, t1 same-day t4

Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit  
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->  
title of csp --> title of csp --> title of csp --> title of csp --> title of csp  
--> title of csp --> title of csp --> title of csp --> title of csp --> title of  
csp --> title of csp --> title of csp --> title of csp --> title of csp --> title  
of csp --> title of csp --> title of csp --> title of csp --> title of csp --> ti  
tle of csp (cost: 28)

29 paths have been expanded and 28 paths remain in the frontier

task, t1 4  
task, t2 3  
task, t3 1  
task, t4 2  
task, t5 4  
task, t6 5  
task, t7 3  
task, t8 1

domain, t1 ends-by Thu 2pm 10  
domain, t2 ends-by Mon 4pm 10  
domain, t3 ends-by Wed 11am 10  
domain, t4 ends-by Fri 2pm 10  
domain, t5 ends-by Tue 3pm 10  
domain, t6 ends-by Wed 9am 10  
domain, t7 ends-by Thu 11am 10  
domain, t8 ends-by Thu 1pm 10  
constraint, t7 same-day t4  
constraint, t3 before t7  
constraint, t4 same-day t2  
constraint, t8 same-day t6  
constraint, t5 before t1  
constraint, t2 same-day t4

Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit  
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->  
title of csp --> title of csp --> title of csp --> title of csp --> title of csp  
--> title of csp --> title of csp --> title of csp --> title of csp --> title of  
csp --> title of csp --> title of csp --> title of csp (cost: 21)

22 paths have been expanded and 21 paths remain in the frontier

```

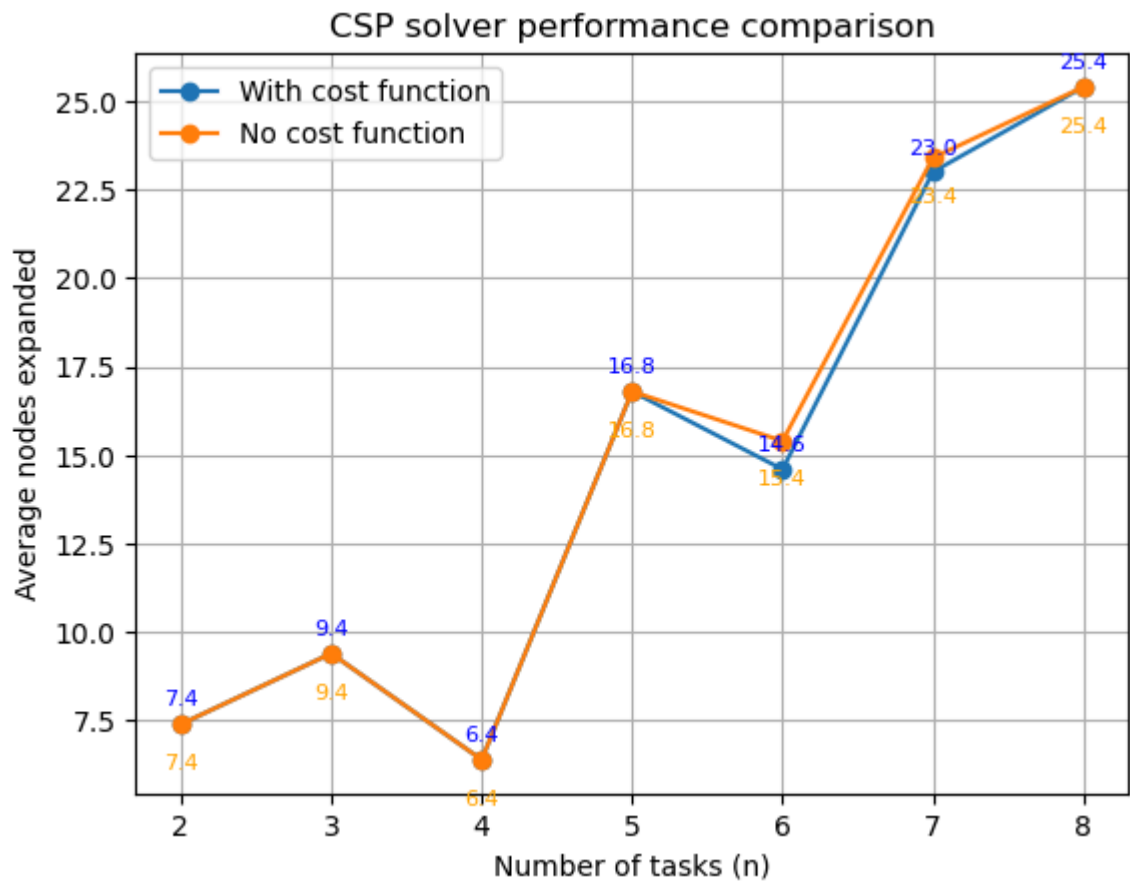
task, t1 4
task, t2 3
task, t3 1
task, t4 2
task, t5 4
task, t6 5
task, t7 3
task, t8 1
domain, t1 ends-by Thu 2pm 10
domain, t2 ends-by Mon 4pm 10
domain, t3 ends-by Wed 11am 10
domain, t4 ends-by Fri 2pm 10
domain, t5 ends-by Tue 3pm 10
domain, t6 ends-by Wed 9am 10
domain, t7 ends-by Thu 11am 10
domain, t8 ends-by Thu 1pm 10
constraint, t7 same-day t4
constraint, t3 before t7
constraint, t4 same-day t2
constraint, t8 same-day t6
constraint, t5 before t1
constraint, t2 same-day t4
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp --> title of csp --> title of csp --> title of csp --> title of csp
--> title of csp --> title of csp --> title of csp --> title of csp --> title of
csp --> title of csp --> title of csp --> title of csp --> title of csp --> title
of csp (cost: 23)
24 paths have been expanded and 23 paths remain in the frontier

```

```

In [16]: plt.plot(ns, avg_nodes_with_cost, marker='o', label='With cost function')
plt.plot(ns, avg_nodes_no_cost, marker='o', label='No cost function')
for i, n in enumerate(ns):
    plt.text(n, avg_nodes_with_cost[i] + max(avg_nodes_with_cost)*0.02,
             f'{avg_nodes_with_cost[i]:.1f}', ha='center', fontsize=8, color='bl
plt.text(n, avg_nodes_no_cost[i] - max(avg_nodes_no_cost)*0.05,
         f'{avg_nodes_no_cost[i]:.1f}', ha='center', fontsize=8, color='oran
plt.xlabel('Number of tasks (n)')
plt.ylabel('Average nodes expanded')
plt.title('CSP solver performance comparison')
plt.legend()
plt.grid(True)
plt.show()

```



(2)

```
In [17]: total_improvement = 0
for i, n in enumerate(ns):
    no_cost = avg_nodes_no_cost[i]
    with_cost = avg_nodes_with_cost[i]
    if no_cost > 0 :
        improvement = ((no_cost - with_cost) / no_cost) * 100
    else: 0
    total_improvement += improvement

average_improvement = total_improvement / len(ns)
print(f"\n Performance Gain: {average_improvement:.1f}%")
```

Performance Gain: 1.0%

In general, the performance of the Cost Function give a small performance gain for the no cost one.

## Question 4 (5 marks)

Compare the Depth-First Search (DFS) solver to the Depth-First Search solver using forward checking with Minimum Remaining Values heuristic (DFS-MRV). For this question, ignore the costs associated with the CSP problems.

- What is the worst case time and space complexity of each algorithm (give a general form in terms of fuzzy scheduling problem sizes)? (1 mark)
- What are the properties of the search algorithms (completeness, optimality)? (1 mark)

- Give an example of a problem that is *easier* for the DFS-MRV solver than it is for the DFS solver, and explain why (1 mark)
- Empirically compare the quality of the first solution found by DFS and DFS-MRV compared to the optimal solution (1 mark)
- Empirically compare DFS-MRV with DFS in terms of the number of nodes expanded (1 mark)

For the empirical evaluations, run the two algorithms on a variety of problems of size `n` for varying `n`. Note that the domain splitting CSP solver with costs should always find an optimal solution.

```
In [18]: # Code for Question 4
# Place your code here
def calculate_solution_cost(csp, solution):          ### as the cost function f
    total_cost = 0
    for v, val in solution.items():
        dur = csp.durations[v]
        soft_dt = csp.soft_day_time[v]
        soft_cost = int(csp.soft_costs[v])
        cost_func = csp.cost_functions[v][0]

        cost = cost_func(val, soft_dt, dur, soft_cost)
        if cost is None:
            cost = 0
        total_cost += cost

    return total_cost

#####-----
import pandas as pd
ns = [2,3,4,5,6,7]
num_epochs = 5
avg_nodes_optimal = []
avg_nodes_dfs = []
avg_nodes_mrv = []
avg_dfs_cost = []
avg_mrv_cost = []
avg_optimal_cost = []

for n in ns:
    nodes_with_cost = []
    nodes_dfs = []
    nodes_mrv = []
    dfs_cost = []
    mrv_cost = []
    optimal_cost = []
    for _ in range(num_epochs):
        spec_text = generate_problem(n)

        # Has cost function
        csp_problem = create_CSP_from_spec(spec_text)
        solver = GreedySearcher(Search_with_AC_from_Cost_CSP(csp_problem))
        final_path = solver.search()          ## test_csp_solver rewriter part
        nodes_with_cost.append(solver.num_expanded) # get nodes number
        if final_path is not None:
            optimal_cost.append(final_path.end().cost)
```

```

#DFS
test_dfs_solver(csp_problem)
solution = dfs_solve1(csp_problem)
if solution:
    total_cost = calculate_solution_cost(csp_problem, solution)
    print("Total soft constraint cost of DFS =", total_cost)
dfs_cost.append(total_cost)
nodes_dfs.append(num_expanded)

#DFSMRV
test_mrv_dfs_solver(csp_problem)
solution = dfs_solve1(csp_problem)
if solution:
    total_cost = calculate_solution_cost(csp_problem, solution)
    print("Total soft constraint cost of DFS-MRV =", total_cost)
mrv_cost.append(total_cost)
nodes_mrv.append(num_expanded)

avg_nodes_optimal.append(sum(nodes_with_cost)/num_epochs)
avg_nodes_dfs.append(sum(nodes_dfs)/num_epochs)
avg_nodes_mrv.append(sum(nodes_mrv)/num_epochs)
avg_dfs_cost.append(sum(dfs_cost)/num_epochs)
avg_mrv_cost.append(sum(mrv_cost)/num_epochs)
avg_optimal_cost.append(sum(optimal_cost)/num_epochs)

print(avg_nodes_optimal)
print(avg_nodes_dfs)
print(avg_nodes_mrv)
print(avg_dfs_cost)
print(avg_mrv_cost)

df_1 = pd.DataFrame({
    "n": ns,
    "Optimal": avg_nodes_optimal,
    "DFS": avg_nodes_dfs,
    "MRV": avg_nodes_mrv,
    'avg_cost_DFS': avg_dfs_cost,
    'avg_cost_DFS_mrv': avg_mrv_cost,
    'avg_cost_optimal': avg_optimal_cost
})
print(df_1)

```

```

task, t1 4
task, t2 3
domain, t1 ends-by Tue 9am 10
domain, t2 ends-by Thu 3pm 10
constraint, t1 same-day t2
constraint, t2 before t1
Solution: title of csp --> title of csp --> title of csp (cost: 2)
  3 paths have been expanded and 2 paths remain in the frontier
Nodes expanded to reach solution: 68
t1: mon 12pm
t2: mon 9am
Nodes expanded to reach solution: 68
Total soft constraint cost of DFS = 0
Nodes expanded to reach solution: 5
t1: mon 12pm
t2: mon 9am
Nodes expanded to reach solution: 8
Total soft constraint cost of DFS-MRV = 0
task, t1 4
task, t2 5
domain, t1 ends-by Fri 10am 10
domain, t2 ends-by Tue 9am 10
constraint, t1 same-day t2
Solution: title of csp --> title of csp --> title of csp --> title of csp --> title of csp (cost: 5)
  6 paths have been expanded and 5 paths remain in the frontier
Nodes expanded to reach solution: 2
t1: mon 9am
t2: mon 9am
Nodes expanded to reach solution: 2
Total soft constraint cost of DFS = 0
Nodes expanded to reach solution: 2
t2: mon 9am
t1: mon 9am
Nodes expanded to reach solution: 2
Total soft constraint cost of DFS-MRV = 0
task, t1 1
task, t2 2
domain, t1 ends-by Fri 2pm 10
domain, t2 ends-by Wed 10am 10
constraint, t2 before t1
constraint, t2 before t1
Solution: title of csp --> title of csp --> title of csp --> title of csp --> title of csp (cost: 5)
  6 paths have been expanded and 5 paths remain in the frontier
Nodes expanded to reach solution: 2
t1: mon 11am
t2: mon 9am
Nodes expanded to reach solution: 2
Total soft constraint cost of DFS = 0
Nodes expanded to reach solution: 2
t2: mon 9am
t1: mon 11am
Nodes expanded to reach solution: 2
Total soft constraint cost of DFS-MRV = 0
task, t1 4
task, t2 1
domain, t1 ends-by Mon 3pm 10
domain, t2 ends-by Mon 9am 10
constraint, t2 same-day t1

```

Solution: title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp (cost: 6)

7 paths have been expanded and 6 paths remain in the frontier

Nodes expanded to reach solution: 2

t1: mon 9am

t2: mon 9am

Nodes expanded to reach solution: 2

Total soft constraint cost of DFS = 10

Nodes expanded to reach solution: 2

t1: mon 9am

t2: mon 9am

Nodes expanded to reach solution: 2

Total soft constraint cost of DFS-MRV = 10

task, t1 5

task, t2 2

domain, t1 ends-by Thu 10am 10

domain, t2 ends-by Fri 10am 10

constraint, t2 before t1

Solution: title of csp --> title of csp --> title of csp --> title of csp --> title of csp (cost: 4)

5 paths have been expanded and 4 paths remain in the frontier

Nodes expanded to reach solution: 2

t1: fri 9am

t2: mon 9am

Nodes expanded to reach solution: 2

Total soft constraint cost of DFS = 280

Nodes expanded to reach solution: 2

t1: fri 9am

t2: mon 9am

Nodes expanded to reach solution: 2

Total soft constraint cost of DFS-MRV = 280

task, t1 1

task, t2 1

task, t3 2

domain, t1 ends-by Fri 12pm 10

domain, t2 ends-by Thu 4pm 10

domain, t3 ends-by Tue 2pm 10

constraint, t3 before t2

constraint, t2 before t3

No (more) solutions. Total of 1 paths expanded.

No solution

No solution

task, t1 4

task, t2 2

task, t3 4

domain, t1 ends-by Tue 9am 10

domain, t2 ends-by Mon 1pm 10

domain, t3 ends-by Mon 3pm 10

constraint, t3 before t2

constraint, t3 same-day t2

constraint, t3 before t1

Solution: title of csp --> title of csp --> title of csp --> title of csp --> title of csp (cost: 5)

6 paths have been expanded and 5 paths remain in the frontier

Nodes expanded to reach solution: 3

t1: fri 9am

t2: mon 1pm

t3: mon 9am

Nodes expanded to reach solution: 3

Total soft constraint cost of DFS = 780



Nodes expanded to reach solution: 3  
 t3: mon 9am  
 t2: mon 1pm  
 t1: tue 9am  
 Nodes expanded to reach solution: 3  
 Total soft constraint cost of DFS-MRV = 60  
 task, t1 1  
 task, t2 1  
 task, t3 2  
 domain, t1 ends-by Wed 12pm 10  
 domain, t2 ends-by Mon 2pm 10  
 domain, t3 ends-by Wed 1pm 10  
 constraint, t3 before t1  
 constraint, t3 same-day t2  
 Solution: title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp (cost: 7)  
 8 paths have been expanded and 7 paths remain in the frontier  
 Nodes expanded to reach solution: 3  
 t1: mon 11am  
 t2: mon 9am  
 t3: mon 9am  
 Nodes expanded to reach solution: 3  
 Total soft constraint cost of DFS = 0  
 Nodes expanded to reach solution: 3  
 t3: mon 9am  
 t2: mon 9am  
 t1: mon 11am  
 Nodes expanded to reach solution: 3  
 Total soft constraint cost of DFS-MRV = 0  
 task, t1 5  
 task, t2 1  
 task, t3 3  
 domain, t1 ends-by Mon 10am 10  
 domain, t2 ends-by Mon 10am 10  
 domain, t3 ends-by Mon 12pm 10  
 constraint, t3 before t2  
 constraint, t3 same-day t1  
 constraint, t1 same-day t3  
 Solution: title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp (cost: 8)  
 9 paths have been expanded and 8 paths remain in the frontier  
 Nodes expanded to reach solution: 3  
 t1: mon 9am  
 t2: mon 12pm  
 t3: mon 9am  
 Nodes expanded to reach solution: 3  
 Total soft constraint cost of DFS = 70  
 Nodes expanded to reach solution: 3  
 t1: mon 9am  
 t3: mon 9am  
 t2: mon 12pm  
 Nodes expanded to reach solution: 3  
 Total soft constraint cost of DFS-MRV = 70  
 task, t1 4  
 task, t2 3  
 task, t3 2  
 domain, t1 ends-by Tue 3pm 10  
 domain, t2 ends-by Wed 10am 10  
 domain, t3 ends-by Wed 11am 10

```

constraint, t2 same-day t3
constraint, t1 before t3
constraint, t2 same-day t1
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp (cost: 6)
  7 paths have been expanded and 6 paths remain in the frontier
Nodes expanded to reach solution: 3
t1: mon 9am
t2: mon 9am
t3: mon 1pm
Nodes expanded to reach solution: 3
Total soft constraint cost of DFS = 0
Nodes expanded to reach solution: 3
t1: mon 9am
t2: mon 9am
t3: mon 1pm
Nodes expanded to reach solution: 3
Total soft constraint cost of DFS-MRV = 0
task, t1 2
task, t2 4
task, t3 2
task, t4 3
domain, t1 ends-by Fri 2pm 10
domain, t2 ends-by Wed 4pm 10
domain, t3 ends-by Tue 9am 10
domain, t4 ends-by Mon 12pm 10
constraint, t4 same-day t3
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp --> title of csp --> title of csp --> title of csp --> title of csp
--> title of csp (cost: 14)
  15 paths have been expanded and 14 paths remain in the frontier
Nodes expanded to reach solution: 4
t1: mon 9am
t2: mon 9am
t3: mon 9am
t4: mon 9am
Nodes expanded to reach solution: 4
Total soft constraint cost of DFS = 0
Nodes expanded to reach solution: 4
t2: mon 9am
t4: mon 9am
t3: mon 9am
t1: mon 9am
Nodes expanded to reach solution: 4
Total soft constraint cost of DFS-MRV = 0
task, t1 2
task, t2 5
task, t3 1
task, t4 4
domain, t1 ends-by Tue 3pm 10
domain, t2 ends-by Fri 4pm 10
domain, t3 ends-by Fri 11am 10
domain, t4 ends-by Thu 11am 10
constraint, t1 same-day t3
constraint, t2 before t1
constraint, t3 before t2
No (more) solutions. Total of 1 paths expanded.
No solution
No solution

```

```

task, t1 3
task, t2 3
task, t3 3
task, t4 5
domain, t1 ends-by Tue 1pm 10
domain, t2 ends-by Fri 3pm 10
domain, t3 ends-by Mon 10am 10
domain, t4 ends-by Tue 3pm 10
constraint, t3 before t4
constraint, t4 same-day t3
No (more) solutions. Total of 1 paths expanded.
No solution
No solution
task, t1 1
task, t2 5
task, t3 1
task, t4 5
domain, t1 ends-by Fri 11am 10
domain, t2 ends-by Tue 11am 10
domain, t3 ends-by Wed 9am 10
domain, t4 ends-by Tue 2pm 10
constraint, t2 same-day t3
constraint, t4 before t3
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp (cost: 9)
  10 paths have been expanded and 9 paths remain in the frontier
Nodes expanded to reach solution: 4
t1: mon 9am
t2: mon 9am
t3: mon 2pm
t4: mon 9am
Nodes expanded to reach solution: 4
Total soft constraint cost of DFS = 0
Nodes expanded to reach solution: 4
t4: mon 9am
t2: mon 9am
t3: mon 2pm
t1: mon 9am
Nodes expanded to reach solution: 4
Total soft constraint cost of DFS-MRV = 0
task, t1 2
task, t2 5
task, t3 1
task, t4 5
domain, t1 ends-by Tue 11am 10
domain, t2 ends-by Wed 2pm 10
domain, t3 ends-by Tue 1pm 10
domain, t4 ends-by Mon 3pm 10
constraint, t1 before t2
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp --> title of csp --> title of csp --> title of csp --> title of csp
--> title of csp --> title of csp --> title of csp (cost: 16)
  17 paths have been expanded and 16 paths remain in the frontier
Nodes expanded to reach solution: 4
t1: mon 9am
t2: fri 9am
t3: mon 9am
t4: mon 9am

```

```

Nodes expanded to reach solution: 4
Total soft constraint cost of DFS = 480
Nodes expanded to reach solution: 4
t2: fri 9am
t4: mon 9am
t1: mon 9am
t3: mon 9am
Nodes expanded to reach solution: 4
Total soft constraint cost of DFS-MRV = 480
task, t1 4
task, t2 4
task, t3 3
task, t4 1
task, t5 5
domain, t1 ends-by Mon 4pm 10
domain, t2 ends-by Thu 12pm 10
domain, t3 ends-by Wed 4pm 10
domain, t4 ends-by Thu 2pm 10
domain, t5 ends-by Wed 2pm 10
constraint, t4 before t3
constraint, t5 same-day t1
constraint, t2 before t1
constraint, t5 before t4
constraint, t4 same-day t1
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp (cost: 10)
 11 paths have been expanded and 10 paths remain in the frontier
Nodes expanded to reach solution: 5
t1: tue 9am
t2: mon 9am
t3: fri 9am
t4: tue 2pm
t5: tue 9am
Nodes expanded to reach solution: 5
Total soft constraint cost of DFS = 650
Nodes expanded to reach solution: 5
t5: tue 9am
t1: tue 9am
t4: tue 2pm
t2: mon 9am
t3: fri 9am
Nodes expanded to reach solution: 5
Total soft constraint cost of DFS-MRV = 650
task, t1 2
task, t2 4
task, t3 3
task, t4 5
task, t5 2
domain, t1 ends-by Thu 3pm 10
domain, t2 ends-by Wed 2pm 10
domain, t3 ends-by Tue 4pm 10
domain, t4 ends-by Fri 9am 10
domain, t5 ends-by Thu 1pm 10
constraint, t4 same-day t5
constraint, t5 same-day t2
constraint, t5 before t1
constraint, t2 before t5
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->

```

```

title of csp --> title of csp --> title of csp (cost: 11)
  12 paths have been expanded and 11 paths remain in the frontier
Nodes expanded to reach solution: 5
t1: tue 9am
t2: mon 9am
t3: mon 9am
t4: mon 9am
t5: mon 1pm
Nodes expanded to reach solution: 5
Total soft constraint cost of DFS = 0
Nodes expanded to reach solution: 5
t4: mon 9am
t5: mon 1pm
t2: mon 9am
t1: tue 9am
t3: mon 9am
Nodes expanded to reach solution: 5
Total soft constraint cost of DFS-MRV = 0
task, t1 3
task, t2 4
task, t3 5
task, t4 2
task, t5 3
domain, t1 ends-by Tue 9am 10
domain, t2 ends-by Mon 1pm 10
domain, t3 ends-by Mon 9am 10
domain, t4 ends-by Wed 12pm 10
domain, t5 ends-by Wed 12pm 10
constraint, t3 same-day t2
constraint, t3 same-day t2
constraint, t2 same-day t3
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp --> title of csp --> title of csp --> title of csp --> title of csp
--> title of csp --> title of csp --> title of csp --> title of csp (cost: 17)
  18 paths have been expanded and 17 paths remain in the frontier
Nodes expanded to reach solution: 5
t1: mon 9am
t2: mon 9am
t3: mon 9am
t4: mon 9am
t5: mon 9am
Nodes expanded to reach solution: 5
Total soft constraint cost of DFS = 50
Nodes expanded to reach solution: 5
t3: mon 9am
t2: mon 9am
t1: mon 9am
t5: mon 9am
t4: mon 9am
Nodes expanded to reach solution: 5
Total soft constraint cost of DFS-MRV = 50
task, t1 4
task, t2 2
task, t3 3
task, t4 3
task, t5 3
domain, t1 ends-by Wed 4pm 10
domain, t2 ends-by Wed 11am 10
domain, t3 ends-by Fri 1pm 10

```

```

domain, t4 ends-by Wed 1pm 10
domain, t5 ends-by Thu 9am 10
constraint, t3 same-day t1
constraint, t4 before t2
constraint, t3 before t2
constraint, t3 same-day t5
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp --> title of csp (cost: 10)
  11 paths have been expanded and 10 paths remain in the frontier
Nodes expanded to reach solution: 5
t1: mon 9am
t2: mon 12pm
t3: mon 9am
t4: mon 9am
t5: mon 9am
Nodes expanded to reach solution: 5
Total soft constraint cost of DFS = 0
Nodes expanded to reach solution: 5
t1: mon 9am
t3: mon 9am
t5: mon 9am
t4: mon 9am
t2: mon 12pm
Nodes expanded to reach solution: 5
Total soft constraint cost of DFS-MRV = 0
task, t1 3
task, t2 1
task, t3 1
task, t4 2
task, t5 1
domain, t1 ends-by Wed 12pm 10
domain, t2 ends-by Mon 10am 10
domain, t3 ends-by Fri 1pm 10
domain, t4 ends-by Tue 1pm 10
domain, t5 ends-by Mon 3pm 10
constraint, t3 before t4
constraint, t2 before t5
constraint, t1 before t2
constraint, t5 same-day t3
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp --> title of csp --> title of csp --> title of csp --> title of csp
--> title of csp --> title of csp --> title of csp --> title of csp --> title of
csp (cost: 18)
  19 paths have been expanded and 18 paths remain in the frontier
Nodes expanded to reach solution: 5
t1: mon 9am
t2: mon 12pm
t3: mon 9am
t4: mon 10am
t5: mon 1pm
Nodes expanded to reach solution: 5
Total soft constraint cost of DFS = 30
Nodes expanded to reach solution: 5
t1: mon 9am
t4: mon 10am
t3: mon 9am
t5: mon 1pm
t2: mon 12pm

```

```
Nodes expanded to reach solution: 5
Total soft constraint cost of DFS-MRV = 30
task, t1 2
task, t2 5
task, t3 5
task, t4 1
task, t5 3
task, t6 2
domain, t1 ends-by Fri 4pm 10
domain, t2 ends-by Thu 12pm 10
domain, t3 ends-by Thu 2pm 10
domain, t4 ends-by Mon 12pm 10
domain, t5 ends-by Wed 1pm 10
domain, t6 ends-by Tue 3pm 10
constraint, t3 before t1
constraint, t4 before t6
constraint, t4 same-day t5
constraint, t5 same-day t1
constraint, t2 before t4
Solution: title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp --> title of csp --> title of csp --> title of csp --> title of csp
(cost: 13)
14 paths have been expanded and 13 paths remain in the frontier
Nodes expanded to reach solution: 6
t1: mon 2pm
t2: mon 9am
t3: mon 9am
t4: mon 2pm
t5: mon 9am
t6: tue 9am
Nodes expanded to reach solution: 6
Total soft constraint cost of DFS = 30
Nodes expanded to reach solution: 6
t2: mon 9am
t3: mon 9am
t6: tue 9am
t4: mon 2pm
t5: mon 9am
t1: mon 2pm
Nodes expanded to reach solution: 6
Total soft constraint cost of DFS-MRV = 30
task, t1 1
task, t2 5
task, t3 1
task, t4 2
task, t5 1
task, t6 4
domain, t1 ends-by Wed 2pm 10
domain, t2 ends-by Wed 1pm 10
domain, t3 ends-by Fri 11am 10
domain, t4 ends-by Mon 9am 10
domain, t5 ends-by Mon 10am 10
domain, t6 ends-by Tue 4pm 10
constraint, t6 same-day t2
constraint, t4 same-day t6
constraint, t3 same-day t1
constraint, t4 same-day t2
constraint, t2 same-day t3
constraint, t2 same-day t3
```

Solution: title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp (cost: 17)

18 paths have been expanded and 17 paths remain in the frontier

Nodes expanded to reach solution: 6

t1: mon 9am

t2: mon 9am

t3: mon 9am

t4: mon 9am

t5: mon 9am

t6: mon 9am

Nodes expanded to reach solution: 6

Total soft constraint cost of DFS = 20

Nodes expanded to reach solution: 6

t2: mon 9am

t6: mon 9am

t4: mon 9am

t3: mon 9am

t1: mon 9am

t5: mon 9am

Nodes expanded to reach solution: 6

Total soft constraint cost of DFS-MRV = 20

task, t1 1

task, t2 4

task, t3 1

task, t4 2

task, t5 1

task, t6 2

domain, t1 ends-by Tue 11am 10

domain, t2 ends-by Thu 11am 10

domain, t3 ends-by Wed 3pm 10

domain, t4 ends-by Mon 3pm 10

domain, t5 ends-by Mon 11am 10

domain, t6 ends-by Wed 3pm 10

constraint, t5 same-day t6

constraint, t6 same-day t2

Solution: title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp (cost: 22)

23 paths have been expanded and 22 paths remain in the frontier

Nodes expanded to reach solution: 6

t1: mon 9am

t2: mon 9am

t3: mon 9am

t4: mon 9am

t5: mon 9am

t6: mon 9am

Nodes expanded to reach solution: 6

Total soft constraint cost of DFS = 0

Nodes expanded to reach solution: 6

t2: mon 9am

t6: mon 9am

t5: mon 9am

t4: mon 9am

t1: mon 9am

t3: mon 9am



```

Nodes expanded to reach solution: 6
Total soft constraint cost of DFS-MRV = 0
task, t1 2
task, t2 4
task, t3 1
task, t4 3
task, t5 2
task, t6 5
domain, t1 ends-by Wed 3pm 10
domain, t2 ends-by Mon 9am 10
domain, t3 ends-by Fri 10am 10
domain, t4 ends-by Mon 1pm 10
domain, t5 ends-by Tue 10am 10
domain, t6 ends-by Wed 4pm 10
constraint, t4 before t1
constraint, t6 before t1
constraint, t4 before t5
constraint, t5 before t2
constraint, t3 before t6
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp --> title of csp --> title of csp --> title of csp (cost: 12)
13 paths have been expanded and 12 paths remain in the frontier
Nodes expanded to reach solution: 7
t1: tue 9am
t2: fri 9am
t3: mon 9am
t4: mon 9am
t5: mon 12pm
t6: mon 10am
Nodes expanded to reach solution: 7
Total soft constraint cost of DFS = 1000
Nodes expanded to reach solution: 6
t6: fri 9am
t1: fri 2pm
t2: tue 9am
t5: mon 12pm
t4: mon 9am
t3: mon 9am
Nodes expanded to reach solution: 6
Total soft constraint cost of DFS-MRV = 1230
task, t1 5
task, t2 4
task, t3 2
task, t4 3
task, t5 3
task, t6 2
domain, t1 ends-by Fri 11am 10
domain, t2 ends-by Wed 1pm 10
domain, t3 ends-by Tue 10am 10
domain, t4 ends-by Wed 12pm 10
domain, t5 ends-by Wed 10am 10
domain, t6 ends-by Thu 12pm 10
constraint, t6 before t5
constraint, t6 same-day t3
constraint, t4 before t5
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp --> title of csp --> title of csp --> title of csp --> title of csp
--> title of csp --> title of csp --> title of csp --> title of csp --> title of

```

```

csp --> title of csp --> title of csp (cost: 20)
  21 paths have been expanded and 20 paths remain in the frontier
Nodes expanded to reach solution: 6
t1: mon 9am
t2: mon 9am
t3: mon 9am
t4: mon 9am
t5: mon 12pm
t6: mon 9am
Nodes expanded to reach solution: 6
Total soft constraint cost of DFS = 0
Nodes expanded to reach solution: 6
t1: mon 9am
t2: mon 9am
t4: mon 9am
t5: mon 12pm
t6: mon 9am
t3: mon 9am
Nodes expanded to reach solution: 6
Total soft constraint cost of DFS-MRV = 0
task, t1 3
task, t2 2
task, t3 5
task, t4 3
task, t5 3
task, t6 1
task, t7 4
domain, t1 ends-by Thu 12pm 10
domain, t2 ends-by Thu 1pm 10
domain, t3 ends-by Mon 12pm 10
domain, t4 ends-by Mon 12pm 10
domain, t5 ends-by Mon 3pm 10
domain, t6 ends-by Mon 1pm 10
domain, t7 ends-by Thu 2pm 10
constraint, t3 same-day t5
constraint, t7 same-day t1
constraint, t4 same-day t5
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp --> title of csp --> title of csp --> title of csp --> title of csp
--> title of csp --> title of csp --> title of csp --> title of csp --> title of
csp --> title of csp --> title of csp --> title of csp --> title of csp (cost: 2
2)
  23 paths have been expanded and 22 paths remain in the frontier
Nodes expanded to reach solution: 7
t1: mon 9am
t2: mon 9am
t3: mon 9am
t4: mon 9am
t5: mon 9am
t6: mon 9am
t7: mon 9am
Nodes expanded to reach solution: 7
Total soft constraint cost of DFS = 20
Nodes expanded to reach solution: 7
t3: mon 9am
t5: mon 9am
t4: mon 9am
t7: mon 9am
t1: mon 9am

```

```

t2: mon 9am
t6: mon 9am
Nodes expanded to reach solution: 7
Total soft constraint cost of DFS-MRV = 20
task, t1 4
task, t2 4
task, t3 4
task, t4 1
task, t5 5
task, t6 3
task, t7 1
domain, t1 ends-by Mon 12pm 10
domain, t2 ends-by Thu 9am 10
domain, t3 ends-by Thu 11am 10
domain, t4 ends-by Mon 3pm 10
domain, t5 ends-by Fri 11am 10
domain, t6 ends-by Thu 11am 10
domain, t7 ends-by Fri 1pm 10
constraint, t3 same-day t5
constraint, t6 before t7
constraint, t4 before t3
constraint, t1 same-day t5
constraint, t7 before t3
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp --> title of csp --> title of csp --> title of csp --> title of csp
--> title of csp --> title of csp --> title of csp --> title of csp (cost: 17)
18 paths have been expanded and 17 paths remain in the frontier
Nodes expanded to reach solution: 7
t1: fri 9am
t2: mon 9am
t3: fri 9am
t4: mon 9am
t5: fri 9am
t6: mon 9am
t7: mon 12pm
Nodes expanded to reach solution: 7
Total soft constraint cost of DFS = 1260
Nodes expanded to reach solution: 7
t5: fri 9am
t1: fri 9am
t3: fri 9am
t2: mon 9am
t6: mon 9am
t7: mon 12pm
t4: mon 9am
Nodes expanded to reach solution: 7
Total soft constraint cost of DFS-MRV = 1260
task, t1 3
task, t2 4
task, t3 3
task, t4 2
task, t5 3
task, t6 5
task, t7 3
domain, t1 ends-by Wed 4pm 10
domain, t2 ends-by Tue 2pm 10
domain, t3 ends-by Wed 4pm 10
domain, t4 ends-by Wed 4pm 10
domain, t5 ends-by Mon 2pm 10

```



```

t7: mon 9am
Nodes expanded to reach solution: 7
Total soft constraint cost of DFS = 0
Nodes expanded to reach solution: 7
t2: mon 9am
t7: mon 9am
t3: mon 9am
t4: mon 9am
t5: mon 9am
t1: mon 9am
t6: mon 11am
Nodes expanded to reach solution: 7
Total soft constraint cost of DFS-MRV = 0
task, t1 1
task, t2 1
task, t3 1
task, t4 1
task, t5 2
task, t6 2
task, t7 2
domain, t1 ends-by Mon 4pm 10
domain, t2 ends-by Fri 9am 10
domain, t3 ends-by Mon 11am 10
domain, t4 ends-by Mon 10am 10
domain, t5 ends-by Thu 12pm 10
domain, t6 ends-by Fri 2pm 10
domain, t7 ends-by Thu 12pm 10
constraint, t5 same-day t1
constraint, t1 before t7
constraint, t4 before t5
constraint, t4 before t6
constraint, t1 before t2
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp --> title of csp --> title of csp --> title of csp --> title of csp
--> title of csp --> title of csp --> title of csp --> title of csp --> title of
csp --> title of csp --> title of csp --> title of csp --> title of csp --> title
of csp --> title of csp --> title of csp --> title of csp --> title of csp (cost:
27)
28 paths have been expanded and 27 paths remain in the frontier
Nodes expanded to reach solution: 7
t1: mon 9am
t2: mon 10am
t3: mon 9am
t4: mon 9am
t5: mon 10am
t6: mon 10am
t7: mon 10am
Nodes expanded to reach solution: 7
Total soft constraint cost of DFS = 0
Nodes expanded to reach solution: 7
t5: mon 10am
t4: mon 9am
t1: mon 9am
t6: mon 10am
t7: mon 10am
t2: mon 10am
t3: mon 9am
Nodes expanded to reach solution: 7
Total soft constraint cost of DFS-MRV = 0

```

[5.4, 6.2, 8.8, 14.2, 17.8, 24.0]							
[15.2, 9.4, 132.4, 5.0, 6.2, 7.0]							
[3.2, 9.4, 132.4, 5.0, 6.0, 7.0]							
[58.0, 226.0, 96.0, 146.0, 210.0, 266.0]							
[58.0, 82.0, 96.0, 146.0, 256.0, 266.0]							
	n	Optimal	DFS	MRV	avg_cost_DFS	avg_cost_DFS_mrv	avg_cost_optimal
0	2	5.4	15.2	3.2	58.0	58.0	2.0
1	3	6.2	9.4	9.4	226.0	82.0	26.0
2	4	8.8	132.4	132.4	96.0	96.0	0.0
3	5	14.2	5.0	5.0	146.0	146.0	58.0
4	6	17.8	6.2	6.0	210.0	256.0	66.0
5	7	24.0	7.0	7.0	266.0	266.0	64.0

#### Answers for Question 4

If you want to submit additional code, put this at the end of the notebook. Here just give the answers (including plots or tables).

(1) For which, the d is the domain of the variables, the n is the number of the variables.

1. Time complexity of DFS:  $O(\text{domain}^n)$
2. Space complexity of DFS:  $O(\text{domain} \cdot n)$
3. Time complexity of DFS-MRV:  $O(\text{domain}^n)$
4. Space complexity of DFS-MRV:  $O(\text{domain} \cdot n)$

(2)

1. Completeness: For DFS and DFS-MRV, they can find the solution in the finite CSP. However, they are not optimal when the domain and variables are infinite.
2. Optimality: For both of them, they are not optimality.

(3)

The question of the SUDUKO can show the advantages of the DFS-MRV than DFS. As the simple DFS works in a very large range to check the solution, when in a situation of large depth and many constraints, it will cost a lot of time because it needs to iterate all of the paths to find the final solution. On the contrast, the DFS-MRV can efficiently decrease the searching time as it iteratively deletes the impossible values in the domain, which leads to a more efficient method of DFS-MRV.

(4)

From the first situation, it can know that the DFS has the problem that can violate the soft constraints. In one of my test, the task 2 starts at 9 am, however its constraint is "domain, t2 ends-by Mon 9am 10". So, it initially has a cost start from 10, while the result of the CSP with arc consistency is lower. The result can be seen from the result cell above, it shows that the cost of the two kinds of DFS is higher than the cost of the ARC consistency one. (The final result table is at the appendix).

(5) According to the nodes that expanded in the three algorithms, the result shows that the number of nodes expanded of the DFS is less than that of the DFS-MRV. As the optimal one is always aimed to find the best solution instead of the first one. The two results are shown above. (The final result table is at the appendix).

## Question 5 (4 marks)

The DFS solver chooses variables in random order, and systematically explores all values for those variables in no particular order.

Incorporate costs into the DFS constraint solver as heuristics to guide the search. Similar to the cost function for the domain splitting solver, for a given variable  $v$ , the cost of assigning the value  $val$  to  $v$  is the cost of violating the soft deadline constraint (if any) associated with  $v$  for the value  $val$ . The *minimum cost* for  $v$  is the lowest cost from amongst the values in the domain of  $v$ . The DFS solver should choose a variable  $v$  with lowest minimum cost, and explore its values in order of cost from lowest to highest.

- Implement this behaviour by modifying the code in `dfs_solver` and place a copy of the code below (2 marks)
- Empirically compare the performance of DFS with and without these heuristics (2 marks)

For the empirical evaluations, again run the two algorithms on a variety of problems of size `n` for varying `n`.

```
In [19]: # Code for Question 5
# Place your code here
num_expanded = 0
display = False

def dfs_solver_cost(constraints, domains, context, var_order):
    """ generator for all solutions to csp (with cost-based ordering)
    context is an assignment of values to some of the variables
    var_order is a list of the variables in csp that are not in context
    """
    global num_expanded, display
    to_eval = {c for c in constraints if c.can_evaluate(context)}

    if all(c.holds(context) for c in to_eval):
        # all of been assigned
        if var_order == []:
            print("Nodes expanded to reach solution:", num_expanded)
            yield context
        else:
            rem_cons = [c for c in constraints if c not in to_eval]

            # Compute minimal cost for each unassigned variable
            var_cost_map = {}
            for v in var_order:
                min_cost = float('inf')
                for val in domains[v]:
                    dur = csp_need_info.durations[v]
                    soft_dt = csp_need_info.soft_day_time[v]
                    soft_cost = int(csp_need_info.soft_costs[v])
                    cost_func = csp_need_info.cost_functions[v][0]
                    cost = cost_func(val, soft_dt, dur, soft_cost)
                    if cost is not None and cost < min_cost:
                        min_cost = cost
                var_cost_map[v] = min_cost
```

```

# Choose variable with smallest minimal cost
var = min(var_cost_map, key=var_cost_map.get)

# Sort values by cost for this variable
sorted_values = []
value_costs = []
for val in domains[var]:
    dur = csp_need_info.durations[var]
    soft_dt = csp_need_info.soft_day_time[var]
    soft_cost = int(csp_need_info.soft_costs[var])
    cost_func = csp_need_info.cost_functions[var][0]
    cost = cost_func(val, soft_dt, dur, soft_cost)
    value_costs.append((val, cost))
for val, _ in sorted(value_costs, key=lambda x: x[1]):
    sorted_values.append(val)

# DFS on sorted values
for val in sorted_values:
    if display:
        print("Setting", var, "to", val)
    num_expanded += 1
    yield from dfs_solver_cost(rem_cons, domains, context|{var:val},

def dfs_solve1_cost(csp, var_order=None):
    """DFS solver: return first solution"""
    global num_expanded
    num_expanded = 0
    global csp_need_info
    csp_need_info = csp
    if var_order == None: # use an arbitrary variable order
        var_order = list(csp.domains)
    for sol in dfs_solver_cost(csp.constraints, csp.domains, {}, var_order):
        return sol # return first one

def test_dfs_solver_cost(csp_problem):
    solution = dfs_solve1_cost(csp_problem)
    if solution is None:
        print('No solution')
    else:
        result_str = ''
        for name in solution.keys():
            result_str += '\n'+str(name)+': '+Day_Time().week_hour_number_to_day
        print(result_str[1:])

####-----
ns = [2,3,4,5,6]
num_epochs = 5
avg_nodes_dfs = []
avg_nodes_dfs_cost = []
avg_cost_DFS = []
avg_cost_DFS_COST = []

for n in ns:
    nodes_with_cost = []
    nodes_no_cost = []
    cost_DFS = []
    cost_DFS_COST = []

```



```

for _ in range(num_epochs):
    spec_text = generate_problem(n)
    csp_problem = create_CSP_from_spec(spec_text)

    ## DFS
    test_dfs_solver(csp_problem)
    solution = dfs_solve1(csp_problem)
    if solution:
        total_cost = calculate_solution_cost(csp_problem, solution)
        print("Total soft constraint cost of DFS =", total_cost)
    nodes_no_cost.append(num_expanded)
    cost_DFS.append(total_cost)

    #DFSV with cost
    test_dfs_solver_cost(csp_problem)
    solution = dfs_solve1_cost(csp_problem)
    if solution:
        total_cost = calculate_solution_cost(csp_problem, solution)
        print("Total soft constraint cost of Cost based DFS=", total_cost)
    nodes_with_cost.append(num_expanded)
    cost_DFS_COST.append(total_cost)

    avg_nodes_dfs.append(sum(nodes_no_cost)/num_epochs)
    avg_nodes_dfs_cost.append(sum(nodes_with_cost)/num_epochs)
    avg_cost_DFS.append(sum(cost_DFS)/num_epochs)
    avg_cost_DFS_COST.append(sum(cost_DFS_COST)/num_epochs)

print(avg_nodes_dfs)
print(avg_nodes_dfs_cost)
print(avg_cost_DFS)
print(avg_cost_DFS_COST)

df_2 = pd.DataFrame({
    'n': ns,
    'avg_nodes_dfs': avg_nodes_dfs,
    'avg_nodes_dfs_cost': avg_nodes_dfs_cost,
    'avg_cost_DFS': avg_cost_DFS,
    'avg_cost_DFS_COST': avg_cost_DFS_COST
})
print(df_2)

```

task, t1 3  
 task, t2 2  
 domain, t1 ends-by Wed 1pm 10  
 domain, t2 ends-by Fri 12pm 10  
 constraint, t1 same-day t2  
 constraint, t2 before t1  
 Nodes expanded to reach solution: 64  
 t1: mon 11am  
 t2: mon 9am  
 Nodes expanded to reach solution: 64  
 Total soft constraint cost of DFS = 0  
 Nodes expanded to reach solution: 64  
 t1: mon 11am  
 t2: mon 9am  
 Nodes expanded to reach solution: 64  
 Total soft constraint cost of Cost based DFS= 0  
 task, t1 4  
 task, t2 3  
 domain, t1 ends-by Thu 2pm 10  
 domain, t2 ends-by Tue 1pm 10  
 constraint, t1 same-day t2  
 constraint, t2 same-day t1  
 Nodes expanded to reach solution: 2  
 t1: mon 9am  
 t2: mon 9am  
 Nodes expanded to reach solution: 2  
 Total soft constraint cost of DFS = 0  
 Nodes expanded to reach solution: 2  
 t1: mon 9am  
 t2: mon 9am  
 Nodes expanded to reach solution: 2  
 Total soft constraint cost of Cost based DFS= 0  
 task, t1 4  
 task, t2 1  
 domain, t1 ends-by Wed 4pm 10  
 domain, t2 ends-by Wed 4pm 10  
 constraint, t1 same-day t2  
 constraint, t1 same-day t2  
 Nodes expanded to reach solution: 2  
 t1: mon 9am  
 t2: mon 9am  
 Nodes expanded to reach solution: 2  
 Total soft constraint cost of DFS = 0  
 Nodes expanded to reach solution: 2  
 t1: mon 9am  
 t2: mon 9am  
 Nodes expanded to reach solution: 2  
 Total soft constraint cost of Cost based DFS= 0  
 task, t1 3  
 task, t2 1  
 domain, t1 ends-by Fri 2pm 10  
 domain, t2 ends-by Tue 11am 10  
 constraint, t1 same-day t2  
 constraint, t2 before t1  
 Nodes expanded to reach solution: 38  
 t1: mon 10am  
 t2: mon 9am  
 Nodes expanded to reach solution: 38  
 Total soft constraint cost of DFS = 0  
 Nodes expanded to reach solution: 38

```

t1: mon 10am
t2: mon 9am
Nodes expanded to reach solution: 38
Total soft constraint cost of Cost based DFS= 0
task, t1 1
task, t2 4
domain, t1 ends-by Wed 1pm 10
domain, t2 ends-by Thu 2pm 10
constraint, t2 same-day t1
Nodes expanded to reach solution: 2
t1: mon 9am
t2: mon 9am
Nodes expanded to reach solution: 2
Total soft constraint cost of DFS = 0
Nodes expanded to reach solution: 2
t1: mon 9am
t2: mon 9am
Nodes expanded to reach solution: 2
Total soft constraint cost of Cost based DFS= 0
task, t1 3
task, t2 4
task, t3 5
domain, t1 ends-by Thu 4pm 10
domain, t2 ends-by Tue 3pm 10
domain, t3 ends-by Fri 4pm 10
constraint, t3 before t2
constraint, t3 before t1
Nodes expanded to reach solution: 1672
t1: tue 9am
t2: tue 9am
t3: mon 9am
Nodes expanded to reach solution: 1672
Total soft constraint cost of DFS = 0
Nodes expanded to reach solution: 1672
t1: tue 9am
t2: tue 9am
t3: mon 9am
Nodes expanded to reach solution: 1672
Total soft constraint cost of Cost based DFS= 0
task, t1 4
task, t2 1
task, t3 4
domain, t1 ends-by Wed 3pm 10
domain, t2 ends-by Wed 9am 10
domain, t3 ends-by Wed 4pm 10
constraint, t2 same-day t1
constraint, t3 same-day t2
Nodes expanded to reach solution: 3
t1: mon 9am
t2: mon 9am
t3: mon 9am
Nodes expanded to reach solution: 3
Total soft constraint cost of DFS = 0
Nodes expanded to reach solution: 3
t1: mon 9am
t2: mon 9am
t3: mon 9am
Nodes expanded to reach solution: 3
Total soft constraint cost of Cost based DFS= 0
task, t1 4

```

```

task, t2 1
task, t3 3
domain, t1 ends-by Thu 10am 10
domain, t2 ends-by Thu 4pm 10
domain, t3 ends-by Tue 1pm 10
constraint, t3 before t2
Nodes expanded to reach solution: 81
t1: mon 9am
t2: mon 12pm
t3: mon 9am
Nodes expanded to reach solution: 81
Total soft constraint cost of DFS = 0
Nodes expanded to reach solution: 81
t1: mon 9am
t2: mon 12pm
t3: mon 9am
Nodes expanded to reach solution: 81
Total soft constraint cost of Cost based DFS= 0
task, t1 5
task, t2 2
task, t3 4
domain, t1 ends-by Tue 1pm 10
domain, t2 ends-by Fri 3pm 10
domain, t3 ends-by Fri 1pm 10
constraint, t3 same-day t1
constraint, t1 same-day t3
Nodes expanded to reach solution: 3
t1: mon 9am
t2: mon 9am
t3: mon 9am
Nodes expanded to reach solution: 3
Total soft constraint cost of DFS = 0
Nodes expanded to reach solution: 3
t1: mon 9am
t2: mon 9am
t3: mon 9am
Nodes expanded to reach solution: 3
Total soft constraint cost of Cost based DFS= 0
task, t1 2
task, t2 2
task, t3 3
domain, t1 ends-by Fri 12pm 10
domain, t2 ends-by Thu 11am 10
domain, t3 ends-by Fri 9am 10
constraint, t1 same-day t2
Nodes expanded to reach solution: 3
t1: mon 9am
t2: mon 9am
t3: mon 9am
Nodes expanded to reach solution: 3
Total soft constraint cost of DFS = 0
Nodes expanded to reach solution: 3
t1: mon 9am
t2: mon 9am
t3: mon 9am
Nodes expanded to reach solution: 3
Total soft constraint cost of Cost based DFS= 0
task, t1 4
task, t2 3
task, t3 4

```

```

task, t4 1
domain, t1 ends-by Mon 10am 10
domain, t2 ends-by Thu 1pm 10
domain, t3 ends-by Mon 4pm 10
domain, t4 ends-by Tue 10am 10
constraint, t2 same-day t3
constraint, t4 same-day t1
constraint, t1 same-day t2
constraint, t3 same-day t4
Nodes expanded to reach solution: 4
t1: mon 9am
t2: mon 9am
t3: mon 9am
t4: mon 9am
Nodes expanded to reach solution: 4
Total soft constraint cost of DFS = 30
Nodes expanded to reach solution: 4
t2: mon 9am
t3: mon 9am
t4: mon 9am
t1: mon 9am
Nodes expanded to reach solution: 4
Total soft constraint cost of Cost based DFS= 30
task, t1 2
task, t2 1
task, t3 4
task, t4 4
domain, t1 ends-by Fri 12pm 10
domain, t2 ends-by Mon 10am 10
domain, t3 ends-by Tue 1pm 10
domain, t4 ends-by Tue 1pm 10
constraint, t4 before t3
constraint, t4 same-day t1
constraint, t1 before t3
Nodes expanded to reach solution: 48
t1: mon 9am
t2: mon 9am
t3: tue 9am
t4: mon 9am
Nodes expanded to reach solution: 48
Total soft constraint cost of DFS = 0
Nodes expanded to reach solution: 48
t1: mon 9am
t2: mon 9am
t3: tue 9am
t4: mon 9am
Nodes expanded to reach solution: 48
Total soft constraint cost of Cost based DFS= 0
task, t1 5
task, t2 3
task, t3 2
task, t4 4
domain, t1 ends-by Mon 4pm 10
domain, t2 ends-by Fri 10am 10
domain, t3 ends-by Wed 9am 10
domain, t4 ends-by Thu 1pm 10
constraint, t2 before t4
constraint, t4 same-day t3
constraint, t4 same-day t3
Nodes expanded to reach solution: 7

```

```

t1: mon 9am
t2: mon 9am
t3: mon 9am
t4: mon 12pm
Nodes expanded to reach solution: 7
Total soft constraint cost of DFS = 0
Nodes expanded to reach solution: 7
t1: mon 9am
t2: mon 9am
t3: mon 9am
t4: mon 12pm
Nodes expanded to reach solution: 7
Total soft constraint cost of Cost based DFS= 0
task, t1 1
task, t2 5
task, t3 5
task, t4 2
domain, t1 ends-by Wed 2pm 10
domain, t2 ends-by Fri 10am 10
domain, t3 ends-by Fri 2pm 10
domain, t4 ends-by Tue 11am 10
constraint, t2 before t1
constraint, t1 before t2
constraint, t1 before t3
constraint, t4 same-day t1
No solution
No solution
task, t1 2
task, t2 4
task, t3 4
task, t4 2
domain, t1 ends-by Wed 3pm 10
domain, t2 ends-by Thu 12pm 10
domain, t3 ends-by Mon 3pm 10
domain, t4 ends-by Thu 11am 10
constraint, t4 same-day t3
Nodes expanded to reach solution: 4
t1: mon 9am
t2: mon 9am
t3: mon 9am
t4: mon 9am
Nodes expanded to reach solution: 4
Total soft constraint cost of DFS = 0
Nodes expanded to reach solution: 4
t1: mon 9am
t2: mon 9am
t3: mon 9am
t4: mon 9am
Nodes expanded to reach solution: 4
Total soft constraint cost of Cost based DFS= 0
task, t1 4
task, t2 4
task, t3 3
task, t4 2
task, t5 1
domain, t1 ends-by Mon 11am 10
domain, t2 ends-by Fri 11am 10
domain, t3 ends-by Tue 12pm 10
domain, t4 ends-by Fri 4pm 10
domain, t5 ends-by Tue 2pm 10

```

```

constraint, t2 before t4
constraint, t1 same-day t4
Nodes expanded to reach solution: 9
t1: mon 9am
t2: mon 9am
t3: mon 9am
t4: mon 1pm
t5: mon 9am
Nodes expanded to reach solution: 9
Total soft constraint cost of DFS = 20
Nodes expanded to reach solution: 9
t2: mon 9am
t3: mon 9am
t4: mon 1pm
t5: mon 9am
t1: mon 9am
Nodes expanded to reach solution: 9
Total soft constraint cost of Cost based DFS= 20
task, t1 3
task, t2 1
task, t3 5
task, t4 5
task, t5 2
domain, t1 ends-by Wed 4pm 10
domain, t2 ends-by Tue 12pm 10
domain, t3 ends-by Mon 9am 10
domain, t4 ends-by Mon 4pm 10
domain, t5 ends-by Fri 4pm 10
constraint, t3 before t5
constraint, t1 before t3
constraint, t2 before t3
Nodes expanded to reach solution: 37
t1: mon 9am
t2: mon 9am
t3: fri 9am
t4: mon 9am
t5: fri 2pm
Nodes expanded to reach solution: 37
Total soft constraint cost of DFS = 1010
Nodes expanded to reach solution: 184
t1: mon 9am
t2: mon 9am
t4: mon 9am
t5: tue 2pm
t3: tue 9am
Nodes expanded to reach solution: 184
Total soft constraint cost of Cost based DFS= 290
task, t1 5
task, t2 2
task, t3 4
task, t4 2
task, t5 5
domain, t1 ends-by Tue 2pm 10
domain, t2 ends-by Tue 4pm 10
domain, t3 ends-by Thu 2pm 10
domain, t4 ends-by Mon 1pm 10
domain, t5 ends-by Wed 11am 10
constraint, t1 before t5
constraint, t5 before t4
Nodes expanded to reach solution: 187

```

```

t1: mon 9am
t2: mon 9am
t3: mon 9am
t4: tue 2pm
t5: tue 9am
Nodes expanded to reach solution: 187
Total soft constraint cost of DFS = 270
Nodes expanded to reach solution: 184
t1: mon 9am
t2: mon 9am
t3: mon 9am
t4: tue 2pm
t5: tue 9am
Nodes expanded to reach solution: 184
Total soft constraint cost of Cost based DFS= 270
task, t1 3
task, t2 3
task, t3 1
task, t4 1
task, t5 3
domain, t1 ends-by Thu 3pm 10
domain, t2 ends-by Thu 10am 10
domain, t3 ends-by Fri 4pm 10
domain, t4 ends-by Thu 9am 10
domain, t5 ends-by Fri 2pm 10
constraint, t3 same-day t4
constraint, t4 same-day t5
constraint, t1 same-day t4
Nodes expanded to reach solution: 5
t1: mon 9am
t2: mon 9am
t3: mon 9am
t4: mon 9am
t5: mon 9am
Nodes expanded to reach solution: 5
Total soft constraint cost of DFS = 0
Nodes expanded to reach solution: 5
t1: mon 9am
t2: mon 9am
t3: mon 9am
t4: mon 9am
t5: mon 9am
Nodes expanded to reach solution: 5
Total soft constraint cost of Cost based DFS= 0
task, t1 2
task, t2 3
task, t3 3
task, t4 2
task, t5 2
domain, t1 ends-by Mon 4pm 10
domain, t2 ends-by Tue 1pm 10
domain, t3 ends-by Wed 1pm 10
domain, t4 ends-by Mon 4pm 10
domain, t5 ends-by Thu 9am 10
constraint, t1 before t5
Nodes expanded to reach solution: 7
t1: mon 9am
t2: mon 9am
t3: mon 9am
t4: mon 9am

```



t5: mon 11am  
 Nodes expanded to reach solution: 7  
 Total soft constraint cost of DFS = 0  
 Nodes expanded to reach solution: 7  
 t1: mon 9am  
 t2: mon 9am  
 t3: mon 9am  
 t4: mon 9am  
 t5: mon 11am  
 Nodes expanded to reach solution: 7  
 Total soft constraint cost of Cost based DFS= 0  
 task, t1 3  
 task, t2 2  
 task, t3 2  
 task, t4 3  
 task, t5 3  
 task, t6 3  
 domain, t1 ends-by Tue 3pm 10  
 domain, t2 ends-by Thu 12pm 10  
 domain, t3 ends-by Wed 2pm 10  
 domain, t4 ends-by Tue 1pm 10  
 domain, t5 ends-by Fri 1pm 10  
 domain, t6 ends-by Tue 9am 10  
 constraint, t4 before t1  
 constraint, t2 same-day t5  
 constraint, t2 before t1  
 constraint, t2 same-day t6  
 constraint, t2 before t3  
 Nodes expanded to reach solution: 831  
 t1: mon 12pm  
 t2: mon 9am  
 t3: mon 11am  
 t4: mon 9am  
 t5: mon 9am  
 t6: mon 9am  
 Nodes expanded to reach solution: 831  
 Total soft constraint cost of DFS = 0  
 Nodes expanded to reach solution: 831  
 t1: mon 12pm  
 t2: mon 9am  
 t3: mon 11am  
 t4: mon 9am  
 t5: mon 9am  
 t6: mon 9am  
 Nodes expanded to reach solution: 831  
 Total soft constraint cost of Cost based DFS= 0  
 task, t1 5  
 task, t2 4  
 task, t3 3  
 task, t4 5  
 task, t5 1  
 task, t6 3  
 domain, t1 ends-by Thu 1pm 10  
 domain, t2 ends-by Tue 9am 10  
 domain, t3 ends-by Thu 4pm 10  
 domain, t4 ends-by Fri 10am 10  
 domain, t5 ends-by Wed 11am 10  
 domain, t6 ends-by Fri 9am 10  
 constraint, t3 same-day t6  
 constraint, t6 same-day t5

```

constraint, t3 same-day t2
constraint, t3 same-day t6
Nodes expanded to reach solution: 6
t1: mon 9am
t2: mon 9am
t3: mon 9am
t4: mon 9am
t5: mon 9am
t6: mon 9am
Nodes expanded to reach solution: 6
Total soft constraint cost of DFS = 0
Nodes expanded to reach solution: 6
t1: mon 9am
t2: mon 9am
t3: mon 9am
t4: mon 9am
t5: mon 9am
t6: mon 9am
Nodes expanded to reach solution: 6
Total soft constraint cost of Cost based DFS= 0
task, t1 1
task, t2 2
task, t3 4
task, t4 3
task, t5 4
task, t6 3
domain, t1 ends-by Thu 2pm 10
domain, t2 ends-by Fri 11am 10
domain, t3 ends-by Wed 3pm 10
domain, t4 ends-by Wed 1pm 10
domain, t5 ends-by Mon 11am 10
domain, t6 ends-by Tue 11am 10
constraint, t6 same-day t5
Nodes expanded to reach solution: 6
t1: mon 9am
t2: mon 9am
t3: mon 9am
t4: mon 9am
t5: mon 9am
t6: mon 9am
Nodes expanded to reach solution: 6
Total soft constraint cost of DFS = 20
Nodes expanded to reach solution: 6
t1: mon 9am
t2: mon 9am
t3: mon 9am
t4: mon 9am
t6: mon 9am
t5: mon 9am
Nodes expanded to reach solution: 6
Total soft constraint cost of Cost based DFS= 20
task, t1 1
task, t2 2
task, t3 3
task, t4 3
task, t5 5
task, t6 4
domain, t1 ends-by Thu 2pm 10
domain, t2 ends-by Mon 2pm 10
domain, t3 ends-by Wed 12pm 10

```

```

domain, t4 ends-by Tue 2pm 10
domain, t5 ends-by Fri 10am 10
domain, t6 ends-by Wed 12pm 10
constraint, t6 same-day t1
constraint, t4 same-day t5
constraint, t5 same-day t4
constraint, t3 same-day t1
Nodes expanded to reach solution: 6
t1: mon 9am
t2: mon 9am
t3: mon 9am
t4: mon 9am
t5: mon 9am
t6: mon 9am
Nodes expanded to reach solution: 6
Total soft constraint cost of DFS = 0
Nodes expanded to reach solution: 6
t1: mon 9am
t2: mon 9am
t3: mon 9am
t4: mon 9am
t5: mon 9am
t6: mon 9am
Nodes expanded to reach solution: 6
Total soft constraint cost of Cost based DFS= 0
task, t1 1
task, t2 2
task, t3 5
task, t4 5
task, t5 4
task, t6 2
domain, t1 ends-by Wed 12pm 10
domain, t2 ends-by Mon 4pm 10
domain, t3 ends-by Wed 2pm 10
domain, t4 ends-by Fri 1pm 10
domain, t5 ends-by Wed 10am 10
domain, t6 ends-by Mon 4pm 10
constraint, t4 same-day t1
constraint, t4 before t1
Nodes expanded to reach solution: 36161
t1: mon 2pm
t2: mon 9am
t3: mon 9am
t4: mon 9am
t5: mon 9am
t6: mon 9am
Nodes expanded to reach solution: 36161
Total soft constraint cost of DFS = 0
Nodes expanded to reach solution: 36161
t1: mon 2pm
t2: mon 9am
t3: mon 9am
t4: mon 9am
t5: mon 9am
t6: mon 9am
Nodes expanded to reach solution: 36161
Total soft constraint cost of Cost based DFS= 0
[21.6, 352.4, 124.6, 49.0, 7402.0]
[21.6, 352.4, 124.6, 77.8, 7402.0]
[0.0, 0.0, 6.0, 260.0, 4.0]

```

	[0.0, 0.0, 6.0, 116.0, 4.0]				
	n	avg_nodes_dfs	avg_nodes_dfs_cost	avg_cost_DFS	avg_cost_DFS_COST
0	2	21.6	21.6	0.0	0.0
1	3	352.4	352.4	0.0	0.0
2	4	124.6	124.6	6.0	6.0
3	5	49.0	77.8	260.0	116.0
4	6	7402.0	7402.0	4.0	4.0

### Answers for Question 5

Write the other answers here.

Here we use the cost function as the main method to calculate the possible solutions and its cost. From the last one, it can know that the node expanded that belongs to the formal DFS is much larger than the cost based DFS. At the same time, the formal DFS even can not find a solution sometimes and has a very large cost. So, the cost based has a better performance. *(The final result table is at the bottom of the result that shown above).*

### Question 6 (3 marks)

The CSP solver with domain splitting splits a CSP variable domain into *exactly two* partitions. Poole & Mackworth claim that in practice, this is as good as splitting into a larger number of partitions. In this question, empirically evaluate this claim for fuzzy scheduling CSPs.

- Write a new `partition_domain` function that partitions a domain into a list of `k` partitions, where `k` is a parameter to the function (1 mark)
- Modify the CSP solver to use the list of `k` partitions and evaluate the performance of the solver using the above metric for a range of values of `k` (2 marks)

```
In [25]: # Code for Question 6
# Place a copy of your code here and run it in the relevant cell
def partition_domain_new(domain, k):
    domain = list(domain)
    n = len(domain)
    if k < 1 or n == 1:
        print('Not use')
        return [domain]
    elif k > n:
        print('Not allowed')
        return []
    size = n // k
    part_start = 0
    partitions = []
    for i in range(k-1):
        part_end = part_start + size
        partitions.append(domain[part_start : part_end])
        part_start += size
    partitions.append(domain[part_start:])
    return partitions

# rewrites real methods to use
class Search_with_AC_from_Cost_CSP_new(Search_problem):
    """ A search problem with domain splitting and arc consistency """
    def __init__(self, csp, k):
```

```

self.cons = Con_solver(csp) # copy of the CSP with access to arc consist
self.domains = self.cons.make_arc_consistent(csp.domains)
self.constraints = csp.constraints
self.cost_functions = csp.cost_functions
self.durations = csp.durations
self.soft_day_time = csp.soft_day_time
self.soft_costs = csp.soft_costs
csp.domains = self.domains # after arc consistency
self.csp = csp
self.k = k

def is_goal(self, node):
    """ node is a goal if all domains have exactly 1 element """
    return all(len(node.domains[var]) == 1 for var in node.domains)

def start_node(self):
    return CSP_with_Cost(self.domains, self.durations, self.constraints,
                        self.cost_functions, self.soft_day_time, self.soft_

def neighbors(self, node):
    """returns the neighboring nodes of node.
    """
    neighs = []
    var = select(x for x in node.domains if len(node.domains[x]) > 1) # chose
    if var:
        partitions = partition_domain_new(node.domains[var], self.k)
        self.display(2, "Splitting", var, "into", partitions)
        to_do = self.cons.new_to_do(var, None)
        for dom in partitions:
            newdoms = node.domains | {var: dom} # overwrite domain of var wi
            cons_doms = self.cons.make_arc_consistent(newdoms, to_do)
            if all(len(cons_doms[v]) > 0 for v in cons_doms):
                # all domains are non-empty
                # make new CSP_with_Cost node to continue the search
                csp_node = CSP_with_Cost(cons_doms, self.durations, self.con
                    self.cost_functions, self.soft_day_time, self.soft_
                neighs.append(Arc(node, csp_node))
            else:
                self.display(2, "...", var, "in", dom, "has no solution")
    return neighs

def heuristic(self, n):
    return n.cost

#####-----
n = 7
ks = [2,3,4,7,8]
num_epochs = 5
avg_nodes_CSP = []
avg_nodes_newCSP = []
avg_cost_CSP = []
avg_cost_newCSP = []

spec_text = generate_problem(n)
for k in ks:
    nodes_newCSP = []
    nodes_CSP = []
    cost_CSP = []
    cost_newCSP = []

```

```

for _ in range(num_epochs):

    # 2 partitions
    csp_problem = create_CSP_from_spec(spec_text)
    solver = GreedySearcher(Search_with_AC_from_Cost_CSP(csp_problem))
    final_path = solver.search()
    if final_path is not None:
        cost_CSP.append(final_path.end().cost)
    else:
        cost_CSP.append(1000)
    nodes_CSP.append(solver.num_expanded) # get nodes number

    # new partition_domain
    csp_problem = create_CSP_from_spec(spec_text)
    solver = GreedySearcher(Search_with_AC_from_Cost_CSP_new(csp_problem, k))
    final_path = solver.search()
    if final_path is not None:
        cost_newCSP.append(final_path.end().cost)
    else:
        cost_newCSP.append(1000)
    nodes_newCSP.append(solver.num_expanded) # get nodes number

    avg_nodes_CSP.append(sum(nodes_CSP)/num_epochs)
    avg_nodes_newCSP.append(sum(nodes_newCSP)/num_epochs)
    avg_cost_CSP.append(sum(cost_CSP)/num_epochs)
    avg_cost_newCSP.append(sum(cost_newCSP)/num_epochs)

####-----

df_3 = pd.DataFrame({
    'k': ks,
    'avg_nodes_CSP': avg_nodes_CSP,
    'avg_nodes_newCSP': avg_nodes_newCSP,
    'avg_cost_CSP': avg_cost_CSP,
    'avg_cost_newCSP': avg_cost_newCSP
})
print(df_3)

```

task, t1 1  
task, t2 2  
task, t3 3  
task, t4 1  
task, t5 5  
task, t6 3  
task, t7 3  
domain, t1 ends-by Thu 4pm 10  
domain, t2 ends-by Thu 2pm 10  
domain, t3 ends-by Thu 11am 10  
domain, t4 ends-by Fri 2pm 10  
domain, t5 ends-by Wed 9am 10  
domain, t6 ends-by Thu 3pm 10  
domain, t7 ends-by Tue 2pm 10  
constraint, t6 same-day t2  
constraint, t3 before t5  
constraint, t6 same-day t4  
constraint, t5 before t4  
Solution: title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp (cost: 18)

19 paths have been expanded and 18 paths remain in the frontier

task, t1 1  
task, t2 2  
task, t3 3  
task, t4 1  
task, t5 5  
task, t6 3  
task, t7 3  
domain, t1 ends-by Thu 4pm 10  
domain, t2 ends-by Thu 2pm 10  
domain, t3 ends-by Thu 11am 10  
domain, t4 ends-by Fri 2pm 10  
domain, t5 ends-by Wed 9am 10  
domain, t6 ends-by Thu 3pm 10  
domain, t7 ends-by Tue 2pm 10  
constraint, t6 same-day t2  
constraint, t3 before t5  
constraint, t6 same-day t4  
constraint, t5 before t4  
Solution: title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp (cost: 18)

19 paths have been expanded and 18 paths remain in the frontier

task, t1 1  
task, t2 2  
task, t3 3  
task, t4 1  
task, t5 5  
task, t6 3  
task, t7 3  
domain, t1 ends-by Thu 4pm 10  
domain, t2 ends-by Thu 2pm 10  
domain, t3 ends-by Thu 11am 10  
domain, t4 ends-by Fri 2pm 10  
domain, t5 ends-by Wed 9am 10

domain, t6 ends-by Thu 3pm 10  
domain, t7 ends-by Tue 2pm 10  
constraint, t6 same-day t2  
constraint, t3 before t5  
constraint, t6 same-day t4  
constraint, t5 before t4  
Solution: title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp (cost: 18)

19 paths have been expanded and 18 paths remain in the frontier

task, t1 1  
task, t2 2  
task, t3 3  
task, t4 1  
task, t5 5  
task, t6 3  
task, t7 3

domain, t1 ends-by Thu 4pm 10  
domain, t2 ends-by Thu 2pm 10  
domain, t3 ends-by Thu 11am 10  
domain, t4 ends-by Fri 2pm 10  
domain, t5 ends-by Wed 9am 10  
domain, t6 ends-by Thu 3pm 10  
domain, t7 ends-by Tue 2pm 10  
constraint, t6 same-day t2  
constraint, t3 before t5  
constraint, t6 same-day t4  
constraint, t5 before t4

Solution: title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp (cost: 18)

19 paths have been expanded and 18 paths remain in the frontier

task, t1 1  
task, t2 2  
task, t3 3  
task, t4 1  
task, t5 5  
task, t6 3  
task, t7 3

domain, t1 ends-by Thu 4pm 10  
domain, t2 ends-by Thu 2pm 10  
domain, t3 ends-by Thu 11am 10  
domain, t4 ends-by Fri 2pm 10  
domain, t5 ends-by Wed 9am 10  
domain, t6 ends-by Thu 3pm 10  
domain, t7 ends-by Tue 2pm 10  
constraint, t6 same-day t2  
constraint, t3 before t5  
constraint, t6 same-day t4  
constraint, t5 before t4

Solution: title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp (cost: 18)

19 paths have been expanded and 18 paths remain in the frontier



```

task, t1 1
task, t2 2
task, t3 3
task, t4 1
task, t5 5
task, t6 3
task, t7 3
domain, t1 ends-by Thu 4pm 10
domain, t2 ends-by Thu 2pm 10
domain, t3 ends-by Thu 11am 10
domain, t4 ends-by Fri 2pm 10
domain, t5 ends-by Wed 9am 10
domain, t6 ends-by Thu 3pm 10
domain, t7 ends-by Tue 2pm 10
constraint, t6 same-day t2
constraint, t3 before t5
constraint, t6 same-day t4
constraint, t5 before t4
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp --> title of csp --> title of csp --> title of csp --> title of csp
--> title of csp --> title of csp --> title of csp --> title of csp --> title of
csp (cost: 18)
19 paths have been expanded and 18 paths remain in the frontier
task, t1 1
task, t2 2
task, t3 3
task, t4 1
task, t5 5
task, t6 3
task, t7 3
domain, t1 ends-by Thu 4pm 10
domain, t2 ends-by Thu 2pm 10
domain, t3 ends-by Thu 11am 10
domain, t4 ends-by Fri 2pm 10
domain, t5 ends-by Wed 9am 10
domain, t6 ends-by Thu 3pm 10
domain, t7 ends-by Tue 2pm 10
constraint, t6 same-day t2
constraint, t3 before t5
constraint, t6 same-day t4
constraint, t5 before t4
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp --> title of csp --> title of csp --> title of csp --> title of csp
--> title of csp --> title of csp --> title of csp --> title of csp --> title of
csp (cost: 18)
19 paths have been expanded and 18 paths remain in the frontier
task, t1 1
task, t2 2
task, t3 3
task, t4 1
task, t5 5
task, t6 3
task, t7 3
domain, t1 ends-by Thu 4pm 10
domain, t2 ends-by Thu 2pm 10
domain, t3 ends-by Thu 11am 10
domain, t4 ends-by Fri 2pm 10
domain, t5 ends-by Wed 9am 10

```

domain, t6 ends-by Thu 3pm 10  
domain, t7 ends-by Tue 2pm 10  
constraint, t6 same-day t2  
constraint, t3 before t5  
constraint, t6 same-day t4  
constraint, t5 before t4  
Solution: title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp (cost: 18)

19 paths have been expanded and 18 paths remain in the frontier

task, t1 1  
task, t2 2  
task, t3 3  
task, t4 1  
task, t5 5  
task, t6 3  
task, t7 3

domain, t1 ends-by Thu 4pm 10  
domain, t2 ends-by Thu 2pm 10  
domain, t3 ends-by Thu 11am 10  
domain, t4 ends-by Fri 2pm 10  
domain, t5 ends-by Wed 9am 10  
domain, t6 ends-by Thu 3pm 10  
domain, t7 ends-by Tue 2pm 10  
constraint, t6 same-day t2  
constraint, t3 before t5  
constraint, t6 same-day t4  
constraint, t5 before t4

Solution: title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp (cost: 18)

19 paths have been expanded and 18 paths remain in the frontier

task, t1 1  
task, t2 2  
task, t3 3  
task, t4 1  
task, t5 5  
task, t6 3  
task, t7 3

domain, t1 ends-by Thu 4pm 10  
domain, t2 ends-by Thu 2pm 10  
domain, t3 ends-by Thu 11am 10  
domain, t4 ends-by Fri 2pm 10  
domain, t5 ends-by Wed 9am 10  
domain, t6 ends-by Thu 3pm 10  
domain, t7 ends-by Tue 2pm 10  
constraint, t6 same-day t2  
constraint, t3 before t5  
constraint, t6 same-day t4  
constraint, t5 before t4

Solution: title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp (cost: 18)

19 paths have been expanded and 18 paths remain in the frontier

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Solution: title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp (cost: 18)

19 paths have been expanded and 18 paths remain in the frontier

task, t1 1

task, t2 2

task, t3 3

task, t4 1

task, t5 5

task, t6 3

task, t7 3

domain, t1 ends-by Thu 4pm 10

domain, t2 ends-by Thu 2pm 10

domain, t3 ends-by Thu 11am 10

domain, t4 ends-by Fri 2pm 10

domain, t5 ends-by Wed 9am 10

domain, t6 ends-by Thu 3pm 10

domain, t7 ends-by Tue 2pm 10

constraint, t6 same-day t2

constraint, t3 before t5

constraint, t6 same-day t4

constraint, t5 before t4

Not allowed

Not allowed

Not allowed

Not allowed

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Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp --> title of csp --> title of csp --> title of csp --> title of csp
--> title of csp --> title of csp (cost: 15)
```

4266 paths have been expanded and 949 paths remain in the frontier

```
task, t1 1
task, t2 2
task, t3 3
task, t4 1
task, t5 5
task, t6 3
task, t7 3
```

```
domain, t1 ends-by Thu 4pm 10
domain, t2 ends-by Thu 2pm 10
domain, t3 ends-by Thu 11am 10
domain, t4 ends-by Fri 2pm 10
domain, t5 ends-by Wed 9am 10
domain, t6 ends-by Thu 3pm 10
domain, t7 ends-by Tue 2pm 10
constraint, t6 same-day t2
constraint, t3 before t5
constraint, t6 same-day t4
constraint, t5 before t4
```

```
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp --> title of csp --> title of csp --> title of csp --> title of csp
--> title of csp --> title of csp --> title of csp --> title of csp --> title of
csp (cost: 18)
```

```
19 paths have been expanded and 18 paths remain in the frontier
```

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```

constraint, t3 before t5
constraint, t6 same-day t4
constraint, t5 before t4
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp --> title of csp --> title of csp --> title of csp --> title of csp
--> title of csp --> title of csp --> title of csp --> title of csp --> title of
csp (cost: 18)
  19 paths have been expanded and 18 paths remain in the frontier
task, t1 1
task, t2 2
task, t3 3
task, t4 1
task, t5 5
task, t6 3
task, t7 3
domain, t1 ends-by Thu 4pm 10
domain, t2 ends-by Thu 2pm 10
domain, t3 ends-by Thu 11am 10
domain, t4 ends-by Fri 2pm 10
domain, t5 ends-by Wed 9am 10
domain, t6 ends-by Thu 3pm 10
domain, t7 ends-by Tue 2pm 10
constraint, t6 same-day t2
constraint, t3 before t5
constraint, t6 same-day t4
constraint, t5 before t4
Not allowed
Not allowed
Not allowed
Not allowed
Not allowed
Not allowed
Not allowed
No (more) solutions. Total of 8 paths expanded.
task, t1 1
task, t2 2
task, t3 3
task, t4 1
task, t5 5
task, t6 3
task, t7 3
domain, t1 ends-by Thu 4pm 10
domain, t2 ends-by Thu 2pm 10
domain, t3 ends-by Thu 11am 10
domain, t4 ends-by Fri 2pm 10
domain, t5 ends-by Wed 9am 10
domain, t6 ends-by Thu 3pm 10
domain, t7 ends-by Tue 2pm 10
constraint, t6 same-day t2
constraint, t3 before t5
constraint, t6 same-day t4
constraint, t5 before t4
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp --> title of csp --> title of csp --> title of csp --> title of csp
--> title of csp --> title of csp --> title of csp --> title of csp --> title of
csp (cost: 18)
  19 paths have been expanded and 18 paths remain in the frontier
task, t1 1

```

```

task, t2 2
task, t3 3
task, t4 1
task, t5 5
task, t6 3
task, t7 3
domain, t1 ends-by Thu 4pm 10
domain, t2 ends-by Thu 2pm 10
domain, t3 ends-by Thu 11am 10
domain, t4 ends-by Fri 2pm 10
domain, t5 ends-by Wed 9am 10
domain, t6 ends-by Thu 3pm 10
domain, t7 ends-by Tue 2pm 10
constraint, t6 same-day t2
constraint, t3 before t5
constraint, t6 same-day t4
constraint, t5 before t4
Not allowed
Not allowed
Not allowed
Not allowed
Not allowed
Not allowed
Not allowed
No (more) solutions. Total of 8 paths expanded.
task, t1 1
task, t2 2
task, t3 3
task, t4 1
task, t5 5
task, t6 3
task, t7 3
domain, t1 ends-by Thu 4pm 10
domain, t2 ends-by Thu 2pm 10
domain, t3 ends-by Thu 11am 10
domain, t4 ends-by Fri 2pm 10
domain, t5 ends-by Wed 9am 10
domain, t6 ends-by Thu 3pm 10
domain, t7 ends-by Tue 2pm 10
constraint, t6 same-day t2
constraint, t3 before t5
constraint, t6 same-day t4
constraint, t5 before t4
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp --> title of csp --> title of csp --> title of csp --> title of csp
--> title of csp --> title of csp --> title of csp --> title of csp --> title of
csp (cost: 18)
19 paths have been expanded and 18 paths remain in the frontier
task, t1 1
task, t2 2
task, t3 3
task, t4 1
task, t5 5
task, t6 3
task, t7 3
domain, t1 ends-by Thu 4pm 10
domain, t2 ends-by Thu 2pm 10
domain, t3 ends-by Thu 11am 10
domain, t4 ends-by Fri 2pm 10

```



domain, t5 ends-by Wed 9am 10  
domain, t6 ends-by Thu 3pm 10  
domain, t7 ends-by Tue 2pm 10  
constraint, t6 same-day t2  
constraint, t3 before t5  
constraint, t6 same-day t4  
constraint, t5 before t4  
Not allowed  
Not allowed  
Not allowed  
Not allowed  
Not allowed  
Not allowed  
Not allowed

No (more) solutions. Total of 8 paths expanded.

task, t1 1  
task, t2 2  
task, t3 3  
task, t4 1  
task, t5 5  
task, t6 3  
task, t7 3  
domain, t1 ends-by Thu 4pm 10  
domain, t2 ends-by Thu 2pm 10  
domain, t3 ends-by Thu 11am 10  
domain, t4 ends-by Fri 2pm 10  
domain, t5 ends-by Wed 9am 10  
domain, t6 ends-by Thu 3pm 10  
domain, t7 ends-by Tue 2pm 10  
constraint, t6 same-day t2  
constraint, t3 before t5  
constraint, t6 same-day t4  
constraint, t5 before t4

Solution: title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp (cost: 18)

19 paths have been expanded and 18 paths remain in the frontier

task, t1 1  
task, t2 2  
task, t3 3  
task, t4 1  
task, t5 5  
task, t6 3  
task, t7 3  
domain, t1 ends-by Thu 4pm 10  
domain, t2 ends-by Thu 2pm 10  
domain, t3 ends-by Thu 11am 10  
domain, t4 ends-by Fri 2pm 10  
domain, t5 ends-by Wed 9am 10  
domain, t6 ends-by Thu 3pm 10  
domain, t7 ends-by Tue 2pm 10  
constraint, t6 same-day t2  
constraint, t3 before t5  
constraint, t6 same-day t4  
constraint, t5 before t4  
Not allowed  
Not allowed  
Not allowed



```

task, t6 3
task, t7 3
domain, t1 ends-by Thu 4pm 10
domain, t2 ends-by Thu 2pm 10
domain, t3 ends-by Thu 11am 10
domain, t4 ends-by Fri 2pm 10
domain, t5 ends-by Wed 9am 10
domain, t6 ends-by Thu 3pm 10
domain, t7 ends-by Tue 2pm 10
constraint, t6 same-day t2
constraint, t3 before t5
constraint, t6 same-day t4
constraint, t5 before t4
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp --> title of csp --> title of csp --> title of csp --> title of csp
--> title of csp --> title of csp --> title of csp --> title of csp --> title of
csp (cost: 18)
  19 paths have been expanded and 18 paths remain in the frontier
task, t1 1
task, t2 2
task, t3 3
task, t4 1
task, t5 5
task, t6 3
task, t7 3
domain, t1 ends-by Thu 4pm 10
domain, t2 ends-by Thu 2pm 10
domain, t3 ends-by Thu 11am 10
domain, t4 ends-by Fri 2pm 10
domain, t5 ends-by Wed 9am 10
domain, t6 ends-by Thu 3pm 10
domain, t7 ends-by Tue 2pm 10
constraint, t6 same-day t2
constraint, t3 before t5
constraint, t6 same-day t4
constraint, t5 before t4
Not allowed
Not allowed
Not allowed
Not allowed
Not allowed
Not allowed
Not allowed
Not allowed
No (more) solutions. Total of 9 paths expanded.
task, t1 1
task, t2 2
task, t3 3
task, t4 1
task, t5 5
task, t6 3
task, t7 3
domain, t1 ends-by Thu 4pm 10
domain, t2 ends-by Thu 2pm 10
domain, t3 ends-by Thu 11am 10
domain, t4 ends-by Fri 2pm 10
domain, t5 ends-by Wed 9am 10
domain, t6 ends-by Thu 3pm 10
domain, t7 ends-by Tue 2pm 10

```

```

constraint, t6 same-day t2
constraint, t3 before t5
constraint, t6 same-day t4
constraint, t5 before t4
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp --> title of csp --> title of csp --> title of csp --> title of csp
--> title of csp --> title of csp --> title of csp --> title of csp --> title of
csp (cost: 18)
  19 paths have been expanded and 18 paths remain in the frontier
task, t1 1
task, t2 2
task, t3 3
task, t4 1
task, t5 5
task, t6 3
task, t7 3
domain, t1 ends-by Thu 4pm 10
domain, t2 ends-by Thu 2pm 10
domain, t3 ends-by Thu 11am 10
domain, t4 ends-by Fri 2pm 10
domain, t5 ends-by Wed 9am 10
domain, t6 ends-by Thu 3pm 10
domain, t7 ends-by Tue 2pm 10
constraint, t6 same-day t2
constraint, t3 before t5
constraint, t6 same-day t4
constraint, t5 before t4
Not allowed
Not allowed
Not allowed
Not allowed
Not allowed
Not allowed
Not allowed
Not allowed
No (more) solutions. Total of 9 paths expanded.
task, t1 1
task, t2 2
task, t3 3
task, t4 1
task, t5 5
task, t6 3
task, t7 3
domain, t1 ends-by Thu 4pm 10
domain, t2 ends-by Thu 2pm 10
domain, t3 ends-by Thu 11am 10
domain, t4 ends-by Fri 2pm 10
domain, t5 ends-by Wed 9am 10
domain, t6 ends-by Thu 3pm 10
domain, t7 ends-by Tue 2pm 10
constraint, t6 same-day t2
constraint, t3 before t5
constraint, t6 same-day t4
constraint, t5 before t4
Solution: title of csp --> title of csp --> title of csp --> title of csp --> tit
le of csp --> title of csp --> title of csp --> title of csp --> title of csp -->
title of csp --> title of csp --> title of csp --> title of csp --> title of csp
--> title of csp --> title of csp --> title of csp --> title of csp --> title of
csp (cost: 18)

```



domain, t2 ends-by Thu 2pm 10  
domain, t3 ends-by Thu 11am 10  
domain, t4 ends-by Fri 2pm 10  
domain, t5 ends-by Wed 9am 10  
domain, t6 ends-by Thu 3pm 10  
domain, t7 ends-by Tue 2pm 10  
constraint, t6 same-day t2  
constraint, t3 before t5  
constraint, t6 same-day t4  
constraint, t5 before t4  
Not allowed  
Not allowed  
Not allowed  
Not allowed  
Not allowed  
Not allowed  
Not allowed  
Not allowed  
No (more) solutions. Total of 9 paths expanded.

task, t1 1  
task, t2 2  
task, t3 3  
task, t4 1  
task, t5 5  
task, t6 3  
task, t7 3  
domain, t1 ends-by Thu 4pm 10  
domain, t2 ends-by Thu 2pm 10  
domain, t3 ends-by Thu 11am 10  
domain, t4 ends-by Fri 2pm 10  
domain, t5 ends-by Wed 9am 10  
domain, t6 ends-by Thu 3pm 10  
domain, t7 ends-by Tue 2pm 10  
constraint, t6 same-day t2  
constraint, t3 before t5  
constraint, t6 same-day t4  
constraint, t5 before t4

Solution: title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp --> title of csp (cost: 18)

19 paths have been expanded and 18 paths remain in the frontier

task, t1 1  
task, t2 2  
task, t3 3  
task, t4 1  
task, t5 5  
task, t6 3  
task, t7 3  
domain, t1 ends-by Thu 4pm 10  
domain, t2 ends-by Thu 2pm 10  
domain, t3 ends-by Thu 11am 10  
domain, t4 ends-by Fri 2pm 10  
domain, t5 ends-by Wed 9am 10  
domain, t6 ends-by Thu 3pm 10  
domain, t7 ends-by Tue 2pm 10  
constraint, t6 same-day t2  
constraint, t3 before t5  
constraint, t6 same-day t4

constraint, t5 before t4

Not allowed

Not allowed

Not allowed

Not allowed

Not allowed

Not allowed

Not allowed

Not allowed

No (more) solutions. Total of 9 paths expanded.

	k	avg_nodes_CSP	avg_nodes_newCSP	avg_cost_CSP	avg_cost_newCSP
0	2	19.0	19.0	0.0	0.0
1	3	19.0	4266.0	0.0	50.0
2	4	19.0	165.0	0.0	250.0
3	7	19.0	8.0	0.0	1000.0
4	8	19.0	9.0	0.0	1000.0

### Answers for Question 6

Write the other answers here.

In this question, we have the two types of the partition domain methods to be compared. From the table shown at the appendix, it can found that the for different k, it will has more nodes to be expanded which might consume more memory. At the same time, the cost for the tasks is continually increase due to the nodes and even cannot find the possible solutions finally.

## Appendix

### 1. Solution costs.

```
In [21]: def calculate_solution_cost(csp, solution):
total_cost = 0
for v, val in solution.items():
    dur = csp.durations[v]
    soft_dt = csp.soft_day_time[v]
    soft_cost = int(csp.soft_costs[v])
    cost_func = csp.cost_functions[v][0]

    cost = cost_func(val, soft_dt, dur, soft_cost)
    if cost is None:
        cost = 0
    total_cost += cost

return total_cost
```

### 4. Other part of Qestion 4.

```
In [22]: print(df_1)
```

	n	Optimal	DFS	MRV	avg_cost_DFS	avg_cost_DFS_mrv	avg_cost_optimal
0	2	5.4	15.2	3.2	58.0	58.0	2.0
1	3	6.2	9.4	9.4	226.0	82.0	26.0
2	4	8.8	132.4	132.4	96.0	96.0	0.0
3	5	14.2	5.0	5.0	146.0	146.0	58.0
4	6	17.8	6.2	6.0	210.0	256.0	66.0
5	7	24.0	7.0	7.0	266.0	266.0	64.0

### 3. Result table of Question5.

In [23]: `print(df_2)`

	n	avg_nodes_dfs	avg_nodes_dfs_cost	avg_cost_DFS	avg_cost_DFS_COST
0	2	21.6	21.6	0.0	0.0
1	3	352.4	352.4	0.0	0.0
2	4	124.6	124.6	6.0	6.0
3	5	49.0	77.8	260.0	116.0
4	6	7402.0	7402.0	4.0	4.0

### 4. Result table of Question6.

In [26]: `print(df_3)`

	k	avg_nodes_CSP	avg_nodes_newCSP	avg_cost_CSP	avg_cost_newCSP
0	2	19.0	19.0	0.0	0.0
1	3	19.0	4266.0	0.0	50.0
2	4	19.0	165.0	0.0	250.0
3	7	19.0	8.0	0.0	1000.0
4	8	19.0	9.0	0.0	1000.0

In [ ]: