

Factor Model with Principal Components

 We propose a beta-neutral factor trading strategy for stocks in the S&P 500, where the factors f are the first K = 5, principal components of a sample of 50 component stocks.

$$r_{excess,i} = \alpha_i + \sum_{j=1}^{K} \beta_{ij} f_j + \epsilon_i$$

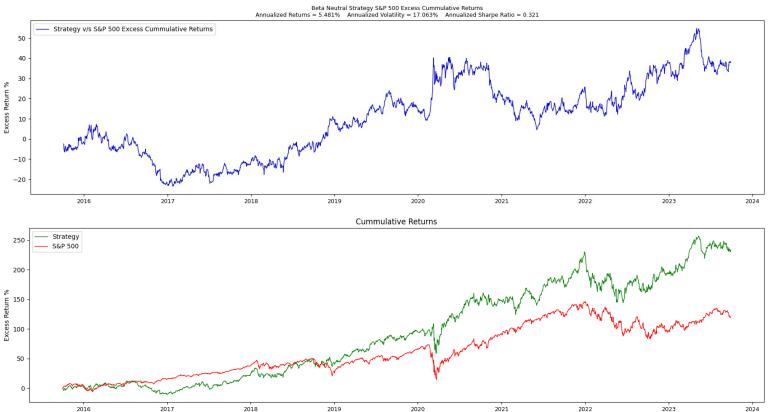
- We calculate the β s of all S&P stocks with the endogenous factors and use a numerical optimizer to calculate weights such that the *net* portfolio β is 0.
- We backtest the strategy from Oct'15 to Sept'23, recalibrating the β s and rebalancing the portfolio monthly. The strategy outperforms a simple *long-only* strategy on the S&P 500 index with an excess Sharpe Ratio of 0.3188

02-Oct-2015 To 30-Sep-2023					
Annualised Metrics	Return	Volatility	Sharpe Ratio	Sortino Ratio	
Strategy vs S&P 500	5.48%	17.18%	0.3188	0.5225	
Strategy Excess	15.70%	20.47%	0.7670	1.2139	
S&P 500 Excess	10.22%	18.70%	0.5467	0.8365	

Out of Sample Strategy Betas				
Alpha	Stock Beta (S&P 500)	US Bond Beta (AGG)		
0.0004	0.6612	0.5868		



Strategy Backtesting Results





Sample Covariance Matrix

- Factor model variance-covariance matrix $\Sigma = \beta \Omega_f \beta' + \Psi_\epsilon$
- If invertible, the expected value of its inverse is a biased estimator for the theoretical inverse*.

$$E(S^{-1}) = \frac{T}{T - N - 2} \Sigma^{-1}$$

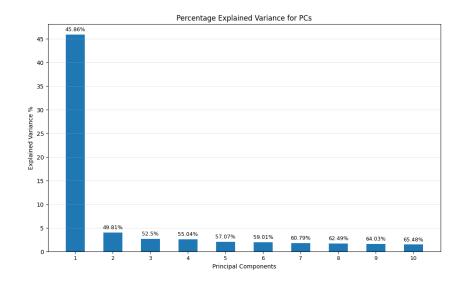
- The sample covariance can be volatile in the sense that the constructed weights for the beta neutral portfolios may give rise to high turnover rates over time.
- To tackle these issues, we select a set of 50 representative stocks proportional to their industry weights in the S&P 500. We calculate the factors using the subset of 50 stocks.

GICS Sector	Number of Stocks	Percentage
Communication Services	23	5%
Consumer Discretionary	52	10%
Consumer Staples	38	8%
Energy	23	5%
Financials	72	14%
Health Care	65	13%
Industrials	76	15%
Information Technology	64	13%
Materials	29	6%
Real Estate	31	6%
Utilities	30	6%
Total	503	100%

Principal Component Factors

- For *n* distinct eigenvalues, we calculate a set of *n* eigenvectors, ordered by most-to-least explained variance. Transforming the returns matrix by multiplication with the eigenvectors returns the stochastic factors for our model a.k.a principal components.
- We select the first k = 5 principal components using PCA decomposition with a cumulative explained variance of 57.07%

$$r_{excess,i} = \alpha_i + \sum_{j=1}^K \beta_{ij} f_j + \epsilon_i$$





Factor-Neutral Strategy

- We omit stocks with missing data from our dataset, reducing our universe to 390 stocks. We perform a linear regression for each stock against the stochastic factors to calculate their factor loadings i.e., β .
- Initializing the portfolio weight vector $\mathbf{w_0}$ with equal weights, we optimize the portfolio weights to minimize the portfolio beta exposure subject to a net zero long/short strategy i.e., $\sum_{1}^{N} w_i = 0$

$$\underset{\mathbf{w}}{\operatorname{argmin}} \left(\sum_{i=1}^{N} \sum_{j=1}^{5} w_{i} \beta_{ij} \right)^{2}$$

• We use SciPy's numerical optimizer with a sequential least squares programming optimization method with constraints to calculate the beta-neutral weight vector \mathbf{w} .



Backtesting

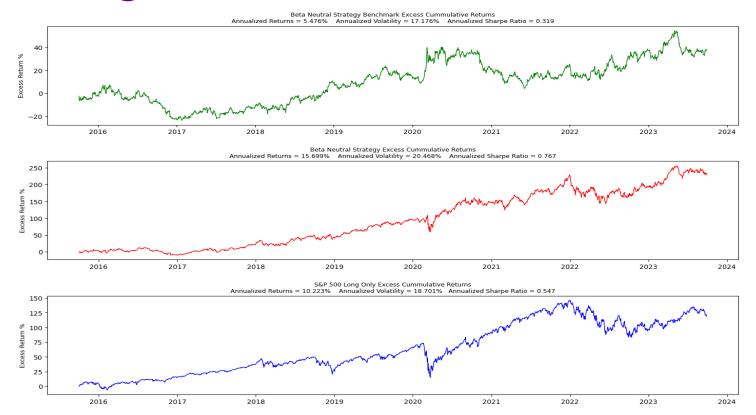
- We perform PCA decomposition on stock returns from 01-October-2003 to 30-September-2015 and select the first 5 principal components as our factors with a cumulative explained variance ratio of 57%.
- We backtest our strategy from 02-October-2015 to 30-September-2023. The betas and portfolio weights are recalibrated monthly by sliding the PCA window forward by 1 month. The portfolio is rebalanced on the first business day of every month.

Out of Sample Strategy Betas			
Alpha	Stock Beta (S&P 500)	US Bond Beta (AGG)	
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- We use the daily yield on the 1-year T-bill as the risk-free rate to calculate excess returns
- We assume zero transaction costs, unlimited leverage and that trades are executed immediately at the EOD close price with no slippage. There is also a tracking error with the benchmark used due to the omission of stocks with missing data.



Backtesting





Appendix

Jupyter Notebook - <u>Github Repo</u>



^{*}The list of S&P 500 stocks is retrieved dynamically from Wikipedia.

^{*}The price dataset is downloaded from Yahoo Finance using an API call within the Jupyter Notebook