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EE17BTECH11051

Q1)

Ans - A random variable is a variable whose value depends on the outcome of some experiment.

Examples of random variables -

i) In the experiment of ~~throwing~~ tossing a coin, X can represent a random variable that ~~takes~~ takes the outcome of ~~head as~~ the tossing.

$\therefore$ $P(X = \text{head}) = \frac{1}{2}$

$P(X = \text{tail}) = \frac{1}{2}$

ii) The outcome of a throwing of a dice is a random variable.

~~iii) The outcome when a card is drawn /~~

iii) The colour of the card when a card is drawn from a deck of cards.

iv) The colour ~~of~~ of a ball while identical shaped balls are placed in a basket.

v) The no. of students passing in a certain exam.

Q2)

Solⁿ a) PMF of X is as follows

X	2	3	4	5	6	7	8	9	10	11	12
P(x)	1/36	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$

Solⁿ b) PMF of X is as follows

X	1	2	3	4	5	6
P(X)	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$

Solⁿ c)

$$P(X=1) = \frac{6}{6\times6\times6} = \frac{6}{216}$$

$$P(X=2) = \frac{{}^3C_2\times 6\times 5}{216} = \frac{3\times6\times5}{216} = \frac{90}{216}$$

$$P(X=3) = \frac{6\times5\times4}{216} = \frac{120}{216}$$

Solⁿ d) $Y = X^2$

$X = \{-2, -1, 0, 1, 2, 3, 5\}$

$P(x_i) = \frac{1}{7}$

$Y = \{0 \ 1 \ 4 \ 9 \ 25\}$

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$$E[Y] = 0 \times \frac{1}{7} + 1 \times \frac{2}{7} + 4 \times \frac{2}{7} + 9 \times \frac{1}{7} + 25 \times \frac{1}{7}$$

$$= \frac{1+2+8+9+25}{7}$$

$$= \frac{45}{7}$$

$$= 6.43$$

Sol^n e) $Y = X^2 + 3X$

$$E[Y] = E[X^2 + 3X]$$

$$= E[X^2] + 3E[X]$$

Q3)

Sol^n a) CDF is defined as $P(X \le x)$. This basic definition can be applied to any function, discrete or continuous.

Sol^n b)
$$f_X(x) = \begin{cases} A(1-x^2) & -1 \le x \le 1 \\ 0 & \text{elsewhere} \end{cases}$$

$$\int_{-\infty}^{\infty} f_X(x)\,dx = 1$$

$$\Rightarrow A \int_{-1}^{1} (1-x^2)\,dx = 1$$

$$\Rightarrow A \left[x - \frac{x^3}{3}\right]_{-1}^{1} = 1$$

$$\Rightarrow A \left[2 - \frac{2}{3}\right] = 1$$

$$\Rightarrow A \left[\frac{4}{3}\right] = 1 \quad \Rightarrow A = \frac{3}{4}$$

$$P(0.5 < X < 1.5) = \int_{\frac{1}{2}}^{1} \frac{3}{4}(1-x^2)\,dx$$

$$= \frac{3}{4}\left[x - \frac{x^3}{3}\right]_{\frac{1}{2}}^{1}$$

$$= \frac{3}{4}\left[1 - \frac{1}{3} - \frac{1}{2} + \frac{1}{8\times 3}\right]$$

$$= \frac{5}{32}$$

$$F_X(x) = \int_{-\infty}^{x} f_X(x)\,dx$$

$$= \frac{3}{4}\int_{-1}^{x}(1-x^2)\,dx$$

$$= \frac{3}{4}\left[x - \frac{x^3}{3}\right]_{-1}^{x}$$

$$= \frac{3}{4}\left[x - \frac{x^3}{3} - \left(-1 + \frac{1}{3}\right)\right]$$

$$= \frac{3}{4}\left[x - \frac{x^3}{3} + \frac{2}{3}\right]$$

$$= \frac{3x}{4} - \frac{x^3}{4} + \frac{2}{4}$$

$$\mu_X = \int_{-\infty}^{\infty} x f_X(x)\,dx$$

$$= \frac{3}{4}\int_{-1}^{1}(x - x^3)\,dx$$

$$= \frac{3}{4}\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_{-1}^{1}$$

$$= 0$$

$$Var(X) = \int_{-\infty}^{\infty} x^2 f_X(x)\,dx$$

$$= \frac{3}{4}\int_{-1}^{1}(x^2 - x^4)\,dx$$

$$= \frac{3}{4}\left[\frac{x^3}{3} - \frac{x^5}{5}\right]_{-1}^{1}$$

$$= \frac{3}{4}\left[\frac{1}{3} - \frac{1}{5} - \left(\frac{-1}{3} + \frac{1}{5}\right)\right]$$

$$= \frac{3}{4}\times 2\left[\frac{1}{3} - \frac{1}{5}\right] = \frac{3}{2}\left[\frac{2}{15}\right] = \frac{1}{5} = 0.2$$

$$\sigma_X = \sqrt{Var(X)}$$

$$= \frac{1}{\sqrt{5}}$$

$$= 0.447$$

~~Sol~~^{n} - c) a) $f_X(x) = \dfrac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$

$\sigma = 3$; $\mu = 10$

$\therefore f_X(x) = \dfrac{1}{\sqrt{18\pi}} e^{\frac{-(x-10)^2}{18}}$

$= 0.133\, e^{-\frac{(x-10)^2}{18}}$

$P(X < 10-\sqrt{3}) = \displaystyle\int_{-\infty}^{10-\sqrt{3}} 0.133\, e^{-\frac{(x-10)^2}{18}}$

$= 1 - Q\left(\dfrac{10-\sqrt{3}-10}{\sqrt{3}}\right)]$

$= [1 - Q(-1)] = 0.1586$

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b) Let the uniform distribution be

$$f_X(x) = \frac{1}{b-a} \quad \forall\, x \in [a, b]$$

Mean $= \frac{b+a}{2} = 10$

$\Rightarrow b = 20 - a$

Variance $= \frac{(b-a)^2}{12} = 3$

$\Rightarrow (b-a)^2 = 36$

$\Rightarrow (20 - a - a)^2 = 36$

$\Rightarrow (20 - 2a)^2 = 36$

$\Rightarrow 20 - 2a = \pm 6$

For $20 - 2a = 6$

$\Rightarrow 2a = 14$

$\Rightarrow a = 7$

$\therefore b = 20 - 7 = 13$

For $20 - 2a = -6$

$\Rightarrow 2a = 26$

$\Rightarrow a = 13$

$\therefore b = 20 - 13 = 7$

$\therefore f_X(x) = \frac{1}{b-a} = \frac{1}{6} \quad \forall\, x \in [7, 13]$

$\therefore P(X < 10 - \sqrt{3})$

$= \int_{-\infty}^{10-\sqrt{3}} f_X(x)\, dx$

$= \frac{1}{6} [x]_{7}^{8.27}$

$= \frac{1.27}{6} = 0.2116$

Q4)

Sol^n:- a) We know

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{X,Y}(x,y)\, dx\, dy = 1$$

This is possible if $f_{XY}(x,y) = \frac{1}{\pi \times 1} = \frac{1}{\pi}$

$$\therefore f_{X,Y}(x,y) = \frac{1}{\pi}$$

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b) $P\{(X,Y) \in A\} = \frac{\text{Area of } A}{\text{Area of circle}}$

$= \frac{1}{4}$.

c) $P\{(X^2+Y^2) \le r^2\} = \frac{\text{Area of circle of radius } r}{\text{Area of original circle}}$

$= \frac{\pi r^2}{\pi}$

$= r^2 \quad \forall\, r \in [0,1]$

d) $f_X(x) = \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} f_{X,Y}(x,y)\, dy$

$= \frac{[y]_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}}{\pi}$

$= \frac{2\sqrt{1-x^2}}{\pi}$

e) $f_{Y|X}(y|x) = \frac{f_{X,Y}(x,y)}{f_X(x)}$

$= \frac{1/\cancel{\pi}}{\frac{2\sqrt{1-x^2}}{\cancel{\pi}}}$

$= \frac{1}{2\sqrt{1-x^2}}$