Programming Assignment - 1
EE 7390

comments

Q 7.48)
Solⁿ: The expected joint and marginal PMFs are very close to the true values.

Q 7.49)
Solⁿ:

X \ Y	j = 0	j = 1
i = 0	$\frac{1}{8}$	$\frac{1}{8}$
i = 1	$\frac{1}{4}$	$\frac{1}{2}$

$$\therefore P(x) = \begin{cases} \frac{1}{4} & ; x = 0 \\ 3/4 & ; x = 1 \end{cases} \qquad P(Y) = \begin{cases} \frac{3}{8} & ; Y = 0 \\ \frac{5}{8} & ; Y = 1 \end{cases}$$

$$\sigma_{xy}^2 = \sum_{\substack{(x,y) \\ \in [(0,1),(0,1)]}} (x - \mu_x)(y - \mu_y)\, p(x,y)$$

$$= \cancel{0.17968}\ 0.0312$$

$$\sigma_x^2 = \frac{3}{16} \qquad \sigma_y^2 = \frac{15}{64}$$

$$\therefore \text{correlation co-eff} = \cancel{\frac{0.17968}{\sqrt{3/16} \times \sqrt{15/64}}} \cancel{= 0.249}$$

$$= \frac{0.0312}{\sqrt{\frac{3}{16}} \times \sqrt{\frac{15}{64}}} = 0.149$$

Q 10.55)

Solⁿ: $f_U(u) = \begin{cases} 1 & 0 < u < 1 \\ 0 & \text{otherwise} \end{cases}$

$$F_U(u) = \begin{cases} 0 & u < 0 \\ u & 0 < u < 1 \\ 1 & u > 1 \end{cases}$$

Given

$$f_X(x) = \begin{cases} 2 - 2x & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\therefore F_x(x) = \begin{cases} 2x - x^2 & 0 \le x \le 1 \\ 0 & x < 0 \\ 1 & x > 1 \end{cases}$$

We require $X = g(u)$

$\Rightarrow$ ~~$u = g(x)$~~ $h(x)$; $h = g^{-1}$

From CDF of X & U.

$$2g(u) - (g(u))^2 = u$$

Let $z = g(u)$

$$\therefore z^2 - 2z + u = 0$$

$$\Rightarrow z = 1 \pm \sqrt{1-u}$$

We need $z = 1 - \sqrt{1-u}$ to satisfy our condition

$$\therefore g(u) = 1 - \sqrt{1-u}$$

Q10.56)

Sol^n: The estimated PDF from code is same as true PDF.

Q10.60)

Sol^n: $X \sim N(1,4)$

$$\therefore f_x(x) = \frac{1}{\sqrt{8\pi}} e^{-\frac{(x-1)^2}{8}}$$

Q10.61)

Sol^n: $Y = X^3 \Rightarrow X = Y^{1/3}$

$$f_x(x) = \begin{cases} 1 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\therefore F_x(x) = \begin{cases} 0 & x < 0 \\ x & 0 \le x \le 1 \\ 1 & x > 1 \end{cases}$$

Now, $F_Y(y) = Pr(Y < y) = Pr(X^3 < y)$

$$= Pr(X < y^{1/3})$$

$$= F_X(y^{1/3})$$

$$= \begin{cases} 0 & y < 0 \\ y^{1/3} & 0 \le y \le 1 \\ 1 & y > 1 \end{cases}$$

$$\therefore f_Y(y) = \frac{d}{dy} F_Y(y) = \begin{cases} \frac{1}{3} y^{-2/3} & 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Estimated pdf from code matches the true one.

Q10.62)

Sol^n: Estimated CDF from code matches true one.

$$F_Y(y) = \begin{cases} 0 & y < 0 \\ y^{1/3} & 0 \le y \le 1 \\ 1 & y > 1 \end{cases}$$

Q16.22)

Sol^n: $X(t) = A\cos(2\pi t)$ $\quad -\infty < t < \infty$

$A \sim N(0,1)$

$E[X(t)] = E[A\cos(2\pi t)] = E[A]\cos(2\pi t) = 0$

$Cov(X(t_1)X(t_2)) = R_X(t_1, t_2) - \mu_{X(t_1)}\mu_{X(t_2)}$

$$= R_X(t_1, t_2)$$

$$= E[X(t_1)X(t_2)]$$

$$= E[A^2\cos(2\pi t_1)\cos(2\pi t_2)]$$

$$= E[A^2]\cos(2\pi t_1)\cos(2\pi t_2)$$

$Var(A) = 1$

$\Rightarrow E[A^2] - E[A]^2 = 1$

$\Rightarrow E[A^2] = 1$

$\therefore cov(X(t_1)X(t_2))$

$$= \cos(2\pi t_1)\cos(2\pi t_2)$$

Q17.49)

Solⁿ True PSD = $\begin{cases} 1 & f > 0 \\ 0 & \text{otherwise} \end{cases}$

As N increases, the value of periodram near $f=0$, goes nearer to 1.