Q1)

Sol^n - $p(x_1, x_2, \ldots x_n) = \frac{1}{Z} \prod_{u \in V} \psi_u(x_u) \prod_{(s,t) \in E} \psi_{s,t}(x_s, x_t)$

$$\mu_s(x_s) = \sum_{x_1, x_2 \ldots x_{s-1}, x_{s+1}, \ldots x_n} p(x_1, \ldots, x_s, \ldots, x_n)$$

$$= \sum_{x_1 - x_{s-1}, x_{s+1} - x_n} \frac{1}{Z} \psi_s(x_s) \prod \psi_u(x_u) \prod_{t \in N(s)} \psi_{st}(x_s, x_t) \prod_{(s,t) \in E_t} \psi_{st}(x_s, x_t)$$

Also, $p(\underline{x}_{V_t}; T_t) \propto \prod_{u \in V_t} \psi_u(x_u) \prod_{(v,w) \in E_t} \psi_{vw}(x_v, x_w)$.

$$\therefore \mu_s(x_s) = K' \psi_s(x_s) \sum_{x_1 \ldots x_{s-1}, x_{s+1} - x_n} p(\underline{x}_{V_t}; x_t) \psi_{st}(x_s, x_t)$$

$$= K' \psi_s(x_s) \sum_{V_1} \sum_{V_2} \ldots \sum_{V_{|N(s)|}} \prod_{t \in N(s)} \psi_{st}(x_s, x_t) p(\underline{x}_{V_t}; T_t)$$

$\because V_1, V_2, \ldots V_{|N(s)|}$ are disconnected parts of a tree, $\therefore$ ~~summation~~ ~~terms~~ term in the summation above are independent of each other.

$$\therefore \mu_s(x_s) = K' \psi_s(x_s) \left(\sum_{V_1} \psi_{st}(x_s, x_t) p(\underline{x}_{V_t}; T_t)\right) \left(\sum_{V_2} \psi_{st}(x_s, x_t) p(\underline{x}_{V_2}, T_t)\right) \cdots \left(\sum_{V_{|N(s)|}} \psi_{st}(x_s, x_t) p(\underline{x}_{V_t}; T_t)\right)$$

$$= K \psi_s(x_s) \prod_{t \in N(s)} \left(\sum_{\underline{x}_{V_t}} \psi_{st}(x_s, x_t) p(\underline{x}_{V_t}; T_t)\right)$$

$$\therefore \mu_s(x_s) = K \psi_s(x_s) \prod_{t \in N(s)} M_{ts}(x_s)$$

$$M_{ts}(x_s) = \sum_{\underline{x}_{V_t}} \psi_{st}(x_s, x_t) p(\underline{x}_{V_t}; T_t)$$