Q3)

Sol^n - a) $P = {}^{n}C_x \; p^x (1-p)^{n-x}$

$$L(x,p) = \prod_{i=1}^{n} \frac{n!}{x_i!\,(n-x_i)!}\, p^{x_i} (1-p)^{n-x_i}$$

$$l(x,p) = \log\left(L(x,p)\right)$$

$$= \sum_{i=1}^{N} \log\left(\frac{n!}{x_i!\,(n-x_i)!}\, p^{x_i} (1-p)^{n-x_i}\right)$$

$$= \sum_{i=1}^{N} \log\left(\frac{n!}{x_i!\,(n-x)!}\right) + \sum_{i=1}^{N} x_i \log(p) + \sum_{i=1}^{N} (n-x_i) \log(1-p)$$

For min,

$$\frac{dl}{dp} = 0$$

$$\Rightarrow \sum_{i=1}^{n} x_i \frac{1}{p} - \sum_{i=1}^{N} (n-x_i) \frac{1}{(1-p)} = 0$$

$\cancel{\Rightarrow \left(\sum x_i - \sum x_i\right)\left[\frac{1}{p} + \frac{1}{1-p}\right]}$ Let $\sum_{i=1}^{N} x_i = x$

$$\Rightarrow \frac{x}{p} - \frac{(Nn - x)}{1-p} = 0$$

$$\Rightarrow x(1-p) - (Nn - x)\,p = 0$$

$$\Rightarrow x - xp + xp - nNp = 0$$

$$\Rightarrow p = \frac{x}{nN} = \frac{\sum_{i=1}^{n} x_i}{nN}$$

b) $f_X(x;\lambda) = \frac{\lambda^x e^{-\lambda}}{x!}$

$L(x;\lambda) = \prod_{i=1}^{N} \frac{\lambda^{x_i} e^{-\lambda}}{x_i!}$

$l(x;\lambda) = \log\left(L(x;\lambda)\right)$

$= \sum_{i=1}^{N} \log\left(\frac{\lambda^{x_i} e^{-\lambda}}{x_i!}\right)$

$= \sum_{i=1}^{N} x_i \log\lambda + \sum_{i=1}^{N}(-\lambda) - \sum_{i=1}^{N} \log(x_i!)$

$\frac{dl}{d\lambda} = 0$

$\Rightarrow \sum_{i=1}^{N} \frac{x_i}{\lambda} - \sum_{i=1}^{N} = 0$

$\Rightarrow \lambda = \frac{\sum_{i=1}^{N} x_i}{N}$

c) $f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x \in R_X \\ 0 & x \notin R_X \end{cases}$

$L(x,\lambda) = \prod_{i=1}^{N} f_X(x_i)$

$l(x,\lambda) = \sum_{i=1}^{N} \log\left(\lambda e^{-\lambda x_i}\right)$

$= \sum_{i=1}^{N} \log(\lambda) - \sum_{i=1}^{N} \lambda x_i$

For min,

$\frac{dl}{d\lambda} = 0$

$\Rightarrow \frac{N}{\lambda} - \sum_{i=1}^{N} x_i = 0$

$\Rightarrow \lambda = \frac{N}{\sum_{i=1}^{N} x_i}$

d) $f_X(x;\mu,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]$;

$$\log(L(x,\mu,\sigma^2)) = \sum_{i=1}^{N} \log\left(\frac{1}{\sqrt{2\pi}\sigma}\right) - \sum_{i=1}^{N} \frac{1}{2\sigma^2}(x_i-\mu)^2 .$$

For min,

$$\frac{\partial}{\partial\mu} \log[L(x,\mu,\sigma^2)] = 0 \quad \& \quad \frac{\partial}{\partial\sigma^2}\log[L(x,\mu,\sigma^2)] = 0$$

Solving, we get, $\mu_{MLE} = \frac{\sum_{i=1}^{N} x_i}{N}$; $\sigma^2 = \frac{\sum_{i=1}^{N}(x_i-\mu_{MLE})^2}{N}$

e)

$$f_X(x,\mu,b) = \frac{1}{2b} e^{-|x-\mu|/b}$$

$$L(x,\theta) = \prod_{i=1}^{N} \frac{1}{2b} e^{-\frac{|x_i-\mu|}{b}} \qquad \theta \in [\mu, b]$$

$$l(L(x,\theta)) = \sum_{i=1}^{N} \log\left(\frac{1}{2b} e^{-\frac{|x_i-\mu|}{b}}\right)$$

$$= \sum_{i=1}^{N} \log\left(\frac{1}{2b}\right) - \sum_{i=1}^{N} \frac{|x_i-\mu|}{b}$$

For min,

$$\frac{\partial l}{\partial b} = 0$$

$$\Rightarrow \frac{N}{\frac{1}{2b}}\left(-\left(\frac{1}{2b}\right)^2 \times 2\right) + \frac{\sum_{i=1}^{N}|x_i-\mu|}{b^2} = 0$$

$$\Rightarrow -\frac{N}{b} + \frac{\sum_{i=1}^{N}|x_i-\mu|}{b^2} = 0$$

$$\Rightarrow b = \frac{\sum_{i=1}^{N}|x_i-\mu|}{N}$$; μ here is μ_{MLE}

$$\frac{\partial l}{\partial \mu} = 0$$

$$\Rightarrow \sum_{i=1}^{N} \frac{\partial}{\partial \mu} |x_i - \mu| = 0$$

$$\Rightarrow \sum_{i=1}^{N} sgn(x_i - \mu) = 0$$

$\therefore$ for this to happen, there are 2 cases -

→ N is odd, then we choose

$\mu_{MLE} = \text{median}(x_1, ..., x_N)$

In this case, $sgn(x_{median} - \mu_{MLE}) = 0$

& other terms will cancel each other out

→ N is even, then we choose

μ_{MLE} to be a value betn the 2

medians of the data.

This way all the terms to the left of μ_{MLE}

in ~~arranged order will~~ sorted order will

cancel out the ones on the right.

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Q2)

Ans - If we have similar, high variance along 2 perpendicular directions, then PCA might fail as ~~~~ finding an axis system to reduce covariance will be difficult.