Investigation of Independent Component Analysis (ICA) for Single-Sensor Hydrophone Recordings of Moving Sources

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Abstract

The performance of any machine learning system depends critically on the quality of the input features. For signals processing applications, independent component analysis (ICA) is one method that can be used to develop features that are tuned to the application by extracting source signals that are statistically independent from each other. While ICA classically requires multiple sensors that simultaneously record data, ICA can, under certain circumstances, be applied to single-sensor systems by using *recordings at multiple times*. In this report, we leverage the physics of acoustic waves to develop ICA approaches that are suitable for single-sensor systems when sources are moving relative to the sensor and each other. Consistent with the quasi-periodic nature of the source signals, we find that the theoretically most promising approach for source separation is ICA performed in the frequency-domain. Using synthetic datasets constructed from pure source audio clips, we evaluated the performance of all approaches except for the most promising one (because, unfortunately, readily-available implementations of the FastICA algorithm are currently unable able to handle complex-valued data). We assessed all approaches by comparing the power spectra of source signals estimated by ICA against the power spectra of ground truth pure source signals. The results of the computational experiments were found to be consistent with theoretical expectations.

Key Technical Outcomes

- Development of an ICA approach that is suitable for single-sensor systems when sources are in motion.
- Theoretical analysis of ICA-based approaches to source separation for source signals that are phase-synchronized and phase-unaligned across multiple recording times.
- Use of power spectra to characterize quasi-periodic source signals and assess the quality of ICA estimates for source signals.
- Validation of theoretical expectations for ICA-based approaches via computational experiments.
- Identification of potential key obstacle to a robust ICA approach for separating quasi-periodic source signals.

1 Introduction

Independent Component Analysis (ICA) is a technique for extracting independent source signals from measurements of mixtures of the source signals[1, 4, 12]. It is commonly used in situations where there are multiple sensors simultaneously collecting measurements for sources whose characteristics (*e.g.*, location) do not change over time. For situations where there is only a single sensor, ICA may still be viable if (1) the sources are moving (relative to the sensor and each other), (2) it is performed on multiple signal clips recorded at different times, and (3) the assumptions of the ICA model are valid for the mixed signal clips.

ICA Source Signals as ML Features

ICA can be useful as a feature engineering method. The source signals estimated by ICA (possibly across mulitple non-overlapping applications of ICA) define a dictionary of potential source signals. Methods, such as simple orthogonal projection or sparse approximation [2, 5, 11], can be used convert unknown signals into a set of features that have are likely to be related to the types of sources present. In a sense, these ICA features are more likely to be *tuned* for the identification of sources and should, in principle, simplify the mappings that any ML algorithm needs to learn in order to perform well.

Quasi-Periodic Source Signals

In the approaches described in this report, we leverage an observation about the source signals of interest: they are quasi-periodic — that is, they are qualitatively but not necessarily precisely repetitive. Mathematically, source signals with this quality have *temporally stable power spectra*. In the following development, this stability of the power spectra is used for both (1) the development of ICA approaches for source separation and (2) the assessment of ICA performance. Figure 1 shows example power spectra of hydrophone recordings from individual vessels.

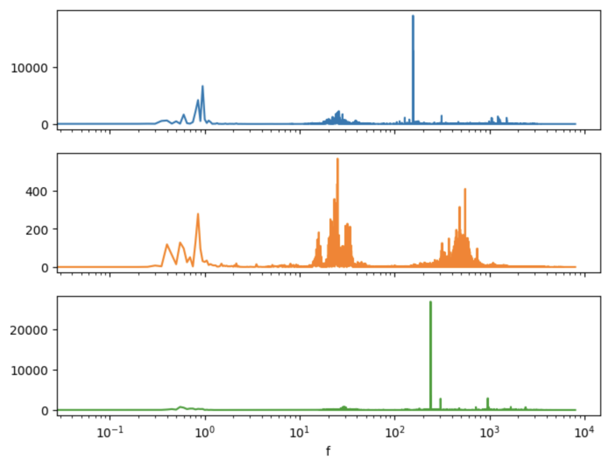


Figure 1: Example power spectra of hydrophone recordings from individual vessels.

Organization of Report

In this report, we investigate the use of ICA for separation of source signals in single-hydrophone recordings of sources that are in motion relative to the hydrophone and each other. We begin with a brief review of basic ICA theory and its application when multiple sensors are used for recording. Second, we present an approach for using ICA when only a single sensor is available by recording clips at multiple times. Next, through controlled computational experiments using synthetic datasets, we probe the conditions required to achieve useful signal separation. Finally, we summarize our findings and comment on promising directions for future research and development.

Notation and Abbreviations

- DFT: discrete Fourier transform
- ICA: independent component analysis

2 ICA Foundations

Independent component analysis is a technique that estimates statistically independent source signals from mixed signals. It is one of several approaches for performing *blind source separation*. In this section, we present the ICA signal model and describe its use for source separation in the classical case when multiple sensors are used to collect measurements.

Signal Model

In ICA, each mixture signal $s_i(t)$ is assumed to be a linear combination of the source signals:

$$s_i(t) = \sum_{j=1}^{N} a_{ij} x_j(t)$$

where $s_i(t)$ is the i-th mixed signal, $x_j(t)$ is the j-th source signal, and a_{ij} is the contribution of the j-th source to the measurement at the i-th sensor. Because the a_{ij} define how the source signals are mixed to obtain the sensor signals, they are called *mixing coefficients*. The mixing equations can be expressed in matrix form as:

$$\vec{s}(t) = \mathbf{A}\vec{x}(t)$$

where $\vec{s}(t) = (s_1(t), \ldots, s_M(t))^T$ and $\vec{x}(t) = (x_1(t), \ldots, x_M(t))^T$ are the sensor and source signal vectors, respectively, and $\mathbf{A}$ is the *mixing matrix*. In general, up to M source signals can be estimated when M mixed signals are available for analysis.

In a noisy environment, the signal model is modified to include a noise term

$$s_i(t) = \sum_{j=1}^{N} a_{ij} x_j(t) + \sigma w_i(t)$$

where $w_i(t)$ is Gaussian white noise with power σ^2.

FastICA

There are several algorithms for solving the ICA equations. The implementation that is readily available in the `scikit-learn` Python is based on FastICA[3, 10], an algorithm that uses fast fixed-point iteration to simultaneously solve for the source signals $x_j(t)$ and the mixing coefficients a_{ij}. It is important to note that the FastICA implementation available in `scikit-learn` only supports real-valued data. Unfortunately, this contraint limits our ability to assess the utility of ICA in situation where it would be beneficial to perform ICA on the Fourier transform of the raw signal.

Classical Application of ICA: Multi-Sensor Systems

ICA is most commonly applied to time-series data gathered by making measurements with *multiple, synchronized* sensors. For these types of systems, the data is real-valued (because we are able to analyze the data in the time domain), so it is straightforward to perform ICA using the FastICA implementation available in `scikit-learn`. Figure 2 illustrates the use of ICA on multi-sensor, time-synchronized data.

3 ICA For Moving Sources Using Single-Sensor Recordings

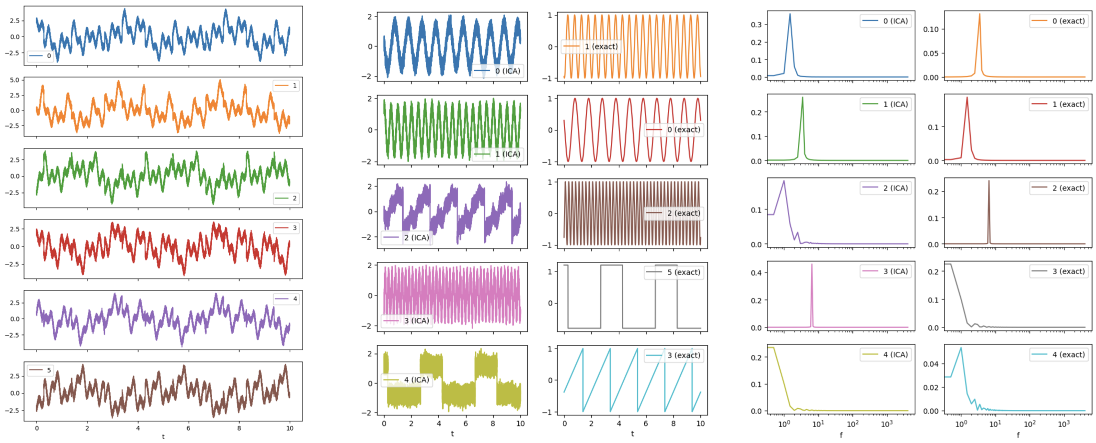
When sources are in motion relative to each other, it is theoretically possible to use ICA for source separation with only a single sensor by *collecting recording clips at multiple times* as long as three conditions are met:

- at each time, the recording is a linear combination of source signals,
- the source signals are synchronized in the domain that ICA is performed in (*i.e.*, time or frequency), and
- the coefficients used to combine source signals change non-trivially between recording times.

For sound waves, the first and third conditions are satisified under weak assumptions. The second condition, however, requires stronger assumptions.

Under the weak assumption that wave propogation is well-modeled by the linear wave equation, the origin of the first condition is the principle of superposition for the wave equation — the pressure at the sensor location is a linear combination of the pressure waves from each of the sources. Note that the condition is satisfied in both the time and frequency domains.

The third condition holds because pressure is proportional to the inverse of the distance between the source and sensor. As a result, the relative magnitudes of the contribution of sources changes between recording times (assuming the time interval between recordings is sufficiently large for sources to have



(a) (b) (c)

Figure 2: ICA applied to the classical source separation problem with multiple sensors simultaneously making measurements. (a) Mixed signals measured simultaneously by five different sensors. (b) Source signals in the time domain: extracted via ICA (left) and ground truth (right). (c) Power spectra of source signals: extracted via ICA (left) and ground truth (right).

moved a significant distance). Note that while there are source trajectories that would cause the relative magnitudes of the pressure to remain constant, these motions are atypical.

The second condition is only satisfied under special assumptions. In the time domain, the second condition only holds if the relative phases of source signals are synchronized across recordings — in other words, the relative phase of the source signals is the same for all signals at all recording times. When this condition is met, ICA can be used to perform source separation in the time domain. Unfortunately, this condition unlikely to be met because sources typically have different periodicities, so source phases are do not remain aligned in time. Since recording times are chosen without knowledge of the sources, it is unlikely that they would happen to be chosen such that source phases are aligned across recordings.

When unaligned source signal phase shifts are present between recordings, the signal model becomes

$$s_i(t) = \sum_{j=1}^{N} a_{ij} x_j(t + \phi_{ij}),$$

and ICA is not directly applicable to the time-domain signal. In this situation, it is useful to use the Fourier transform to convert signals to the frequency domain because phase shifts in the time domain become multiplication by a complex exponential in the frequency domain. For this approach to be viable, we require two additional assumptions — one for the type of Fourier analysis and one for the structure of the source signals.

- The Fourier transform must be performed using the complex exponential Fourier basis. When sines and cosines are used as basis functions, phase shifts in time manifest as mixing between the coefficients of the basis functions which is incompatible with the signal model for ICA.
- The source signals must be qualitatively repetitive so that the Fourier spectra of the source signals remains stable across all recordings.

When both of these conditions are met, the signal model in the *frequency domain* is

$$\hat{s}_i(f) = \sum_{j=1}^{N} a_{ij} \hat{x}_j(f),$$

where $\hat{s}_i(f)$ and $\hat{x}(f)$ are the Fourier transforms of $s_i(t)$ and $x(t)$, respectively, and a_{ij} are mixing coefficients that *implicitly encode the relative phase shifts between source signals.* This equation has the same form as the ICA signal model, so source separation can be achieved by applying ICA in the frequency domain. It is important, however, to emphasize that in this model, all quantities are *complex-valued.*

4 Computational Experiments

To evaluate the effectiveness of and failure cases for source separation using ICA, we performed a series of computational experiments:

- ICA on time-domain signals when source signals are phase-synchronized across recording times;
- ICA on time-domain signals when relative phase shifts are present between source signals at different recording times; and
- ICA on frequency-domain signals using a sine and cosine basis when relative phase shifts exist between source signal.

Unfortunately, due to limitations of the FastICA implementation in `scikit-learn`, we were unable to perform a computational experiment for the most promising approach for source separation in the presence of relative phase shifts between source signals: ICA on frequency-domain signals using a complex exponential basis.

Methodology

Each experiment was performed using the following procedure. Each of these steps is described in more detail its own subsection.

1. Generate a synthetic dataset of mixed signals from recordings of individual sources.
2. Preprocess the mixed signals to extract segments to use as input to ICA.
3. Perform ICA on the preprocessed mixed signals.
4. Assess the quality of source separation by qualitatively comparing the power spectra of estimated and ground truth source signals. *Note*: unlike the simple demonstration presented for classical ICA, the complexity of the source signals precludes direct comparison in the time domain.

Dataset Generation

For each experiment, we constructed a synthetic dataset by combining audio clips for pure sources in the following manner (for more details, see the help message for the `generate-synthetic-dataset.py` script).

- First, we select random initial positions (relative to the sensor) for all of the sources. If distances from the pure sources are known, those distances are used; otherwise, source distances are selected randomly. Azimuths for all sources are randomly selected.
- Next, we randomly select the heading and velocity for each source.
- At multiple recording start times that allow for the sources to travel far enough to change the strength of their signals at the sensor, we use the theoretical solution to the pressure wave equation to estimate the signal strength for each source.
- For experiments where there are relative phase shifts between source signals, a random temporal shift is selected for each source signal at each recording time.
- For each recodring time, a mixed signal recording is constructed by combining the pure source signals with (1) scaling implied by the estimated signal strength at the new source position and (2) a temporal shift (if applicable).

Signal Preprocessing

Before performing ICA, each recording is preprocessed using the following procedure.

- We performed experiments with and without bandpass filtering of the raw mixed signal. Our hypothesis was that bandpass filtering to remove low and high frequencies likely to have strong non-source contributions (*e.g.*, wind noise) would improve ICA performance. When bandpass was applied, we used a Butterworth filter with the parameters:
  - Passband: [5, 1000] Hz
  - Stopband Edges: [1, 1500] Hz
  - Maximum Loss in Passband (dB): 1

– Minimum Attenuation in the Stopband (dB): 100

- A 20 second snippet of the recording is extracted for ICA. The duration of the snippet is chosen to be (1) short enough that the signal strength from sources does not change significantly during the snippet[1], (2) long enough that the resolution of the discrete Fourier transform (DFT) of the signal is high enough to capture the structure of the power spectra of the source signals[2], and (3) long enough for a reasonably large number of "quasi-cycles" of the source signal to be included for analysis[3].

ICA Analysis

ICA was performed on the preprocessed audio snippets by using the `scikit-learn` `FastICA` class with default parameters except for:

- `n_components`: set to the number sources to estimate
- `whiten`: set to `"unit-variance"`.

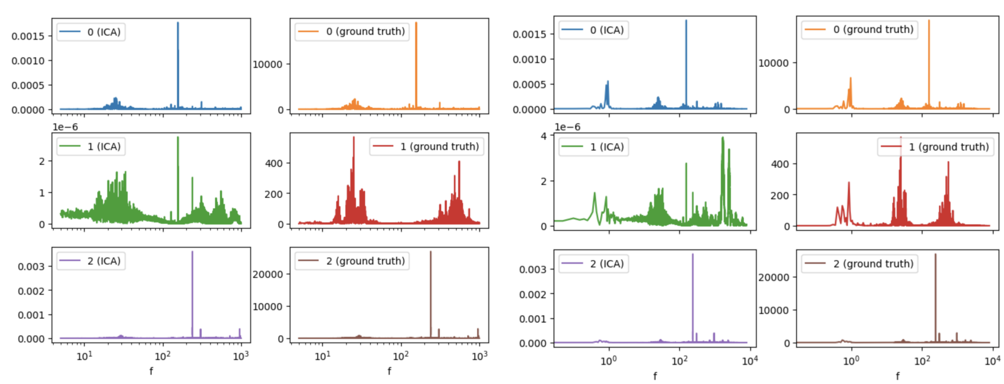
Evaluation of Source Signal Estimates

To assess the quality of the source signals extracted by ICA, we visually compared the power spectra of the estimated sources and the ground truth audio clips used to construct the synthetic mixed signals.

Results and Discussion

Time-Domain ICA For Phase-Synchronized Source Signals

For source signals that are phase-synchronized across recording times, ICA was found to be effective for estimating source signals from multiple, time-separated recordings collected using a single, stationary sensor. Figure 3 compares the power spectrum of the estimated and ground truth source signals without bandpass filtering. Figure 4 compares the power spectrum of the estimated and ground truth source signals with bandpass filtering.



(a) (b)

Figure 3: Comparison of power spectra for time-domain ICA without bandpass filtering applied to single-sensor, multiple-time synthetic signals constructed from source signals that are phase-synchronized across recording times. (a) Power spectra of source signals in the passband: ICA estimates (left) and ground truth (right). (b) Power spectra of source signals at all frequencies: ICA estimates (left) and ground truth (right).

[1] Assuming a typical sensor-to-source distance of 500m and a source speed of 1m/s, the ratio of the pressure amplitude the source after 20s of travel lies in the range 0.96 to 1.04.

[2] For a 20s sample, the spacing between frequencies in the DFT is $1/20 = 0.05$Hz, which for a 16kHz sampling rate corresponds to a resolution of $(16000/2)/0.05 = 160,000$ points for the power spectrum.

[3] Since the bandpass filter rejects frequencies below 5Hz, a 20s sample includes about 100 cycles of the lowest frequency for analysis.

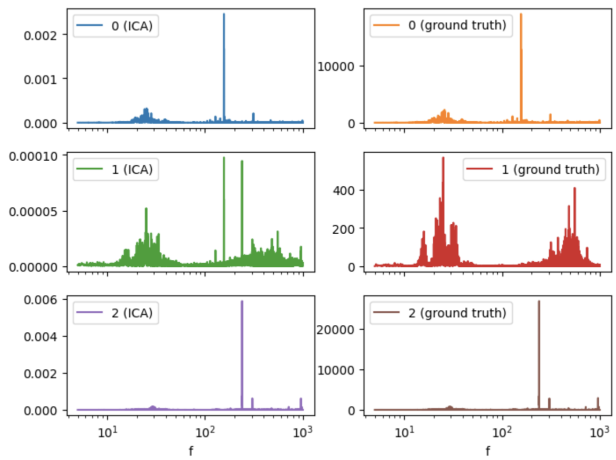
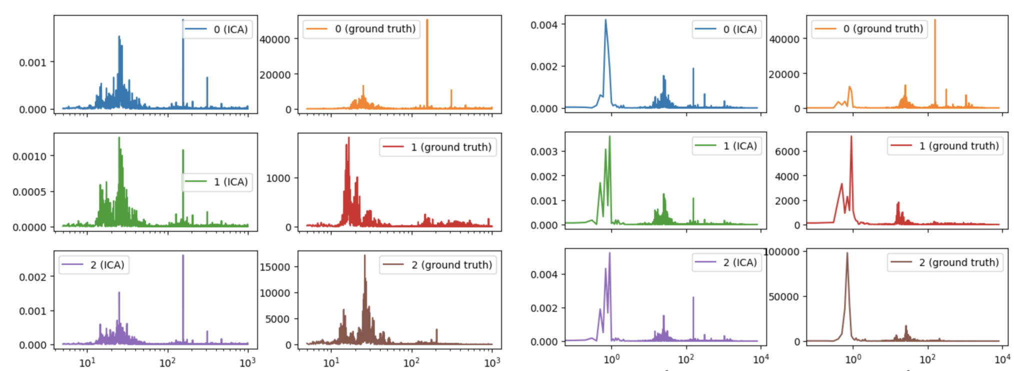


Figure 4: Comparison of power spectra for time-domain ICA with bandpass filtering applied to single-sensor, multiple-time synthetic signals constructed from source signals that are phase-synchronized across recording times: ICA estimates (left) and ground truth (right).

Applying a bandpass filter before performing ICA appeared to improve performance and reduce the chance that ICA was using ambient noise to estimate signals. Visually, the power spectra of the ICA estimated source signals appear to more cleanly match the ground truth source power spectra. The power spectra peaks below 1Hz appear to be difficult for ICA to fit and may be the origin of the poorer quality source signals when bandpass filtering is not applied.

Time-Domain ICA For Phase-Unalignd Source Signals

For source signals that are not phase-synchronized across recording times, ICA was found to be less effective for estimating source signals from multiple, time-separated recordings collected using a single, stationary sensor. Figure 5 compares the power spectrum of the estimated and ground truth source signals without bandpass filtering. Figure 6 compares the power spectrum of the estimated and ground truth source signals with bandpass filtering.



(a) (b)

Figure 5: Comparison of power spectra for time-domain ICA without bandpass filtering applied to single-sensor, multiple-time synthetic signals constructed from source signals that are phase-unaligned across recording times. (a) Power spectra of source signals in the pass-band: ICA estimates (left) and ground truth (right). (b) Power spectra of source signals at all frequencies: ICA estimates (left) and ground truth (right).

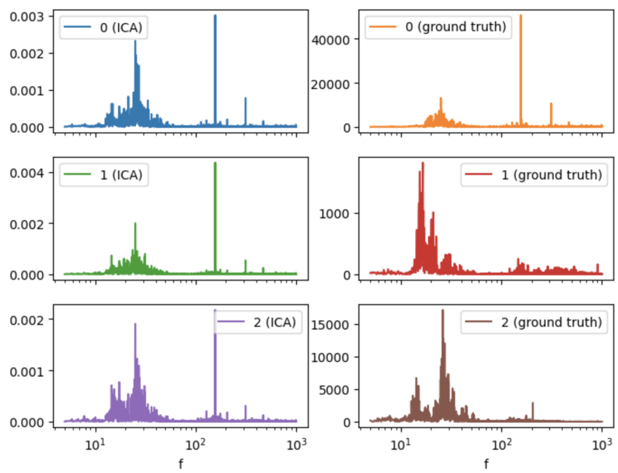


Figure 6

Comparison of power spectra for time-domain ICA with bandpass filtering applied to single-sensor, multiple-time synthetic signals constructed from source signals that are phase-unaligned across recording times: ICA estimates (left) and ground truth (right).

Applying a bandpass filter before performing ICA did not appear to significantly affect performance. In both cases, ICA appears to be able to estimate some sources reasonably well, but does a poorer job for other sources. The origin of the discrepancy in the quality of source signal estimates might be the level relative phase-synchronization between sources across recording times. ICA might be able to estimate the source signals for the subset of source signals that are cloesest to being phase-synchronized. Source signals less phase-synchronized might be less well estimated. These results are consistent with the theoretical expectations for ICA.

Frequency-Domain ICA for Phase-Unalignd Source Signals

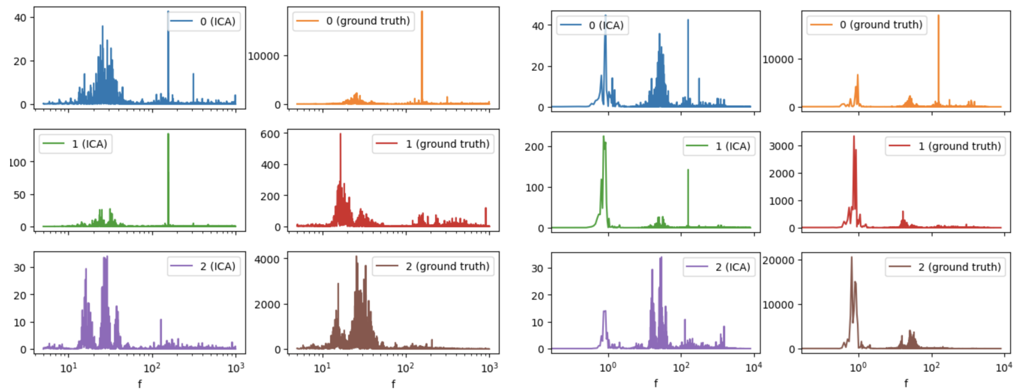
For source signals that are not phase-synchronized across recording times, we were only able to perform experiments applying ICA to the DFT of the mixed signals using a sine and cosine basis because the FastICA implementation in `scikit-learn` is currently unable to handle complex-valued datasets. Figure 7 compares the power spectrum of the estimated and ground truth source signals without bandpass filtering. Figure 8 compares the power spectrum of the estimated and ground truth source signals with bandpass filtering.

As expected from theory, ICA applied to the DFT with a sine and cosine basis did not perform particularly well. Visual inspection of the power spectra indicates that the estimated source signals contained mixtures of the ground truth source signals. Bandpass filtering did not significantly increase the quality of the estimated source signals.

Unfortunately, the most promising approach based on application of ICA to the DFT of the mixed signals using a complex exponential basis *could not be evaluated due to restriction of the* `scikit-learn` *FastICA implementation to real-valued data.* While a few ICA implementations for complex-valued data were found, none were deemed mature enough for this investigation (*e.g.*, performance was too slow to be useful, it was unclear if the code had been sufficiently debugged, *etc.*). Should a suitable implementation of ICA for complex-valued data become available in the future, this approach would be very interesting to further investigate and explore.

5 Summary and Conclusions

In this report, we presented ICA-based approaches to source separation for single-sensor systems with moving sources by leveraging the physics of acoustic waves and the quasi-periodic nature of the source signals of interest. Using computational experiments, we evaluated the approaches on synthetic data constructed from pure source recordings and found ICA performance to be consistent with theoretical expectations.



(a) (b)

Figure 7: Comparison of power spectra for frequency-domain ICA without bandpass filtering applied to single-sensor, multiple-time synthetic signals constructed from source signals that are phase-unaligned across recording times. (a) Power spectra of source signals in the passband: ICA estimates (left) and ground truth (right). (b) Power spectra of source signals at all frequencies: ICA estimates (left) and ground truth (right).

Summary of ICA-based Source Separation Approaches and Results

- Time-domain ICA. Good source separation is achieved when source signals are phase-synchronized across recording times. When source signals are phase-unaligned across recording times, source separation is less effective.
- Frequency-domain ICA with sine and cosine basis. This approach is not suitable for phase-unaligned recording times because the relative phase shifts in the source signals lead to coefficient mixing, which is inconsistent with the ICA signal model. Computational experiments confirm that source separation is not particularly effective for this approach.
- Frequency-domain ICA with complex exponential basis. This is the most promising approach for source separation when source signals are phase-unaligned between recording times because (1) the frequency-domain signal model is consistent with the ICA signal model and (2) the relative phase shifts manifest as multiplicative changes to the mixing coefficients. Unfortunately, it was not possible to perform computational experiments to evaluate this approach because the `scikit-learn` FastICA implementation is limited to real-valued data.

Future Directions

- To assess the robustness of time-domain ICA, it would be beneficial to perform time-domain ICA on a wider range of synthetic datasets constructed over a wider range of vessel types, combinations, and number.
- Since time-domain ICA for phase-synchronized source signals showed good performance on synthetic datasets, it would be useful to perform experiments on non-synthetic datasets containing multiple vessels to determine whether or not it is reasonable to assume that real-world source signals are phase-synchronized. If so, time-domain ICA may be sufficient for feature engineering purposes.
- To assess the viability of frequency-domain ICA using a complex exponential basis, it would be interesting to implement ICA for complex signals. Complex-valued ICA algorithms have been described in the literature [6, 7, 8, 9], but they do not appear to have been ported to Python.

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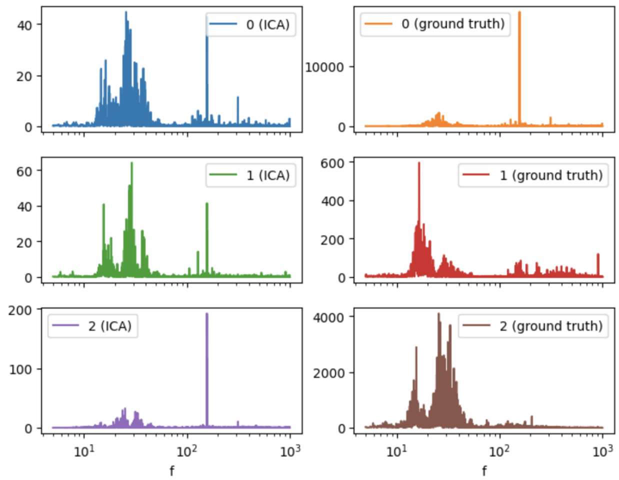

Figure 8

Comparison of power spectra for frequency-domain ICA with bandpass filtering applied to single-sensor, multiple-time synthetic signals constructed from source signals that are phase-unaligned across recording times: ICA estimates (left) and ground truth (right).

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