System Planning Toolbox

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Abstract

The System Planning Toolbox performs the engineering analysis required for the coordination of communication networks under national and international regulatory criteria with a focus on the Radiocommunication Sector Space Services. The toolbox is implemented in MATLAB using an object-oriented approach, and calculates equivalent power flux density (EPFD) up, down, and inter-satellite for comparison with Article 22 limits.

1 Introduction

The System Planning Toolbox performs the engineering analysis required for the coordination of communication networks under regulatory criteria of the U.S. Federal Communications Commission (FCC) and the International Telecommunications Union (ITU) with a focus on the Radiocommunication Sector Space Services. The communication network, or system, may consist of an arbitrary Earth and space segment, with the space segment consisting of geostationary or non-geostationary satellites. The toolbox:

- Implements Earth and space transmitting and receiving antenna patterns as defined in the ITU antenna pattern library
- Models space station orbits as two-body orbits optionally including J2 effects
- Models Earth station position using WGS84
- Models multiple beams per space station, accounting for beam multiplexing, and implements simplified beam assignment algorithms
- Calculates equivalent power flux density (EPFD) up, down, and inter-satellite

2 Getting Started

The System Planning Toolbox is implemented in MATLAB using an object-oriented approach and consists of the packages and classes shown in summary form in table 1. Note that package names beginning with + follow the MATLAB convention for name space creation. All classes have a corresponding test class, not shown, which implement unit tests of public, and some private, methods.

Table 1: System Planning Toolbox Packages and Classes

```
com
|__ springbok
    |__ antenna
    |   |__ Antenna.m
    |   |__ EarthStationAntenna.m
    |   |__ SpaceStationAntenna.m
    |__ pattern
    |   |__ EarthPattern.m
    |   |__ Pattern.m
    |   |__ PatternE*.m (Earth)
    |   |__ PatternS*.m (Space)
    |   |__ ReceivePattern.m
    |   |__ SpacePattern.m
    |   |__ TransmitPattern.m
    |__ station
    |   |__ Beam.m
    |   |__ EarthStation.m
    |   |__ Emission.m
    |   |__ SpaceStation.m
    |   |__ Station.m
    |__ system
    |   |__ Link.m
    |   |__ Network.m
    |   |__ Performance.m
    |   |__ Propagation.m
    |   |__ System.m
    |__ twobody
        |__ Coordinates.m
        |__ EarthConstants.m
        |__ EquinoctialOrbit.m
        |__ KeplerianOrbit.m
        |__ Orbit.m
        |__ TwoBodyOrbit.m
```

```
com
|__ springbok
    |__ simulation
    |   |__ +example
    |   |   |__ Plot.m
    |   |   |__ Simulate.m
    |   |   |__ getIntLeoEarthSegment.m
    |   |   |__ getIntLeoSpaceSegment.m
    |   |   |__ getSystems.m
    |   |   |__ getWntGsoEarthSegment.m
    |   |   |__ getWntGsoSpaceSegment.m
    |   |__ +spacex
    |       |__ Plot.m
    |       |__ Simulate.m
    |       |__ getEdges.m
    |       |__ getIntLeoEarthSegment.m
    |       |__ getIntLeoSpaceSegment.m
    |       |__ getOrbitsAndCells.m
    |       |__ getSystemsHigh.m
    |       |__ getSystemsLow.m
    |       |__ getWntGsoEarthSegment.m
    |       |__ getWntGsoSpaceSegment.m
    |__ test
        |__ +gso_gso
        |   |__ *.m (Functions)
        |__ +gso_leo
            |__ *.m (Functions)
```

```
com
|__ celestrak
  |__ sgp4v
|       |__ *.m (Functions)
|__ springbok
|   |__ operator
|   |   |__ HohmannTransfer.m
|   |   |__ OrbitDetermination.m
|   |   |__ SatelliteCatalog.m
|   |   |__ SimulationConstants.m
|   |__ sgp4v
|       |__ Sgp4Coordinates.m
|       |__ Sgp4Orbit.m
|       |__ Sgp4OrbitTest.m
|__ utility
    |__ Article22Utility.m
    |__ PatternUtility.m
    |__ PlotUtility.m
    |__ SException.m
    |__ TestUtility.m
    |__ TimeUtility.m
    |__ TypeUtility.m
    |__ jsonlab-1.2 (Third-party toolbox)
```

The system planning toolbox is delivered as a compressed archive which can be expanded in any directory, referred to here as <TOOLBOX_HOME>. To use the toolbox, start MATLAB, navigate to the toolbox home directory, and add the classes to the MATLAB search path as follows:

```
>> cd <TOOLBOX_HOME>
>> addpath(genpath('src/matlab'))
```

The communication network, or system, may consist of an arbitrary Earth segment, which is represented as an array of Earth stations, and an arbitrary space segment, which is represented as an array of space stations. The primary purpose of the system is to assign a space station beam to each Earth station to form an array of networks which can be characterized by their link performance.

Earth and space stations are conceived as having a separately specified transmit and receive antenna, with an associated antenna pattern, an emission, describing the power and frequency characteristics of the transmission, and a beam, in the case of Earth stations, and an array of beams, in the case of space stations, which are used for assignment of stations into networks, and multiplexing. A network represents a single space station and beam assigned to a single Earth station, the assignment made by, for example, maximum elevation of the space station as observed by the Earth station, and corresponding up and down links, each of which contain corresponding transmit and receive stations, and a transmit beam.

Figure 1 presents the object hierarchy of an arbitrary system. Note that the Network and Link classes are created during beam assignment, and the remaining classes are typically created from the bottom up and left to right when creating a system. The `Simulate` function in the `example` package illustrates this process, and the following sections describe the classes in this typical creation order.

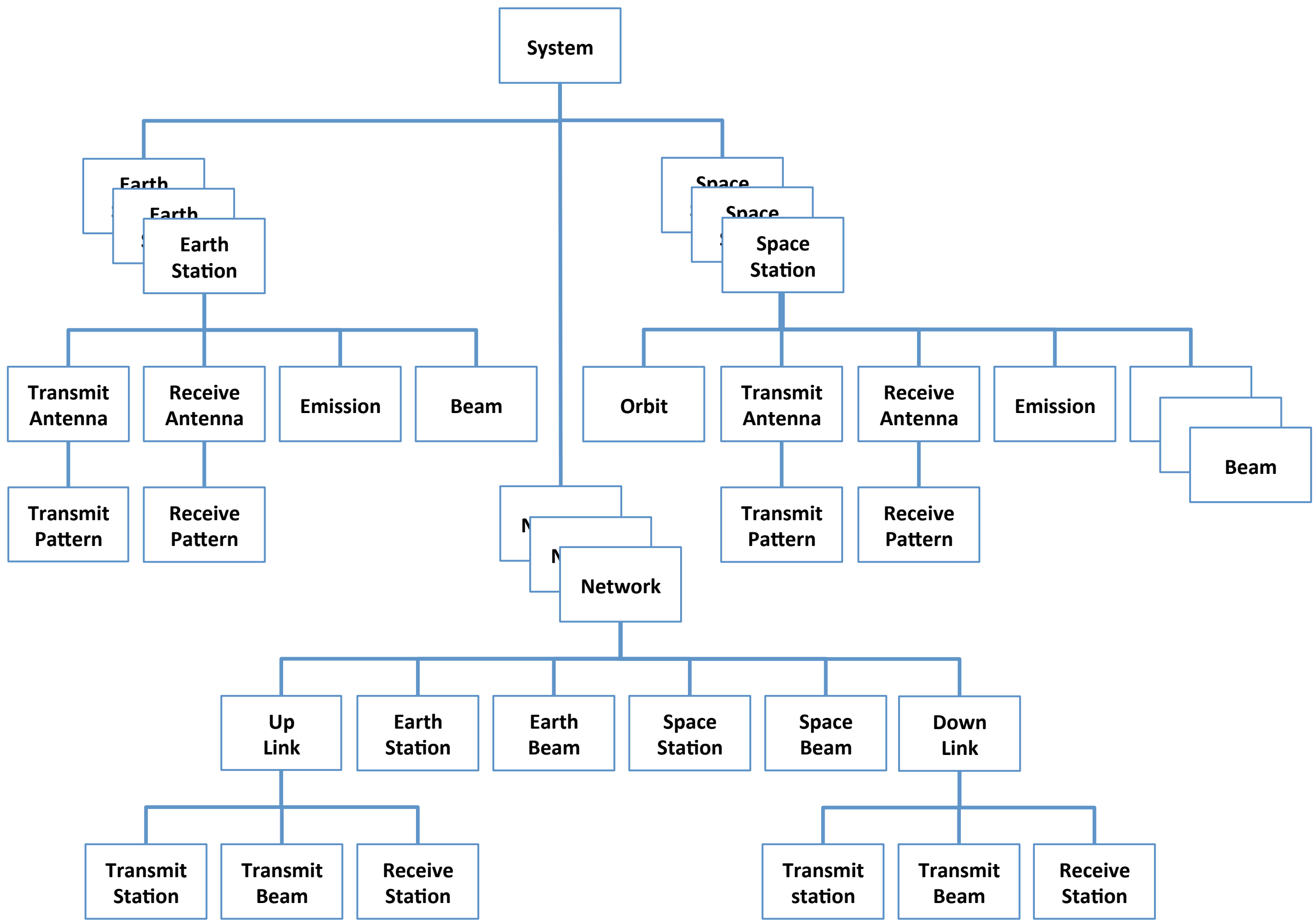

Figure 1: System Object Diagram

2.1 Pattern Classes

All reference antenna patterns published by the ITU are implemented in the `pattern` package [F]. The reference antenna pattern classes all inherit from parent class `Pattern`, and depending on their purpose, from either `EarthPattern` or `SpacePattern`, and from either or both of `TransmitPattern` or `ReceivePattern`. A typical `Pattern` is created as follows:

```
GainMax = 34;     % Maximum antenna gain [dB]
Efficiency = 0.7; % Antenna efficiency, fraction

pattern = PatternEREC013V01(GainMax, Efficiency);
```

Note that the reference antenna pattern class constructors do not all take the same arguments.

2.2 Antenna Classes

The antenna classes in the `antenna` package inherit from parent class `Antenna`, and correspond to either Earth or space stations. Although the functionality of the `EarthStationAntenna` and `SpaceStationAntenna` class is largely identical, these classes enforce correct use of the reference antenna patterns. With a `Pattern` created as in the previous section, a typical transmit `EarthStationAntenna` is created as follows:

```
name = 'NGSO ES Tx'; % Antenna name
gain = GainMax;      % Antenna gain
pattern_id = 1;      % Antenna pattern identifier

transmitAntenna = EarthStationAntenna(name, gain, pattern_id, pattern);
```

and a receive `EarthStationAntenna` is created as follows:

```
name = 'NGSO ES Rx'; % Antenna name
gain = GainMax;      % Antenna gain
pattern_id = 2;      % Antenna pattern identifier
noise_t = 150;       % Antenna noise temperature

receiveAntenna = EarthStationAntenna(name, gain, pattern_id, pattern, noise_t);
```

Note that a receive antenna is created with the additional argument `noise_t`, and that the `gain`, `name` and `pattern_id` properties are descriptive only, and so can take any value without altering functionality. As before, the reference antenna pattern class constructors do not all take the same arguments.

2.3 Emission Class

The `Emission` class in the `station` package contains parameters for describing the transmit waveform characteristics. A Typical `Emission` is created as follows:

```
design_emi = '1K20G1D--'; % Emission designator
pwr_ds_max = -60;         % Maximum power density
pwr_ds_min = NaN;         % Minimum power density
freq_mhz = 13000;         % Center frequency
c_to_n = NaN;             % Required C/N

emission = Emission( ...
  design_emi, pwr_ds_max, pwr_ds_min, freq_mhz, c_to_n);
```

Note that the `design_emi` property, which is consistent with ITU filings, is descriptive only, and so can take any value without altering functionality.

2.4 Beam Class

The `Beam` class in the `station` package provides functionality for multiple beam assignment and beam multiplexing. A typical `Beam` is created as follows:

```
name = 'IntLeoEarthSegment'; % Beam name
multiplicity = 1;            % Maximum number of divisions allowed
dutyCycle = 100;             % Duty cycle [%]

beam = Beam(name, multiplicity, dutyCycle);
```

Note that the `name` property is descriptive only, and so can take any value without altering functionality.

2.5 Earth Station Class

The `EarthStation` class in the `station` package inherits from parent class `Station`, and contains instances of a transmit and receive `Antenna`, an `Emission`, and a `Beam`. With instances created as in the previous sections, an `EarthStation` is created as follows:

```
stationId      = 'wanted'; % Identifier for station
varphi         = 0;        % Geodetic latitude [rad]
lambda         = lla(2);   % Longitude [rad]
doMultiplexing = 0;        % Flag indicating whether to do multiplexing,
                           %   or not

earthStation = EarthStation(stationId, transmitAntenna, receiveAntenna, ...
                            emission, beam, varphi, lambda, doMultiplexing);
```

Note that the station_id property is descriptive only, and so can take any value without altering functionality.

2.6 Orbit Classes

The orbit classes in the `twobody` package inherit from the parent classes `Orbit` and `TwoBodyOrbit`, and provide for the calculation of geocentric equatorial inertial position using either Keplerian or equinoctial orbital elements. A typical `KeplerianOrbit` is created as follows:

```
a         = 1 + 1500 / EarthConstants.R_oplus; % Semi-major axis [er]
e         = 0.001;                             % Eccentricity [-]
i         = 80.0 * pi / 180;                   % Inclination [rad]
Omega     = 30.0 * pi / 180;                   % Right ascension of the
                                               %   ascending node [rad]
omega     = 60.0 * pi / 180;                   % Argument of perigee [rad]
M         = 130.0 * pi / 180;                  % Mean anomaly [rad]
epoch     = epoch_0;                           % Epoch date number
method    = 'halley';                          % Method to solve Kepler's equation:
                                               %   'newton' or 'halley'

orbit = KeplerianOrbit(a, e, i, Omega, omega, M, epoch, method);
```

2.7 Space Station Class

The `SpaceStation` class in the `station` package inherits from parent class `Station`, and contains instances of a transmit and receive `Antenna`, an `Emission`, an array of `Beams`, and an `Orbit`. With instances created as in previous sections, a typical `SpaceStation` is created as follows:

```
stationId = 'interfering'; % Identifier for station

spaceStation = SpaceStation( ...
  stationId, transmitAntenna, receiveAntenna, emission, beams, orbit);
```

Note that the station_id property is descriptive only, and so can take any value without altering functionality.

2.8 System Class

The `System` class in the `system` package contains instances of `EarthStations`, `SpaceStations`, and `Networks`. However, the `Network` instances are created during beam assignment, rather than during creation of the `System` instance. With instances created as in previous sections, a typical `System` is created as follows:

```
losses = {}; % Propagation loss models to apply
dNm = datenum(2014, 10, 20, 19, 5, 0); % Current date number

system = System(earthStations, spaceStations, losses, dNm);
```

Note that the `losses` property is descriptive only, and so can take any value without altering functionality.

2.9 Simulation

Simulation of system performance involves assigning beams to create networks, and computing link performance, possibly in the presence of an interfering system, at each of a set of times which is sufficient to represent the motion of the Earth and space stations. The `Simulate` function in the `example` package demonstrate the process of simulating the performance of a wanted geostationary system consisting of a single space and Earth station, in the presence of an interfering non-geostationary system consisting of 18 Earth and 225 space stations in which the Earth stations are located on a grid which includes the wanted Earth station location. The typical order of calculations is as follows:

```
dNm_0 = datenum(2014, 10, 20, 19, 5, 0);
nSec = 1000;
parfor iDNm = [1 : nSec]
  dNm(iDNm) = dNm_0 + iDNm / 86400;

  if ~mod(iDNm, 4)
    interferingSystem.assignBeams([], [], dNm(iDNm));

  end % if

  performanceUp(iDNm) = wantedSystem.computeUpLinkPerformance( ...
      dNm(iDNm), interferingSystem, 1, 1, ref_bw);
  performanceDn(iDNm) = wantedSystem.computeUpLinkPerformance( ...
      dNm(iDNm), interferingSystem, 1, 1, ref_bw, 'DoIS', 1);
  performanceIS(iDNm) = wantedSystem.computeDownLinkPerformance( ...
      dNm(iDNm), interferingSystem, 1, 1, ref_bw);

end % for
```

The following sections describe the engineering basis for the classes required by this simulation procedure, again, in the order the classes are created, in order to establish notation, and provide references for additional information.

3 Patterns, Antennas, Beams, and Emissions

3.1 Antenna Fundamentals

As a radio wave arrives at an antenna, the antenna collects the power contained in its effective aperture area, A_e. For a perfect, loss-less antenna, this effective aperture area would be equal to the actual projected area, A. In practice, the antenna will have some losses, which are taken into account through the antenna efficiency, η:

$$A_e = \eta A \tag{1}$$

where A_e = effective aperture area (m^2), and A = antenna area (m^2). Therefore, the effective area of a circular reflector can be expressed as:

$$A_e = \eta \pi r^2 \tag{2}$$

where r = radius of antenna (m).

The relationship between the peak gain and effective area of an antenna is given by:

$$G = \frac{4\pi A_e}{\lambda^2} \tag{3}$$

where λ = wavelength (m). Note that the wavelength of the electromagnetic wave is related to the frequency as follows:

$$\lambda = \frac{c}{f} \tag{4}$$

where c = speed of light (299792458 m/s), and f = frequency (Hz). The off-axis gain of an antenna is typically expressed as a function of the off-axis angle, Φ, that is, $G(\Phi)$. Corresponding reference antenna patterns published by the ITU are described in the next section.

3.2 Reference Antenna Patterns

Figure 2 shows a typical reference antenna pattern excerpted from Appendix 7 of the Radio Regulations.

Reference patterns are defined by several regions: the main beam is the area of highest gain near the pointing direction of the antenna, the first side lobe region is represented by a constant gain value, the log roll-off region is represented by the decreasing gain as a function of off-axis angle, and the back lobe is represented by a constant value for the far off-axis gain. Figure

The relationship $\varphi(\alpha)$ is used to derive a function for the horizon antenna gain (dBi), $G(\varphi)$ as a function of the azimuth α, by using the actual earth station antenna pattern, or a formula giving a good approximation. For example, in cases where the ratio between the antenna diameter and the wavelength is equal to or greater than 35, the following equation is used:

$$G(\varphi) = \begin{cases} G_{amax} - 2.5 \times 10^{-3} \left(\dfrac{D}{\lambda} \varphi \right)^2 & \text{for } 0 < \varphi < \varphi_m \\ G_1 & \text{for } \varphi_m \le \varphi < \varphi_r \\ 29 - 25 \log \varphi & \text{for } \varphi_r \le \varphi < 36° \\ -10 & \text{for } 36° \le \varphi \le 180° \end{cases} \qquad (97)$$

$$G_1 = \begin{cases} -1 + 15 \log (D/\lambda) & \text{dBi} \quad \text{for} \quad D/\lambda \ge 100 \\ -21 + 25 \log (D/\lambda) & \text{dBi} \quad \text{for} \quad 35 \le D/\lambda < 100 \end{cases}$$

$$\varphi_m = \frac{20\,\lambda}{D} \sqrt{G_{amax} - G_1} \qquad \text{degrees}$$

$$\varphi_r = \begin{cases} 15.85\,(D/\lambda)^{-0.6} & \text{degrees} \quad \text{for} \quad D/\lambda \ge 100 \\ 100\,(\lambda/D) & \text{degrees} \quad \text{for} \quad 35 \le D/\lambda < 100 \end{cases}$$

Where a better representation of the actual antenna pattern is available, it may be used.

In cases where D/λ is not given, it may be estimated from the expression:

$$20 \log \frac{D}{\lambda} \approx G_{amax} - 7.7$$

where:

G_{amax}: main beam axis antenna gain (dBi)

D: antenna diameter (m)

λ: wavelength (m)

G_1: gain of the first side lobe (dBi).

Figure 2: Reference antenna pattern from Appendix 7

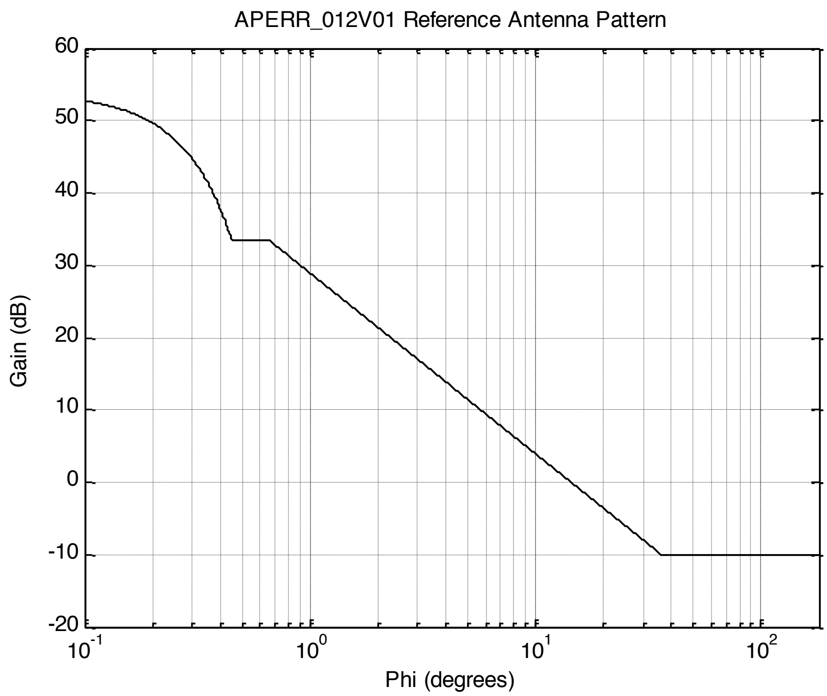

Figure 3: Reference antenna pattern from Appendix 7

3 shows the plot of a typical reference antenna pattern and illustrates these four gain regions.

This example pattern from Appendix 7 requires the antenna peak gain and the ratio of the antenna diameter to the wavelength (D/λ) as input in order to calculate the gain at an off-axis angle. Given the frequency and antenna diameter, the antenna equations discussed previously can be manipulated to derive the input parameters needed by this Appendix 7 model:

$$A_e = \eta\pi\left(\frac{D}{2}\right)^2 \tag{5}$$

$$G = \frac{4\pi A_e}{\lambda^2} \tag{6}$$

where D = diameter of the antenna (m). Note that gain is typically expressed in dB, in which case the gain is given by:

$$G_{dB} = 10\log_{10}\frac{4\pi A_e}{\lambda^2} \tag{7}$$

Given only the antenna gain (and efficiency, which can be assumed to be $\approx$ 65%), compute D/λ from:

$$G = \eta\left(\frac{\pi D}{\lambda}\right)^2 \tag{8}$$

or

$$G_{dB} = 10\log_{10}\pi^2\eta + 20\log_{10}\left(\frac{D}{\lambda}\right) \tag{9}$$

so

$$\frac{D}{\lambda} = 10^{\frac{G_{dB}-10\log_{10}\pi^2\eta}{20}} \tag{10}$$

As an alternative, Appendix 7 provides an approximation of D/λ based on the peak gain of the antenna:

$$20\log_{10}(D/\lambda) \approx G_{dB} - 7.7 \tag{11}$$

So D/λ can be approximated from:

$$\frac{D}{\lambda} \approx 10^{\frac{G_{dB}-7.7}{20}} \tag{12}$$

This and the previous three subsections provide all of the objects required to created a space or Earth station instance. The following two sections describe the engineering basis for modeling these stations, and for creating the corresponding classes.

4 Space and Earth stations

The space and Earth station classes in the `station` package contain a transmit and receive antenna, an emission, and an array or single beam, along with methods for computing position at a given time.

4.1 Space Stations

Even though elaborate models have been developed to compute the motion of artificial Earth satellites to high accuracy, the main features of their orbits may still be described by a reasonably simple approximation. [M&G-2]

4.1.1 Newton's Law of Gravitation

A satellite is considered whose mass is negligible compared to the Earth's mass $M_\oplus$. Assuming the Earth to be spherically symmetric, the acceleration $\ddot{r}$ of the satellite is given by Newton's law of gravity

$$\ddot{\boldsymbol{r}} = -\frac{GM_\oplus}{r^2}\frac{\boldsymbol{r}}{r} \tag{2.1}$$

where $-\boldsymbol{r}/r$ denotes a unit vector pointing from the satellite to the center of the Earth, which forms the origin of the coordinate system.

G is known with limited accuracy from torsion balance experiments

$$G = (6.67259 \pm 0.00085) \cdot 10^{-11}\mathrm{m}^3\mathrm{kg}^{-1}\mathrm{s}^{-2} \tag{2.2}$$

$GM_\oplus$ is known with considerable accuracy from laser ranging measurements of Earth satellites

$$GM_\oplus = (398600.4405 \pm 0.001)\mathrm{km}^3\mathrm{s}^{-2} \tag{2.3}$$

4.1.2 The Two-Body Problem

The study of the motion of a satellite in the spherically symmetric $1/r^2$ force field of a central mass is usually referred to as *Kepler's problem*, or as the *two-body* problem. [M&G-2.1] It was first solved in the second half of the 17th century by Isaac Newton.

4.1.3 Plane Motion

The fact that the force exerted on the satellite always points to the Earth's center in the two-body problem has the immediate consequence that the orbit is confined to a fixed plane for all times. [M&G-2.1.1] The satellite cannot leave the orbital plane, since the force is always anti-parallel to the position vector and, therefore, does not give rise to any acceleration perpendicular to the plane.

A mathematical description of this fact can be written as

$$\boldsymbol{r} \times \dot{\boldsymbol{r}} = \boldsymbol{h} = \text{constant} \tag{2.7}$$

The vector $\boldsymbol{h}$ is the angular momentum per unit mass or the specific angular momentum. It is related to the angular momentum vector $\boldsymbol{l}$ by $\boldsymbol{l} = m\boldsymbol{h}$, where m is the mass of the satellite.

4.1.4 Law of Areas

Equation (2.7) implies Kepler's second law, or the law of areas, since

$$\Delta A = \frac{1}{2}|\boldsymbol{r} \times \dot{\boldsymbol{r}}\Delta t| = \frac{1}{2}|\boldsymbol{h}|\Delta t \tag{2.8}$$

is the area swept by the radius vector during the time Δt, and since h is a constant, the radius vector sweeps over equal areas in equal time intervals.

4.1.5 The Form of the Orbit

Multiplying both sides of the equation of motion (2.1) by $\boldsymbol{h}$, rearranging, using vector multiplicative identities, and integrating yields

$$\boldsymbol{h} \times \dot{\boldsymbol{r}} = -GM_{\oplus}\left(\frac{\boldsymbol{r}}{r}\right) - \boldsymbol{A} \tag{2.12}$$

where $\boldsymbol{A}$ is called the *Runge-Lenz* or *Laplace vector* (an additive constant of integration determined by the initial position and velocity). [M&G-2.1.2] Multiplying by $\boldsymbol{r}$, and introducing ν, the *true anomaly*, as the angle between $\boldsymbol{A}$ and the position vector $\boldsymbol{r}$, p, the *semi-latus rectum*, $p = h^2/GM_{\oplus}$, and e, the *eccentricity*, $e = A/GM_{\oplus}$, yields

$$r = \frac{p}{1 + e\cos\nu} \tag{2.17}$$

which is a conic section in polar coordinates describing an ellipse, if $e < 1$, a parabola, if $e = 1$, or hyperbola, if $e > 1$. See Fig. 2.2.

This discussion focuses on satellites in Earth orbit which exhibit an elliptic orbit with $e < 1$.

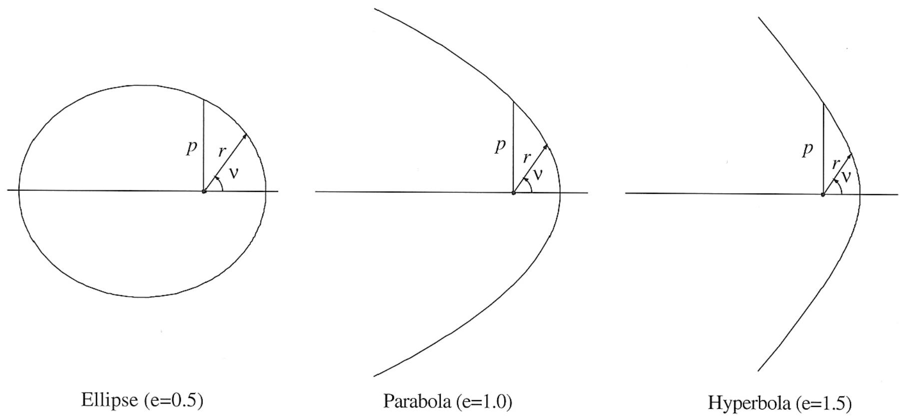

Fig. 2.2. Conic sections with eccentricities $e = 0.5$, $e = 1.0$, and $e = 1.5$ with the same semi-latus rectum p

4.1.6 The Energy Integral

Squaring both sides of (2.12) and simplifying yields

$$v^2 = GM_\oplus \left(\frac{2}{r} - \frac{1}{a} \right) \tag{2.22}$$

which is called the *vis-viva law*. It is equivalent to the energy statement that the sum of the kinetic energy, $(1/2)mv^2$, and the potential energy, $-GmM_\oplus/r$, is constant during motion. [M&G-2.1.3]

4.1.7 Kepler's Equation

Introduce the angle, E, between the center of a circle circumscribing the orbit and the point of intersection of the orbit and a line perpendicular to the line of apsides through the satellite position, called the *eccentric anomaly*. [M&G-2.2.1] See Fig. 2.5. Then

$$\begin{aligned} \hat{x} &= r \cos \nu =: a(\cos E - e) \\ \hat{y} &= r \sin \nu =: a\sqrt{1 - e^2} \sin E \end{aligned} \tag{2.30}$$

or equivalently

$$r = a(1 - e \cos E) \tag{2.31}$$

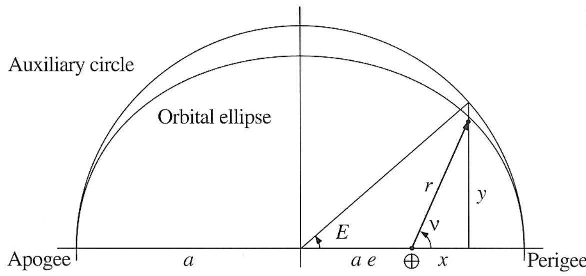

Fig. 2.5. The definition of the eccentric anomaly E

Using these definitions, and the definition of the specific angular momentum vector h, yields

$$(1 - e\cos E)\dot{E} = n \tag{2.34}$$

where n, called the *mean motion*, is introduced to simply notation and given by

$$n = \sqrt{\frac{GM_{\oplus}}{a^3}} \tag{2.35}$$

Integrating with respect to time yields *Kepler's Equation*

$$E(t) - e\sin E(t) = n(t - t_p) \tag{2.36}$$

where t_p denotes the time of perigee passage. The right hand side

$$M = n(t - t_p) \tag{2.37}$$

is called the *mean anomaly.* It changes by 360° during one revolution but – in contrast to the true and eccentric anomalies – increases uniformly with time. The orbital period

$$T = \frac{2\pi}{n} \tag{2.39}$$

Kepler's equation can be solved by iterative methods only. [M&G-2.2.2] A common way is to start with an approximation of $E_0 = M$ or $E_0 = \pi$ and employ Newton's method.

4.1.8 The Orbit in Space

Introduce the unit vector $\boldsymbol{P} = \boldsymbol{A}/|\boldsymbol{A}|$, which points towards the perigee, and the perpendicular unit vector $\boldsymbol{Q}$, corresponding to a true anomaly of $\nu = 90°$, to express the position by

$$\boldsymbol{r} = \hat{x}\boldsymbol{P} + \hat{y}\boldsymbol{Q} = a(\cos E - e)\boldsymbol{P} + a\sqrt{1 - e^2}\sin E\boldsymbol{Q} \tag{2.43}$$

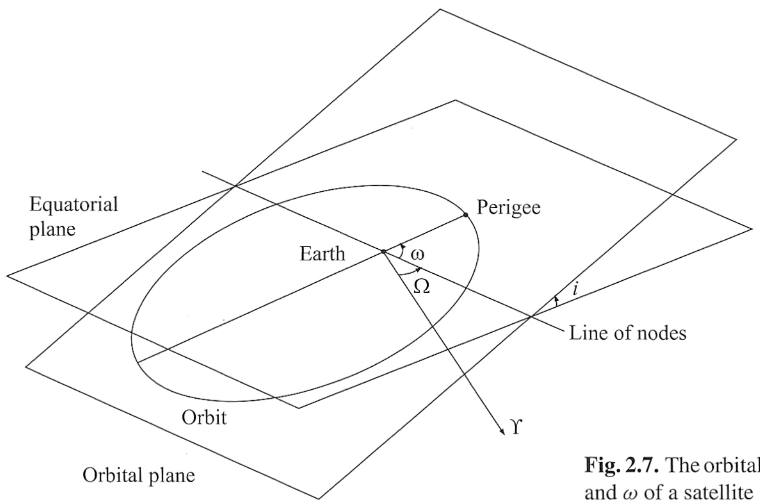

Fig. 2.7. The orbital elements i, Ω, and ω of a satellite

and the velocity by

$$\dot{\boldsymbol{r}} = \dot{x}\boldsymbol{P} + \dot{y}\boldsymbol{Q} = \frac{\sqrt{GM_{\oplus}a}}{r}(-\sin E\boldsymbol{P} + \sqrt{1-e^2}\cos E\boldsymbol{Q}) \tag{2.44}$$

See Fig 2.7. [M&G-2.2.3]

4.1.9 Two-line Element Sets

A Two-Line Element (TLE) set is a data format encoding a list of orbital elements of an Earth-orbiting object for a given point in time, the epoch. Using suitable prediction formula, the position and velocity, together called the state, of the object at any point in the past or future can be estimated.

The format uses two lines of 80-column ASCII text to store the data, reflecting its origin in punch card format with one line per card. The United States Air Force tracks objects in Earth orbit and creates a corresponding TLE for each non-classified object, which is made available on the Space-track website (https://www.space-track.org/). A TLE may also include a title line that precedes the element data. Figure 2 shows an example TLE for the International Space Station. Figure 4 shows how the data in a two line element set is decoded.

Table 2: Example two line element set

```
ISS (ZARYA)
1 25544U 98067A   04236.56031392  .00020137  00000-0  16538-3 0  9993
2 25544  51.6335 344.7760 0007976 126.2523 325.9359 15.70406856328903
```

Line 0

01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23
I	S	S		(	Z	A	R	Y	A	)												
1																						

24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46

47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69

Field	Columns	Content	Example
1	01–24	Satellite name	ISS (ZARYA)

Line 1

01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23
1		2	5	5	4	4	U		9	8	0	6	7	A				0	4	2	3	6
1		2					3		4		5			6				7		8		

24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46
.	5	6	0	3	1	3	9	2			.	0	0	0	2	0	1	3	7			0
										9											10	

47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69
0	0	0	0	-	0			1	6	5	3	8	-	3		0			9	9	9	3
							11									12		13				14

Field	Columns	Content	Example
1	01	Line number	1
2	03–07	Satellite Catalog Number	25544
3	08	Elsat Classification (U=Unclassified)	U
4	10–11	International Designator (Last two digits of launch year)	98
5	12–14	International Designator (Launch number of the year)	067
6	15–17	International Designator (continued)	A
7	19–20	Element Set Epoch Year (last two digits of year)	04
8	21–32	Element Set Epoch Day (day of the year and fractional portion of the day)*	236.56031392
9	34–43	1st Derivative of the Mean Motion with respect to Time	0.00020137
10	45–52	2nd Derivative of the Mean Motion with respect to Time (decimal point assumed	00000-0
11	54–61	BSTAR drag term (decimal point assumed)	16538-3
12	63	Element Set Type	0
13	65–68	Element Number	999
14	69	Checksum	3

*Note: spaces are acceptable in columns 21 & 22

Line 2

01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23
2		2	5	5	4	4			5	1	.	6	3	3	5		3	4	4	.	7	7
1		2						3									4					

24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46
6	0		0	0	0	7	9	7	6		1	2	6	.	2	5	2	3		3	2	5
			5								6									7		

47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69
.	9	3	5	9		1	5	.	7	0	4	0	6	8	5	6	3	2	8	9	0	3
						8											9					10

Field	Columns	Content	Example
1	01	Line number	2
2	03–07	Satellite Catalog Number	25544
3	09–16	Orbit Inclination (degrees)	51.6335
4	18–25	Right Ascension of Ascending Node (degrees)	344.7760
5	27–33	Eccentricity (decimal point assumed)	0007976
6	35–42	Argument of Perigee (degrees)	126.2523
7	44–51	Mean Anomaly (degrees)	325.9359
8	53–63	Mean Motion (revolutions/day)	15.70406856
9	64–68	Revolution Number at Epoch	32890
10	69	Checksum	3

Figure 4: Two line element set decoding

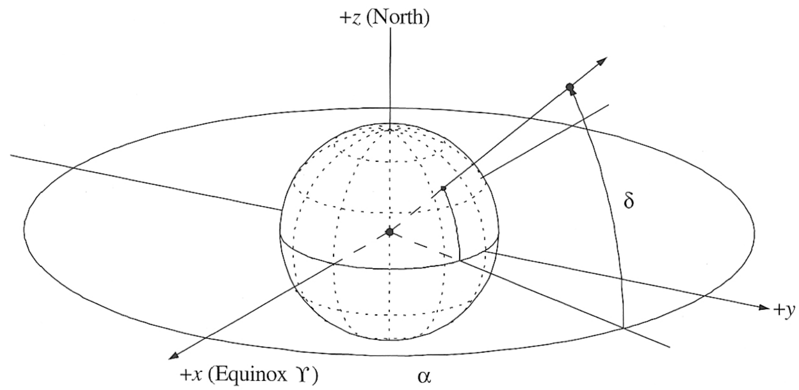

Fig. 2.6. The equatorial coordinate system

4.1.10 The Equatorial Coordinate System

The most common coordinate system for describing Earth-bound satellite orbits is the geocentric *equatorial coordinate system*, which is aligned with the Earth's rotation axis and equator. Its origin is the center of the Earth, the z-axis points to the north pole and the equatorial plane forms the x-y reference plane. The x-axis is aligned with the northern, *vernal equinox*, or the First Point of Aries.

In order to describe the orientation of the orbital plane and the perigee with respect to the equatorial coordinate system, three angles are commonly employed

i The *inclination* gives the angle of intersection between the orbital plane and the equator. An inclination of more than 90° means that the satellite's motion is retrograde, its direction of revolution around the Earth being opposite to that of the Earth's rotation.

Ω The *right ascension of the ascending node* indicates the angle between the vernal equinox and the point on the orbit at with the satellite crosses the equator from south to north.

ω The *argument of perigee* is the angle between the direction of the ascending node and the direction of the perigee.

See Fig 2.6.

In the orbital plane system, which is defined by the unit vectors $\boldsymbol{P}$, $\boldsymbol{Q}$, $\boldsymbol{W} = \boldsymbol{h}/h$, the coordinates are given by

$$(\hat{x}, \hat{y}, \hat{z}) = (r\cos\nu, r\sin\nu, 0) \tag{2.47}$$

where

$$\boldsymbol{P} = \begin{pmatrix} +\cos\omega\cos\Omega - \sin\omega\cos i\sin\Omega \\ +\cos\omega\sin\Omega + \sin\omega\cos i\cos\Omega \\ +\sin\omega\sin i \end{pmatrix} \tag{2.52}$$

$$\boldsymbol{Q} = \begin{pmatrix} -\sin\omega\cos\Omega - \cos\omega\cos i\sin\Omega \\ -\sin\omega\sin\Omega + \cos\omega\cos i\cos\Omega \\ +\cos\omega\sin i \end{pmatrix} \tag{2.53}$$

$$\boldsymbol{W} = \begin{pmatrix} +\sin i\sin\Omega \\ -\sin i\cos\Omega \\ +\cos i \end{pmatrix} \tag{2.54}$$

are referred to as *Gaussian vectors.*

In the equatorial system, the coordinates are given by

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \boldsymbol{R}_z(-\Omega)\boldsymbol{R}_x(-i)\boldsymbol{R}_z(-\omega)\begin{pmatrix} \cos\nu \\ \sin\nu \\ 0 \end{pmatrix} \tag{2.50}$$

where

$$\boldsymbol{R}_x(\phi) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & +\cos\phi & +\sin\phi \\ 0 & -\sin\phi & +\cos\phi \end{pmatrix}$$

$$\boldsymbol{R}_y(\phi) = \begin{pmatrix} +\cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ +\sin\phi & 0 & +\cos\phi \end{pmatrix}$$

$$\boldsymbol{R}_z(\phi) = \begin{pmatrix} +\cos\phi & +\sin\phi & 0 \\ -\sin\phi & +\cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.55+}$$

are rotation matrices about each coordinate axis.

4.1.11 Force Model

The inverse square law describes the gravitational attraction of a point-like mass, and can always be shown to be true for extended bodies, provided that they are built up of concentric shells of constant density. [M&G-3.1] Since this is a basic model of the structure of the Earth, Keplerian orbits provide a reasonable first approximation of satellite motion.

However, the trajectory of the satellite is not a closed ellipse with an unchanging orientation in space, but an open curve that continuously evolves with time, both in shape and orientation. There are two types of orbit perturbations:

- The perturbations of gravitational origin which derive from a potential for which total energy is conserved, including the geopotential and the effects of the Sun and Moon, and
- The non-gravitational perturbations that do not derive from a potential and that dissipate energy, including atmospheric drag and solar radiation pressure

Various perturbations of a satellite orbit are compared in Fig. 3.1.

4.1.12 Geopotential

Due to daily rotation, the Earth is not, however, a perfect sphere, but has the form of an oblate spheroid with an equatorial diameter that exceeds the polar diameter by about 20 km. [M&G-3.1] The resulting equatorial bulge exerts a force that pulls the satellite back to the equatorial plane whenever it is above or below this plane and thus tries to align the orbital plane with the equator. This perturbation is about three orders of magnitude smaller than the central attraction. Due to its angular momentum the orbit behaves like a gyroscope, and reacts with a precessional motion of the orbital plane, and a shift of the line of nodes by several degrees per day.

Assuming that the total mass of the Earth is concentrated in the center of the coordinate system, the gravitational law

$$\ddot{\boldsymbol{r}} = -\frac{GM_\oplus}{r^2}\frac{\boldsymbol{r}}{r} \tag{2.1}$$

can be used to calculate the acceleration of a satellite at $\boldsymbol{r}$. [M&G-3.2] For the following discussion of a more realistic model, it is common to use an equivalent representation involving the gradient of the corresponding gravity potential U

$$\ddot{\boldsymbol{r}} = \nabla U \quad \text{with} \quad U = GM_\oplus \frac{1}{r} \tag{3.3}$$

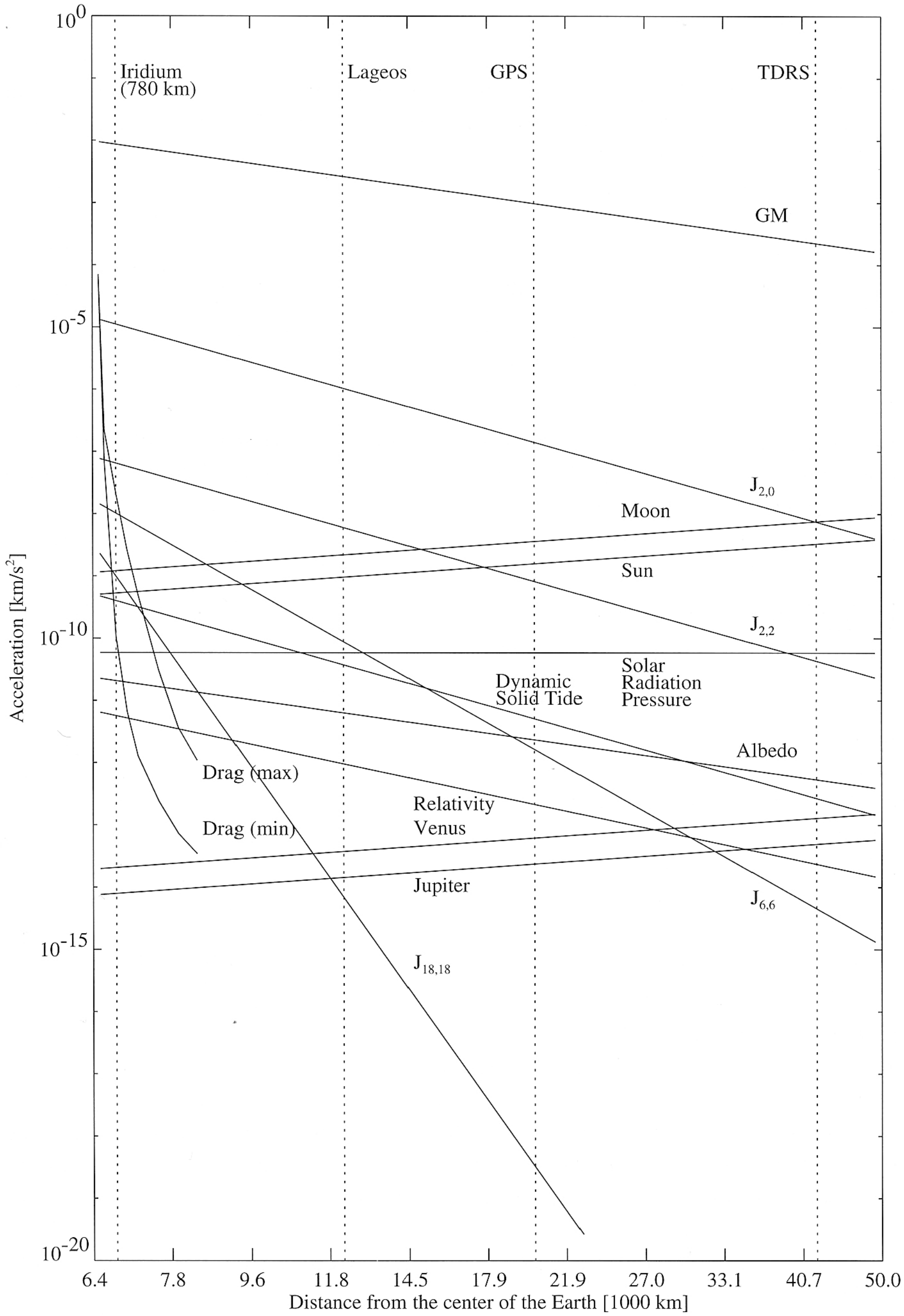

Fig. 3.1. Order of magnitude of various perturbations of a satellite orbit. See text for further explanations.

This expression for the potential may be generalized to an arbitrary mass distribution by summing up the contributions created by individual mass elements $dm = \rho(\boldsymbol{s})d^3\boldsymbol{s}$ according to

$$U = G \int \frac{\rho(\boldsymbol{s})d^3\boldsymbol{s}}{|\boldsymbol{r} - \boldsymbol{s}|} \tag{3.4}$$

where $\rho(\boldsymbol{s})$ is the density at point $\boldsymbol{s}$ inside the Earth, and $|\boldsymbol{r} - \boldsymbol{s}|$ is the satellite's distance from $\boldsymbol{s}$.

4.1.13 Expansion in Spherical Harmonics

In order to evaluate the integral in the above equation, the inverse of the distance may be expanded in a series of Legendre polynomials. [M&G-3.2.1] For $r > s$, which holds for all points $\boldsymbol{r}$ outside a circumscribing sphere, one has

$$\frac{1}{|\boldsymbol{r} - \boldsymbol{s}|} = \frac{1}{r} \sum_{n=0}^{\infty} \left(\frac{s}{r}\right)^n P_n(\cos\gamma) \quad \text{with} \quad \cos\gamma = \frac{\boldsymbol{r} \cdot \boldsymbol{s}}{rs} \tag{3.5}$$

Here

$$P_n(u) = \frac{1}{2^n n!} \frac{d^n}{du^n}(u^2 - 1)^n \tag{3.6}$$

is the Legendre polynomial of degree n, and γ is the angle between $\boldsymbol{r}$ and $\boldsymbol{s}$.

By introducing the longitude λ (positive towards the East) and the geocentric latitude ϕ of the point $\boldsymbol{r}$ according to

$$\begin{aligned} x &= r\cos\phi\cos\lambda \\ y &= r\cos\phi\sin\lambda \\ z &= r\sin\phi \end{aligned} \tag{3.7}$$

as well as the corresponding quantities λ' and ϕ' for $\boldsymbol{s}$, one can make use of the addition theorem of Legendre polynomials, which states that

$$P_n(\cos\gamma) = \sum_{m=0}^{n} (2 - \delta_{0m} \frac{(n-m)!}{(n+m)!} P_{nm}(\sin\phi) P_{nm}(\sin\phi') \cos(m(\lambda - \lambda')) \tag{3.8}$$

where P_{nm} (the associated Legendre polynomial of degree n and order m) is defined as

$$P_{nm} = (1 - u^2)^{m/2} \frac{d^m}{du^m} P_n(u) \tag{3.9}$$

Table 3: Geopotential coefficients up to degree and order three [M&G-Table-3.2]

C_{nm}	$m = 0$	1	2	3
$n = 0$	$+\ 1.00$			
1	0.00	0.00		
2	$-1.08 \cdot 10^{-3}$	0.00	$+1.57 \cdot 10^{-6}$	
3	$+2.53 \cdot 10^{-6}$	$+2.18 \cdot 10^{-6}$	$+3.11 \cdot 10^{-7}$	$+1.02 \cdot 10^{-7}$
S_{nm}	$m = 0$	1	2	3
$n = 0$	0.00			
1	0.00	0.00		
2	0.00	0.00	$-9.03 \cdot 10^{-7}$	
3	0.00	$+2.68 \cdot 10^{-7}$	$-2.12 \cdot 10^{-7}$	$+1.98 \cdot 10^{-7}$

One is now able to write the Earth's gravity potential in the form

$$U = \frac{GM_\oplus}{r} \sum_{n=0}^{\infty} \sum_{m=0}^{n} \frac{R_\oplus^n}{r^n} P_{nm}(\sin\phi)(C_{nm}\cos(m\lambda) + S_{nm}\sin(m\lambda)) \tag{3.10}$$

with coefficients

$$\begin{aligned} C_{nm} &= \frac{2-\delta_{0m}}{M_\oplus} \frac{(n-m)!}{(n+m)!} \int \frac{s^n}{R_\oplus^n} P_{nm}(\sin\phi')\cos(m\lambda')\rho(\boldsymbol{s})d^3\boldsymbol{s} \\ S_{nm} &= \frac{2-\delta_{0m}}{M_\oplus} \frac{(n-m)!}{(n+m)!} \int \frac{s^n}{R_\oplus^n} P_{nm}(\sin\phi')\sin(m\lambda')\rho(\boldsymbol{s})d^3\boldsymbol{s} \end{aligned} \tag{3.11}$$

which describe the dependence of U on the Earth's internal mass distribution. Geopotential coefficients with $m = 0$ are called *zonal* coefficients, since they describe the potential that does not depend on the longitude. All S_{n0} vanish due to their definition, and the notation

$$J_n = -C_{n0} \tag{3.12}$$

is commonly used for the remaining zonal terms. The other geopotential coefficients are known as *tesseral* and *sectorial* coefficients for $(m < n)$ and $(m = n)$, respectively. See Table 1.

Because the internal mass distribution of the Earth is not known, the geopotential coefficients cannot be calculated from the defining equation, but have to be determined indirectly using satellite tracking, surface gravimetry, and satellite altimeter data. [M&G-3.2.3] Various government and research organization undertake to develop these models. For example, a cooperation between the National Aeronautics and Space Administration (NASA) Goddard Space Flight Center (GSFC), the University of Texas Center for Space Research (CSR), the Centre National d'Études Spatiales (CNES) led to the Joint Gravity Model (JGM) series, with JGM-3 being published in 1996.

4.1.14 Secular Perturbations

First-order secular variations of the classical elements affect the right ascension of the ascending node, the argument of perigee, and the mean motion, or, equivalently, the initial value of the mean anomaly, to produce a linear change with time given by:

$$\dot{\Omega} = -\frac{3nJ_2}{2p^2}\cos(i) \tag{13}$$

$$\dot{\omega} = \frac{3nJ_2}{4p^2}(4 - 5\sin^2(i)) \tag{14}$$

$$\dot{M}_0 = \frac{3nJ_2(1-e^2)}{4p^2}(2 - 3\sin^2(i)) \tag{15}$$

where the semi-latus rectum $p = a*(1-e^2)$ with semi-major axis a expressed in Earth radii. [V-9.6.1]

4.2 Earth Stations

4.2.1 Earth Rotation

The rotation of the Earth is represented by the right ascension, in degrees, of the Greenwich meridian as a function of time given by

$$\Theta = 280.4606 + 360.9856473d \tag{16}$$

where d is the time in days since noon on January 1, 2000.

4.2.2 Geodetic Datums

The World Geodetic System 1972 (WGS1972) and 1984 (WGS1984) have been established by the United States Department of Defense (DoD) and the Defense Mapping Agency (DMA) for use with the TRANSIT and GPS satellite navigation systems. [M&G-5.5]

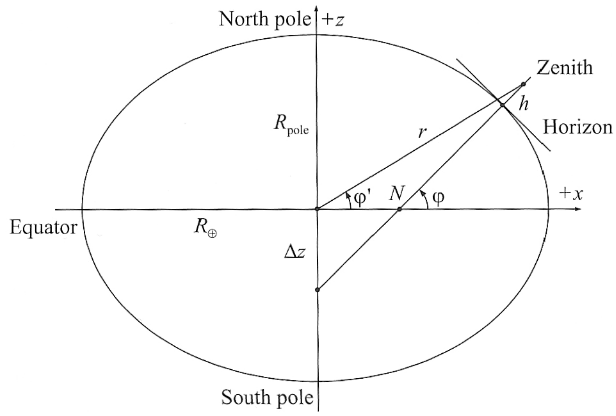

Fig. 5.12. Geocentric and geodetic latitude

Unlike the geocentric latitude ϕ' that specifies the inclination of the position vector with respect to the equatorial plane, the geodetic latitude ϕ gives the angle between the Earth's equator and the normal to the reference ellipsoid, or the elevation of the North Celestial Pole about the local tangent plane. See Fig. 5.12.

The relation between Cartesian and geodetic coordinates is give by

$$\boldsymbol{r} = \begin{pmatrix} (N+h)\cos\varphi\cos\lambda \\ (N+h)\cos\varphi\sin\lambda \\ ((1-f)^2 N + h)\sin\phi \end{pmatrix} \tag{5.83}$$

where

$$N = \frac{R_\oplus}{\sqrt{1 - f(2-f)\sin^2\varphi}} \tag{5.84}$$

For WGS84 $R_\oplus = 6,378,137$ m and $1/f = 298.257223563$.

5 Link Performance

The basic equation of a link relates the received carrier power to the noise level of the receive system, which is the carrier-to-noise ratio, C/N. In the clear sky condition (i.e., no additional propagation losses due to precipitation), the C/N is given by:

$$\begin{aligned} C/N_{UP} &= PD_{ES,W} + G_{ES,W}(\theta_W) \\ &\quad -SL + G_{SS,W}(0) - 10\log_{10}(T) - k \qquad (17) \\ C/N_{DN} &= PD_{SS,W} + G_{SS,W}(0) \\ &\quad -SL + G_{ES,W}(\theta_W) - 10\log_{10}(T) - k \qquad (18) \end{aligned}$$

where subscript X refers to link direction, C/N_X = link carrier-to-noise ratio (dB), $PD_{ES,W}$ = power density of earth station wanted signal (dBW/Hz), $PD_{SS,W}$ = power density of space station wanted signal (dBW/Hz), $G_{ES,W}(\theta_W)$ = co-pol gain of wanted earth station in direction of wanted space station (dBi), SL = space loss (dB), $G_{SS,W}$ = gain of wanted space station in direction of wanted earth station (dBi), T = receive noise temperature (K), and k = Boltzmans constant (-228.6 dBW/Hz-K).

The carrier-to-noise ratio is just the transmit power less the propagation loss plus the receive gain compared to receiver thermal noise. If the C/N of the link exceeds the required level, the link is said to have positive link margin. Links with positive margin have sufficient power to transmit information at the desired rate and accuracy, and the link is said to be closed. Negative margin means the link does not close since it has insufficient power to meet the performance requirements.

The carrier-to-interference ratio (C/I) is calculated from the following equations:

$$\begin{aligned} C_{DN} &= PD_{SS,W} + G_{SS,W}(0) - SL + G_{ES,W}(\theta_W) \qquad (19) \\ C_{UP} &= PD_{ES,W} + G_{ES,W}(\theta_W) - SL + G_{SS,W}(0) \qquad (20) \\ I_{DN} &= PD_{SS,I} + G_{SS,I}(0) - SL + G_{ES,W}(\theta_I) \qquad (21) \\ I_{UP} &= PD_{ES,I} + G_{ES,I}(\theta_I) - SL + G_{SS,W}(0) \qquad (22) \end{aligned}$$

where subscript X refers to link direction, C_X = link carrier power density (dBW/Hz), I_X = co-pol interference power density (dBW/Hz), $PD_{ES,I}$ = power density of earth station interfering signal (dBW/Hz), $PD_{SS,I}$ = power density of space station interfering signal (dBW/Hz), $G_{SS,I}$ = gain of interfering space station in direction of wanted earth station (dBi), $G_{ES,W}(\theta_I)$ = co-pol gain of wanted earth station in direction of interfering space station (dBi), $G_{ES,I}(\theta_I)$ = co-pol gain of interfering earth station in direction of wanted space station (dBi) and

$$C/I_X = C_X - I_X + FDR + PD \qquad (23)$$

with parameters expressed in dB, and

$$C/(N+I)_X = 10\log_{10}\left(\left(\frac{C}{(N+I)_X}\right) \times FDR \times PD\right) \tag{24}$$

with parameters expressed as power ratios, where FDR = frequency dependent rejection (dB, assumed 0 for this analysis), and PD = polarization discrimination (dB). Note that polarization discrimination (PD) is derived from the group polarization symbol given in the ITU filing data. Using the C/I ratio, the total $C/(N+I)$ and availability both with and without interference are calculated assuming the rain fade statistics given by Recommendation ITU-R P.618-9.

5.1 Noise Temperature and Figure of Merit

Noise power at the receiver input is due both to the internal sources such as the inherent movement of electrons, and external sources such as contribution from the antenna. The noise power, N, or noise spectral density, N_0, is related to the noise temperature, T, as follows:

$$N = kTB \tag{25}$$

$$N_0 = kT \tag{26}$$

where N = Noise power (W), N_0 = Noise spectral density (W/Hz), T = Noise temperature (K), B = Noise bandwidth (Hz), and k = Boltzmanns constant (1.3806510-23 J/K).

In calculating link budgets the noise spectral density or the noise temperature is often used instead of the total noise power so that there is no need to specify the bandwidth in which the noise is measured.

The noise caused by a receiver is usually expressed in terms of an equivalent amplifier noise temperature T_R, which is defined as the temperature of a noise source that, when connected to the input of a noiseless receiver, gives the same noise at the output as the actual receiver.

5.2 Free Space Loss

The free-space propagation loss in dB is given by:

$$FSL = 20\log_{10}\left(\frac{4\pi d}{\lambda}\right) \tag{27}$$

where d = distance between space station and earth station (m), and λ = wavelength (m).

6 References

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V Vallado, D. A., *Fundamentals of Astrodynamics and Applications, Second Edition*, Microcosm Press and Kluwer Academic Publishers, 2004.