1. Data Representation [15 points]

Interpolation and cells [5 points]

a) relationship between interpolation over a grid and nature of the cells in the grid [3 points]

* important aspects to mention (or convey some - even implicit - understanding of):
    i) interpolation = continuous (ideally smooth) reconstruction of discrete data
    ii) mesh is discrete data + connectivity that must be interpolated to be visualized
    iii) the interpolation in this case is defined piecewise, with a different interpolation constraint in each cell
    iv) there are two conditions to satisfy for this cell-wise interpolating function:
        a) interpolating property: it matches the given value at each cell vertex
        b) global continuity: the global function obtained by putting together these individual cell-wise interpolating functions must be continuous (otherwise this defies the whole purpose of the smooth reconstruction.
    v) the number of vertices in each cell must be matched by the number of degrees of freedom of the interpolating function (ex: a triangle in 2D has three vertices which is matched by a linear function of the form a*x + b*y + c, with a, b, and c degrees of freedom).

I would suggest:
+1 point for each of i), ii), and iii)
+2 points for each of iv) and v)

cap the sum at 3

b) What interpolation in a triangle? in a hexahedron? + justification [2 points]

i) triangle: (affine) linear interpolation because there are 3 vertices and 3 degrees of freedom in an affine linear function (1, x, y).
ii) hexahedron: trilinear interpolation because there are 8 vertices and 8 degrees of freedom in a trilinear function (1, x, y, z, xy, xz, yz, xyz).

both are incidentally the simplest interpolation solution for these special cases.

I would suggest:
+1 point for each of i) and ii).

Barycentric coordinates [5 points]

c) system of equations for barycentric coordinates in N dimensions. Explain: [3 points]

i) interpolating property (the P_i's are N+1 points in R^N)

$$\mathbf{P} = \sum_{i=1}^{N+1} \beta_i(\mathbf{P}_i)\ \forall P \in I\!R^N$$

ii) Convexity condition:

$$\sum_{i=1}^{N+1} \beta_i(\mathbf{P}_i) = 1$$

+1 for each equation stated correctly (if they used N instead of N+1, count it as correct, too)
+1 for having used N+1 instead of N in the sum
+1 for mentioning the interpolation property in i)
+1 for mentioning the convexity condition in ii)

cap sum at 3)

d) in what cells is it applicable? [2 points]

i) in simplices
ii) cells that are spanned by N+1 vertices in N dimensions
iii) such that the convex hull of these vertices is not fully contained in a subspace of dimension <= N-1
iv) in 2D: triangles, in 3D: tetrahedra, in Nd: generalization to Nd

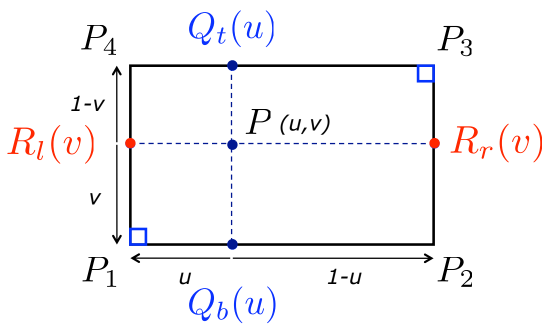
I would suggest:
+2 for each of i) and ii)
+1 for each of iii) and iv)

cap the sum at 2

Bilinear interpolation [5 points]

e) explain bilinear interpolation [3 points]

i) bilinear interpolation can be summarized as compounded linear interpolations.
ii) draw this figure:



iii) Interpolate linearly for fixed u along top and bottom edges and then interpolate linearly between the two values obtained that way.
iv) Interpolate linearly for fixed value of v along left and right edges and then interpolate linearly between the two values obtained that way.
v) iii) and iv) are equivalent
vi) f(u,v) = (1-v)((1-u)f1 + u f2) + v((1-u)f4 + u f3)
vii) f(u,v) = (1-u)((1-v)f1 + v f4) + u((1-v)f2 + v f3)
viii) f(u,v) = f1 + u(f2-f1) + v(f4-f1) + uv(f1 - f2 + f3 - f4)
ix)

$$f(u,v) = \begin{pmatrix} 1-u & u \end{pmatrix} \begin{pmatrix} f_4 & f_3 \\ f_1 & f_2 \end{pmatrix} \begin{pmatrix} v \\ 1-v \end{pmatrix}$$

(we saw the expression in class before the midterm).

I would suggest:
+2 for each of i), ii), iii), iv), vi), vii), viii), ix)
+1 for v)

cap sum at 3

f) prove linearity of bilinear interpolation along edges and explain link to global continuity of the reconstruction. [2 points]

i) insert u=0 (resp. u=1) in vi), vii), viii), or ix) and show that expression becomes a linear combination of f1 and f4 (resp. f2 and f3).
ii) insert v=0 (resp. v=1) in vi), vii), viii), or ix) and show that expression becomes a linear combination of f1 and f2 (resp. f4 and f3).
iii) linearity along edges means in particular that the function along the edge is determined only by the values at its two vertices. Hence two cells that have an edge in common will have matching bilinear interpolation along this edge.
iv) Since the bilinear function itself is smooth and therefore continuous, the global reconstruction is continuous.

+1 for i) or ii)
+2 for iii) or iv)

cap sum at 2.

2. Human Vision

a) Mach band illusion [3 points]

The illusion shown (perceived variation in brightness while the brightness is in fact constant) is called

i) "simultaneous contrast"
ii) it is an example of the fundamentally relative nature of the way our visual system processes the visual input
iii) the effect is created by having each band located between a lighter band to its left and a darker one to its right. Each neighbor contributes to skewing the perceived brightness in a different direction: the left neighbor makes the left hand side of the band appear darker, while the right neighbor makes it appear lighter. Combined, these effects create the illusion.

+2 for each of i), ii) and iii)
cap total at 3

b) Process [2 points]

i) receptive fields
ii) ganglia play the role of edge detectors
iii) ganglia compound the information coming from a set of photoreceptors

+2 for each of i), ii) and iii)
cap sum at 2

Note: question c) and d) were very closely related. Grade them together for 5 points

c) & d) box illusion [5 points]

i) perceived perspective explains the illusion
ii) farther objects appear smaller so same apparent size is intuitively interpreted as larger
iii) we try to make sense of a 3D scene from a 2D projection in the image
iv) we are hardwired to interpret perspective in that way
v) feedback from higher cognitive layers in the visual system
vi) drawing uses converging lines to show perspective
vii) parallel lines to convey equidistant lines from the viewer
viii) right box is somewhat more distant from the middle box than middle box from front box

+3 points for each of i) through v)

+2 points for each of vi), vii) and viii)

cap sum at 5

3. Colors [15 points]

a) Trichromatic color theory [4 points]

basic principle:
i) we perceive colors as a combination of red, green, and blue signals
ii) we perceive colors as a combination of short, medium, and long wavelength signals

perceptual:
iii) color space is 3D / any visible color can be expressed by only 3 coefficients
iv) certain types of color blindness are characterized by the inability to perceive one of these three colors

anatomical:
v) 3 types of cones: S, M, and L among the photoreceptors provide anatomical basis for the theory

+ 2 points for i) OR ii) (only 2 points for giving both)
+ 2 points for each of iii) and iv)
+ 2 points for v)

cap sum at 4

b) Double ended color scale [4 points]

i) double ended color scale is a color scale with two colors on the extremities that meet at a neutral middle color
ii) blue-white-red is an example
iii) useful when the goal is to emphasize the extremes
iv) useful when the goal is to distinguish positive from negative values in a continuous range

+ 2 points for each of i), iii), and iv)
+ 1 point for naming an example such as ii), unless answer i) was already provided (in which case, no additional point)
+ 2 points for naming an example corresponding to idea iii) or iv)

cap sum at 4

c) Mystery color scale [3 points]

description: from left to right of the scale
i) increase in value
ii) increase in saturation
iii) two rotations around the color spectrum / hue

name:
iv) such color scales are named redundant

+1 point for each of i), ii), and iii)
+1 point for iv)

cap sum at 3

d) Color visualization of vector dataset [4 points]

open question, every well motivated answer should be given full point if:
- the arguments named make use of what was discussed in class about color perception and color visualization
- the student says something about the limitations of his/her proposed solution

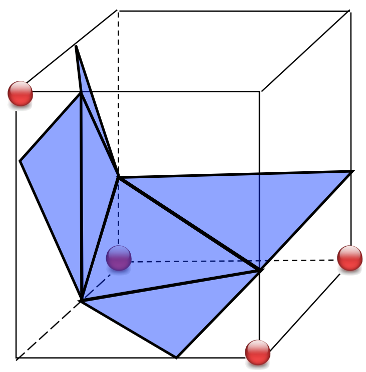
- 1 point for encoding only part of the information (say, only angle or only x component) unless this problem is acknowledged
- 1 point for treating vx and vy differently unless the issue of symmetry is mentioned in some way.
- 2 points for misunderstanding the problem (e.g., not understanding that the task is to visualize a 2D vector field)
- 2 points for naming a color scale without justification

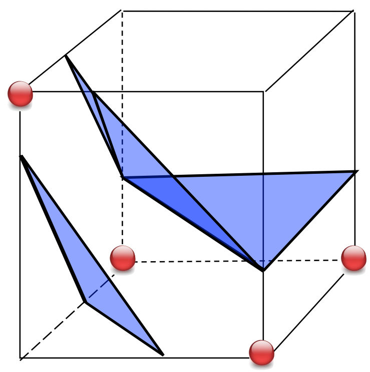
cap sum between 0 and 4

4. Isosurfaces [15 points]

a) MC's ambiguity [8 points]

i) ambiguity here means that based on the information available at the vertices there are more than one way to triangulate the isosurface inside the voxel
ii) limitations of the linear model a trilinear cell
iii) two examples of alternative triangulations:



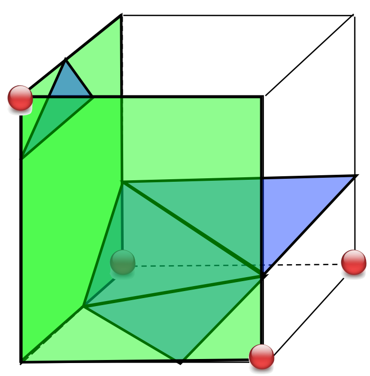


+4 points for each of i) and iii) (one of those cases or some other one that would be correct)
+2 points for ii)

cap sum at 8

b) 2D ambiguity and saddle method [7 points]

i) front and left faces are ambiguous.



ii) both faces have positive vertices on opposite ends of the diagonal
iii) the saddle method can be used to resolve this ambiguity in the 2D case
iv) the saddle method consists in checking the value of the function at the position corresponding to a zero value of the gradient (derivative) of the function.
v) if the function is positive at the saddle point a positive tunnel connecting the positive vertices on opposite ends is the correct configuration. If it is negative, the correct configuration has both vertices separated by a negative tunnel that cuts across.

+3 points for indicating the correct faces
+1 points for indicating only one of the two faces
+2 points for naming the saddle method
+4 points for each of iv) and v)

cap sum at 7

5. Volume Rendering [15 points]

a) Role of transfer function [6 points]

i) a transfer function maps scalar values to color and opacity properties
ii) a transfer function maps scalar values to optical properties
iii) these properties are needed to describe the contribution of each voxel in the volume rendering integral

iv) a transfer function determines what parts of the volume will be visible (opacity control)
v) a transfer function permits to emphasize certain aspects of the volume / features in the data
vi) transfer functions are challenging to design because they are defined in value space instead of being defined in the physical domain
vii) this is a problem because visibility decisions have to be made for all locations sharing the same value instead of applying only to a particular spatial region where something of interest is known to be present.
viii) example of the difficulty to characterize an edge in value space contrasted with the relative simplicity to do so in the physical domain.

+ 2 point for each of (i) OR ii)), iii), iv), v)
+ 2 points for each of vi), vii), and viii)

cap sum at 4

b) Volume rendering to display surfaces [4 points]

i) volume rendering can be seen as a generalization of isosurfacing
ii) a transfer function can be designed to render only specific values, thus creating a typical isosurfacing result
iii) the superiority of volume rendering in this context is that it can extract "thick" isosurfaces by giving non-zero opacity to values that are close to the actual isovalue

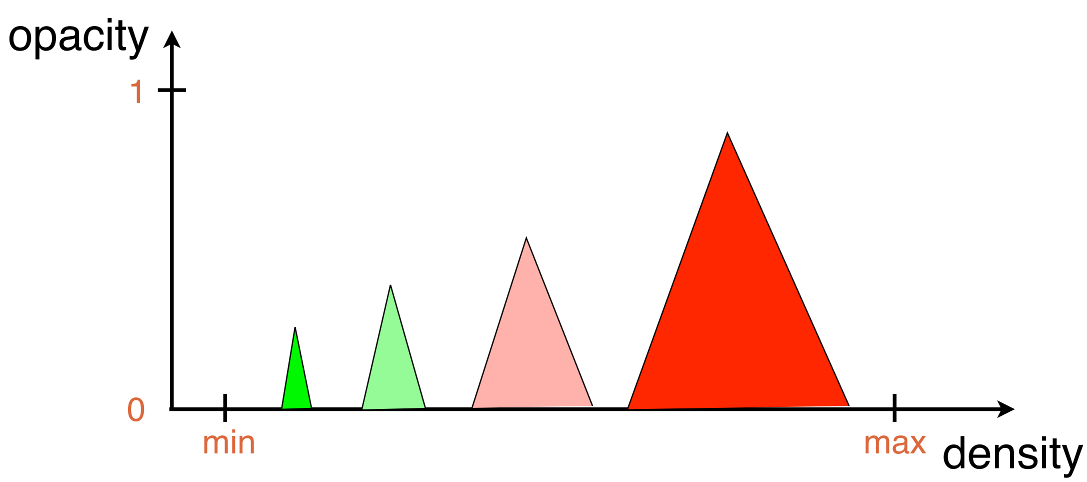
+2 points for i) and iii)
+4 point for ii)

cap sum at 4

c) Transfer function design [5 points]

known properties:
- density decreases with distance to the center
- boundary sharpness increases with distance to center

i) We want to create a rendering that reveals all the boundaries present in the data
ii) This requires us to properly capture individual boundaries in the transfer function
iii) we must also make sure that the outer surfaces do not occlude the inner surfaces
iv) to prevent occlusion we must assign lower opacity to the outer regions and higher opacity to the inner regions
v) to capture the boundaries we assign non-zero opacity to values close to the values associated with each boundary
vi) the corresponding intervals should become wider as we move toward the center since the boundaries become fuzzier
vii) the corresponding transfer function looks something like this (we move toward the center of the sphere from left to right since the density increases toward the center).



+2 points for each of i) through vii)
cap sum at 5