Table 1: For instructor's use

Question	Points Scored	Possible Points
1		12
2		12
3		12
4		12

Here is some extra space. **Show all of your work on the questions!** If you need more paper just ask. Good luck!!

Question 4. Bayes Nets

A professor is heading back into the lab late at night and is trying to guess whether his student is working on their paper. He has noticed that whenever this student is working, their car is in the garage and there is music playing in the lab. Let W, C, M be three binary random variables corresponding to the events "student Working on paper", "Car in parking lot", and "Music playing". Through careful observation, the professor has determined the following conditional probabilities:

(1) $P(C|W, M) = P(C|W) =$

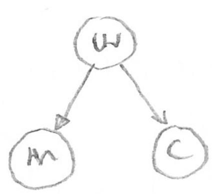
	$W = 1$	$W = 0$
$C = 1$	0.8	0.1
$C = 0$	0.2	0.9

(2) $P(M|W) = P(C|W)$

(3) $P(W = 1) = 0.8$

Note that equation (2) says that C and M have the same conditional distribution (e.g. $P(C = 1|W = 1) = P(M = 1|W = 1) = 0.8$.)

(a) Draw a Bayesian network model (i.e. a directed graphical model) for this problem.



(b) Compute the marginal distributions $P(C)$ and $P(M)$, before any evidence is availble

$P(C, W) = P(C|W)P(W) = P(M|W)P(W) = P(M, W)$ so joint pdf is also same

$$= \begin{array}{c} c \backslash w \\ \end{array} \begin{bmatrix} 0.64 & 0.02 \\ 0.16 & 0.18 \end{bmatrix} \begin{array}{c} \rightarrow \\ \rightarrow \end{array} \begin{pmatrix} 0.66 \\ 0.34 \end{pmatrix} \begin{array}{c} C=1 \\ C=0 \end{array}$$

↑ $P(C)$ and $P(M)$

(c) Upon entering the garage, the professor notices that the student's car is parked there. Calculate $P(W|C = 1)$.

Bayes Rule:

$$P(W|C=1) = \frac{P(C=1|W)\,P(W)}{P(C=1)} = \frac{1}{0.66} \times \begin{bmatrix} 0.8 \times 0.8 \\ 0.2 \times 0.1 \end{bmatrix} = \begin{array}{cc} \begin{bmatrix} 0.97 \\ 0.03 \end{bmatrix} & \begin{array}{c} W=1 \\ W=0 \end{array} \end{array}$$

$P(C=1)$ (from marginal) $\quad P(W) \quad P(C=1|W)$

(d) Calculate the updated probability that music is playing in the lab

We need to take $C=1$ into account

$$P(M,W|C=1) = P(M|W,C=1)\,P(W|C=1) = P(M|W)\,P(W|C=1)$$

$$= \begin{array}{c|cc} C \backslash W & 1 & 0 \\ \hline 1 & 0.8 & 0.1 \\ 0 & 0.2 & 0.9 \end{array} \times \begin{array}{cc} \multicolumn{2}{c}{W|C=1} \\ \begin{bmatrix} 0.97 \\ 0.03 \end{bmatrix} & \begin{array}{c} 1 \\ 0 \end{array} \end{array}$$

$$= \begin{bmatrix} 0.8 \times 0.97 & 0.1 \times 0.03 \\ 0.2 \times 0.97 & 0.9 \times 0.03 \end{bmatrix} = \begin{array}{c|cc} M \backslash W & 1 & 0 \\ \hline 1 & 0.78 & 0.003 \\ 0 & 0.19 & 0.027 \end{array} \rightarrow \begin{array}{cc} \multicolumn{2}{c}{P(M|C=1)} \\ \boxed{\begin{array}{c} 0.783 \\ 0.217 \end{array}} & \begin{array}{c} 1 \\ 0 \end{array} \end{array}$$

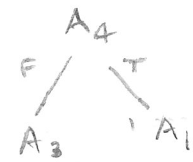
Question 5. Decision Tree Learning

Consider the following dataset for training a decision tree:

	A_1	A_2	A_3	A_4	Output
1	1	1	1	0	1
2	1	0	1	0	1
3	0	0	1	1	1
4	0	0	0	0	1
5	1	0	0	1	0
6	1	1	1	1	0
7	1	0	0	0	0
8	0	0	1	1	0

change to 1

$H = B(1/2) = 1$



(a) The attribute A_4 is selected for the first (root) node of the decision tree. Calculate the information gain based on splitting on A_4.

$H(o|A_4) = \frac{1}{2}B(1/4) + \frac{1}{2}B(3/4) = 0.8113$

$\text{Gain} = 1 - 0.8113 = 0.1887$

(b) Consider the **TRUE** branch of the A_4 split. Identify whether an additional split is needed, and if so which attribute to select next.

$A_1: \frac{1}{2}B(0) + \frac{1}{2}B(\frac{1}{2}) = 0.5$ ← CHOOSE A_1

$A_2: 1/4\,B(0) + 3/4\,B(1/3) = 0.6887$

$A_3: 3/4\,B(1/3) + 1/4\,B(0) = 0.6887$

c) FALSE BRANCH

$A_1: 3/4\,B(2/3) + 1/4\,B(1) = 0.6887$

$A_2: 1/4\,B(1) + 3/4\,B(2/3) = 0.6887$

$A_3: 1/2\,B(1) + \frac{1}{2}B(1/2) = 0.5$ ← choose A_3

d) $A_3: 5/8\,B(3/5) + 3/8\,B(1/3)$

$\rightarrow 3/4\,B(2/3) + 1/4\,B(0) = 0.6887$ ✓

Question 6. Markov Decision Processes

Consider the grid world shown below. The agent starts in state S_1 and is trying to reach the positive goal G_2. There is also a negative terminal G_1 which the robot wants to avoid. The reward for each of the two goals and the four states is shown in the top right of each square. Assume that there are *three* action choices in each state: *Left, Up,* and *Down.* Use the standard noise model for actions in which the probability of moving in the desired direction is 0.8 and there is 0.1 chance to move in each of the orthogonal directions (this is the same action model we used in class and is used in your book). Assume that rewards are not being discounted (i.e. discount factor of 1.0).

+1.0 G_2	-0.01 S_4 ←	-0.01 S_3 ←	(shaded)
(shaded)	-1.0 G_1	-0.01 S_2 ↑	-0.01 S_1 ↓

(a) Assume that the utilities for each of the states S_1 to S_4 are initialized to 0.1. Perform two rounds of value iteration, and show the utilties for each of the four states at each round.

Round 1:

$U(S_1) = -0.01 + \max_{LUD} \{ \overset{\leftarrow}{0.8 \times (0.1)} + \overset{\uparrow}{0.1 \times (0.1)} + \overset{\downarrow}{0.1 \times (0.1)}$ 0.1 — 0.09 — L TIED w/ U & D

All other utilities will be same by symmetry U & D

$U(S_2) = -0.01 + \max_{LUD} \{ \overset{\leftarrow}{0.8 \times (-1)} + \overset{\uparrow\downarrow}{0.2 \times (0.1)}$ −0.78 L

$\overset{\uparrow}{0.8 \times (0.1)} + \overset{\leftarrow}{0.1 \times (-1)} + \overset{\downarrow}{0.1 \times (0.1)}$ −0.01 U TIED w/ D — −0.02

D will be same by symmetry

$U(S_3) = -0.01 + \max_{LUD} \{$ ALL OPTIONS SAME, IDENTICAL TO S_1 — 0.09 — L TIED w/ U & D

$U(S_4) = -0.01 + \max_{LUD} \{ \overset{\leftarrow}{0.8 \times (1.0)} + \overset{\uparrow}{0.1 \times (0.1)} + \overset{\downarrow}{0.1 \times (-1)}$ 0.71 — L — 0.7

$\overset{\uparrow\rightarrow}{0.9 \times (0.1)} + \overset{\leftarrow}{0.1 \times (1.0)}$ 0.19 U

$\overset{\downarrow}{0.8 \times (-1)} + \overset{\leftarrow}{0.1 \times (1)} + \overset{\rightarrow}{0.1 \times (0.1)}$ −0.69 D

Round 2:

$U(S_1) = -0.01 + \max_{LUD} \{$ L IS NOT COMPETITIVE W/ NEGATIVE UTIL L

$0.9 \times (0.09) + 0.1 \times (-0.02) = 0.079$ U $\quad$ 0.069 D $\quad$ BREAK TIE ALPHABET

D will be same by symmetry

$U(S_2) = -0.01 + \max_{LUD} \{$ L CAN'T WIN

$0.8 \times (0.09) + 0.1 \times (-1) + 0.1 \times (0.09) = -0.019$ (U) $\quad$ -0.029

D CAN'T WIN BECAUSE $0.09 > -0.02$

$U(S_3) = -0.01 + \max_{LUD} \{ 0.8 \times (0.7) + 0.1 \times (-0.02) + 0.1 \times (0.09) = 0.567$ (L) $\quad$ 0.557

U & D CAN'T WIN

$U(S_4) = -0.01 + \max_{LUD} \{ 0.8 \times (1) + 0.1 \times (0.7) + 0.1 \times (-1) = 0.77$ (L) $\quad$ 0.76

U & D CAN'T WIN

(b) For each state in the figure above, draw an arrow indicating the optimal policy following the 2nd iteration of value iteration.

See figure, ties broken alphabetically in S_1

(c) Now assume that the robot is moving on an infinite plane with no obstacles or goals, and the reward at each state is 0.5. Using a discounting factor of 0.9, what is the expected utility of the optimal policy?

The question is asking about infinite horizon case

$$U = \sum_{t=0}^{\infty} \gamma^t R(S_t) = 0.5 \sum_{t=0}^{\infty} (0.9)^t = \frac{0.5}{1-0.9} = 5$$

Note: Since all directions equal, all actions equal. Then t is just how many cells visited

See (17.1) in book

Question 8. Consider the following paragraph. Your goal is to translate the statements into propositional logic and then use resolution to demonstrate that an assertion is true. Specifically, your goal is to write Horn clauses so that we can leverage efficient forward/backward chaining methods for entailment.

> To be in kindergarten or first grade you must be a child. Male children are boys and female children are girls. Children of civic-minded parents are also scouts. Boys who are scouts are boy-scouts. Girls who are scouts are girl-scouts. Boy-scouts and girl-scouts are responsible.

Assertion: *A female child of civic-minded parents is responsible*

(a) Identify a set of literals and write a set of Horn clauses to specify the knowledge base associated with the text passage above.

K - kindergarten FG - first grade C - child etc.

$K \Rightarrow child$

$FG \Rightarrow child$

$child \wedge male \Rightarrow boy$

$child \wedge female \Rightarrow girl$

$civic \wedge child \Rightarrow scout$

$scout \wedge boy \Rightarrow boy\text{-}scout$

$scout \wedge girl \Rightarrow girl\text{-}scout$

$boy\text{-}scout \Rightarrow responsible$

$girl\text{-}scout \Rightarrow responsible$

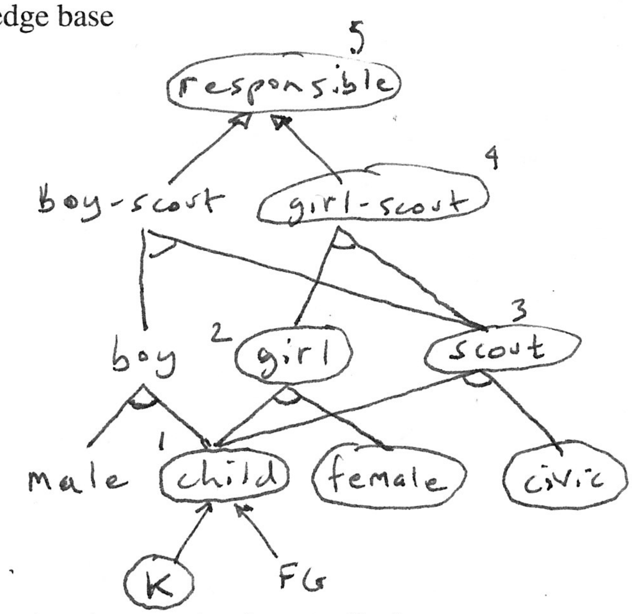
(b) Draw an AND-OR graph which represents the knowledge base

Asserted:

K

female

civic



(c) In the graph above, use forward chaining to demonstrate that the assertion is true. Circle all symbols which are inferred and number them to indicate the order in which they were inferred during chaining.