4) What does the vector A_0A represent?

5) What does the vector B_0B represent?

6) What does the vector C_0C represent?

Using the data in Figure 4.11, calculate:

7) $\tan \alpha$.

8) $\tan \beta$.

9) $u_1(x_1+d\,x_1, x_2) - u_1(x_1, x_2)$ and give an interpretation of this result.

10) $u_2(x_1+d\,x_1, x_2) - u_2(x_1, x_2)$ and give an interpretation of this result.

11) $u_1(x_1, x_2+d\,x_2) - u_1(x_1, x_2)$ and give an interpretation of this result.

12) $u_2(x_1, x_2+d\,x_2) - u_2(x_1, x_2)$ and give an interpretation of this result.

13) Calculate the components of the tensor based on elongations per unit length. Give the expressions of this tensor denoted by e_{ij}.

14) In the case of an overall rotation of the plane with an angle δ without strain, calculate the components of the tensor.

15) Prove that the second rank tensor T_{ij} can be written in the form of a symmetric tensor S_{ij} and of an antisymmetric tensor A_{ij}.

16) Give the expressions of S_{ij} and A_{ij} in the case of tensor e_{ij}.

17) Provide an interpretation of the terms S_{ij} and A_{ij}.

4.4.1.1.*Solutions*

1) Recall the principle of the model used for the calculation of the strain tensor using Figure 4.11 as a reference.

Solutions:

The hypothesis of the continuous medium is generally adopted in order to describe the strain of a material. At a point of the material localized by a

vector **r**, which in 2D has the components x_1 and x_2 and located at point A in the figure, consider small displacements represented by a translation vector **u(r)**, which is represented by vector A_0A in the figure.

During the deformation of the material, the variations of $u_1(x_1, x_2)$ along x_1 and x_2 are evaluated, as the quantities $du_1(x_1, x_2) = (\partial u_1/\partial x_1)dx_1 + (\partial u_1/\partial x_2)dx_2$ and $du_2(x_1, x_2) = (\partial u_2/\partial x_1)dx_1 + (\partial u_2/\partial x_2)dx_2$, which corresponds to four possible components for the mathematical representation of this deformation.

Taking Figure 4.6 of section 4.3.1 as a reference, the square denoted by ABCD is represented in Figure 4.11 as the unit area element in 2D considered at the point of coordinates x,y of edges dx and dy.

2) Provide an interpretation of the square in Figure 4.11 in the model used for the calculation of the strain tensor using Figure 4.6 as a reference.

Solutions:

A unit area element around point A_0 is considered in order to evaluate the strains of the material by determining how the vector **u(r)** is modified during a deformation. This is calculated from the derivatives of this vector with respect to coordinates by considering its modification at neighboring points B_0 and C_0 separated from A_0 by the distances dx_1 and dx_2, respectively. The square corresponds to the infinitesimal area in the surroundings of A_0 with its origin at A_0.

3) Provide an interpretation of the rhomboid in Figure 4.11 in the model used for the calculation of the strain tensor using Figure 4.6 as a reference.

Solutions:

When the material is deformed, at the point localized by **r** of components x_1, x_2 at A_0, and in its surroundings (B_0, C_0 and D_0), each component of **u(r)**, $u_1(x_1, x_2)$ and $u_2(x_1, x_2)$ is *a priori* subjected to different modifications depending on the position $x'_1 = x_1 + \alpha dx_1$ and $x'_2 = x_2 + \beta dx_2$ ($\alpha = 0$ or 1, $\beta = 0$ or 1). This different variation depending on the initial position A_0, B_0, C_0 or D_0, during the deformation, is represented on the figure by the vectors B_0B, C_0C and D_0D. The rhomboid is a schematic representation of the unit area deformed around point A separated from A_0 by $u(r)$.

4) What does the vector A_0A represent?

Solutions:

The vector A_0A in the figure is a translation vector $u(r)$ with its origin at A_0. Its components are $u_1(x_1, x_2)$ and $u_2(x_1, x_2)$.

5) What does the vector B_0B represent?

Solutions:

The vector B_0B in the figure is a translation vector $u(r+dr)$ with its origin at B_0. Its components are $u_1(x_1+d\,x_1, x_2)$ and $u_2(x_1+d\,x_1, x_2)$.

6) What does the vector C_0C represent?

Solutions:

The vector C_0C in the figure is a translation vector $u(r+dr)$ with its origin at C_0. Its components are $u_1(x_1, x_2+d\,x_2)$ and $u_2(x_1, x_2+d\,x_2)$.

7) Using the data in the figure, calculate tan α.

Solutions:

According to the figure, tan $\alpha = du_2(x_1, x_2)/(d\,x_1+du_1(x_1, x_2))$. Since $du_1(x_1, x_2)$ is small compared to $d\,x_1$ and for small angles, then $\alpha = du_2(x_1, x_2)/dx_1 = \partial u_2/\partial x_1$.

8) Using the data in the figure, calculate tan β.

Solutions:

According to the figure, tan $\beta = du_1(x_1, x_2) / (d\,x_2+du_2(x_1, x_2))$. Since $du_2(x_1, x_2)$ is small compared to $d\,x_2$ and for small angles, then $\beta = du_1(x_1, x_2)/dx_2 = \partial u_1/\partial x_2$.

9) $u_1(x_1+d\,x_1, x_2) - u_1(x_1, x_2)$ and give an interpretation of this result.

Solutions:

$$u_1(x_1+d\,x_1, x_2) - u_1(x_1, x_2) = du_1(x_1, x_2) = (\partial u_1/\partial x_1)dx_1.$$

This difference represents the elongation of the component u_1 of vector u parallel to OX_1. The elongation per unit length is the strain along OX_1.

10) $u_2(x_1+d\,x_1, x_2) - u_2(x_1, x_2)$ and give an interpretation of this result.

Solutions:

$u_2(x_1+d\,x_1, x_2)- u_2(x_1, x_2)= du_2(x_1, x_2)= (\partial u_2/\partial x_1)dx_1.$

This difference represents the elongation of the component u_2 of the vector u by rotation through an angle α. The elongation per unit length is the angle of rotation on the side dx_1.

11) $u_1(x_1, x_2+d\,x_2) - u_1(x_1, x_2)$ and give an interpretation of this result.

Solutions:

$u_1(x_1, x_2+d\,x_2)- u_1(x_1, x_2)= du_1(x_1, x_2)= (\partial u_1/\partial x_2)dx_2.$

This difference represents the elongation of the component u_1 of vector u by rotation with an angle β. The elongation per unit length is the angle of rotation on the side dx_2.

12) $u_2(x_1, x_2+d\,x_2) - u_2(x_1, x_2)$ and give an interpretation of this result.

Solutions:

$u_2(x_1, x_2+d\,x_2)- u_2(x_1, x_2)= du_2(x_1, x_2)= (\partial u_2/\partial x_2)dx_2.$

This difference represents the elongation of the component u_2 of vector u parallel to OX_2. The elongation per unit length is the strain along OX_2.

13) Calculate the components of the tensor based on elongations per unit length. Give the expressions of this tensor denoted by e_{ij}.

Solutions:

Given the calculations from 9 to 12, the following relations can be written:

$(\partial u_1/\partial x_1) = e_{11}$ and $(\partial u_2/\partial x_2)=e_{22}.$

Similarly: $(\partial u_1/\partial x_2) = e_{12}$ and $(\partial u_2/\partial x_1) = e_{21}$.

14) In the case of an overall rotation of the plane with an angle δ without strain, calculate the components of the tensor.

Solutions:

$e_{11}=e_{22}=0$ and $e_{12} = - e_{21} = \delta$ (as $\alpha >0$, but $\beta<0$).

15) Prove that the second rank tensor T_{ij} can be written in the form of a symmetric tensor S_{ij} and of an antisymmetric tensor A_{ij}.

Solutions:

Given $S_{ij}= (1/2(T_{ij}+T_{ji})$; it can be verified that $S_{ji} = S_{ij}$.

Given $A_{ij}= (1/2(T_{ij}-T_{ji})$; it can be verified that $A_{ji} = -A_{ij}$.

where $T_{ij} = S_{ij} +A_{ij}$.

16) Give the expressions of S_{ij} and A_{ij} in the case of tensor e_{ij}.

Solutions:

$S_{ij}= (1/2(e_{ij}+e_{ji})$ and $A_{ij}= (1/2(e_{ij}-e_{ji})$.

17) Provide an interpretation of the terms of S_{ij} and A_{ij}.

Solutions:

For an overall rotation: $S_{11}=S_{22}=0$ and $S_{12}= S_{21}=0$.

$A_{11}=A_{22}=0$ and $A_{12}= e_{12}$ and $A_{21}= -e_{21}$ (axial vector equivalent to a second rank antisymmetric tensor: a rotation axis and an angle about this axis).

In this case, the symmetric tensor S_{ij} represents the strain:

$S_{12}= (1/2(e_{12}+e_{21})$, $S_{21}= (1/2(e_{21}+e_{12})$ and $S_{11}= e_{11}$ and $S_{22}= e_{22}$.

As shown in questions (9) and (12), S_{11} and S_{22} represent an elongation strain or an expansion.

In this case, S_{12} and S_{21} represent a shear or a sliding strain (except for block rotation).

4.4.2. *A piezoelectric accelerometer*

In a piezoelectric accelerometer, a case (see Figure 4.12) is supported by a piezoelectric material that supplies an electric charge Q proportional to the corrective force, hence to the mass displacement. An accelerometer in longitudinal compression mode is considered, as illustrated in Figure 4.12. When the case is subjected to an acceleration a, a force of inertia F compresses the piezoelectric material in parallel to the direction of the acceleration which is labeled as axis 3. The dimensions at rest of a piezoelectric layer are t for the thickness, L for the length and W for the width. Its capacitance is denoted by C, and the corresponding dielectric permittivity is ε_{33}. The objective is to determine the static characteristics of the accelerometer.
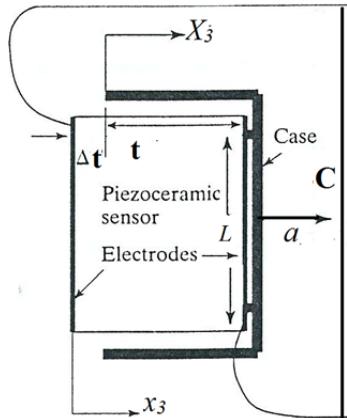

Figure 4.12. *Diagram of a piezoelectric accelerometer*

4.4.2.1.*Questions*

1) Find the vector expression of the strain tensor S of the layer.

2) The polarization of the layer being along the axis of deformation, find the vector expression of the polarization vector D of the layer.

3) Find the vector expression of the stress tensor T of the layer.

4) Relying on equations [4.63] and [4.64] expressed as $S = sT + dE$ and $D = dT + \varepsilon E$, deduce the equations verified by the piezoelectric layer for its mechanical strain and its electric polarization. (It is worth noting that there is no external electric stress, which means that $E = 0$.).

5) Based on the equations of question 4) show that: $\dfrac{Q}{\Delta t / t} = \dfrac{d_{33} L W}{s^E_{33}}$

6) Given: $t = 0.5$ mm; $L = 38.1$ mm; $W = 12.7$ mm; $\Delta t / t = 10^{-6}$; $d_{33} = 298.8 \ 10^{-12}$ CN-1; $s^E_{33} = 12.5 \ 10$ -12 $m^2 N^{-1}$; $\varepsilon_{33} = 11.95 \ 10^{-9}$ Fm^{-1}. Calculate: the charge Q, the capacitance C of the layer and the voltage V across the layer.

4.4.2.2. *Solutions*

1) Find the vector expression of the strain tensor S of the layer.

$$S = \begin{pmatrix} S_1 = & 0 \\ S_2 = & 0 \\ S_3 = & \Delta t / t \\ S_4 = & \\ S_5 = & \\ S_6 = & \end{pmatrix}$$

2) The polarization of the layer being along the axis of deformation, find the vector expression of the polarization vector D of the layer.

$$D = \begin{pmatrix} D_1 = & 0 \\ D_2 = & 0 \\ D_3 = & Q / LW \end{pmatrix}$$

3) Find the vector expression of the stress tensor T of the layer.

$$T = \begin{pmatrix} T_1 = & 0 \\ T_2 = & 0 \\ T_3 = & F/LW \\ T_4 = & 0 \\ T_5 = & 0 \\ T_6 = & 0 \end{pmatrix}$$

4) Relying on equations [4.63] and [4.64] expressed as $S = sT + dE$ And $D = dT + \varepsilon E$, deduce the equations verified by the piezoelectric layer for its mechanical strain and its electric polarization. (It is worth noting there is no external electric stress, which means that $E = 0$.)

1) $S_3 = s_{33}^E T_3 \Rightarrow \dfrac{\Delta t}{t} = s_{33}^E \dfrac{F}{LW}$ and 2) $D_3 = d_{33} T_3 \Rightarrow \dfrac{Q}{LW} = d_{33} \dfrac{F}{LW}$

5) Based on the equations of question 4 show that: $\dfrac{Q}{\Delta t/t} = \dfrac{d_{33} LW}{s_{33}^E}$

Equations 1 and 2 from the previous solution lead to:

$$\dfrac{\dfrac{Q}{LW}}{\dfrac{\Delta t}{t}} = \dfrac{d_{33} \dfrac{F}{LW}}{s_{33}^E \dfrac{F}{LW}} \Rightarrow \dfrac{Q}{\dfrac{\Delta t}{t}} = \dfrac{d_{33} LW}{s_{33}^E}$$

6) Given: $t = 0.5$ mm; $L = 38.1$ mm; $W = 12.7$ mm; $\Delta t/t = 10^{-6}$; $d_{33} = 298.8 \ 10^{-12} \ CN^{-1}$; $s_{33}^E = 12.5 \ 10^{-12} \ m^2 N^{-1}$; $\varepsilon_{33} = 11.95 \ 10^{-9} \ Fm^{-1}$. Calculate: the charge Q, the capacitance C of the layer and the voltage V across the layer.

$$Q = \dfrac{\Delta t}{t} \bigg/ \dfrac{d_{33} LW}{s_{33}^E} \qquad \text{AN : Q} = 1{,}1157 \ 10^{-8} \ C$$

$$C = \dfrac{\varepsilon \times \text{surface}}{\text{distance}} \Rightarrow C = \dfrac{\varepsilon_{33} \times LW}{t} \qquad \text{A.N. C} = 1{,}156 \ 10^{-8} \ F$$

$Q = CV \Rightarrow V = Q/C = 1$ V.

4.4.3. *Piezoelectric transducer*

Piezoelectric materials can be stacked in the form of layers either mechanically in series or electrically in parallel as shown in Figure 4.13.
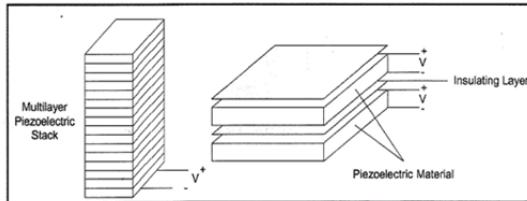

Figure 4.13. *Diagram of a piezoelectric transducer*

The stack on the right of this figure is considered for this exercise. Equations [4.63] and [4.64] are expressed as $S=sT+dE$ and $D=dT+\varepsilon E$, respectively. The objective is to determine the force, the displacement and the capacitance as a function of external excitations in the absence of stress and strain. The following conventions are adopted for the usual axes Ox, Oy and Oz by denoting them as 1, 2 and 3, respectively, as shown in Figure 4.14 and the stresses and strains are all considered zero, except for S_3 and T_3, respectively. Similarly, the electric field and the electric displacement are parallel to axis 3, which means that $E_1 = E_2 = 0$ and $D_1 = D_2 = 0$.
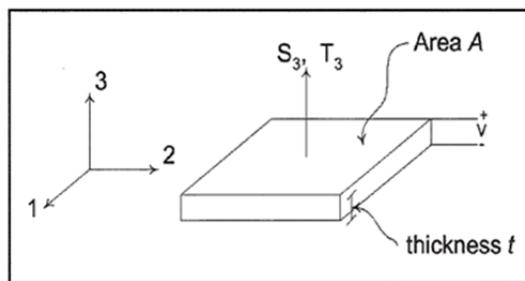

Figure 4.14. *Piezoelectric layer*

4.4.3.1. *Questions*

1) Express S_3 and D_3 as a function of T_3 and E_3. $x_i = \int_0^t S_3 dx_3$

2) The elongation of the ith layer is given by:

Deduce x_i as a function of s_{33}, d_{33}, T_3 and E_3.

3) Express E_3 and T_3 as a function of the given data.

4) Deduce from question 3, the final expression of x_i.

5) Assume that the transducer is composed of n layers and that the total length of the transducer is L. What is its total elongation X in this case?

6) The amount of charges due to the ith layer is given by:

$$q_i = \iint_{surface} D_3 dx_1 dx_2$$

Deduce from this expression, q_i as a function of d_{33}, ε_{33}, T_3 and E_3. Then express the total charge Q on the faces of the transducer as a function of d_{33}, L, t, F, n, A, V and ε_{33}.

7) Express the relation between X and Q and F and V in matrix form. Give an interpretation of the various elements of this 2 x 2 matrix.

8) Consider the situation of a zero force or of a zero strain on the transducer as shown in Figure 4.15(a) and (b).

Find in each case the expression of the force and of the displacement of the transducer and of the charge.

9) Deduce from results of question 8) that the capacitance C can be expressed in the following form: $(1 - k_{33}^2) (n\varepsilon_{33}A) /t$. Find in each case the expression of k_{33}^2.

10) Calculate the stress for an electric field of 0.5 MV/m and a strain of 10 MPa applied to the piezoelectric material. The inverse of Young's modulus is, $1/Y = s = 20 \ 10^{-12}$ m²/N and the piezoelectric stress coefficient is $d_{33} = 650 \ 10^{-12}$ m/V.

11) Find the geometry of a stack of piezoelectric layers subjected to a blocking force of 1000 N and a free displacement of 30 μm. The inverse of Young's modulus is, $1/Y = s = 20 \ 10^{-12}$ m²/N, the piezoelectric stress

coefficient is $d_{33} = 650 \ 10^{-12}$ m/V, the layer thickness is 254 μm, and the maximum allowed electric field is 0.3 MV/m.
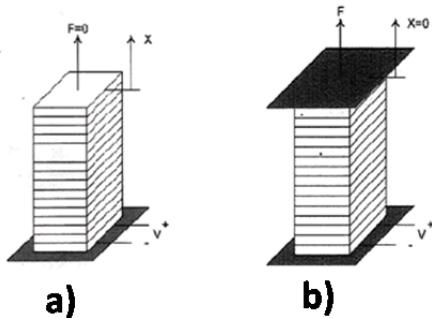
**Figure 4.15.** *a) Zero forces and b) zero strain*

4.4.3.2. *Solutions*

1) Express S_3 and D_3 as a function of T_3 and E_3.

From

$$\underline{S} = s\underline{T} + d\underline{E}$$
$$\underline{D} = d\underline{T} + \varepsilon\underline{E}$$

It can be deduced that:

$$S_3 = S_{33}^E T_3 + d_{33}E \text{ and } D_3 + d_{33}T_3 + \varepsilon_{33}^T E$$

2) The elongation of the ith layer is given by: $x_i = \int_0^t S_3 dx_3$. Deduce x_i as a function of s_{33}, d_{33}, T_3 and E_3.

For the ith layer: $x_i = \int_0^t (s_{33}^E T_3 + d_{33}E_3)dx_3 = (s_{33}^E T_3 + d_{33}E_3)t$

3) Express E_3 and T_3 as a function of the given data:

$$E_3 = \frac{V}{t} \text{ and } T_3 = \frac{F}{A}$$

If there are no stress and strain, then:

4) Deduce the final expression of x_i

Result:

$$x_i = s_{33}^E \frac{F}{A} t + d_{33} \frac{V}{t} t = s_{33}^E \frac{F}{A} t + d_{33}V$$

5) Assume that the transducer is composed of n layers and that the total length of the transducer is L. What is its total elongation X in this case?

Result:

$$X = \sum_i x_i = nx_i = \frac{L}{t}(s_{33}^E \frac{F}{A} t + d_{33}V) = s_{33}^E \frac{FL}{A} + d_{33}V \frac{L}{t}$$

6) The amount of charges due to the ith layer is given by:

$$q_i = \iint_{surface} D_3 dx_1 dx_2$$

Deduce from this expression, q_i as a function of d_{33}, ε_{33}, T_3 and E_3. Then express the total charge Q on the faces of the transducer as a function of d_{33}, L, t, F, n, A, V and ε_{33}.

Result:

$$q_i = \iint_S (d_{33}T_3 + \varepsilon_{33}^T E_3) dx_1 dx_2 = d_{33}T_3 A + \varepsilon_{33}^T E_3 A$$

Hence:

$$Q = \sum_i q_i = nq_i = \frac{L}{t} d_{33} \frac{F}{A} A + \frac{L}{t} \varepsilon_{33}^T \frac{V}{t} A = \frac{L}{t} d_{33}F + \varepsilon_{33}^T n \frac{V}{t} A$$

7) Express the relation between X and Q and F and V in matrix form. Give an interpretation of the various elements of this 2 x 2 matrix.

$$X = \frac{S_{33}^E L}{A} F + \frac{d_{33}L}{t} V \text{ and } Q = \frac{d_{33}L}{t} F + \frac{\varepsilon_{33}^T nA}{t} V$$

It can be written that:

Hence, the matrix form: $\begin{pmatrix} X \\ Q \end{pmatrix} = \begin{pmatrix} \dfrac{s_{33}^{E}L}{A} & \dfrac{d_{33}L}{t} \\ \dfrac{d_{33}L}{t} & \dfrac{\varepsilon_{33}^{T}nA}{t} \end{pmatrix} \begin{pmatrix} F \\ V \end{pmatrix}$

The various terms can be interpreted as follows:

The first term (M_{11}) is the displacement under the effect of the mechanical force in the absence of electric stress (or electric field): it is the compliance or the mechanical elasticity coefficient or the inverse Young's modulus ($X=CF$ or $F=YX$)) (m/N).

The second term (M_{12}) is the displacement under the effect of the electric stress (or electric field): it is the piezoelectric coupling coefficient (m/V).

The third term (M_{21}) corresponds to the appearance of electric charges under the effect of a mechanical force in the absence of electric stress (or electric field): it is the piezoelectric coupling coefficient (m/V).

The fourth term (M_{22}) is the displacement under the effect of electric constraint (or electric field): it is the capacitance ($Q=CV$; C in Farads).

8) Consider the situation of a zero force or of a zero strain on the transducer as shown in Figure 4.15(a) and (b).

Find in each case the expression of the force and of the displacement of the transducer and of the charge.

a) Zero force: $F= X = \dfrac{d_{33}L}{t}V = nd_{33}V$ and $\qquad Q = \dfrac{\varepsilon_{33}^{T}nA}{t}V = C^{T}V$

b) Zero strain, X=0: $0 = \dfrac{s_{33}^{E}L}{A}F + \dfrac{d_{33}L}{t}V \Rightarrow F = -\dfrac{d_{33}A}{s_{33}^{E}t}V$

and $Q = \dfrac{d_{33}L}{t}F + \dfrac{\varepsilon_{33}^{T}nA}{t}V = -\dfrac{d_{33}^{2}AL}{s_{33}^{E}t^{2}}V + \dfrac{\varepsilon_{33}^{T}nA}{t}V = (-\dfrac{d_{33}^{2}}{s_{33}^{E}} + \varepsilon_{33}^{T})\dfrac{nA}{t}V$

9) Deduce from the results of question 8 that the capacitance C can be expressed in the following form: $(1 - k_{33}{}^{2})\,(n\varepsilon_{33}A)\,/t$. Find in each case the expression of $k_{33}{}^{2}$.

a) $k_{33}^2 = 0$

b) $Q = CV \Rightarrow C = (-\frac{d_{33}^2}{s_{33}^E} + \varepsilon_{33}^T)\frac{nA}{t} = (1 - \frac{d_{33}^2}{\varepsilon_{33}^T s_{33}^E})\frac{nA\varepsilon_{33}^T}{t} = (1 - k_{33}^2)C^T$

10) Calculate the stress for an electric field of 0.5 *MV/m* and a strain of 10 *MPa* applied to the piezoelectric material. The inverse of Young's modulus is, $1/Y = s = 20 \times 10^{-12}$ *m²/N*, and the piezoelectric stress coefficient is $d_{33} = 650 \times 10^{-12}$ *m/V*.

$$\frac{X}{L} = s_{33}^E \frac{F}{A} + d_{33} \frac{V}{t} \Rightarrow S = s_{33}^E T + d_{33} E$$

Hence: $S = (20 \ 10^{-12} \times 10 \ 10^6) + (650 \ 10^{-12} \times 0.5 \ 10^6) = 525 \times 10^{-6}$.

11) Find the geometry of a stack of piezoelectric layers subjected to a blocking force of 1000 *N* and a free displacement of 30 μm. The inverse of Young's modulus is, $1/Y = s = 20 \times 10^{-12}$ *m²/N*, the piezoelectric stress coefficient is $d_{33} = 650 \times 10^{-12}$ *m/V*, the layer thickness is 254 μm, and the maximum allowed electric field is 0.3 *MV/m*.

The free displacement is: $\frac{X}{L} = d_{33}\frac{V}{t} \Rightarrow L = \frac{X}{d_{33}E}$

Hence: $L = (30 \times 10^{-6} / (650 \times 10^{-12} \times 0.3 \times 10^6) = 0.154$ m.

The blocking force is: $F = -\frac{d_{33}A}{s_{33}^E t}V = -\frac{d_{33}A}{s_{33}^E}E \Rightarrow A = \left|\frac{Fs_{33}^E}{d_{33}E}\right|$

Hence: $A = (1000 \times 20 \times 10^{-12)} / (650 \ 10^{-12} \times 0.3 \times 10^6) = 1.03 \times 10^{-4}$ m².

The calculated number of layers is: $n = L/t = (0.154/254 \times 10^{-6}) = 607$.

4.4.4. *Piezoelectric sensor*

Piezoelectric materials can be stacked as double layers (see Figure 4.16(a)) that constitute a force sensor as in a scanning probe microscope used for studying the topography of nanomaterial surfaces.

The objective is to determine the force, the displacement and the capacitance as a function of external excitations in the absence of stress and strain. Equations [4.63] and [4.64] are expressed as $S=sT+dE$ and $D=dT+\varepsilon E$, respectively. By convention, the Ox, Oy and Oz axes are denoted by 1, 2 and

3, respectively, as shown in Figure 4.16(b), and all the stresses and strains are considered to be zero, except for S_1 and T_1, respectively. Similarly, the electric field and the electric displacement are parallel to axis 3, meaning that $E_1 = E_2 = 0$ and $D_1 = D_2 = 0$.


a)   b)
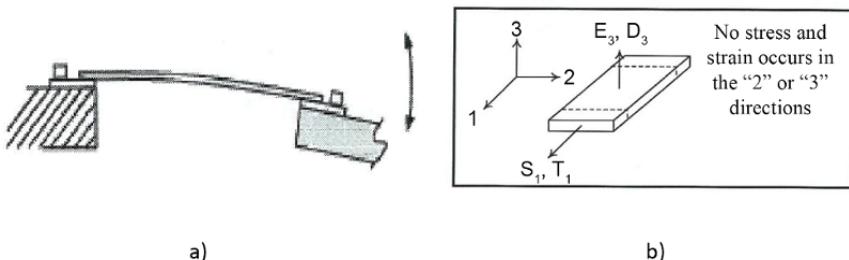
Figure 4.16. *a) Piezoelectric sensor and b) piezoelectric layer*

4.4.4.1. *Questions*

1) Based on the following general constitutive equations of a piezoelectric material: $\begin{aligned} \underline{S} &= s\underline{T} + d\underline{E} \\ \underline{D} &= d\underline{T} + \varepsilon\underline{E} \end{aligned}$ express S_1 and D_3 as a function of T_1 and E_3.

Consider the stack shown in Figure 4.17 and the associated electric circuit such that the top and bottom layers are subjected to an electric field whose direction is, respectively, parallel and antiparallel to the direction of the layer polarization. It is assumed that $L \gg t$.
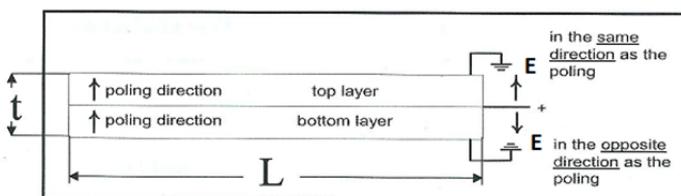
Figure 4.17. *Piezoelectric double layer*

2) Consider that the stress T is zero. Calculate S_1 for each layer. Compare the sign and comment the diagrams in Figure 4.18(a) and (b).
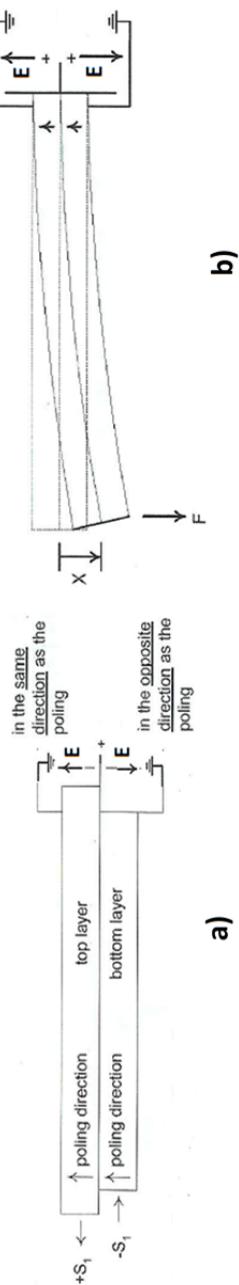
Figure 4.18. *a) Piezoelectric double layer and b) bending of layers*

The bending of the layer is parameterized by the X variable, which gives the vertical displacement of the contact plane between the two layers.

Given: $X = 4 \dfrac{s_{11}^E L^3}{wt^3} F + 3 \dfrac{d_{31} L^2}{t^2} V$

3) What is the free displacement X of the two layers ($F=0$)?

4) Calculate the force F required to cancel the displacement X ($X=0$).

5) What is the effect of the ratio L/t on X and F calculated in solutions 3 and 4?

4.4.4.2. *Solutions*

1) Based on the following general constitutive equations of a piezoelectric material: $\begin{aligned} \underline{S} &= s\underline{T} + d\underline{E} \\ \underline{D} &= d\underline{T} + \varepsilon\underline{E} \end{aligned}$ express S_1 and D_3 as a function of T_1 and E_3.

Based on the general equation and since only T_1, S_1, E_3 and D_3 are different from zero, the relations can be written as follows:

$$S_1 = s_{11}^E T_1 + d_{13} E_3$$
$$D_3 = d_{31} T_1 + \varepsilon_{33}^T E_3$$

2) Consider that the stress T is zero. Calculate S_1 for each layer. Compare the sign and comment the diagrams in Figure 4.18(a) and (b).

In the absence of stress T, equation 1 of the first question, for the top and bottom layers, leads, respectively, to:

$$S_1 = +d_{13} E_3$$
$$S_1 = -d_{13} E_3$$

Comparison: the signs are opposite, as the electric potential difference is in opposition.

Comments: an expansion of the top layer and a contraction of the bottom layer can be noted in Figure 4.18(a), which is in agreement with the results of the calculations. The bending of the double layer minimizes the stresses and prevents the fracture of the two layers.

3) What is the free displacement X of the two layers ($F=0$)?

The calculation leads to:

$$X = 3\frac{d_{31}L^2}{t^2}V$$

4) Calculate the force F required for cancelling out the displacement X ($X=0$).

The calculation yields:

$$F = -\frac{(3\frac{d_{31}L^2}{t^2}V)}{(4\frac{s_{11}^E L^3}{wt^3})} = -(\frac{d_{31}wt}{s_{11}^E L})V$$

5) What is the effect of the ratio L/t on X and F calculated in solutions 3 and 4?

X depends on L/t while F is inversely proportional to L/t. Increasing L/t leads to an increase of X and a decrease of the force, F.

4.5. Appendix: crystal symmetry

In condensed phase, only the rotation axes of orders 1, 2, 3, 4 and 6 enable a periodic pavement of the space compatible with crystalline lattices. The symmetry elements used by the crystallographers to define the symmetry about a point in space, for example, the central point of the unit cell, are: a point (center) of symmetry, rotation axes, mirror planes, and combinations thereof. Using these elements of symmetry, it is shown in crystallography that there are 32 symmetry point groups (Table 4.8) based on symmetry operations. The various symmetry elements are divided into

seven crystalline systems as follows: triclinic, monoclinic, orthorhombic, tetragonal, rhombohedral, hexagonal and cubic.

Crystalline system	Number of point groups	Herman-Mauguin	Schoenflies
Triclinic	2	$1, \bar{1}$	C_1, C_i
Monoclinic	3	$2, m, 2/m$	C_2, C_s, C_{2h}
Orthorhombic	3	$222, 2mm, mmm$	$D_2, C_{2v}, D_{2h},$
Rhombohedral	5	$3, 32, 3m$ $\bar{3}, \bar{3}m$	C_3, D_3, C_{3v} S_6, D_{3d}
Hexagonal	7	$6, 622, 6mm, 6/m,$ $\bar{6}, \bar{6}2m, 6/mmm$	C_6, D_6, C_{6v}, C_{6h} C_{3h}, D_{3h}, D_{6h}
Tetragonal (quadratic)	7	$4, \bar{4}, 422, 4mm,$ $\bar{4}2m, 4/m, 4/mmm$	$C_4, S_4, D_4, C_{4v},$ D_{2d}, C_{4h}, D_{4h}
Cubic	5	$23, m3, 432,$ $\bar{4}3m, m3m$	T, T_h, O T_d, O_h

Table 4.8. *The point groups of 32 classes of symmetry and the equivalence between Schoenflies and Hermann–Mauguin symbols*

There are special point groups that can be added to the above groups. Among them, the continuous point groups have an infinite number of symmetry elements, the axial and spherical groups of symmetry used for listing the groups characterized by the presence of several rotational symmetry axes of high order n associated with regular polyhedrons, Platonic solids, such as the triangular tetrahedron, the cube with six square faces, the octahedron with eight triangular faces, the dodecahedron with 12 regular pentagonal faces with three pentagonal faces joined at one point and the icosahedron with 20 equilateral triangular faces with five faces joined at one point.

Table 4.9 summarizes the two conventions used to represent a symmetry element in a crystal.

Symmetry operation	Herman-Mauguin	Schoenflies
Identity	1	C_1 or E
Rotation ($2\pi/n$)	n	C_n
Mirror	m	σ, C_s, S_1
Inversion	$\bar{1}$	i, C_i, S_2
Rotation-Reflection	-	S_{2n}
Rotation-Inverison	$\bar{n}$	-

Table 4.9. *Schoenflies and Hermann–Mauguin notation*

The convention used in condensed phase is that of Hermann–Mauguin, which is preferred in crystallography to that of Schoenflies which is used in the spectroscopy study of atoms and molecules: the cyclic groups are denoted by their order n (rotational axis of symmetry of order n), a plane of symmetry is denoted by m and the group is denoted by nm ($C_{2v} \equiv 2m$); if the plane is perpendicular to the axis of symmetry, n/m ($C_{2h} \equiv 2/m$) is used; the presence of a center of symmetry is indicated by a bar above the order of the group ($C_i = S_2 \equiv \bar{1}$, $S_4 \equiv \bar{4}$, $S_6 = C_i \otimes C_3 \equiv \bar{3}$). For example, $O_h \equiv m3m$ and $D_{3h} \equiv \bar{3}m2$, etc.

Appendix

Propagation of a Light Ray

In light diffraction problems, the amplitude of a wave $E_z(x,y)$ on a surface located in a plane z is determined by the amplitude $E_0(x,y)$ of a wave in the plane $z = 0$.

In Chapter 3 of [DAH 16], equation 3.6 gives the expression of an electromagnetic wave traveling along Oz, in the form $u(z-vt)=acos(k(z-ct))$, where v is the speed of propagation of the wave in a medium of index n, a is its amplitude and $k=2\pi n/\lambda$. In a vacuum, $n=1$ and in a three-dimensional space the monochromatic plane wave of the angular frequency $\omega=kc$ and the wave vector $k = (k_x, k_y, k_z)$ can be expressed in the form $E = E_0 \; exp \; i(2\pi/\lambda (\alpha x+\beta y+\gamma z) - \omega t)$, where the components of the wave vector have the form: $k_x=2\pi\alpha/\lambda$, $k_y=2\pi\beta/\lambda$ and $k_x=2\pi\gamma/\lambda$, with α, β, γ being the direction cosines of the wave vector **k** and λ being the wavelength. This expression can be obtained by solving the Helmholtz equation (equation 3.7, [DAH 16]):

$$\Delta \vec{E} + \frac{\omega^2}{c^2}\vec{E} = \vec{0} \qquad\qquad [A.1]$$

where $\Delta\vec{E} = \vec{\nabla}^2\vec{E}$, using Green's function $G_k(\vec{r},\vec{r}_0)$ of the Helmholtz equation that verifies that:

$$(\nabla^2 + k^2)G_k(\vec{r},\vec{r}_0) = \delta(\vec{r} - \vec{r}_0) \qquad\qquad [A.2]$$

given that $G_k(\vec{r},\vec{r}_0) = -\frac{exp(ik|\vec{r}-\vec{r}_0|)}{4\pi|\vec{r}-\vec{r}_0|}$, where the unit vector is $\vec{e}_r = \frac{(\vec{r}-\vec{r}_0)}{|\vec{r}-\vec{r}_0|}$. The solution is readily obtained, since for the k mode of the electric field generated by a source placed at $\vec{r}_0$, $\vec{E}_k(\vec{r},\vec{r}_0) = -\frac{exp(ik|\vec{r}-\vec{r}_0|)}{4\pi|\vec{r}-\vec{r}_0|}\vec{e}_r$.

The propagation of a wave through obstacles (slits or opaque objects) can be determined using the Green–Ostrogradsky theorem and the Green function $G_k(\vec{r}, \vec{r}_0)$ of the Helmholtz equation.

Based on the relation:

$$\vec{\nabla}(u\vec{\nabla}v - v\vec{\nabla}u) = u\vec{\nabla}^2 v - v\vec{\nabla}^2 u = u\Delta v - v\Delta u \qquad [A.3]$$

where u and v are solutions to the Helmholtz equation and the Green–Ostrogradsky theorem, which enables a volume integral to be transformed into a surface integral:

$$\int (u\Delta v - v\Delta u)d\tau = \int (u\vec{\nabla}^2 v - v\nabla^2 u)d\tau = \oint (u\vec{\nabla}v - v\vec{\nabla}u)d\vec{s} \quad [A.4]$$

It can be written that:

$$\int \left(\vec{E}(\vec{r})\Delta G_k(\vec{r}, \vec{r}_0) - G_k(\vec{r}, \vec{r}_0)\Delta\vec{E}(\vec{r}) \right) d\tau = \int \vec{E}(\vec{r})\delta(\vec{r} - \vec{r}_0)d\tau$$

$$- \int \vec{E}(\vec{r})k^2 G_k(\vec{r}, \vec{r}_0)d\tau + \int G_k(\vec{r}, \vec{r}_0)k^2\vec{E}(\vec{r})d\tau = \vec{E}(\vec{r}_0) \qquad [A.5]$$

hence

$$\vec{E}(\vec{r}_0) = \oint \left(\vec{E}(\vec{r})\vec{\nabla}G_k(\vec{r}, \vec{r}_0) - G_k(\vec{r}, \vec{r}_0)\vec{\nabla}\vec{E}(\vec{r}) \right) d\vec{s}$$

$$= \oint (\vec{E}(\vec{r})\{\vec{\nabla}G_k(\vec{r}, \vec{r}_0)d\vec{s} - ikG_k(\vec{r}, \vec{r}_0)ds\}) \qquad [A.6]$$

where the surface integral surrounds the point located at $\vec{r}_0$ and $\vec{r}$ indicates the position of the surface element $d\vec{s}$, (Figure A.1) so that $\vec{\nabla}\vec{E}(\vec{r}).d\vec{s} = ik\vec{E}(\vec{r})ds$, for a wave that propagates in the volume surrounded by an integration surface ds. For a wave that propagates outside the surface, the relation is $\vec{\nabla}\vec{E}(\vec{r}).d\vec{s} = -ik\vec{E}(\vec{r})ds$.

Given that $G_k(\vec{r}, \vec{r}_0) = -\dfrac{exp(ik|\vec{r}-\vec{r}_0|)}{4\pi|\vec{r}-\vec{r}_0|}$, then:

$$\vec{\nabla}G_k(\vec{r}, \vec{r}_0) = ikG(\vec{r}, \vec{r}_0)\vec{e}_r + \frac{exp(ik|\vec{r} - \vec{r}_0|)}{4\pi|\vec{r} - \vec{r}_0|^2}\vec{e}_r$$

$$= \left(\frac{ik}{|\vec{r}-\vec{r}_0|} - \frac{1}{|\vec{r}-\vec{r}_0|^2} \right)(\vec{r} - \vec{r}_0)G_k(\vec{r}, \vec{r}_0) \qquad [A.7]$$
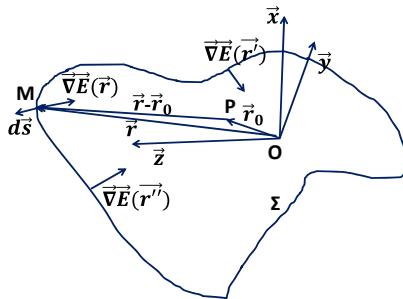
Figure A.1. *Kirchhoff's integral over a surface Σ passing through a point M surrounding point P. For a color version of this figure, see www.iste.co.uk/dahoo/metrology1.zip*

At a large distance compared to the wavelength such that $k|\vec{r} - \vec{r}_0| \gg 1$, the solution is:

$$\vec{E}(\vec{r}_0) = \oint \left(\vec{E}(\vec{r}) \left\{ \frac{ik}{|\vec{r}-\vec{r}_0|} (\vec{r} - \vec{r}_0) G_k(\vec{r}, \vec{r}_0) d\vec{s} - ik G_k(\vec{r}, \vec{r}_0) ds \right\} \right) \quad [A.8]$$

or:

$$\vec{E}(\vec{r}_0) = -\frac{ik}{4\pi} \oint \vec{E}(\vec{r}) \frac{exp(ik|\vec{r}-\vec{r}_0|)}{|\vec{r}-\vec{r}_0|^2} \left((\vec{r} - \vec{r}_0) d\vec{s} - |\vec{r} - \vec{r}_0| ds \right) \quad [A.9]$$

This solution corresponds to the formulation of the Huygens–Fresnel principle or the Huygens–Fresnel equation obtained using Kirchhoff's integral, which is the Fresnel–Kirchhoff diffraction formula.

When a monochromatic plane wave traveling in parallel to Oz meets a screen (E_0) with a slit (Figure A.2), a diffraction pattern appears in the observation plane. For the sake of simplicity, assume that the slit plane is perpendicular to the direction of propagation of the plane wave and that the amplitude and the gradient of the electric field of the wave are constant on the surface of the slit in the vicinity of M (Figure A.2) and zero everywhere else. The amplitude on the other side of the slit at a point P located at $\vec{r}_0$ is given by the Fresnel–Kirchhoff diffraction formula. If the slit is in the Oxy plane (Figure A.2), and the dimensions are small compared to the position of point P at $\vec{r}_0$, the amplitude of the field at P is given by:

$$\vec{E}(\vec{r}_0) = -\frac{ik\vec{E}(\vec{r})}{4\pi R} (cos\theta + 1) \oint exp(ik|\vec{r} - \vec{r}_0|) dx dy \quad [A.10]$$

where R is the distance between the central point of the slit and point P (in a first approximation, it is the same distance for all the points of the slit), and θ is the angle between the Oz axis and the vector $\vec{MP} = \vec{r}_0 - \vec{r}$. The origin of coordinates is considered in the plane containing the diffraction slit.
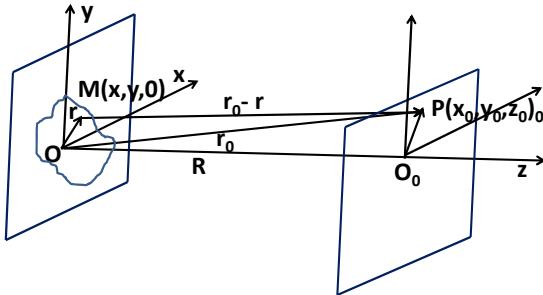
Figure A.2. *Diagram of diffraction in a plane containing P through a slit containing M. For a color version of this figure, see www.iste.co.uk/dahoo/metrology1.zip*

The module of the vector $\vec{r}_0 - \vec{r}$ is given by:

$$|\vec{r}_0 - \vec{r}| = \sqrt{r_0^2 - 2\vec{r}_0.\vec{r} + r^2}.$$

Considering the conditions of Fraunhofer diffraction in the far field, with an origin O in the slit plane (Figure A.2), where R is the distance between the observation plane and the diffraction plane, the following relation can be obtained:

$$exp(ik|\vec{r} - \vec{r}_0|) = exp\left(ik\sqrt{r_0^2 - 2\vec{r}_0.\vec{r} + r^2}\right) =$$
$$e^{ikR}exp\left(-ik(\alpha x + \beta y)\right) \tag{A.11}$$

where: $\alpha = \frac{x_0}{R}$ and $\beta = \frac{y_0}{R}$.

According to the Gaussian approximation of paraxial rays, the field at P is:

$$\vec{E}(\vec{r}_0) = A \oiint exp\left(-ik(\alpha x + \beta y)\right)dxdy \tag{A.12}$$

Fraunhofer diffraction at large distance can be calculated using this formula. For a circular hole, the formula is:

$$\vec{E}(\vec{r}_0) = A \oiint exp\left(-ik(\alpha\rho cos\varphi)\right)\rho d\rho d\varphi$$

$$= 2\pi A \int_0^a J_0(k\alpha\rho)\rho d\rho = 2\pi A a^2 \frac{J_1(k\alpha\rho)}{k\alpha\rho} \qquad \text{[A.13]}$$

where $J_0(x)$ and $J_1(x)$ are zero and the first-order Bessel functions.
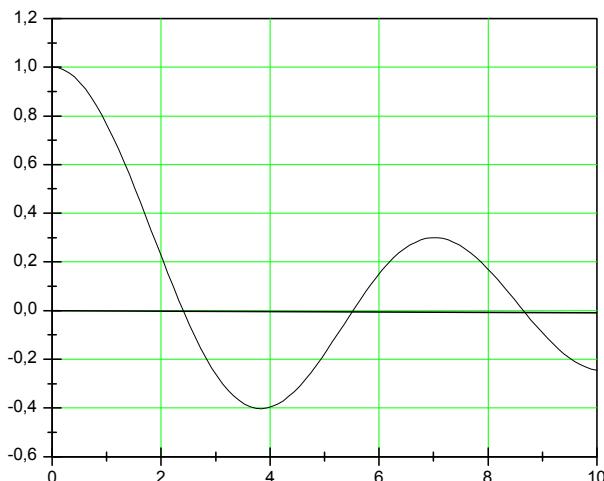
Figure A.3. *Distribution of the diffraction amplitude through a circular hole. For a color version of this figure, see www.iste.co.uk/dahoo/metrology1.zip*

Fresnel diffraction is obtained for an observation plane that is closer to the slit. In this case, second-order terms in x should be considered. Moreover, it is wise to use a coordinate system so as to get rid of the terms of order 1 in x and y. Assume that the slit is in the Oxy plane and the Oz axis is a line perpendicular to the plane that passes through the point of observation r_0. In this system of coordinates, $\alpha = \beta = \vec{r}.\vec{r}_0 = 0$. Under these conditions, the following expression is obtained:

$$exp(ik|\vec{r} - \vec{r}_0|) = exp\left(ik\sqrt{r_0^2 - 2\vec{r}_0.\vec{r} + r^2}\right) = e^{ikR} exp\left(\frac{ik(x^2+y^2)}{2R}\right) \text{[A.14]}$$

In the case of the Gaussian approximation of paraxial rays, the field at P is given by:

$$\vec{E}(\vec{r}_0) = A \int exp \left(\frac{ikx^2}{2R} \right) dx \int exp \left(\frac{iky^2}{2R} \right) dy \qquad \text{[A.15]}$$

which can be used to calculate the Fresnel diffraction pattern.

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