Master d'Informatique Fondamentale – 2021
Semantics and Verification (M1)

ENS DE LYON

TD – Semantics and Verification

VIII– More Büchi Automata

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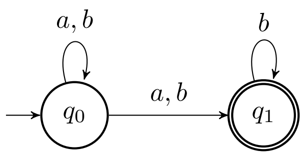
- We denote the words characterised by an LTL formula φ by $[\![\varphi]\!] = \{\sigma \mid \sigma \vDash \varphi\}$.
- For a set of finite words W, we denote by $\overrightarrow{W}$ the set of words having an infinite number of prefixes in W: $\overrightarrow{W} = \{\sigma \in \Sigma^\omega \mid \exists^\infty \hat{\sigma} \in W, \hat{\sigma} \subseteq \sigma\}$
- A regular safety property P is a safety property such that P_{bad} is regular (i.e., recognizable by a DFA)

Let A be a set. Recall that

- For $u \in A^*$, $\mathrm{ext}(u) = \{\sigma \in A^\omega \mid u \subseteq \sigma\}$
- We can equip A^ω with the topology $\mathcal{U} = \{\mathrm{ext}(U) \mid U \subseteq A^*\}$

Decomposition of ω-regular Linear Time Properties

Exercise 1.
Let $L = (a+b)b^\omega$ and the following NBA $\mathcal{A}$ recognizing L:



Show that there is a word $\sigma \notin \mathcal{L}_\omega(\mathcal{A})$ such that σ has an infinite number of prefixes recognized by $\mathcal{A}$ as a DFA (i.e., $\exists^\infty \hat{\sigma} \subseteq_{\text{finite}} \sigma, \hat{\sigma} \in \mathcal{L}(\mathcal{A})$).

Exercise 2.
We want to show the decomposition theorem for ω-regular properties.
Let $U \subseteq \Sigma^*$.

1. Show that if $A \subseteq \Sigma^\omega$ is ω-regular, then $\mathrm{pref}(A) \subseteq \Sigma^*$ is regular. (You may use that $\mathrm{pref}(U)$ is regular if $U \subseteq \Sigma^*$ is regular.)
2. Show that $\mathrm{cl}(U)$ is a safety property induced by $P_{bad} = U^c$.
3. Deduce that if $P \subseteq \mathcal{P}(\mathrm{AP})^\omega$ is an ω-regular safety property, then P is a *regular* safety property.
4. Let P_{bad} be a set of bad prefix of $\mathrm{cl}(U)$ and $\mathcal{A}$ be a complete deterministic automaton recognizing P_{bad} such that for all final state q_F of $\mathcal{A}$ and all $a \in \Sigma, \delta(q_F, a) = q_F$. Show that $\mathcal{L}_\omega(\mathcal{A}) = (2^{\mathrm{AP}})^\omega \backslash \mathrm{cl}(U)$.
5. Show that if U is regular then $\mathrm{cl}(U)$ is ω-regular *(we recall that if $\mathrm{cl}(U)$ is a regular safety property, we can always find P_{bad} and $\mathcal{A}$ satisfying the properties of the previous question. Try to adapt $\mathcal{A}$ to recognize $\mathrm{cl}(U)$).*
6. Show that for every ω-regular linear time property P, there is a ω-regular safety property P_{safe} and a ω-regular liveness property P_{live} such that $P = P_{safe} \cap P_{live}$.

From LTL to NBAs

Exercise 3.

Let φ be a the LTL formula over $\mathrm{AP} = \{a, b, c\}$ given by $\varphi = \Box a \wedge (b\,\mathcal{U}\,\neg c)$. Construct an (G)NBA $\mathcal{A}$, such that, $\mathcal{L}_\omega(\mathcal{A}) = [\![\varphi]\!]$.

Characterization of DBAs

Exercise 4.

1. Let $\mathrm{AP} = \{a, b\}$. Show that $\{\sigma \mid \exists^\infty t, a \in \sigma(t)\} \subseteq (2^{\mathrm{AP}})^\omega$ is not closed.
2. Show that there is a DBA $\mathcal{A}$ such that $\mathcal{L}_\omega(\mathcal{A})$ is not a safety property.
3. Show that for every regular safety property P, there is a DBA $\mathcal{A}$ such that $P = \mathcal{L}_\omega(\mathcal{A})$.