

# A Novel Triad Twisted String Actuator for Controlling a Two Degrees of Freedom Joint: Design and Experimental Validation

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**Abstract**—Actuated universal joints, or equivalent joint systems, are found in a number of robotic applications, in particular mobile snake robots, snake-arm robots and robotic tails. These joints have two degrees of freedom on two axes, each perpendicular to a third axis and to themselves. Such joints use a variety of actuation methods, including direct drive motors, linear screw drives, cable based systems, and hydraulics/pneumatics. In this paper the authors design and validate a mechanism that uses the Twisted String Actuator (TSA) in an antagonistic triad to actuate the universal joint, using orientation sensors and load cells to create a robust cascading closed loop control system. This results in a light, compact, high-performance actuation system that avoids the extra mass and hardware complexity that alternative actuation methods present, with the additional challenge of nonlinearity.

**Index Terms**—???

## I. INTRODUCTION

Actuated Universal Joint (AUJ) mechanisms are found in a wide range of robotic applications, such as confined space inspection using snake-arm robots **buckingham2010**, highly manoeuvrable mobile snake robots **luo2018**, and biomimetic robot tails for stability **rone2018**. Mobile snake robots must usually incorporate electric actuators inline with their joints. This results in an AUJ having to shift the mass of the follower segments and all the actuators inside the follower segments, which results in high torque requirements. Snake-arm robots and robotic tails can reduce the mass and size of the AUJ by moving their actuators away from the AUJs and use cables to transfer the force to the joints, or use hydraulic or pneumatic actuators which tend to be lighter than equivalent electric motors. This comes at the expense of increased mass and bulk at the base of the arm or tail.

First developed by **wurtz2010** **wurtz2010** in **wurtz2010**, the Twisted String Actuator (TSA) uses two or more strings between two fixtures as a linear actuator. When one fixture is rotated (typically by an electric motor), the looped string twists into a helix, decreasing the distance between them. TSA actuators have been used for a hand orthosis **muehlbauer2021twisted**, elbow joint **park2020control** and foldable robot arm **suthar2021design** among other functions.

The primary advantage of TSA over similar linear actuators such as a leadscrew is the reduction the TSA provides is not proportional (or even slightly inversely proportional) to the mass of the actuator. By decreasing the string cross-section radius (plus half the gap between any two strings that intersects the axis of rotation when there are more than two strings) the reduction increases for a constant  $l_u$  and  $\theta_s$ , resulting in a greater reduction with no increase, or even a slight decrease, in actuator mass. While the reduction in a leadscrew can be increased by decreasing the lead on the thread, which also has no increase in mass, this has a limited range and can quickly run up against manufacturing tolerances or material strength requirements. In order to achieve greater or more robust reductions, the screw radius has to be increased, or the driving motor has to have a larger reduction before driving the screw, both of which usually result in more material (typically steel) and therefore more mass.

However, TSA does have some disadvantages, the most significant of which is a non-linear reduction equation, which is also dependent on the motor angle  $\theta_s$  (and therefore actuator position). The reduction decreases in a non-linear fashion as the angle increases, with the derivative decreasing as the angle increases. There is also the compliance of the strings to consider, depending on the thickness and material chosen, which becomes a significant factor under high forces. Both of these issues can be addressed with accurate modelling **nedelchev2020** and/or a robust control strategy, as demonstrated in **wurtz2010**. What is more of an issue is the unidirectional force of the TSA, which can only impart force in tension. This means that for an AUJ, which is a two degree of freedom joint, a minimum of three TSA are required for unless spring return mechanisms are used, which would impart additional force on the TSA and therefore reduce performance. However, the potential high force to mass ratio of the TSA due to the non-proportional reduction may adequately compensate for the additional actuator requirement.

The focus of this research is to investigate if the TSA is a suitable candidate for control of an AUJ considering both the benefits and drawbacks. To this end, the first objective is to simulate a model and then construct a physical experimental prototype to validate the proposed control system. Then, the second objective is to compare the results from both the simulation and experimental **protoype** with other inline actuation methods for an

AUJ to analyse and discuss the potential advantages and disadvantages of TSA actuation against alternatives.

### A. Twisted String Actuator

First developed by **wurtz2010** in 2010, TSA uses two or more strings between two fixtures as a linear actuator. When one fixture is rotated (typically by an electric motor), the strings twist into a helix, decreasing the distance between the fixtures. Given the unwound length  $l_u$  and the cross-section radius of the string  $r_s$ , the actuator length is given by

$$l_s(\theta_s) = \sqrt{l_u^2 - \theta_s^2 r_s^2} \quad (1)$$

where  $\theta_s$  is the motor angle. This equation assumes an infinite string stiffness, so is only reasonably accurate under low tension. Although theoretically the stroke of the TSA can be the entire domain of  $[0, l_u]$ , in reality the thickness of the string prevents a geometric helix from forming once the helix pitch  $q < 4r_s$  (or  $q < 2nr_s$  for  $n$  strings). This limits the lower bound of the stroke as follows,

$$l_{min} = \frac{l_u}{\sqrt{\frac{\pi^2}{2} + 1}} \approx 0.46 l_u \quad (2)$$

or approximately 46% of  $l_u$  for a two string TSA.

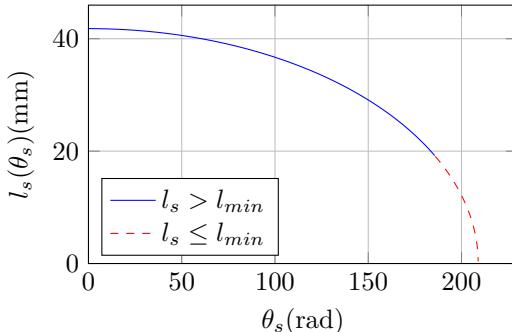


Fig. 1: TSA string length against motor angle with coefficients from table I.

### B. Antagonistic Triad

As mentioned in the introduction, because the TSA provides only tensile force, a minimum of three actuators are required. These can be arranged in a triangular configuration to create an “antagonistic triad”, akin to the antagonistic pairs of muscles found in animals. Where a revolute joint would be found between the connecting ends of the actuator, a universal joint is found instead. The geometric structure of the system can be described with two equilateral triangles of inradius  $r$  on two planes separated in the  $z$  axis. The centroids are then connected via a universal joint from each plane normal to an intersecting point, let the vector  $\boldsymbol{\theta} = [\theta_1 \ \theta_2]$  denote the rotation of the second plane relative to the first, in the

$x$  and  $y$  axes around the intersecting point, and let  $l_1$  and  $l_2$  denote the normal distance from the intersection to the first and second plane centroids respectively. When  $\boldsymbol{\theta} = [0 \ 0]$  the triangles are parallel to each other. The distance between the vertex pairs of each triangle is then denoted as  $[\lambda_1 \ \lambda_2 \ \lambda_3]$  for the “top”, “left” and “right” vertices of the triangles. When  $\boldsymbol{\theta}$  is changed, this will change  $\lambda_1$ ,  $\lambda_2$  and  $\lambda_3$  respectively. Figure 2 illustrates this antagonistic triad.

To calculate the lengths of the strings for a given  $\boldsymbol{\theta}$  of the universal joint, we define a vector function  $\Lambda(\boldsymbol{\theta}) = [\lambda_1(\boldsymbol{\theta}) \ \lambda_2(\boldsymbol{\theta}) \ \lambda_3(\boldsymbol{\theta})]$  as follows.

$$\begin{aligned} \lambda_1(\boldsymbol{\theta}) &= \sqrt{(l_1 + l_2 \cos \theta_1 \cos \theta_2 + r \cos \theta_1 \sin \theta_2)^2 \\ &\quad + (r - r \cos \theta_2 + l_2 \sin \theta_2)^2 \\ &\quad + (l_2 \cos \theta_2 \sin \theta_1 + r \sin \theta_1 \sin \theta_2)^2} \\ \lambda_2(\boldsymbol{\theta}) &= \sqrt{(a - b + c)^2 + (l_1 - d)^2 + e^2} \\ \lambda_3(\boldsymbol{\theta}) &= \sqrt{(a + b - c)^2 + (l_1 + d)^2 + e^2} \end{aligned} \quad (3)$$

where:

$$\begin{aligned} a &= -\frac{\sqrt{3}r(\cos \theta_1 - 1)}{2} \\ b &= l_2 \cos \theta_2 \sin \theta_1 \\ c &= \frac{r \sin \theta_1 \sin \theta_2}{2} \\ d &= \frac{\sqrt{3}r \sin \theta_1}{2} + l_2 \cos \theta_1 \cos \theta_2 - \frac{r \cos \theta_1 \sin \theta_2}{2} \\ e &= \frac{r \cos \theta_2}{2} - \frac{r}{2} + l_2 \sin \theta_2 \end{aligned}$$

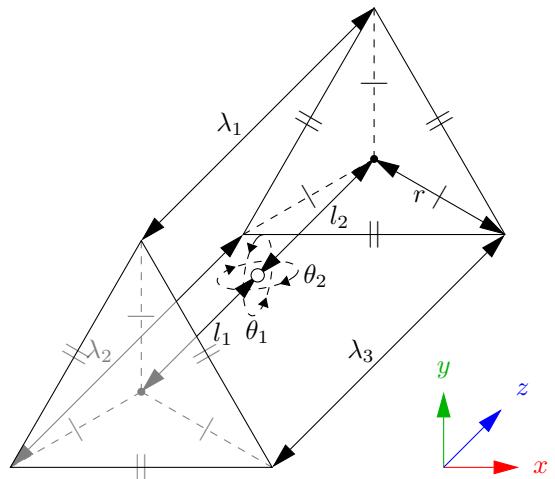


Fig. 2: Kinematic diagram of the antagonistic triad, where the universal joint rotation is defined by  $\theta_{1,2}$  on the  $x$  and  $y$  axes respectively, and the string lengths are defined by  $\lambda_{1,2,3}$  for the “Top”, “Left” and “Right” strings.  $r$  and  $l_{1,2}$  define the anchor points of the strings.

## II. CONTROL SYSTEM

The control system is a four layer cascade design, using feedback of the joint position from the accelerometers and TSA force from the load cells. It uses a second order set-point trajectory  $\mathbf{q}$  as input, which can either be pre-defined

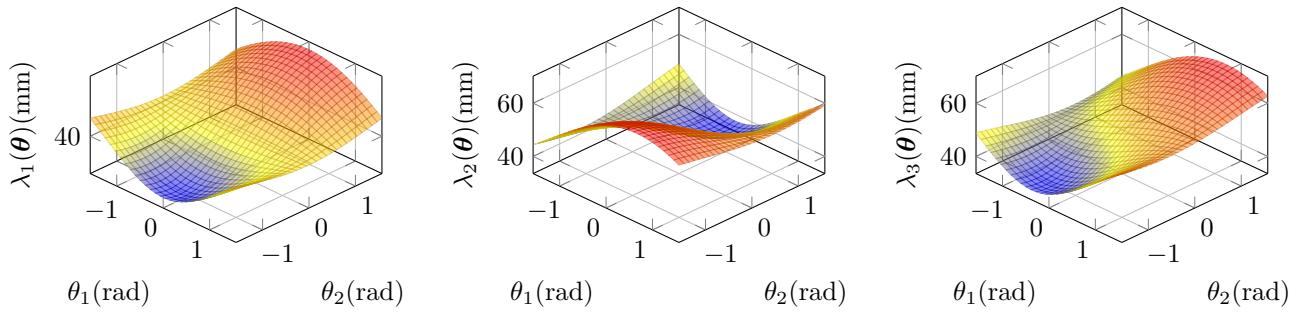


Fig. 3: Surface plots of each element of the vector function  $\Lambda(\theta)$ , assuming coefficient values from table I.

or generated dynamically from user input. Feedback is provided by the AUJ angular position  $\theta$ , angular velocity  $\dot{\theta}$ , and TSA tension force  $\hat{f}$ .

- 1)  $C_1$  AUJ Position PID Controller
- 2)  $C_2$  Inverse Dynamics
- 3)  $C_3$  TSA Force Optimisation Algorithm
- 4)  $C_4$  TSA Force P Controller

Functions  $C_{1\dots 4}$  are then combined into a cascade function  $C_4(C_3(C_2(C_1(\dots), \dots), \dots), \dots)$ .

#### A. Actuated Universal Joint Position PID Controller with Acceleration Feedforward

Firstly, a PID controller is used to generate a control signal  $\mathbf{u}$  with the input  $\mathbf{q}$  as the setpoint, and the AUJ angular position  $\theta$  and velocity  $\dot{\theta}$  as feedback, plus the addition of a feedforward term for the input acceleration  $\ddot{\mathbf{q}}$ , i.e.

$$\mathbf{u} = \mathbf{k}_p(\mathbf{q} - \theta) + \mathbf{k}_i \left( \int_0^t \mathbf{q} - \theta \right) dt + \mathbf{k}_d (\dot{\mathbf{q}} - \dot{\theta}) + \ddot{\mathbf{q}}. \quad (4)$$

In the discrete implementation used for fixed step simulation and experimental model control, the integral term is replaced by the trapezoidal rule.

$$K_I \int_0^t \epsilon \approx \sum_{i=0}^N \frac{\epsilon(t_i) + \epsilon(t_{i-1})}{2} \Delta t \quad (5)$$

#### B. Inverse Dynamics

The control signal  $\mathbf{u}$  from the PID controller is then converted to the desired AUJ torque  $\tau$  as follows

Firstly the affine transformation matrix  $T$  for a coordinate frame between the AUJ pivot and the centre of mass of the follower segment can be defined. The order of  $R_x$  and  $R_y$  can be reversed, but this requires other terms to be reversed as well.

$$T(\theta) = R_x(\theta_1) R_y(\theta_2) P_z(l_2) \quad (6)$$

Then the linear velocity jacobian  $J_v$  is simply the jacobian of the translation vector of  $T$ .

$$J_v(\theta) = \begin{bmatrix} \frac{\partial t_{14}}{\partial \theta_1} & \frac{\partial t_{24}}{\partial \theta_1} & \frac{\partial t_{34}}{\partial \theta_1} \\ \frac{\partial t_{14}}{\partial \theta_2} & \frac{\partial t_{24}}{\partial \theta_2} & \frac{\partial t_{34}}{\partial \theta_2} \end{bmatrix} \quad (7)$$

The angular velocity jacobian  $J_\omega$  is calculated using the first joint angle relative to the base frame, and the second in the frame of the first. If  $R_x$  is the first rotation in  $T$  then the first column of the jacobian is  $[100]^\top$  (to represent the pitch angle) and the second column is  $R_x [010]^\top$ .

$$J_\omega(\theta) = \begin{bmatrix} 1 & r_{x_{11}} \\ 0 & r_{x_{21}} \\ 0 & r_{x_{31}} \end{bmatrix} \quad (8)$$

Then the mass matrix  $D$  can be created from the jacobian, the mass for the follower segment  $m$  and its inertia tensor  $I \in \mathbb{R}^{3 \times 3}$ , and  $R_x$  and  $R_y$  to express the inertia in the correct frame.

$$D(\theta) = m J_v^\top J_v + J_\omega^\top (R_x R_y) I (R_x R_y)^\top J_\omega^\top \quad (9)$$

The centrifugal/coriolis matrix  $C$  is created from the christoffel symbols of  $D$ , along with the AUJ velocity vector  $\dot{\theta}$ .

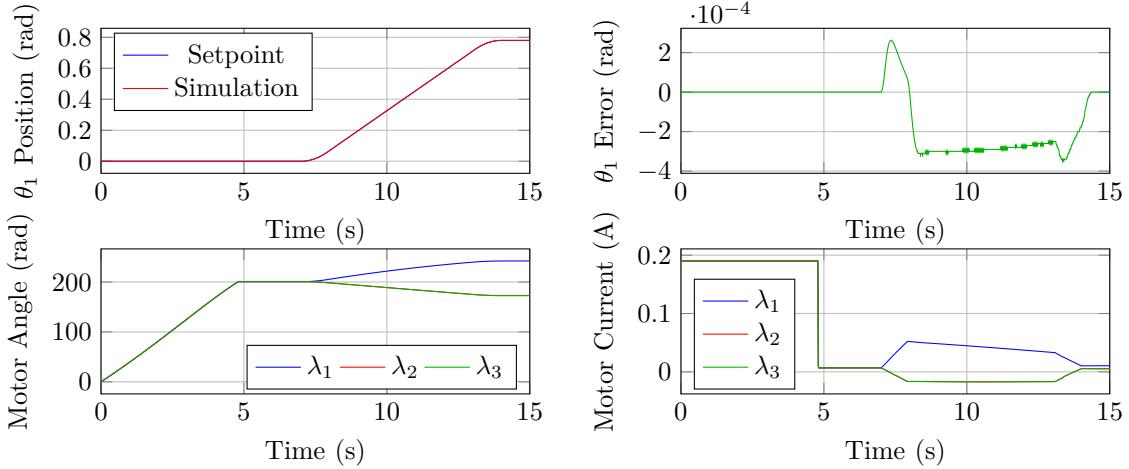
$$C(\theta, \dot{\theta})_{k,j} = \sum_{i=1}^N \frac{1}{2} \frac{\partial d_{kj}}{\partial \theta_i} + \frac{\partial d_{ki}}{\partial \theta_j} - \frac{\partial d_{ij}}{\partial \theta_k} \dot{\theta}_i \quad (10)$$

Then the gravity term  $G$ . As the gravity vector direction is the same as the z axis as in figure 2, the height is equal to  $-l_2 \cos \theta_1 \cos \theta_2$ , therefore the potential energy is  $mg(-l_2 \cos \theta_1 \cos \theta_2)$ . The jacobian of this then becomes the gravity term.

$$G(\theta) = \begin{bmatrix} \frac{\partial mg - l_2 \cos \theta_1 \cos \theta_2}{\partial \theta_1} \\ \frac{\partial mg - l_2 \cos \theta_1 \cos \theta_2}{\partial \theta_2} \end{bmatrix} \quad (11)$$

$D$ ,  $C$  and  $G$  are then combined to form the dynamics equation  $C_2$ , along with the AUJ position and velocity vectors.  $C_1$  is used as the acceleration term. This results in a setpoint joint torque which can be used in the optimisation algorithm.

$$\tau = D(\theta) \mathbf{u} + C(\theta, \dot{\theta}) \dot{\theta} + G(\theta). \quad (12)$$

Fig. 4: Simulation results for a trajectory of  $\theta_1$  from 0 to  $\frac{\pi}{4}$ .

### C. Twisted String Actuator Force Optimisation Algorithm

This uses a modified algorithm from **dessen1986** to select an optimal force vector from the desired joint torque. A force matrix  $F$  is created from the torque input  $\tau$ , jacobian  $J_\Lambda$  from the vector function  $\Lambda$  as defined in equation 3, and minimum force constant  $f_{min}$ .  $f_{ii}$  is equal to  $f_{min}$ , while the other elements in the column are based on a calculation using  $J_{\Lambda_{-i,*}}$  where  $-i$  is a row removed from the matrix.

$$\begin{aligned} J_\Lambda &= \begin{bmatrix} \frac{\partial \lambda_1}{\partial \theta_1} & \frac{\partial \lambda_2}{\partial \theta_1} & \frac{\partial \lambda_3}{\partial \theta_1} \\ \frac{\partial \lambda_1}{\partial \theta_2} & \frac{\partial \lambda_2}{\partial \theta_2} & \frac{\partial \lambda_3}{\partial \theta_2} \end{bmatrix} \\ F(\tau, \theta) &= \begin{cases} f_{i,i} = f_{min} \\ f_{-i,i} = -J_{\Lambda_{-i,*}}^{-T} (J_{\Lambda_{-i,*}}^T f_{min} + \tau) \end{cases} \quad (13) \\ &= \begin{bmatrix} f_{min} & f_{12} & f_{13} \\ f_{21} & f_{min} & f_{23} \\ f_{31} & f_{32} & f_{min} \end{bmatrix} \end{aligned}$$

The following algorithm then selects one column of  $F$  to be the output force vector  $f$ .

```

1:  $s \leftarrow [\mathbf{T} \quad \mathbf{T} \quad \mathbf{T}]$ 
2: if  $f_{23} > f_{min}$  then  $s_2 \leftarrow \mathbf{F}$  else  $s_3 \leftarrow \mathbf{F}$  end if
3: if  $f_{31} > f_{min}$  then  $s_3 \leftarrow \mathbf{F}$  else  $s_1 \leftarrow \mathbf{F}$  end if
4: if  $f_{12} \geq f_{min}$  then  $s_1 \leftarrow \mathbf{F}$  else  $s_2 \leftarrow \mathbf{F}$  end if
5: for  $i = 1$  to 3 do
6:   if  $s_i \rightarrow \mathbf{T}$  then  $f \leftarrow f_{*,i}$  end if
7: end for

```

### D. Twisted String Actuator Force Proportional Controller

The selected forces are then used as an input to a P controller using the measured load cell forces  $\hat{f}$  as feedback. The output from this can then be used to control the top, left and right TSA motors.

1) *Simulation Current Control:* In the simulation, each TSA was modelled as a state-space system which takes motor current  $u$  as an input and outputs  $y$  as the TSA tension force. **wurtz2010** defines it as such, where  $J$

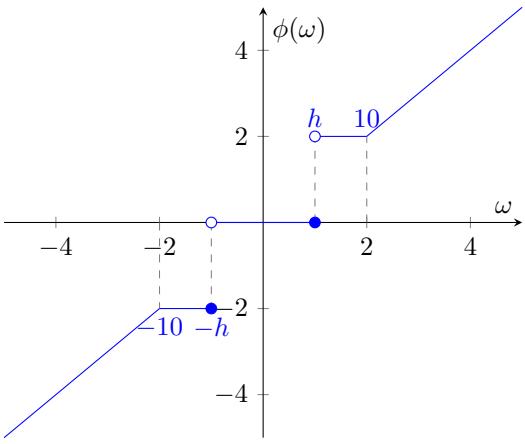
is the motor inertia,  $C$  is the motor coulomb friction (modified from viscous friction as the 1724TSR only has dry friction),  $K_t$  is the motor torque constant, and  $K_L$  is the load stiffness. As the original definition is for a fixed load  $l_u$  distance from the motor a modified model is required which takes into account the varying length between the motor and load defined by  $\Lambda(\theta)$ . A saturation function is used to prevent incorrect compression forces when the string is slack. All of the motor coefficients were taken from the Faulhaber 1724TSR datasheet **1724tsr** as this is the motor used in the experimental model. An estimated value is used for the load stiffness, this was chosen to be a high number as the model is expected to be very stiff.

$$\begin{aligned} h(\theta_s) &= \frac{\theta r_s^2}{\sqrt{l_u^2 - \theta_s^2 r_s^2}} \\ k(\theta_s, \theta) &= \lambda_n(\theta) - \sqrt{l_u^2 - \theta_s^2 r_s^2} \\ \dot{x} &= \begin{bmatrix} x_2 \\ -\frac{K_L}{J} h(x_1) k(x_1, \theta) - \frac{C}{J} \text{sgn}(x_2) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{K_t}{J} u \end{bmatrix} \\ y &= K_L \text{sat}_0^\infty k(x_1, \theta) \end{aligned} \quad (14)$$

The state space model was then adapted to include constraints on motor velocity and acceleration set by the motor controller in order to keep the motor within design limits, by replacing  $\dot{x}$  with  $\dot{x}'$  which contains saturation functions for maximum motor velocity  $v_s$  and acceleration  $\alpha_s$ .

$$\dot{x}' = \begin{bmatrix} \text{sat}_{\omega_s} \dot{x}_1 \\ \text{sat}_{\alpha_s} \dot{x}_2 \end{bmatrix} \quad (15)$$

2) *Experimental Velocity Control with Deadband Compensation:* In the experimental model, current control did not result in a stable output, so instead the P controller output would be the motor velocity as  $\omega$  (each motor controller has a hardware velocity PI controller). Due to a controller deadband within  $\pm 10 \text{ min}^{-1}$ , an adjustable deadband compensator is used,



$$\phi_i(\omega_i) = \begin{cases} 10 & 10 \leq \omega_i < h \\ -10 & -h < \omega_i \leq -10 \\ 0 & h \leq \omega_i \leq -h \\ \omega_i & \text{otherwise} \end{cases} \quad (16)$$

where  $\phi_i$  is the compensator for the controller  $i$ . An adjustment value  $h \in [0, 10]$  changes the threshold at which the compensator switches on and off, allowing for a small deadband to remain.

The result from the TSA is then a compressive force acting between each of the three TSA and its corresponding endpoint on the Antagonistic Triad, imparting a torque on the axes of the universal joint.

3) *Direct Current Control (DCC)*: This mode takes a preset velocity  $\dot{\theta}_{set} \in [0, 4220]$  and uses a hardware PI controller with velocity feedback  $\dot{\theta}_{act}$  to generate a control voltage. This voltage  $\mathbf{V}$  is then multiplied by the signum of the result from the cascade function in order to ensure the motor spins in the right direction, and then passed to a current limiter with the current error (the result with the actual current  $I_{act}$  subtracted) as the limit, before being sent to the motor. This ensures the motor stops spinning when the target current is reached.

$$\epsilon_c = \dot{\theta}_{set} - \dot{\theta}_{act}$$

$$\mathbf{V} = \left( K_{P_c} \epsilon_c + K_{D_c} \int_0^t \epsilon_c \right) \operatorname{sgn}(C_4(\dots)) \omega(C_4(\dots) - I_{act}) \quad (17)$$

Where  $\omega(\dots) \in [0, 1]$  is an unknown hardware limiting function that controls the motor speed depending on the current error.

4) *P Current Controller*: This strategy is a more direct method of current control, using a software P controller to directly set the voltage of the motor, using the MCDC 3002 as simply a passive amplifier. In this case the current error is passed directly to a P controller which has its output limited to prevent damage to the motors.

$$V = K_{P_c} C_4(\dots) - I_{act} \quad (18)$$

TABLE I: Model coefficients.

Coefficient	Value	Coefficient	Value
$l_1$	41.8 mm	$J$	$1 \times 10^{-6} \text{ kg m}^{-2}$
$l_2$	0 mm	$K_L$	$1000 \text{ N m}^{-1}$
$r$	13 mm	$f_{min}$	3 N
$l_u$	41.8 mm	$\omega_s$	$441.9 \text{ rad s}^{-1}$
$r_s$	200 $\mu\text{m}$	$I_s$	0.19 A
$m$	72.619 13 g	$K_t$	$0.0263 \text{ N m A}^{-1}$
$C$	0.1315 N mm	$\tau_s$	4.5 mN m
$\alpha_s$	$1 \times 10^5 \text{ rad s}^{-2}$		
Coefficient	Value		
$I$	$\begin{bmatrix} 3 \times 10^{-5} & 0 & 0 \\ 0 & 3.2 \times 10^{-5} & 0 \\ 0 & 0 & 1.4 \times 10^{-5} \end{bmatrix} \text{ kg m}^{-2}$		

TABLE II: PID gains in the simulation and experiment.

Gain	Value	
	Simulation	Experiment*
$k_p$	800	$3 \times 10^4$
$k_i$	3000	350
$k_d$	50	50
$k_{ps}$	19	100

\* Tracking mode, see section III-B.

5) *Velocity Control*: This strategy simply uses the result from the cascade function as a velocity setpoint using the hardware velocity PI controller as in the DCC.

$$\mathbf{V} = K_{P_c} \epsilon_c + K_{D_c} \int_0^t \epsilon_c \quad (19)$$

### III. SIMULATION & EXPERIMENTAL RESULTS

To design and refine the parameters of the control system, a Simscape Multibody™ model of the antagonistic triad and control system was created in MATLAB®/Simulink™. This allowed for model design coefficients  $l_1, l_2$  and controller gains  $k_p, k_i, k_d, k_{ps}$  to be modified in order to have the most stable control within design limits.

#### A. Experimental Setup

For the experimental validation, a physical prototype of the mechanism was constructed with coefficients from table I as design parameters. This was mounted vertically, in order for the Inertial Measurement Unit (IMU) to measure the orientation of the universal joint. The TSA mechanisms consist of a high torque motor attached to the base segment, with a string clamp attached to the motor shaft. On the follower segment, a load cell is mounted on top of a universal joint to ensure a purely axial load, with a capstan bolt attached to the load cell. The string itself is attached to the clamp at both ends using two grub screws for extra security and easy adjustment, and looped through the hole in the capstan bolt.

The motors to be used would be Faulhaber 1724T024SR micromotors, selected for their compact size but high torque. These would be controlled by Fauhaber

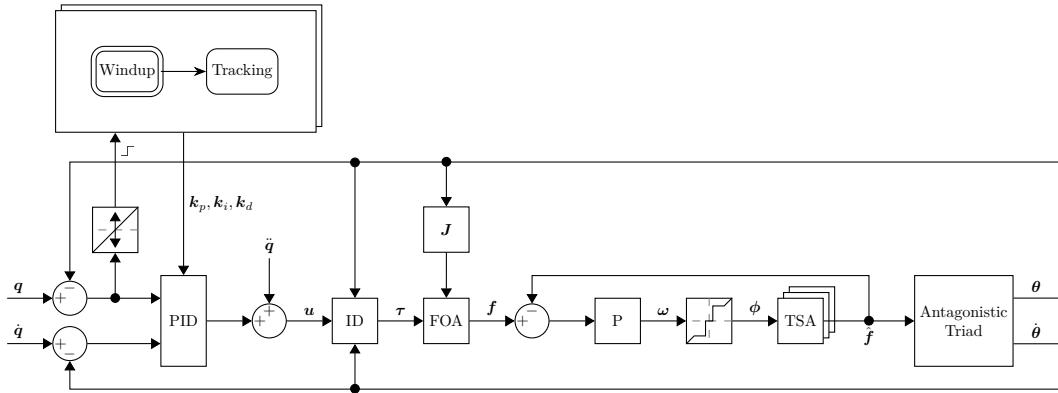


Fig. 5: Block diagram of the complete experimental control system, excluding the hardware velocity controllers for the motors.

MCDC3002 motor controllers, which could interface with a National Instruments MyRIO via the USB port, using a USB to serial converter. The load cells would be Futek LCM100 miniature load cells, selected for their small size. The signals from these would be amplified using Flyde FE-359-TA instrumentation amplifiers and decoded using an external AD7606 ADC before being fed into the MyRIO using SPI. The orientation of the AUJ would be

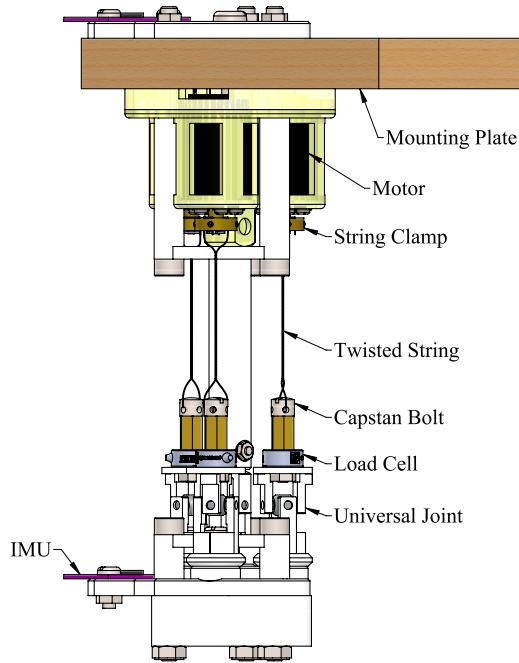


Fig. 6: Schematic of the experimental model with labelled components.

### B. Windup & Tracking States

When the mechanism is started with the TSA in a completely unwound state, before it can begin tracking a motion trajectory, the TSA strings must “wind up” to closely match the initial state of  $f$ . During this phase,

TABLE III: PID constants for both states.

Gain	Value	
	Windup	Tracking
$k_p$	800	$3 \times 10^4$
$k_i$	3000	350

the outer PID gains  $k_p, k_i$  are unsuitable and can result in damage to the mechanism. To mitigate this, two sets of PID gains are chosen, one for the windup state, and another for the tracking state, which the windup state transitions to once suitable stability is achieved. This transition trigger is defined on a per-axis basis, as the first zero crossing of the angle error (as  $q = 0$  this is effectively  $\theta$ ).

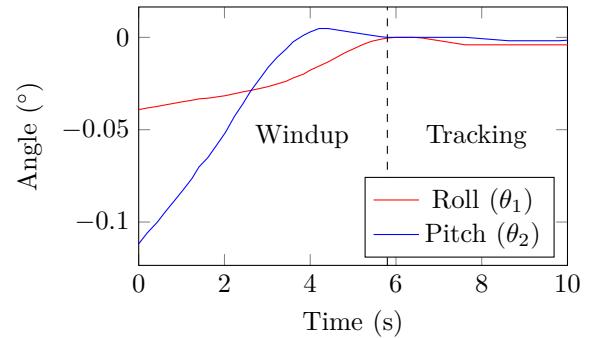


Fig. 7: The initial startup of the mechanism showing the transition between “windup” and “tracking” states.

### C. Results

Figure 8 plots the tracking response of both the simulation and experiment. Three trajectories were created to test the capabilities of the mechanism. Two were only in one axis of the Universal Joint, and the third was in both axes. The deflection angle range was limited due to the low value of  $l_u$ , but can easily be extended by increasing this value. A low  $l_u$  was chosen as it resulted in easier installation.

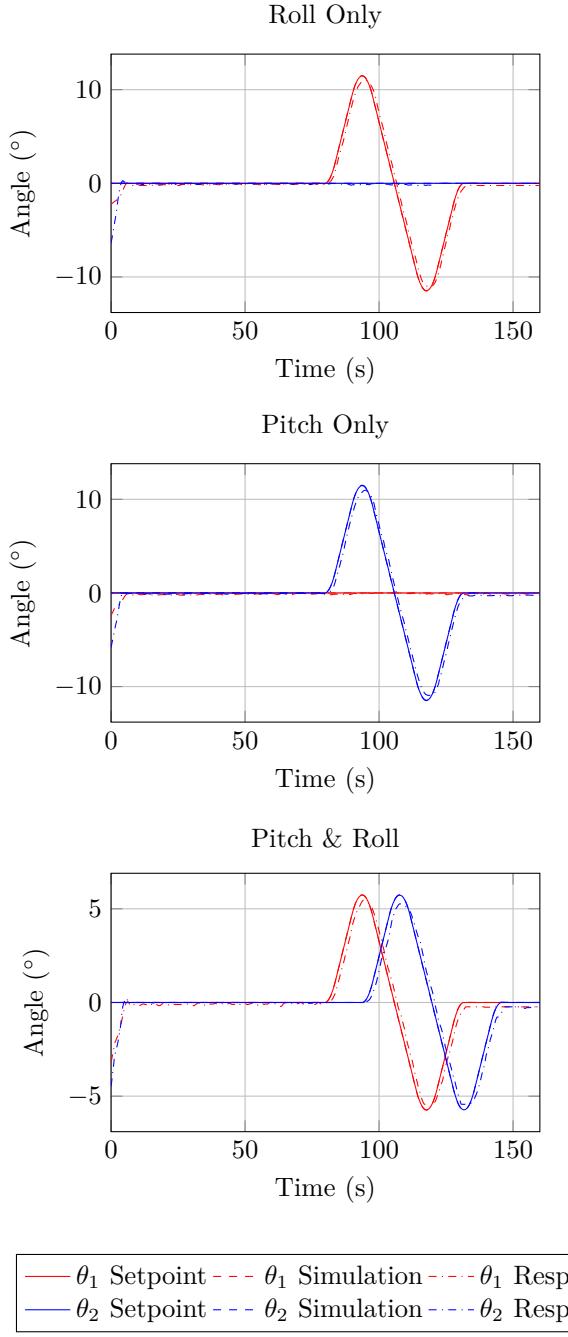


Fig. 8: Plots of the response for three different trajectories, one on only the roll axis, one on only the pitch axis, and one on both axes.

#### D. Twisted String Actuator Control System & Motor Characterisation

Before verifying robust control of the AUJ, the control of each individual TSA needed to be tested in order to characterise the performance of the motors and to ensure the inner loop control system was robust. This would involve selecting the control strategy that gave the best performance. A test trajectory consisting of a smooth ramp followed by a sine wave was fed into the inner loop of the cascade function as  $[f(t) \ f(t) \ f(t)]$ . Each control

strategy was tested, and in the end the velocity control strategy proved most optimal, as is shown in figure 12.

#### IV. PERFORMANCE COMPARISON

To compare the performance of a TSA AUJ against alternatives, we can measure two metrics, the maximum tension force  $f_{max}$  and maximum stroke velocity  $\dot{p}_{max}$ . The alternatives chosen for comparison are leadscrews of various rod diameters  $d_m$  and pitches  $\lambda$ , and a “direct drive” where the motor is rotating the universal joint directly without any reduction or motion transformation.

##### A. Twisted String Actuator

For the TSA metrics, the equations from **wurtz2010**, in particular  $h(\theta)$  and  $k(\theta)$  as used for the state space, which can be used to determine  $f_{max}$  and  $\dot{p}_{max}$ . By extracting coefficient  $r_s$  as an input to make  $f(p, r_s)$  and  $\dot{p}(\theta, p, r_s)$  the performance of different string thicknesses can be compared for a given unwound length  $l_u$  and  $\tau_{max}, \dot{\theta}_{max}$  over the range of the contraction length  $p$ .

$$h^{-1}(\theta) = \frac{\sqrt{l_n^2 - r_s^2\theta^2}}{r_s^2\theta} \quad (20)$$

$$f(p) = h^{-1}(k^{-1}(p)) = \pm \frac{\sqrt{(l_u - p)^2}}{r_s \sqrt{p(2l_u - p)}} \quad (21)$$

$$f_{max} = f(p)\tau_{max}$$

$$k(\dot{\theta}, \theta) = \frac{\dot{\theta}r_s^2\theta}{\sqrt{l_n^2 - r_s^2\theta^2}}$$

$$\dot{p}(\dot{\theta}, p) = k(\dot{\theta}, k^{-1}(p)) = \pm \frac{\dot{\theta}r_s \sqrt{p(2l_n - p)}}{\sqrt{(l_n - p)^2}} \quad (21)$$

$$\dot{p}_{max} = \dot{p}(\dot{\theta}_{max}, p)$$

##### B. Leadscrew

For the leadscrew metrics, the raising torque calculation **shigley2004mechanical** can be used as the absolute value of  $f_{max}$ , since the TSA only operates in tension, which can be used to determine the same metrics. The performance of different screw diameters  $d_m$  and leads  $\lambda$  can then be compared for a given  $\tau_{max}$  and coefficient of friction  $\mu$ .  $\dot{p}_{max}$  is then calculated by multiplying  $\lambda$  with  $\dot{\theta}_{max}$ . The performance of different  $\lambda$  can then be compared for a given  $\dot{\theta}_{max}$ .

$$|\tau(f)| = \frac{d_m f (\lambda + \pi d_m \mu)}{2(\pi d_m - \lambda \mu)} \quad |f(\tau)| = \frac{2\tau(\pi d_m - \lambda \mu)}{d_m (\lambda + \pi d_m \mu)} \quad (22)$$

$$f_{max} = |f(\tau_{max})|$$

$$\dot{p}(\dot{\theta}) = \lambda \frac{\dot{\theta}}{2\pi} \quad \dot{p}_{max} = \dot{p}(\dot{\theta}_{max}) \quad (23)$$

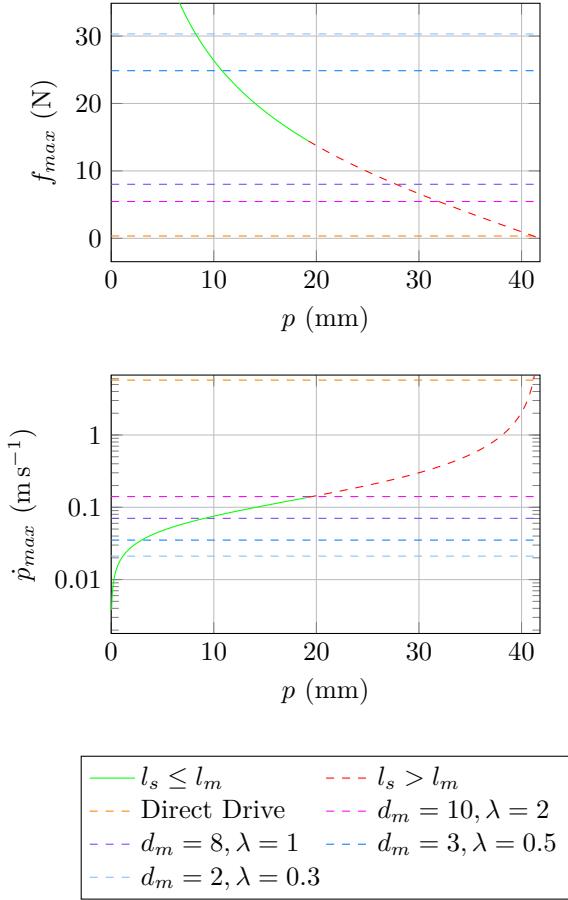


Fig. 9: Performance comparison of the TSA configuration using coefficients from table I to various leadscrew configurations with different  $d_m$  and  $\lambda$ , and the direct drive, where  $0.1 = 0.1$  for the leadscrews.

### C. Direct Drive

The direct drive metrics are trivially calculated using the lever force and tangential velocity that would be generated at the endpoint of a linear actuator able to impart equivalent angular velocity and torque on the universal joint.

$$f_{max} = \frac{\tau_{max}}{\sqrt{l_2^2 + r^2}} \quad (24)$$

$$\dot{p}_{max} = \dot{\theta}_{max} \sqrt{l_2^2 + r^2} \quad (25)$$

### D. Comparison

As the values for  $\tau_{max}$  and  $\dot{\theta}_{max}$  for the TSA depend on  $p$ , but remain constant for the leadscrew, the performance of the TSA is going to be better or worse than a given leadscrew depending on the value  $p$ . Figure 10 compares the TSA configuration using the coefficients from table I against a number of common leadscrew configurations that are practical for the dimensions of the AUJ. The TSA outperforms or underperforms different leadscrew configurations depending on  $p$ .

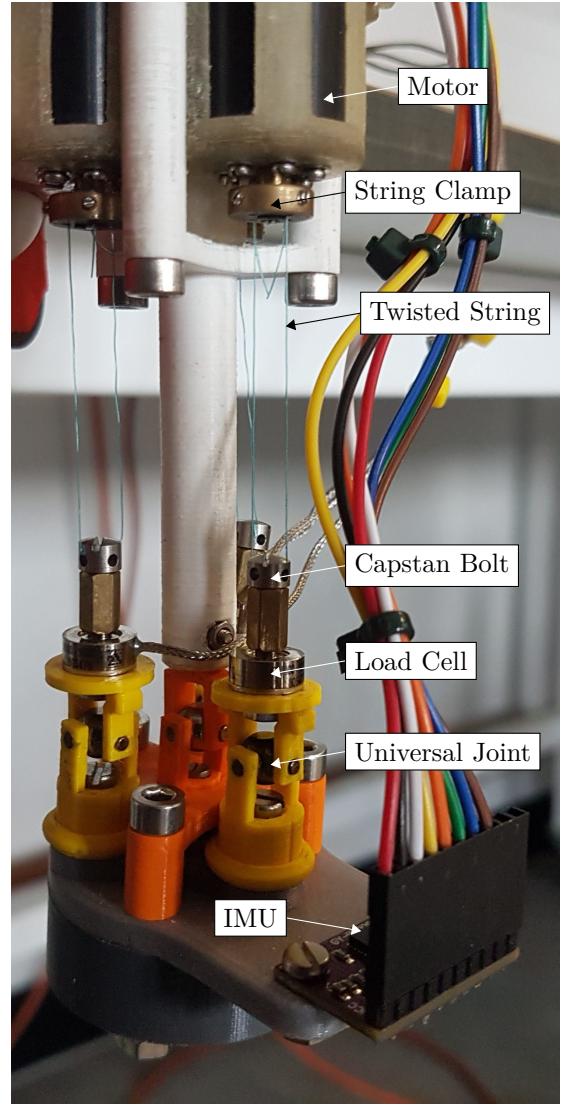


Fig. 10: Annotated photograph of the experimental model.

### V. CONCLUSION

This research has demonstrated the robust control of the orientation of a universal joint using TSA in an antagonistic triad configuration. It has also compared the performance of the system to alternative actuation methods. Future developments would include improvements to the orientation sensors, the data from the IMU proved to be unreliable and of poor resolution, so either a superior IMU should be used or an alternative method for sensing the universal joint orientation should be investigated, such as Linear Variable Differential Transformers (LVDTs), hall effect sensors or potentiometers. Eventually, we wish to develop a system comprised of multiple segments, to demonstrate its suitability for applications such as mobile snake robots or snake-arm robots.

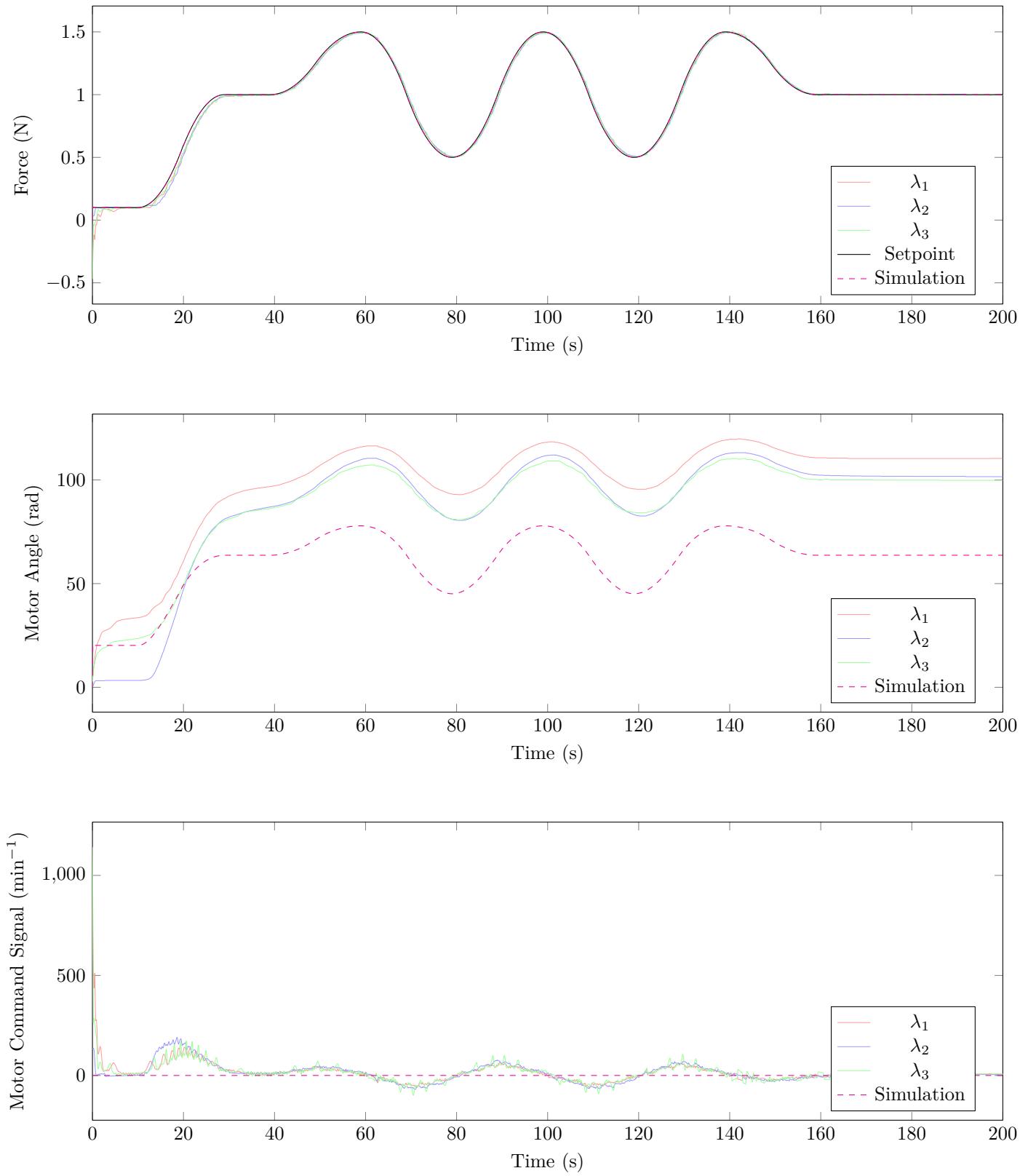


Fig. 11: Load cell plot from each string using the velocity control strategy.

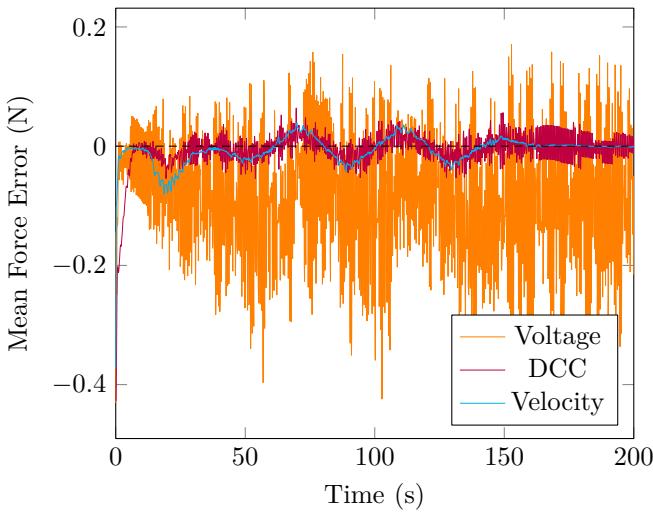


Fig. 12: Mean tracking error for each control strategy.



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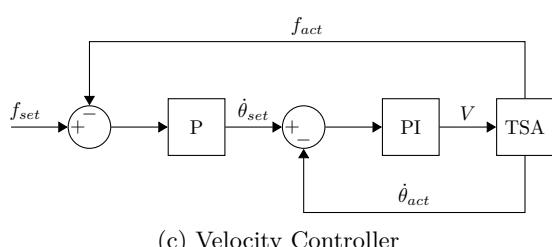
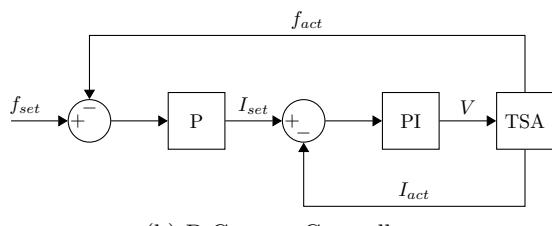
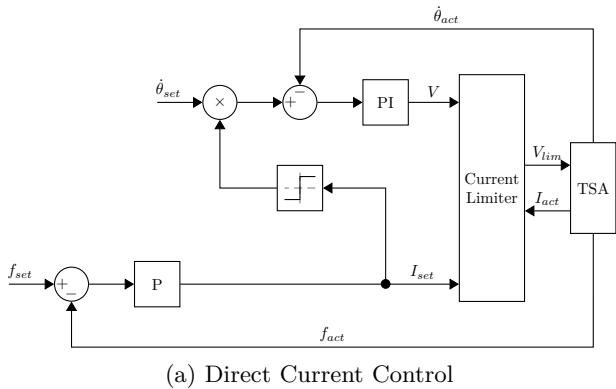


Fig. 13: Block diagrams of each control strategy.



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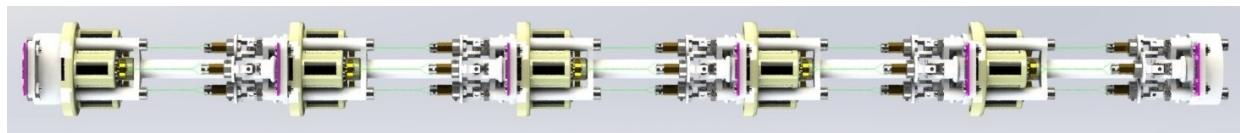


Fig. 14: Visualisation of several TSA AUJ chained together to form a robotic snake or tail.