Thesis Title

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3D Three Dimensional

COM Centre of Mass

Abstract

This is abstract text.

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Chapter 1

Creating a Configurable Payload for Instability Experiments

1.1 Introduction

In order to generate a diverse set of test data for the experiments in chapter **??**, a configurable payload was conceived, an object that could be configured to have a wide range of masses and COM. A series of test points can then be generated which have a specific mass and COM, and a matching algorithm can be used to find the configuration that mostly closely matches these parameters. The experiments in chapter **??** can then be run with each of these test points to generate the test data.

1.2 Payload Design

The payload consists of a matrix of cubes of various materials packed tightly into a Three Dimensional (3D) printed container. The cubes are designed to be changed after each experimental run to alter the mass and COM of the payload. A lid on the container prevents the cubes from falling out during the experiment, and the exterior design of the box may accommodate additional features to improving the handling of the payload by the robot arm.

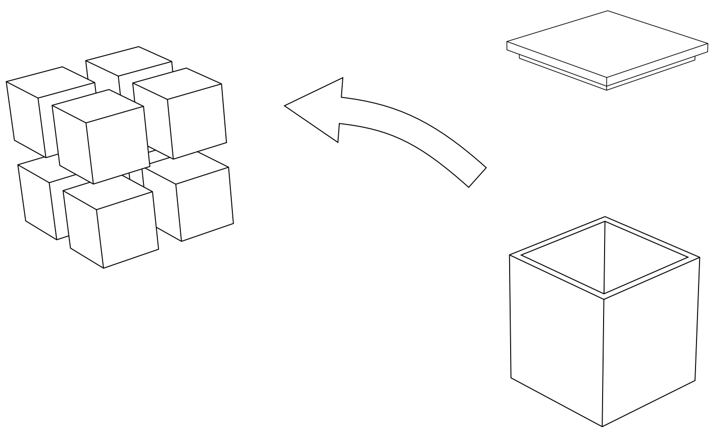

Figure 1.1: Concept drawing of the configurable payload.

1.3 Payload Configuration Space

In order to find a configuration that closely matches a desired test point, first the payload has to be abstracted mathematically, so the mass and COM can be calculated for a given configuration. Firstly we can consider a positive real set of material densities $\mathcal{P} \in \mathbb{R}^+$, each element the density (in $\mathrm{kg\,m^{-3}}$) of a material to be used:

$$\mathcal{P} = \{\rho_1, \rho_2 \ldots \rho_n \mid \rho_i > 0\} \tag{1.1}$$

Each configuration can then be defined as an $n \times n \times n$ matrix $\boldsymbol{C}$, such that each element is an element of $\mathcal{P}$, where n^3 is the number of cubes in the matrix:

$$\boldsymbol{C} = (c_{ijk}) \in \mathbb{R}^{n^n} \mid (c_{ijk}) \in \mathcal{P} \tag{1.2}$$

1.3.1 Mass and COM functions

To calculate the mass of the configuration, we can take the sum of all the cube densities multiplied by their volume a^3, where a is the cube edge length, plus the container mass m_c:

$$M(\boldsymbol{C}) = \left(\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} c_{ijk}a^3\right) + m_c \tag{1.3}$$

To calculate the COM, we can take the sum of each cube mass multiplied by its position relative to the centroid of the center cube (c_{222}), which can be calculated from the cube indexes ijk, plus the container COM $\boldsymbol{r}_c$ if non-zero:

$$R(\boldsymbol{C}) = \frac{\left(\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} c_{ijk}a^4\left(\begin{bmatrix} i & j & k\end{bmatrix} - n + 1\right)\right) + \boldsymbol{r}_c}{M(\boldsymbol{C})} \tag{1.4}$$

1.4 Test Points

With $\mathcal{Z}$ as the set of all permutations of $\boldsymbol{C}$, test points can be derived from subsets of $\mathcal{Z}$ defined either by logical expressions on $\mathcal{Z}$, or the nearest neighbours of $\mathcal{Z}$ from a set of coordinates.

1.4.1 Extrema Set ($\mathcal{E}$)

The extrema set is designed to test the extremas of the space of $\mathcal{Z}$ for both $M(\mathcal{Z})$ and $R(\mathcal{Z})$. The extrema set if defined from a set of logical constraints. The first two constraints of the set find the maximum and minimum values of the payload mass using $M(\mathcal{Z})$, and the next four constraints use the payload COM using $M(\mathcal{C})$ to get the maximum and minimum values of the x and y component of the COM. Finally, the last four constraints define the diagonal maximum and minimum values where the COM components match $x = y$ or $x = -y$.

$$\mathcal{E} = \left\{ \boldsymbol{x} \subset \mathcal{Z} \mid \begin{cases} M(\boldsymbol{x}) = \max\{M(\mathcal{Z})\} \\ M(\boldsymbol{x}) = \min\{M(\mathcal{Z})\} \\ R(\boldsymbol{x})_x = \max\{R(\mathcal{Z})_x\} \\ R(\boldsymbol{x})_x = \min\{R(\mathcal{Z})_x\} \\ R(\boldsymbol{x})_y = \max\{R(\mathcal{Z})_y\} \\ R(\boldsymbol{x})_y = \min\{R(\mathcal{Z})_y\} \\ R(\boldsymbol{x})_x = \max\{R(\mathcal{Z})_x\} \wedge R(\boldsymbol{x})_x = R(\boldsymbol{x})_y \\ R(\boldsymbol{x})_x = \min\{R(\mathcal{Z})_x\} \wedge R(\boldsymbol{x})_x = R(\boldsymbol{x})_y \\ R(\boldsymbol{x})_x = \max\{R(\mathcal{Z})_x\} \wedge R(\boldsymbol{x})_x = -R(\boldsymbol{x})_y \\ R(\boldsymbol{x})_x = \min\{R(\mathcal{Z})_x\} \wedge R(\boldsymbol{x})_x = -R(\boldsymbol{x})_y \end{cases} \right\} \tag{1.5}$$

Mass Limited $\mathcal{E}_1$

When this set was generated, it was found that $M(\mathcal{E}_1)$ was greater than the chosen robot arm could safely lift. Therefore, $\mathcal{E}_1$ was changed to $\begin{bmatrix} m_{max} & 0 & 0 & 0 \end{bmatrix}$ where m_{max} is the safe mass limit that the robot arm can lift.

1.4.2 Cube Set ($\mathcal{C}$)

The cube set is defined by the vertices of a cube of size b centred around the COM origin $\begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$.

$$\mathcal{C} = \left\{ \boldsymbol{x} \subset \mathcal{C} \mid \begin{bmatrix} \pm\frac{b}{2} & \pm\frac{b}{2} & \pm\frac{b}{2} \end{bmatrix} = \boldsymbol{x} \right\} \tag{1.6}$$

1.4.3 Balanced Set ($\mathcal{B}$)

The balanced set is defined by q points in $\mathcal{C}$ subject to the constraint $R(\boldsymbol{x})_x = 0 \wedge R(\boldsymbol{x})_y = 0$. This can be defined as a "balanced" set as the COM x and y components are both zero. The points are evenly spaced between the maximum and minimum mass as defined in section **??**.

$$\begin{aligned} m_r &= \frac{\max\{M(\mathcal{Z})\} - \min\{M(\mathcal{Z})\}}{q+1} \\ \boldsymbol{z} &= \begin{bmatrix} m_r & 2m_r & \cdots & qm_r \end{bmatrix} \\ \mathcal{B} &= \{\boldsymbol{x} \subset \mathcal{C} \mid z_i = \boldsymbol{x} \wedge R(\boldsymbol{x})_x = 0 \wedge R(\boldsymbol{x})_y = 0\} \end{aligned} \tag{1.7}$$

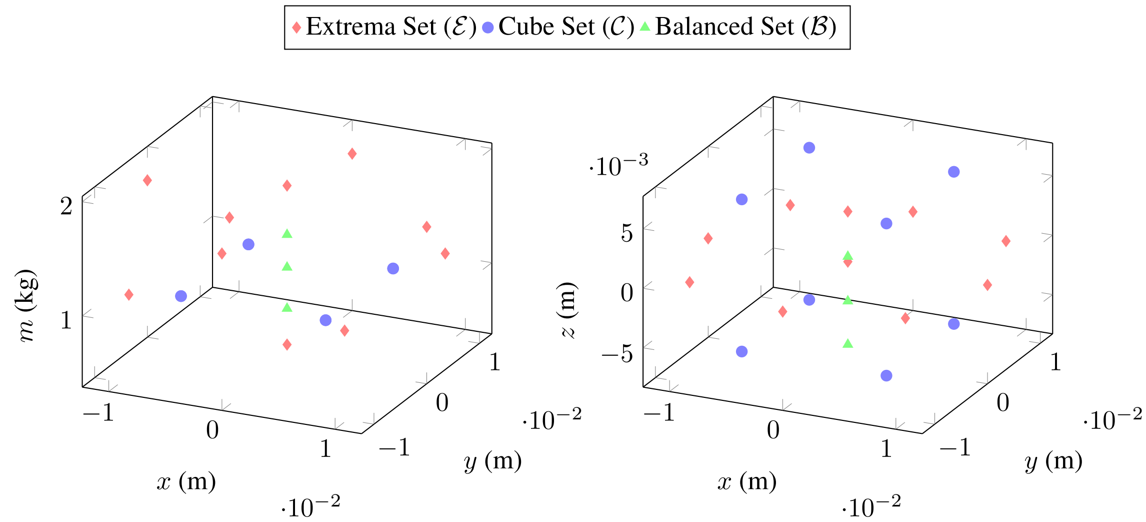

Figure 1.2: Mass and COM coordinates for each test point set.

1.5 Test Point Matching Search Methods

While we are guaranteed an exact result for elements of $\mathcal{E}$ as every element is defined by constraints on known configurations, for $\mathcal{C}$ and $\mathcal{B}$ as the elements are defined numerically, there is no guarantee that any element will have an exact match in solution space. The cardinality of $\mathcal{Z}$ is also important. It is defined as $|\mathcal{Z}| = |\mathcal{P}|^{n^n}$ which increases super exponentially with n. For example, when $|\mathcal{P}| = 4$, $n = 2$ results in 256 permutations and $n = 3$ results in approximately 1.8×10^{16} permutations. It's very clear that when $n > 2$ for non-trivial cardinalities of $\mathcal{P}$, any kind of brute-force method is not computationally tractable. The only exception is for $\mathcal{E}_1$ and $\mathcal{E}_2$, where the solution is trivial as $\mathcal{E}_1 = \boldsymbol{C} \mid \min(\mathcal{P}) \forall c_{ijk}$ and $\mathcal{E}_2 = \boldsymbol{C} \mid \max(\mathcal{P}) \forall c_{ijk}$. Therefore, we investigated both a brute-force nearest neighbour method for when $n = 2$, and a simulated annealing method for when $n = 3$.

1.5.1 Simulated Annealing ($n = 3$)

1.5.2 Brute-Force Nearest Neighbour ($n = 2$)

If $\mathcal{Z}$ is suitably small, as it is when $n = 2$ then a brute-force method can be used which is guaranteed to find the nearest element to the target within a finite time. This can be done by calculating the L2 norms between the target vector t and all the elements of $\mathcal{Z}$ and finding the minimum. If several elements of $\mathcal{Z}$ have the minimum norm, then one is chosen at random from this set.

$$NN(t, \mathcal{Z}) = \min\{\|x\|_2 \mid \forall x \in \mathcal{Z}\} \tag{1.8}$$

1.5.3 Selected Method

Target		Nearest		L2 Norm Error
m	$\boldsymbol{r}$	m	$\boldsymbol{r}$	
Cube Set ($\mathcal{C}$)				
$*$	$\begin{bmatrix}-0.007 & -0.007 & -0.007\end{bmatrix}$	1.061	$\begin{bmatrix}-0.006 & -0.006 & -0.006\end{bmatrix}$	8.616×10^{-4}
$*$	$\begin{bmatrix}-0.007 & -0.007 & 0.007\end{bmatrix}$	1.061	$\begin{bmatrix}-0.006 & 0.006 & -0.006\end{bmatrix}$	1.340×10^{-2}
$*$	$\begin{bmatrix}-0.007 & 0.007 & -0.007\end{bmatrix}$	1.061	$\begin{bmatrix}0.006 & -0.006 & -0.006\end{bmatrix}$	1.894×10^{-2}
$*$	$\begin{bmatrix}-0.007 & 0.007 & 0.007\end{bmatrix}$	1.061	$\begin{bmatrix}0.006 & 0.006 & -0.006\end{bmatrix}$	1.340×10^{-2}
$*$	$\begin{bmatrix}0.007 & -0.007 & -0.007\end{bmatrix}$	1.061	$\begin{bmatrix}-0.006 & -0.006 & 0.006\end{bmatrix}$	1.340×10^{-2}
$*$	$\begin{bmatrix}0.007 & -0.007 & 0.007\end{bmatrix}$	1.061	$\begin{bmatrix}-0.006 & 0.006 & 0.006\end{bmatrix}$	1.894×10^{-2}
$*$	$\begin{bmatrix}0.007 & 0.007 & -0.007\end{bmatrix}$	1.061	$\begin{bmatrix}0.006 & -0.006 & 0.006\end{bmatrix}$	1.340×10^{-2}
$*$	$\begin{bmatrix}0.007 & 0.007 & 0.007\end{bmatrix}$	1.061	$\begin{bmatrix}0.006 & 0.006 & 0.006\end{bmatrix}$	8.616×10^{-4}
Balanced Set ($\mathcal{B}$)				
0.516	$\begin{bmatrix}0.000 & 0.000 & *\end{bmatrix}$	0.832	$\begin{bmatrix}0.000 & 0.000 & -0.003\end{bmatrix}$	3.156×10^{-1}
1.258	$\begin{bmatrix}0.000 & 0.000 & *\end{bmatrix}$	1.192	$\begin{bmatrix}-0.000 & -0.000 & 0.000\end{bmatrix}$	6.630×10^{-2}
2.000	$\begin{bmatrix}0.000 & 0.000 & *\end{bmatrix}$	1.478	$\begin{bmatrix}-0.000 & -0.000 & -0.007\end{bmatrix}$	5.219×10^{-1}

Table 1.1: Table of the target and actual vectors for $\mathcal{C}$ and $\mathcal{B}$ with the L2 norm error. $*$ notation indicates "don't care" and is excluded from the search algorithm.

1.6 Conclusion and Discussion

Chapter 2

Creating a Configurable Payload for Instability Experiments

2.1 Introduction

2.2 Control Experiments

2.2.1 Results

2.3 Conclusion and Discussion

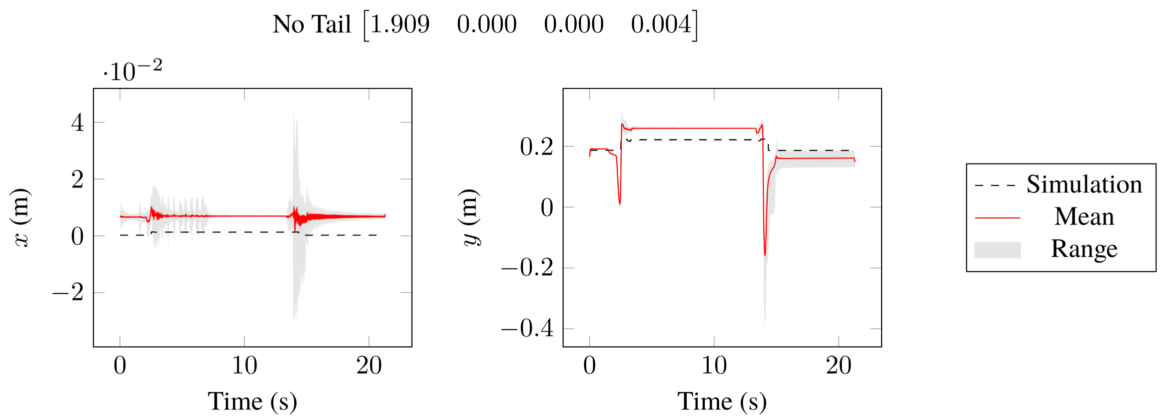

Figure 2.1: COM x and y position of the test rig along the test trajectory for the mass maximum element of the extrema set with no tail.

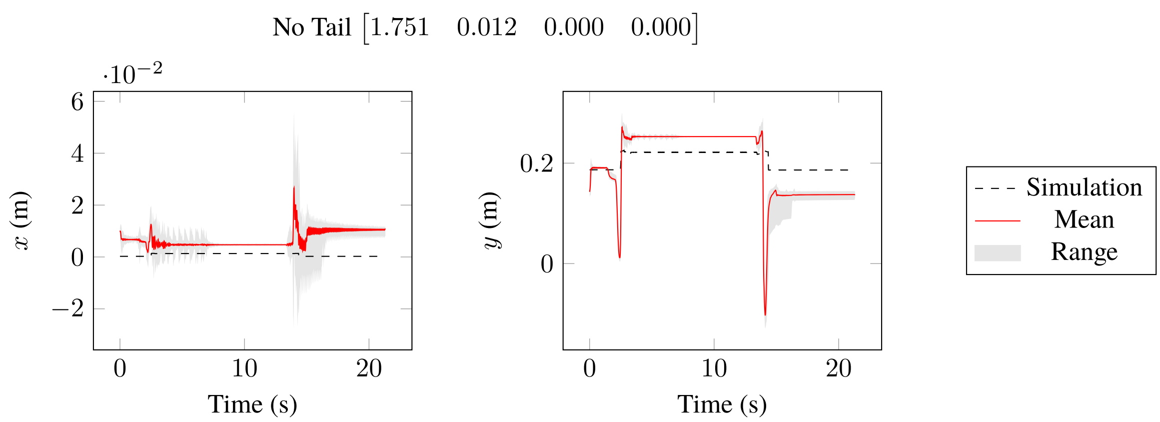

Figure 2.2: COM x and y position of the test rig along the test trajectory for the mass minimum element of the extrema set with no tail.

Appendices