Thesis Title

A thesis submitted to the University of Manchester for the degree of
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Contents

Word Count: X

List of figures

List of tables

List of publications

Publications go here.

3D Three Dimensional

COM Centre of Mass

Abstract

This is abstract text.

Declaration of originality

I hereby confirm that no portion of the work referred to in the thesis has been submitted in support of an application for another degree or qualification of this or any other university or other institute of learning.

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Acknowledgements

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Chapter 1

Creating a Configurable Payload for Instability Experiments

1.1 Introduction

In order to generate a diverse set of test data for the experiments in chapter **??**, a configurable payload was conceived, an object that could be configured to have a wide range of masses and COM. A series of test points can then be generated which have a specific mass and COM, and a matching algorithm can be used to find the configuration that mostly closely matches these parameters. The experiments in chapter **??** can then be run with each of these test points to generate the test data.

1.2 Payload Design

The payload consists of a matrix of cubes of various materials packed tightly into a Three Dimensional (3D) printed container. The cubes are designed to be changed after each experimental run to alter the mass and COM of the payload. A lid on the container prevents the cubes from falling out during the experiment, and the exterior design of the box may accommodate additional features to improving the handling of the payload by the robot arm.

1.3 Payload Configuration Space

In order to find a configuration that closely matches a desired test point, first the payload has to be abstracted mathematically, so the mass and COM can be calculated for a given configuration. Firstly we can consider a positive real set of material densities $\mathcal{P}$, each element the density (in $\mathrm{kg\,m^{-3}}$) of a material to be used:

$$\mathcal{P} = \{\rho_1, \rho_2 \ldots \rho_n \mid \rho_i > 0\} \tag{1.1}$$

Each configuration can then be defined as an $n \times n \times n$ matrix $\boldsymbol{C}$, such that each element is an element of $\mathcal{P}$, where n^3 is the number of cubes in the matrix:

$$\boldsymbol{C} = (c_{ijk}) \in \mathbb{R}^{n^n} \mid (c_{ijk}) \in \mathcal{P} \tag{1.2}$$

1.3.1 Mass and COM functions

To calculate the mass of the configuration, we can take the sum of all the cube densities multiplied by their volume a^3, where a is the cube edge length, plus the container mass m_c:

$$M(\boldsymbol{C}) = \left(\sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} c_{ijk} a^3 \right) + m_c \tag{1.3}$$

To calculate the COM, we can take the sum of each cube mass multiplied by its position relative to the centroid of the center cube (c_{222}), which can be calculated from the cube indexes ijk, plus the container COM $\boldsymbol{r}_c$ if non-zero:

$$R(\boldsymbol{C}) = \frac{\left(\sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} c_{ijk} a^4 \left(\begin{bmatrix} i & j & k \end{bmatrix} - n + 1 \right) \right) + \boldsymbol{r}_c}{M(\boldsymbol{C})} \tag{1.4}$$

1.4 Test Points

1.4.1 Extrema Set ($\mathcal{E}$)

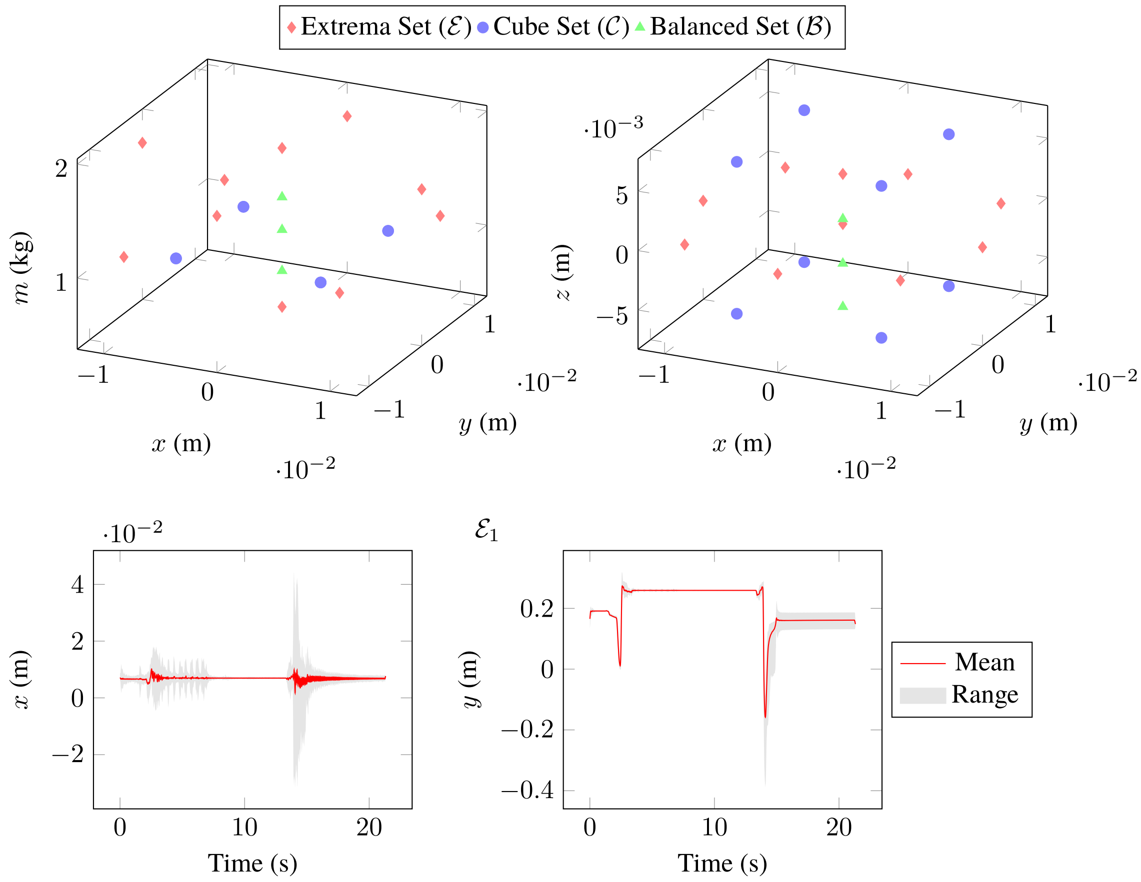
1.4.2 Cube Set ($\mathcal{C}$)

1.4.3 Balanced Set ($\mathcal{B}$)

1.5 Test Point Matching Search Methods

While we are guaranteed an exact result for $\mathcal{E}$ as every element is defined by constraints on known configurations, for $\mathcal{C}$ and $\mathcal{B}$ as the elements are defined numerically, there is no guarantee that any element will have an exact match in solution space. Therefore, we need to use a search algorithm in order to find the closest match (or matches) to the desired test point. We can define the solution space for our test points to search as a mapping from $\boldsymbol{S}() \Big[M(\boldsymbol{C})$

The total number of possible permutations of $\boldsymbol{C}$ can be determined by $|\mathcal{P}|^{n^n}$ which increases super exponentially with n. For example, when $|\mathcal{P}| = 4$, $n = 2$ results in 256 permutations and $n = 3$ results in approximately 1.8×10^{16} permutations. It's very clear that when $n > 2$ for non-trivial cardinalities of $\mathcal{P}$, any kind of brute-force method is not computationally tractable. Therefore, we investigated both a brute-force nearest neighbour method for when $n = 2$, and a simulated annealing method for when $n = 3$.



1.5.1 Simulated Annealing ($n = 3$)

1.5.2 Brute-Force Nearest Neighbour ($n = 2$)

1.5.3 Selected Method

1.6 Conclusion and Discussion

Appendices