A Scoping Review of Robotic Tails for Land-based Mobile Robot Locomotion

Damian Crosby[1,*], Joaquin Carrasco[2], William Heath[2] and Andrew Weightman[1]

[1]Department of Mechanical, Aerospace and Civil Engineering, School of Engineering, Faculty of Science and Engineering, The University of Manchester, Manchester, United Kingdom

[2]Department of Electrical and Electronic Engineering, School of Engineering, Faculty of Science and Engineering, The University of Manchester, Manchester, United Kingdom

Correspondence*:
Damian Crosby
damian.crosby@manchester.ac.uk

ABSTRACT

The aim of this review was to systematically identify and review literature relating to the design, development and implementation of robotic tails that improve land-based mobile robot locomotion, including static and dynamic balance and jumping, to inform the design of future systems. A systematic literature review was conducted to identify papers relating to land-based robots utilising tails to improve performance in IEEE Xplore, Web of Science and Scopus between January 1980 and December 2018. 68 papers were identified, of which 47 papers included a physical robot or prototype, and 33 distinct physical robotic systems were determined. In order of prevalence, robotic tails have been utilised for aerial reorientation, locomotion stability, induced turning, turning stability and velocity change stability. The most common tail structure for individual robots was rigid (79%) with the majority of tails identified composed of a single tail segment (70%) and actuation predominately by revolute electric motors (91%). Control systems were predominately closed outer-loop type. The literature illustrates robotic tails can improve performance but existing work has been limited to low degree of freedom systems. The authors propose that increased robot performance should be contrasted with the additional energy consumption and storage requirements needed to facilitate this.

Keywords: keyword, keyword, keyword, keyword, keyword, keyword, keyword, keyword

1 INTRODUCTION

The motivation for mobile robotics has predominately been driven by the need for systems which can explore hazardous and extreme environments which are too dangerous for people. For example nuclear decommissioning, where radiation is potentially fatal, or planetary exploration, where it is not possible to send and retrieve astronauts. Mobile robots have been successfully developed and utilised to explore nuclear sites such as Sellafield and Fukushima as well as the Martian surface, however obstacles and challenging terrain limit activities and can lead to the loss of robots which are often non recoverable. Mobile robots have evolved from wheeled machines to legged systems, which can run, jump or hop. These abilities enable mobile robotic systems to better adapt and navigate adverse terrain; In other words mobile robotic systems are becoming increasingly agile. As mobile robots move more towards increased agility, dynamic abilities and biomimetics, this has influenced the direction of research into investigating strategies for improving dynamic performance and stability by exploring the use of robotic tails to improve performance and robustness. Saab & Rone Saab et al. (2018b) recently published a state-of-the-art review of robotic tails in which the authors considered the design, modelling, analysis and implementation of robotic tails for mobile robots. The authors highlighted that robotic tails can be utilised for enhancing stability, manoeuvrability and propulsion of mobile robots, accomplished by enabling inertial adjustment. The review summarises challenges for future development with respect to mechanical design, modelling and control.

In this paper we present the results of a systematic review of literature relating to robotic tails for land-based mobile robot locomotion. This research complements the work of Saab & Rone Saab et al. (2018b) identifying an additional 41 papers and 16 robotic systems. Furthermore, we provide details of tail structure and classification, control, actuation, mass, length, and tail functionality. In this paper we define "tail" as anything that is referred to in the literature as such that meets the inclusion criteria and the topic of the review. This includes tails that are static or unactuated, as they can still influence robot locomotion.

2 LITERATURE SEARCH

A computerised literature search was undertaken of the electronic databases: IEEE Xplore, Web of Science and Scopus between January 1980 and December 2018, searching for Tail or Appendage in the document title. Papers were excluded if they concerned water walking, swimming or flying robots, as the use of tails in fluid dynamics was not in the scope of the review. The language was limited to English. To identify relevant studies the titles and abstracts of the literature within the databases were scanned with the search terms:

> (Tail* OR Appendage) is contained in Document Title AND Robot* NOT (Fish OR Swim) NOT (Surgery OR Medic* OR Tumour) NOT (Helicopter OR Unmanned Aerial Vehicle OR UAV) NOT Underwater NOT (Chemical OR Chemistry) NOT Tailor* is contained in Document Title

The Chemical OR Chemistry search terms were included to exclude "tail" in the molecular sense (i.e. the and tail of a polar molecule). The Tailor* negative search term was included to exclude false positives caused by Tail*. An additional search was also conducted using Tail AND Tails AND Tailed AND Tailor* to verify that no relevant records contained both Tail* and Tailor* stems in separate words in the Document Title.

To ensure the author identified all relevant literature, that may have been missed by the database search, the authors screened the reference list of the identified papers and cross referenced with the database results.

2.1 Selection and Exclusion Criteria

The following selection and exclusion criteria were utilised to identify literature to meet the aim of the review:

1. Concerning the use of robotic tails with mobile land-based robots to aid locomotion including static and dynamic stability while rolling (i.e. wheeled), walking, hopping/jumping and in free fall.
2. Excluding robotic tails for aquatic locomotion (including water walking), or to aid aerial robotic flight.

2.2 Study Selection Process

The first author (DC) conducted the initial database searches screening all the titles and abstracts. Duplicate records were removed using MATLAB code, which compared the identified paper titles from each database search result, followed by manual inspection of the results by the first author (DC). The full-text of the selected studies were then independently screened against the selection and exclusion criteria by two authors (DC and AW). Any disagreements were resolved through another author acting as a reviewer (WH).

2.3 Data Extraction and Presentation

After literature that met the inclusion criteria had been identified it was analysed and synthesised to identify approaches and corresponding technical details. The following information was extracted and tabulated:

- The Paper Structure, which is one of three categories:
  - Abstract Model: Papers that discussed tailed robots as an abstract mathematical concept, based on free body models that were were based on first principles.
  - Simulation: Papers that discussed a detailed virtual model of a tailed robot similar to a physical prototype.
  - Experimental: Papers that used a physical prototype or existing robot to generate experimental data.
- The Locomotion of the robot or abstract mechanical model, which contains one or more of the following keywords:
  - Walking: Leg-based locomotion (bipedal, quadrupedal etc.) on a solid surface with no aerial phase (e.g. Human).
  - Hopping: Leg-based locomotion with an aerial phase (e.g. Kangaroo).
  - Wheeled: Wheel-based locomotion (e.g. Car).
  - Tracked: Track-based locomotion (e.g. Tank).
- The Control System Architecture which categorises the control systems into 4 different types based on a simplified control schema. The types are defined as (and illustrated in Figure 5):
  1. A fully "blind" open-loop system with no feedback control whatsoever, the system is controlled by a fixed pattern or model that runs in a sequence and takes no inputs.
  2. An inner-loop system where position data from the tail actuators are used as inputs to the control system to control the tail position.

3. An outer-loop system where sensor data from the robot (IMU, accelerometer, gyroscope etc.) are used as inputs to the control system to control the tail position.
4. A multi-loop system where both sensor data from the robot and position data from the tail actuators are used as inputs to the control system to control the tail position.

- The Tail Structure, categorised into three types and described in Figure 3.
  - Rigid: Tail is constructed from one or more rigid bodies connected by joints. The joints move in order to move the tail, the bodies do not deform.
  - Flexible: Tail is constructed from one or more flexible bodies connected together rigidly. The bodies deform continuously in order to move the tail.
  - Pseudo-Flexible: Tail is constructed from a large number of rigid bodies connected by joints, so many that they approximate a flexible body (this is also referred to in some publications as “serpentine”, for example in Rone and Ben-Tzvi (2015, 2017); Saab and Ben-Tzvi (2016) and Saab et al. (2018a)).
- The Number of Segments in the tail, which corresponds to the number of bodies connected together with joints in the tail, not including the rest of the robot. Segments can be Active or Passive, depending on if they are directly controlled by an actuator.
- The Tail Dimension Class, which categorises the tails based on the movement space M of the tip of the tail (or end effector). If a volume V is conceptualised, centred around the base of the tail such that $M \subseteq V$, then M can be found on four distinct operations of V. The classes are defined as (and illustrated in Figure 4):
  - Class 1: M is on a curve on the surface of V.
  - Class 2: M is on a section of V.
  - Class 3: M is on the surface of V.
  - Class 4: M is a volume within (or equal to) V.
- The Tail Dimension Class, which categorises the tails based on the movement space of the tip of the tail (or end effector). The classes are defined as (and illustrated in Figure 4):
  1. A single revolute joint moves the tip along a circular arc on a plane.
  2. Multiple parallel revolute joints move the tip within a trimmed portion of a plane.
  3. Two perpendicular revolute joints move the tip within a trimmed portion of a spherical surface.
  4. Multiple perpendicular or parallel revolute joints move the tip within a volume.
- The Tail Degrees of Freedom which is typically the number of active segments multiplied by the dimensions each one can be actuated in.
- The Actuator that is used to move the active segments.
- The Tail Mass, in kilograms.
- The Body Mass, in kilograms.
- The Tail Length, in metres.

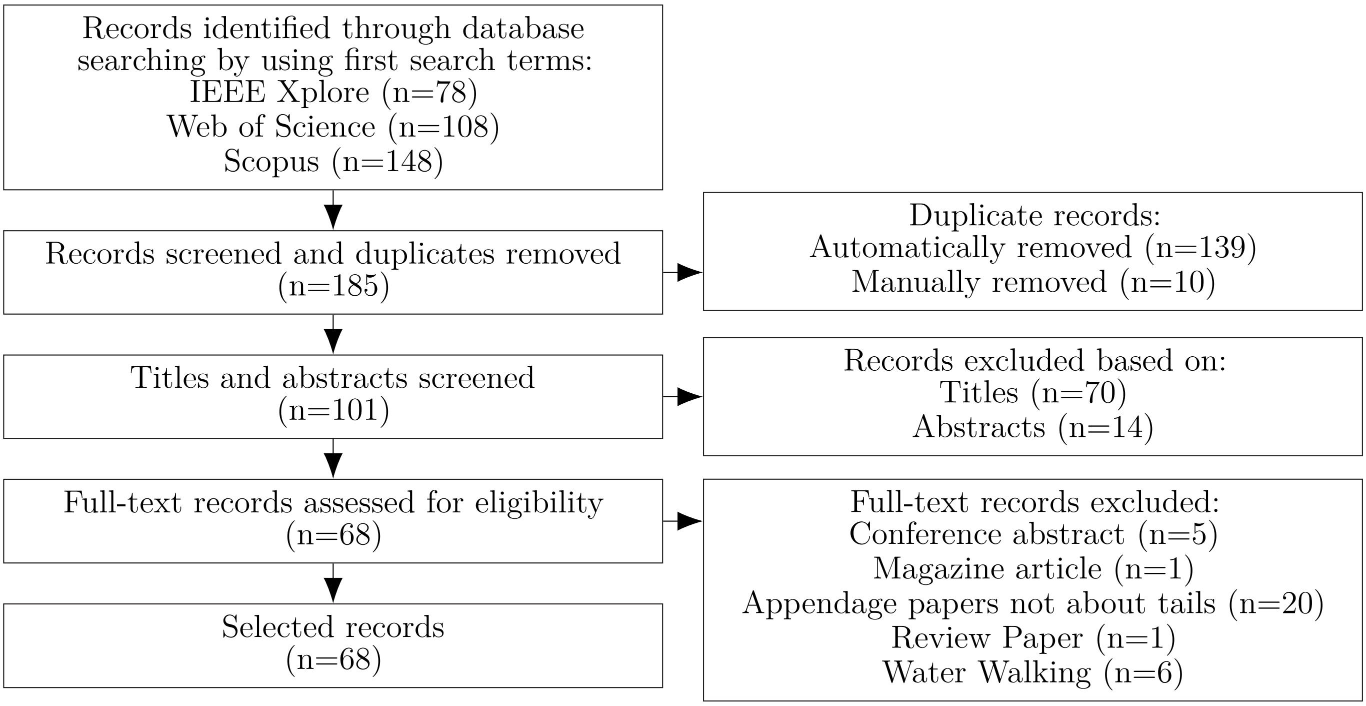


Figure 1. Flowchart of the study selection process.

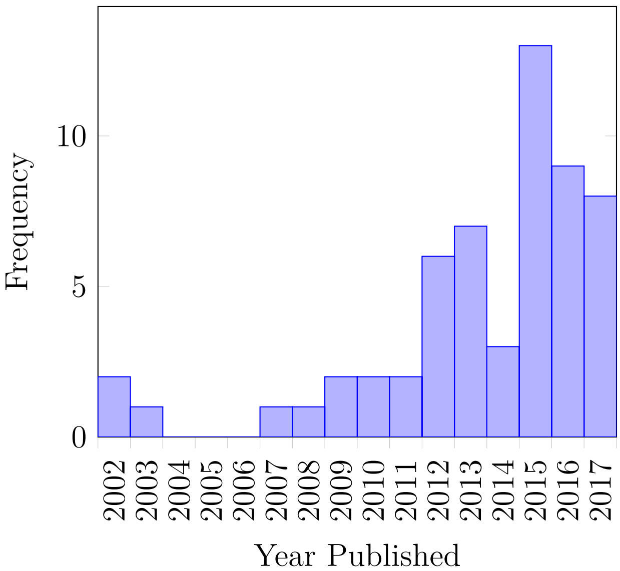


Figure 2. Histogram of the years the 68 selected records were published.

3 RESULTS

Figure 1 illustrates the flowchart of the study selection process and the papers identified. In total 185 studies were identified after duplicates were removed, this reduced to 101 papers after the titles and abstracts had been screened. Out of the 101 papers 33 were excluded because:

- The paper was a short abstract for a conference (5).
- The paper was a magazine article (1).

- The paper included the Appendage keyword in the Document Title but was not about tails (20).
- The paper was the review paper Saab et al. (2018b) (1).
- The paper concerned robots walking on water, like a pond skater (6).

This process led to the remaining 68 studies being analysed with data extracted, as detailed in Table 5. 33 unique physical robots have been identified from the records (that were physically experimented on in an Experimental paper), as detailed in Table 2. Since there were some robots which had multiple records associated with them, some of the records had some duplicate data, which was merged together for each individual physical robot. The data is displayed in Table 3. Papers which did not have a physical robot have this data displayed separately in Table 4. Figure 2 is a histogram of robotic tail papers published as a function of year, the first paper was published in 2002 and approximately 71% (42 papers) of the papers have been published in the last five years since 2013.

3.1 Paper Structure

Out of the 68 studies identified, 44 were Experimental papers, 16 were Abstract Model papers, and 8 were Simulation papers. Experimental papers typically develop a control system which is first verified on a simulated model (either an Abstract Model or a more complex Simulation) then build a prototype or use an existing robot to experimentally verify the control system.

3.2 Physical Robots

As explained previously, all Experimental papers included a physical robot or prototype. In total, 33 unique physical robots were found (images can be found in the supplementary material, all images have been sourced from selected papers unless specified). Out of these, 9 were used in multiple papers. 23 named robots were identified from the literature, the rest of the robots had no name and had only a single paper associated with them apart from Kim and Shell (2017) and Kim and Shell (2018). Table 2 lists the physical robots by name, with the papers they were referenced in and the year the first paper mentioning the robot was published. Table 3 lists the physical robots by their properties. Table 4 lists all the papers that do not have physical robots connected with them.

3.3 Research Objectives of Robots Identified in the Literature

Table 6 shows diagrams of non-unique research objectives that involved the tail operating in free space (i.e. not in contact with the ground or other objects). All of them involved counteracting or reducing torques.

3.3.1 Legged, Wheeled and Tracked

Of the 43 Experimental papers that included walking, wheeled, tracked or hopping robots, 11 had the objective of correcting any torques induced on the robot so it lands with the correct orientation (Briggs et al. (2012); Chang-Siu et al. (2011); De and Koditschek (2015); Johnson et al. (2012); Libby et al. (2012, 2016); Guan-Horng et al. (2014); Wenger et al. (2016); Jianguo et al. (2013, 2015b,a)), 8 had the objective of correcting torques induced by the unstable motion of the robot to prevent it falling over (Berenguer and Monasterio-Huelin (2008); Heim et al. (2016); Saab et al. (2018c); Simon et al. (2018); Takita et al. (2002a,b, 2003); Xiuli et al. (2016)), 4 had the objective of minimising roll torques to prevent the robot falling over during a turn (Aiello and Crespo (2013); Kohut et al. (2013); Patel and Braae (2013); Patel and Boje (2015)), and 3 had the objective of initiating a yaw torque on the robot, enabling it to have a smaller turning circle (Casarez et al. (2013); Kohut et al. (2012); Pullin et al. (2012)). 7 papers dealt with “tail-dragging” robots (Casarez and Fearing (2018); Guarnieri et al. (2009); Kim and Shell (2017, 2018); Kwak and Bae (2015); McInroe et al. (2016); Ren et al. (2009)), that had the tail acting as an appendage for additional stability, locomotion or object manipulation. Briggs et al. (2012) also considered use of a tail (along with rejecting angular momentum) to rebalance the robot following a disturbance, in their example, a “wrecking ball” impacting the torso of the robot. Sato et al. (2016) used the tail to allow their hopping robot to jump higher. The remaining 2 papers (Brill et al. (2015); De and Koditschek (2015)) did not state a specific objective.

3.3.2 No Locomotion

For the 11 papers that dealt with robots with no locomotion, the objectives were more varied. 5 had the objective of testing mechanisms for later inclusion on a legged robot (Rone and Ben-Tzvi (2014); Rone et al. (2017, 2018); Saab et al. (2018a); Iwamoto and Nishikawa (2018)). Chang-Siu et al. (2013) and Jusufi et al. (2010) had the objective of re-orienting a robot when dropped, Santiago et al. (2016) had a tail that was designed to vary its stiffness using a novel mechanism, in order for it to be used as both a “hard” appendage when used as a ground support, and a “soft” appendage for other functions. Kessens and Dotterweich (2017) used the tail as a self-righting mechanism, and Jovanova et al. (2018) considered a novel actuation system for a robot appendage based on a scorpion tail. The desire for mobile robotic systems which can explore hazardous and extreme environments has led to the development of systems which have greater functionality, adaptability, autonomy and dynamic ability. The enablers for the development of the next generation of mobile robotic systems include:

1. Increased simulation capabilities such that designs can be optimised before prototyping,
2. Advances in embedded computing power improving sensing and intelligent control systems. The capability of mobile robots, which can walk, run, hop and jump has created a need for the investigation and development of systems which can improve dynamic performance and robustness of outcome.

Many research groups have developed robotic tail models and physical systems for the purpose of improving the dynamic performance of mobile robots. Mobile robots utilising tail actuators have demonstrated improvements in performance, and they predominately have a limited number of degrees of freedom (Briggs et al. (2012); Heim et al. (2016); Libby et al. (2016); Sato et al. (2016)). There are several distinct areas for future research.

3.4 Robot Physical Properties

3.4.1 Walking Robots

Of the 12 walking robots, 3 were Bipedal (McInroe et al. (2016); Takita et al. (2002a,b, 2003); Berenguer and Monasterio-Huelin (2008)), 3 were Quadrupedal (Briggs et al. (2012); Heim et al. (2016); Xiuli et al. (2016)), 5 were Hexapedal (Kohut et al. (2012, 2013); Libby et al. (2016); Casarez et al. (2013); Casarez and Fearing (2018)), and 1 was Octopedal (Pullin et al. (2012)). (McInroe et al. (2016)) was Bipedal, but used the tail as a “third leg”, technically making it Tripedal.

3.5 Tail Physical Properties

3.5.1 Tail Structure

Table 3 illustrates that a rigid tail, made up of rigid bodies connected by joints, is the commonest physical tail structure with 31 robots, followed by a flexible structure, made up of flexible bodies that act as joints, with 4 robots, and pseudo-flexible, made up of a large number of mostly passive

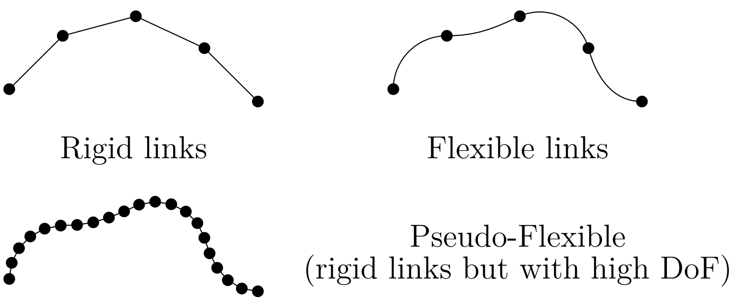


Figure 3. Tail structure classification. Black dots indicate individual joints which may be active or passive.

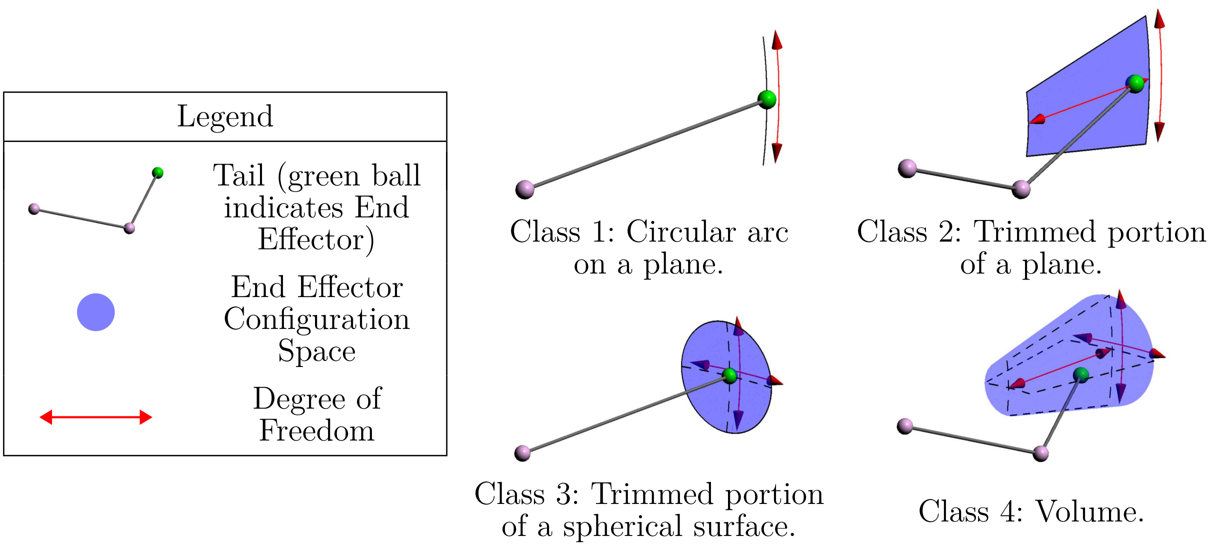


Figure 4. Tail Dimension Class visual illustration.

rigid joints that closely approximate a flexible body, with 3 robots (Figure 3 gives an illustration of this difference). Most of the non rigid robots were static experiments with no locomotion, apart from (Kim and Shell (2017, 2018)), though several (Rone et al. (2018); Saab et al. (2018a,c)) were testing static systems with an eventual aim of mounting on a legged robot.

3.5.2 Tail Segmentation

Table 3 illustrates that 22 robots had one tail segment, and 9 robots had more than one tail segment. Out of the 9 robots with more than one tail segment: 5 had 2-segments, 1 had 3-segments, 1 had 4-segments and 2 had 6-segments. Kim and Shell (2017, 2018) was a piece of unactuated flexible rope, which could be considered to have a nearly infinite number of segments. A common justification for an increased number of segments was the increased reaction torque available for a given length, as found in Rone and Ben-Tzvi (2016). This came at the cost of requiring additional actuators, except in Sato et al. (2016) which used a passive system, though the gains were marginal in that case (a 7% increase in jump height).

3.5.3 Tail Dimension Class

Tables 3 and 1 illustrate that 17 robots had a tail dimension class of 1, where the range of motion for the tail end effector is restricted to a circular arc on a plane, typically a simple “pendulum” design of a mass on the end (or along) a rod of negligible mass with a rotary joint that allowed the robot to adjust its moment of inertia in one axis when performing a manoeuvre. 9 robots had a similar design but with an extra degree of freedom to turn the the single revolute joint into two perpendicular revolute joints, giving the tail a dimension class of 3, where the end effector range is restricted to the trimmed surface of a sphere, typically for the purpose of allowing the robot to induce torques in two axes instead of one (such as aerial reorientation in both pitch and roll axes). Inducing torques in all three axes did not appear to be considered, as in stability applications maintaining yaw angle was not required. More complicated multi-segment designs were also found in 9 robots, which all had a tail dimension class of 2, where the end effector is restricted to a planar cross-section of a volume, or 4, where the end effector is free to move within a volume, typically for increased reaction torques as mentioned previously (Figure 4 gives an illustration of the different classes). Finally, 3 robots had a static tail. Of the 12 physical robots which have been developed for walking, 9 had a tail dimension class of 1, and 3 robots had a class of 3. For the other types of locomotion, hopping, wheeled, wheeled and hopping, and tracked there were too few papers and different class categories to determine correlations.

3.5.4 Tail Degrees of Freedom

As can be seen from Table 1, 17 robots had 1 degree of freedom, 11 robots had 2 degrees of freedom, 1 robot had 3 DoF, 2 robots had 4 DoF, 1 robot had 6 DoF and 1 robot had 7 DoF. 1 robot had a static tail (Ren et al. (2009); Ren and Hong (2010)) and 1 robot (Kim and Shell (2017, 2018)) had an unactuated rope which had infinite degrees of freedom. Higher degrees of freedom than 2 exclusively corresponded to multi-segment designs with the corresponding performance improvements, whereas 2 degrees of freedom was a mix of additional torque axis (dimension class 3) and multi-segment designs (dimension class 2).

3.6 Tail Actuation

As can be seen from Table 3, 26 robots, or 91% used a Revolute Motor to actuate the tail. 4 robots used revolute motors to move cables via pulleys. For the other 9%, 1 robot had a static tail (Ren et al. (2009); Ren and Hong (2010)), 1 robot had an unactuated completely passive tail (Kim and Shell (2017, 2018)), and 1 robot also used stored energy via a spring instead of stored electrical energy (Jusufi et al. (2010)).

3.7 Control Systems

3.7.1 Controller/Model

As can be seen in Figure 5, each system can be described as having a Controller/Model, where the commands for controlling the tail actuators (whether real or virtual) are generated. These can be described as “fixed” or “variable”. Fixed systems (type 1 and type 2) do not accept external input from the robot, running a periodic sequence or pattern, or following remote commands sent by a user. This is a simple control system to implement, and in some highly deterministic stability applications or experiments it is sufficient for satisfactory performance. Variable systems (type 3 and type 4) use sensor data from the robot to influence the output of the Controller/Model, typically when using the tail to correct or induce force, in a quasi feedback loop (Figure 5 shows a block diagram of each controller/model). As can be seen in Table 5, 7 papers described a type 1 (open-loop) system, 15 papers described a type 2 (inner-loop) system (typically due to the use of servo motors, which turn any system they are implemented in into at least inner-loop), 18 papers described a type 3 (outer-loop) system, and 13 papers described a type 4 (multi-loop) system. 5

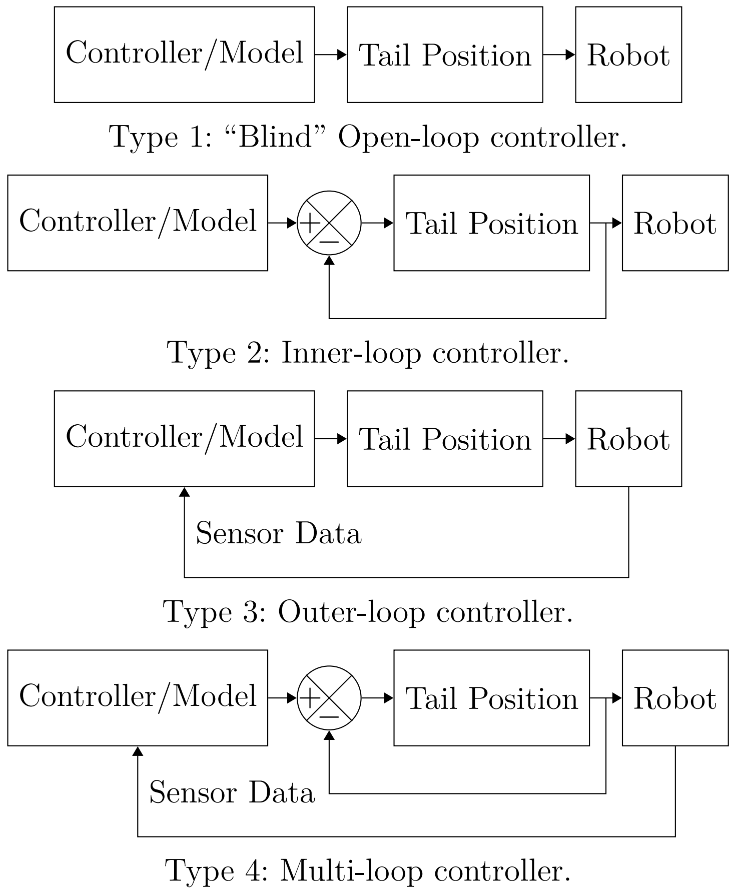


Figure 5. Control system classification for robotic tails.

papers were either static or uncontrolled systems, and 10 papers did not consider, or did not have enough information to determine, a control system.

There didn't appear to be any noticeable correlations between the control system and other properties of the robots, as it depended on the experimental setup, and whether the system was designed to apply to determined torques induced by robot actions (such as a walking or hopping motion) or undetermined torques from the environment (such as driving off a ledge or navigating uneven terrain).

3.7.2 Feedback Control Systems

For position feedback of the tail joints (type 2 and type 4), P (Berenguer and Monasterio-Huelin (2008)), PD (Berenguer and Monasterio-Huelin (2008); Chang-Siu et al. (2013); Guan-Horng et al. (2014); Machairas and Papadopoulos (2015a); Sato et al. (2016)), PI (Patel and Braae (2014)), PID (Kwak and Bae (2015); Pullin et al. (2012); Casarez and Fearing (2017); Saab et al. (2018c)) and State Feedback (Patel and Braae (2014)) control systems were used.

For variable Controller/Model systems (type 3 and type 4), P (Chang-Siu et al. (2013); Mutka et al. (2013)), PD, (Chang-Siu et al. (2011, 2013); Graichen and Hentzelt (2015); Jianguo et al. (2013, 2015b); Johnson et al. (2012); Libby et al. (2012, 2016); Machairas and Papadopoulos (2015a); Rone and Ben-Tzvi (2017); Xiaoyun et al. (2015)), PI (Patel and Braae (2013, 2014)) PID (Pullin et al. (2012)) and State Feedback (Patel and Braae (2013)) control systems relating sensor data to tail joint position. Kohut et al. (2013) used a simple Bang/Bang control system due to the variable friction present on the model.

Regarding performance of different control systems, Berenguer and Monasterio-Huelin (2007) outlined a simulation comparing a P and PD control law, and found a marginal but noticeable increase in performance in the PD control law (a 6% increase in “crossed distance” and a 9% reduction in mechanical energy), and Jianguo et al. (2015b) compared PD and sliding mode control, again finding an increase in performance (a 75% reduction in overshoot for the tail controller) for sliding mode control.

3.8 Locomotion/Tail Dimension Class

Table 1 shows the relationship between the robot locomotion and the tail dimension class. Class 1 was the most prevalent in all of the mobile robots, followed by class 3, whereas static experiments typically used more complex tails. Tail dimension class was generally associated with the axes the tail was designed to induce torques on, with class 1 only able to induce torque on a single axis, and class 2 being a multi segment version of class 1. Class 3 and 4 could induce torques on 2 or more axes, allowing for enhanced functionality, such as being able to control both the pitch and roll angle of the robot in aerial reorientation.

4 DISCUSSION

4.1 Potential Future Research

4.1.1 Dynamically Changing Plant

All the studies appear to consider robots (when without their tails) that had a static mass, centre of mass and tensor of inertia. Future work could consider robots with these properties changing dynamically. It is likely that mobile robotic systems will pick up and manoeuvre payloads, carry unstable payloads such as a container filled with fluid, or will have an appendage such as a manipulator arm that is independent of locomotion. As such there will be a motivation for robotic tails to be utilised to compensate for this dynamic behaviour.

4.1.2 Energy Consumption and Storage

The use of energy in control systems is a well understood topic that has been the subject of many publications, such as Ortega et al. (2001), hence the control of the tail may be associated with the consumption of energy. However, the literature reviewed does not indicate there has been significant consideration regarding the energy consumption of the robotic tails. There are some calculations presented regarding peak power output in order to determine actuator specifications (Johnson et al. (2012)), but nothing considering actual energy consumption. There is likely to be a trade-off between energy consumption and the improved robot agility that a robotic tail enables. Furthermore, there are few details in the literature regarding the energy storage needs to enable the increased actuation of a robotic tail, clearly additional storage needs will add mass and therefore impact the dynamic behaviour of the mobile robotic system. The authors would encourage the community to provide more details regarding their systems to enable comparisons between different actuator, sensor and controller configurations.

4.2 Actuator Technologies

The choice of actuator for mobile robotics systems is crucial for achieving the potential increased agility desired. DC brushed and brushless electric motors offer good speed and torque characteristics but will add significant mass and for large numbers of degrees of freedom increase control complexity.

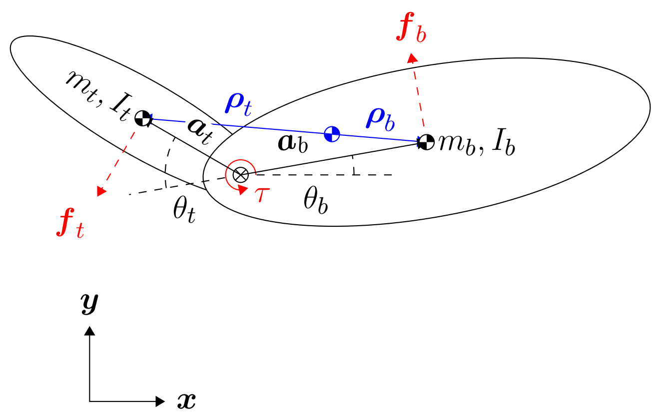


Figure 6. A planar rigid body diagram of a generic robot with a 1 DoF tail.

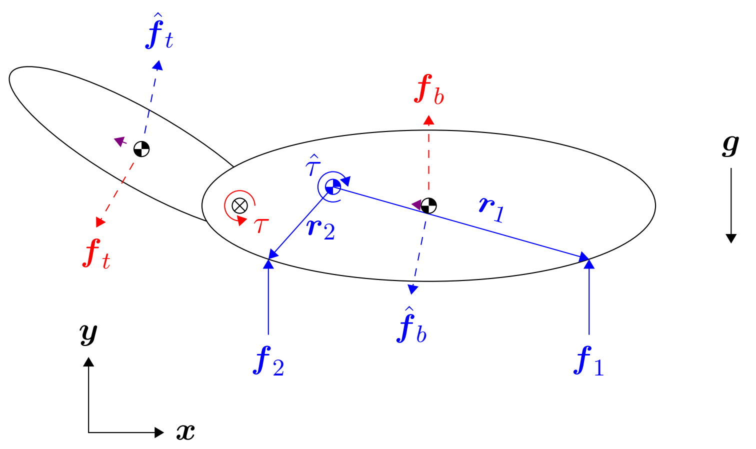
Stepper motors decrease control complexity in comparison to DC brushed/brushless motors but will add significant mass and have limited capability for high-speed operation. Relative to electric motor solutions artificial muscles are much lighter whilst having suitable force characteristics, their main limitations include hysteretic behaviour and bandwidth. The search for low mass, high force/torque, high bandwidth actuators will no doubt continue.

5 CONCLUSION

The desire for mobile robotic systems which can explore hazardous and extreme environments has led to the development of systems which have greater functionality, adaptability, autonomy and dynamic ability. The capability of mobile robots which can walk, run, hop and jump, has created a need for the investigation and development of systems which can improve dynamic performance and robustness of outcome. Many research groups have developed robotic tail models and physical systems for the purpose of improving the dynamic performance of mobile robots. Mobile robots utilising tail actuators have demonstrated improvements in performance, predominately these have a limited number of degrees of freedom. Barriers that may inhibit the development of robotic tail systems for mobile robots include the additional storage/drain on system energy supply, high performance low mass actuation for multiple degrees of freedom and complexity of control. There is clearly the potential for further research in this field, which could see improved dynamic performance, and robustness for mobile robotic systems. Robotic tails offer great potential to improve the dynamic performance of mobile robotic platforms. Research in this area has grown over the last 10 years with modelling/simulation and experimental approaches adopted, demonstrating robotic tails can improve performance. The authors hope that this scoping review will provide a useful reference for those research groups working in this area and those who wish to contribute in the future.

6 RIGID BODY MODELS

In order to understand the diverse range of functions a robot tail can perform, a planar rigid body model can be used in order to simplify and abstract the dynamics of each application. In essence, any robot with a tail can be described as two bodies, the main robot body with mass m_b and inertia I_b, and the tail with mass m_t and inertia I_t, joined by a pivot which can generate a torque τ in one or more axes. For a robot with a multi-segment tail, any configuration of the joints in the tail can be abstracted into a single pseudo-body and a base pivot torque with suitable dynamics calculations. The other coefficients of the system are $\boldsymbol{a}_b$ and $\boldsymbol{a}_t$, which denote the vectors from each body's COM



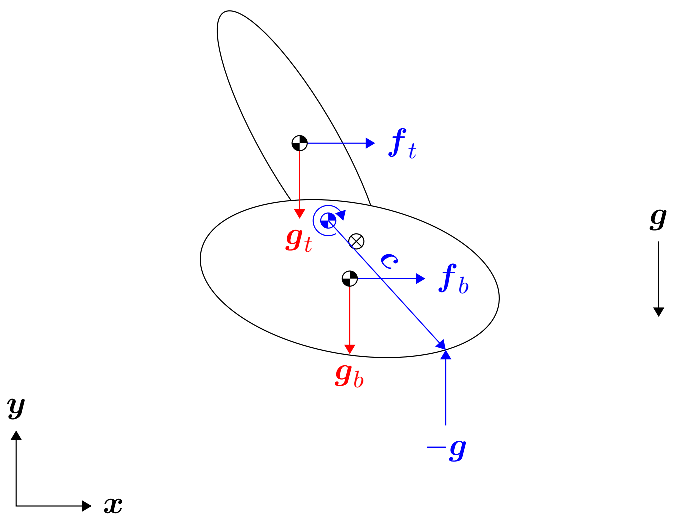
to the pivot. The kinematic state of the system can then be described with θ_b and θ_t, denoting the absolute rotation of the robot body and the relative rotation of the tail body to the robot body, and $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$, denoting the vectors from the origin $\boldsymbol{\rho}$ to each body. The origin can equal the COM of the model with the constraint $m_b\boldsymbol{\rho}_b + m_t\boldsymbol{\rho}_t = \boldsymbol{0}$. When no other external forces are present, the resultant torque $\boldsymbol{T}$ of the model is only influenced by the pivot torque. θ_b can then be calculated by integration from the sum of the moments of inertia and the resultant torque, as in equation 1 where $\boldsymbol{z}$ is the normal unit vector to the plane.

$$\begin{aligned}
\boldsymbol{f}_b &= \boldsymbol{a}_b \times \tau\boldsymbol{z} \\
\boldsymbol{f}_t &= \boldsymbol{a}_t \times \tau\boldsymbol{z} \\
\boldsymbol{T} &= \boldsymbol{\rho}_b \times \boldsymbol{f}_b + \boldsymbol{\rho}_t \times \boldsymbol{f}_t \\
\ddot{\theta}_b &= \frac{I_b + I_t}{\boldsymbol{T}}
\end{aligned} \tag{1}$$

In all of the functions examined here, $\dot{\theta}_b$ is either being minimised or controlled. If it is being minimised it is due to a disturbance by unbalanced external forces that would otherwise push the robot into an unstable state. If it is being controlled it is deliberately changing the orientation of the robot in order to achieve an objective, such as skidding in order to change direction or landing vertically on a wall to climb up it .

6.1 Aerial Reorientation

Many robots use jumping as a means of locomotion, or to reach otherwise inaccessible areas . Other robots are designed to drive off ledges . The “jump” can be described as impulse forces $\boldsymbol{f}_n$ on one or more points on the main robot body (such as the foot of a leg), where $\boldsymbol{r}_n$ describe the vectors from the robot COM to the points. As long as $\sum_{i=1}^{n} \boldsymbol{f}_i \times (m_b + m_t)\boldsymbol{g} > (m_b + m_t)\boldsymbol{g}$, where $\boldsymbol{g}$ is the gravity vector, then the robot will be lifted into the air. However, if $\sum_{i=1}^{n} \boldsymbol{f}_i \times \boldsymbol{r}_i \neq \boldsymbol{0}$, then $\boldsymbol{T} \neq \boldsymbol{0}$, and therefore $\ddot{\theta}_b \neq 0$. This could result in the robot landing with an orientation that prevents locomotion or causes damage to the robot. This is certain to occur to some extent in a practical application, as even if all the contact points are evenly spaced from the COM ($\sum_{i=1}^{n} \boldsymbol{r}_i = \boldsymbol{0}$), all the elements of $\boldsymbol{f}_n$ are never going to be exactly equal due to differences in actuation.



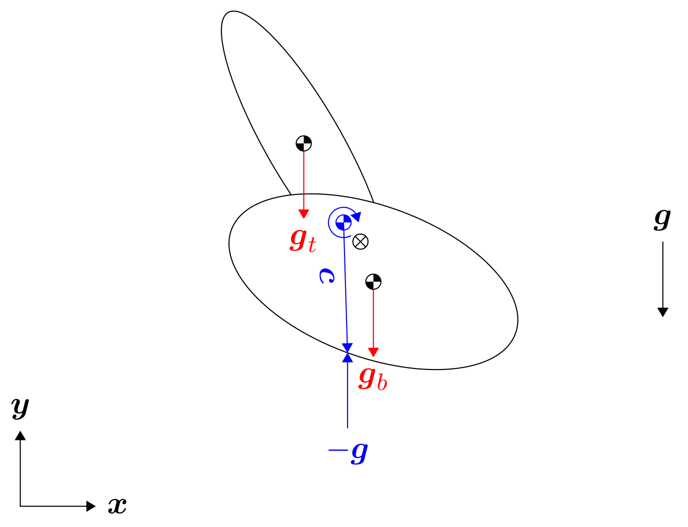
In order to reduce this rotation so the robot remains stable, the pivot torque can provide an opposing torque on the model such that θ_b remains within an acceptable interval for stability upon landing. This can be done by keeping $\boldsymbol{T}$ as small as possible. Or rotation may want to be controlled so the robot's orientation is changed to a new interval that is stable for a new environment, such as climbing a wall. In which case, a controller can output a desired $\boldsymbol{T}$ for an angle error $q - \theta_b$, where q is the desired body angle.

$$\begin{aligned}
\hat{\tau} &= \sum_{i=1}^{n} \boldsymbol{f}_i \times \boldsymbol{r}_i \\
\hat{\boldsymbol{f}}_b &= \boldsymbol{\rho}_b \times \hat{\tau}\boldsymbol{z} \\
\hat{\boldsymbol{f}}_t &= \boldsymbol{\rho}_t \times \hat{\tau}\boldsymbol{z} \\
\boldsymbol{T} &= \boldsymbol{\rho}_b \times \left(\boldsymbol{f}_b + \hat{\boldsymbol{f}}_b\right) + \boldsymbol{\rho}_t \times \left(\boldsymbol{f}_t + \hat{\boldsymbol{f}}_t\right) = \boldsymbol{\rho}_b \times \boldsymbol{f}_b + \boldsymbol{\rho}_t \times \boldsymbol{f}_t + \hat{\tau}
\end{aligned} \tag{2}$$

6.2 Centrifugal Force Compensation

When a robot is turning at speed, the effects of centrifugal force can be significant. Depending on the height between the COM and the ground, this can result in a significant torque on the robot which can cause it to tip over during the turn once θ_b is outside a stable interval. The centrifugal force can be modelled as a force $\boldsymbol{f}_{b,t}$ on the body and tail of the model in the opposite direction to the turn. These depend on the mass of the body, the velocity of the robot v and the radius r of the turn. There is also a ground reaction force $-\boldsymbol{g}$ which acts on the contact point with the ground furthest in the centrifugal force direction.

In order to maintain θ_b within a stable interval, the tail can be used to shift the COM of the robot in the opposite direction of the centrifugal force. This may normally cause θ_b to be outside the other bound of the stable interval when the robot is not in a turn, but during the turn can be used to maintin stability as long as the position of the tail is reset once the turn is complete. This again can be done by keeping $\boldsymbol{T}$ as small as possible.



$$
\begin{aligned}
\boldsymbol{f}_b &= m_b \frac{v^2}{r} \boldsymbol{x} \\
\boldsymbol{f}_t &= m_t \frac{v^2}{r} \boldsymbol{x} \\
\boldsymbol{g}_b &= m_b \boldsymbol{g} \\
\boldsymbol{g}_t &= m_t \boldsymbol{g} \\
-\hat{\boldsymbol{g}} &= (m_b + m_t) \times -\boldsymbol{g} \\
\boldsymbol{T} &= \boldsymbol{\rho}_b \times (\boldsymbol{g}_b + \boldsymbol{f}_c) + \boldsymbol{\rho}_t \times (\boldsymbol{g}_t + \boldsymbol{f}_c) + \boldsymbol{c} \times -\boldsymbol{g}
\end{aligned}
\tag{3}
$$

6.3 Walking Stability

This function has a particularly broad scope, but can be generally considered in a similar fashion to centrifugal force compensation, but without the centrifugal force. Instead the instability can be caused by the suddenly changing location of ground reaction forces due to rough terrain , or an unpredictable external disturbance such as an impact Briggs et al. (2012). A simple example of this function can be modelled on an inverted double pendulum, analogous to walking on a tightrope. Even minute changes in ground reaction force or very lateral small disturbances, will, if uncompensated, inevitably lead to θ_b being outside a stable interval.

Maintaining stability is again, similar to centrifugal force compensation, shifting the COM of the robot to counteract the change or disturbance.

$$
\begin{aligned}
\boldsymbol{g}_b &= m_b \boldsymbol{g} \\
\boldsymbol{g}_t &= m_t \boldsymbol{g} \\
-\boldsymbol{g} &= (m_b + m_t) \times -\boldsymbol{g} \\
\boldsymbol{T} &= \boldsymbol{\rho}_b \times \boldsymbol{g}_b + \boldsymbol{\rho}_t \times \boldsymbol{g}_t + \boldsymbol{c} \times -\boldsymbol{g}
\end{aligned}
\tag{4}
$$

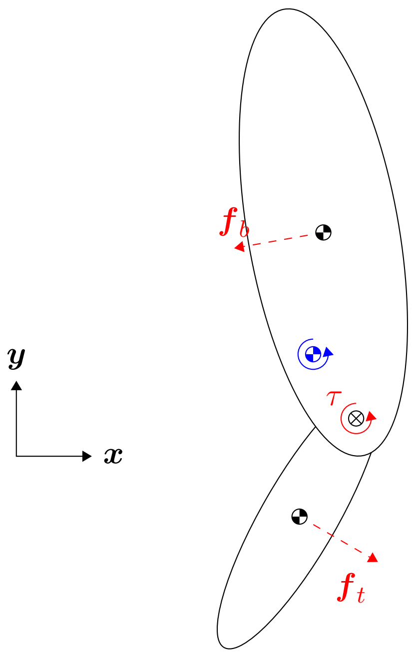
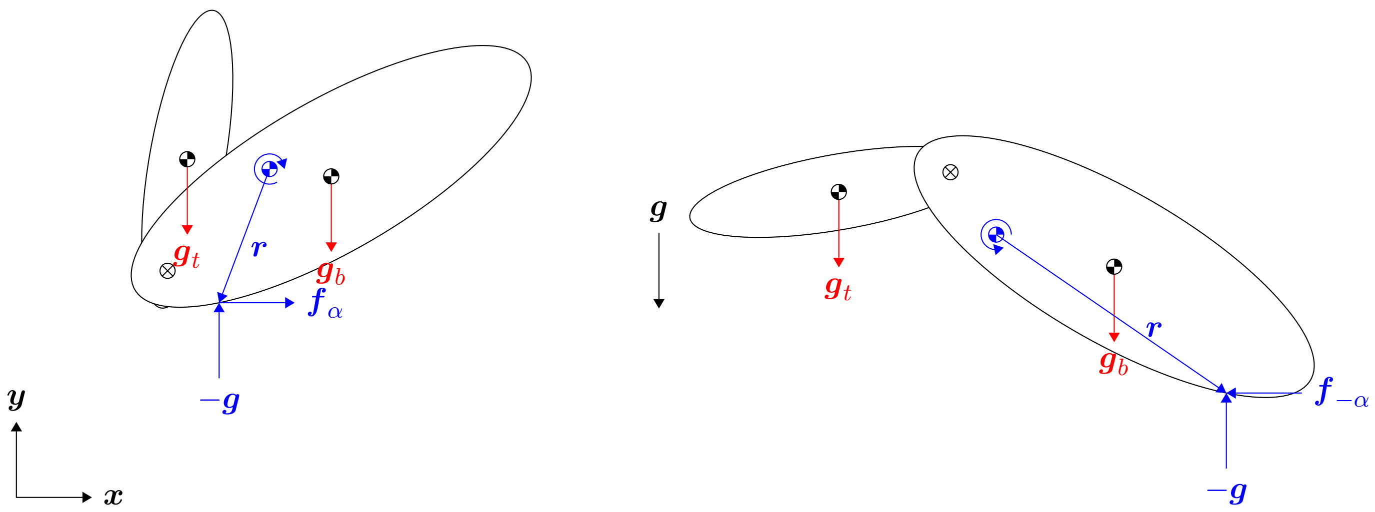


Figure 7.



424 6.4 Induced Steering

425 This function is very similar to aerial reorientation in its model, but with no gravity to consider as the robot is on the ground. In this case the objective is to rotate the entire robot around the z axis for the purposes of changing the direction of locomotion. This can be achieved by changing the resultant torque T by changing τ for a target θ_b. This is useful for small, light robots where the friction force is low, allowing the robot to “skid” across a surface.

$$\boldsymbol{T} = \boldsymbol{\rho}_b \times \boldsymbol{f}_b + \boldsymbol{\rho}_t \times \boldsymbol{f}_t \tag{5}$$

430 6.5 Start/Stop Inertia Compensation

431 Much like when a robot turns at speed, when the robot starts and stops there are forces on the robot which can induce enough torque to cause instability. These are the acceleration $\boldsymbol{f}_\alpha$ and

braking $\boldsymbol{f}_{-\alpha}$ forces which act on the contact point between the robot and the ground furthest in the direction of the force. This will cause the robot to tip forward or back, potentially putting θ_b outside the stable interval. These forces are generated by the actuation of legs, wheels, or other locomotion device, and depend on the robot's rate of acceleration and decelration.

Maintaining a stable interval involves a similar method to centrifugal force compensation, moving the tail to shift the COM.

$$\boldsymbol{T} = \boldsymbol{\rho}_b \times \boldsymbol{g}_b + \boldsymbol{\rho}_t \times \boldsymbol{g}_t + \boldsymbol{r} \times \left(\boldsymbol{f}_{\alpha,-\alpha} - \boldsymbol{g}\right) \tag{6}$$

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Table 1. Comparison of Robot Locomotion to Tail Dimension Class.

Locomotion	Tail Dimension Class					Total
	1	2	3	4	0 (Static)	
Walking	9	0	3	0	0	12
Hopping	1	2	1	0	0	4
Wheeled	2	1	2	1	1	7
Wheeled, Hopping	1	0	0	0	0	1
Tracked	1	0	0	0	0	1
None	1	1	2	4	0	8
Total	15	4	8	5	1	33

Table 2. Table of all the physical robots, the year it was first seen in a selected paper and the selected papers it was found in.

References	Robot Name	First Published
Kessens and Dotterweich (2017)	3DoF No Catch	2017
Heim et al. (2016)	Cheetah-Cub	2016
Saab et al. (2018a)	DMST	2018
Xiuli et al. (2016)	Dcat	2016
Patel and Braae (2014, 2013)	Dima	2013
Patel and Boje (2015)	Dima II	2015
Guarnieri et al. (2009)	Helios VIII	2009
Ren and Hong (2010); Ren et al. (2009)	IMPASS	2009
Casarez and Fearing (2018)	LoadRoACH	2018
Briggs et al. (2012)	MIT Cheetah	2012
Jianguo et al. (2015a,b, 2013)	MSU Tailbot	2013
McInroe et al. (2016)	MuddyBot	2016
Pullin et al. (2012)	OctoRoACH	2012
De and Koditschek (2018); Shamsah et al. (2018); Wenger et al. (2016); De and Koditschek (2015); Brill et al. (2015)	Penn Jerboa	2015
Saab et al. (2018c)	R3-RT	2018
Kwak and Bae (2015)	RoMiRAMT	2015
Kohut et al. (2013, 2012)	TAYLRoACH	2012
Libby et al. (2012); Chang-Siu et al. (2011)	Tailbot	2011
Takita et al. (2003, 2002b,a)	Titrus III	2002
Rone et al. (2018)	USRT	2018
Casarez and Fearing (2017)	VelociRoACH	2017
Libby et al. (2016); Johnson et al. (2012)	XRL/RHex	2012
Berenguer and Monasterio-Huelin (2008)	Zappa	2008
Simon et al. (2018)	-	2018
Casarez et al. (2013)	-	2013
Chang-Siu et al. (2013)	-	2013
Jusufi et al. (2010)	-	2010
Guan-Horng et al. (2014)	-	2014
Aiello and Crespo (2013)	-	2013
Sato et al. (2016)	-	2016
Santiago et al. (2016)	-	2016
Kim and Shell (2018, 2017)	-	2017
Jovanova et al. (2018)	-	2018

Table 3. Table of the physical robots. "Tail DoF" refers to the total number of DoF, including passive joints. Robots with passive DoF are denoted with a superscript. Number of decimal places reflects the precision found in the references.

Reference	Locomotion	Tail Structure	Number of Segments	Tail Dimension Class	Tail DoF	Actuator	Tail Mass (kg)	Body Mass (kg)	Tail Length (m)
Kessens and Dotterweich (2017)	None	Rigid	2	4	3	Revolute Motor	0.33	0.99	N/A
Heim et al. (2016)	Walking	Rigid	1	1	1	Revolute Motor	0.053	1.197	0.128, 0.168
Saab et al. (2018a)	None	Pseudo-Flexible	2	3	2	Revolute Motor[3]	3.5	9.525	0.3
Xiuli et al. (2016)	Walking	Rigid	1	1	1	Revolute Motor	0.25	5.312	0.3
Patel and Braae (2014, 2013)	Wheeled	Rigid	1	3	2	Revolute Motor	0.4	5	0.5
Patel and Boje (2015)	Wheeled	Rigid	1	3	2	Revolute Motor	0.4	5	0.5
Guarnieri et al. (2009)	Tracked	Rigid	1	1	1	Revolute Motor	N/A	N/A	0.5
Ren and Hong (2010); Ren et al. (2009)	Wheeled	Rigid	1	0	0[1]	Static Tail	N/A	N/A	0.889
Casarez and Fearing (2018)	Walking	Rigid	1	1	1	Revolute Motor	N/A	N/A	0.09
Briggs et al. (2012)	Walking	Rigid	1	3	2	Revolute Motor	0.74	35	0.54
Jianguo et al. (2015a,b, 2013)	Wheeled, Hopping	Rigid	1	1	1	Revolute Motor	0.017	0.0252	0.127
McInroe et al. (2016)	Walking	Rigid	1	3	2	Revolute Motor	N/A	N/A	0.02
Pullin et al. (2012)	Walking	Rigid	1	1	1	Revolute Motor	0.017	0.035	0.1
De and Koditschek (2018); Shamsah et al. (2018); Wenger et al. (2016); De and Koditschek (2015); Brill et al. (2015)	Hopping	Rigid	1	3	2	Revolute Motor	0.15	2.269	0.21
Saab et al. (2018c)	None	Pseudo-Flexible	2	2	2	Revolute Motor	N/A	8.8	N/A
Kwak and Bae (2015)	Wheeled	Rigid	2	2	2	Revolute Motor	N/A	N/A	N/A
Kohut et al. (2013, 2012)	Walking	Rigid	1	1	1	Revolute Motor	0.004	0.045	0.115
Libby et al. (2012); Chang-Siu et al. (2011)	Wheeled	Rigid	1	1	1	Revolute Motor	0.017	0.16	0.127
Takita et al. (2003, 2002b,a)	Walking	Rigid	1	3	2	Revolute Motor	0.1	0.1	N/A
Rone et al. (2018)	None	Rigid	6	4	6	Revolute Motor[3]	0.51	6.507	0.48
Casarez and Fearing (2017)	Walking	Rigid	1	1	1	Revolute Motor	0.008	0.0767	0.09
Libby et al. (2016); Johnson et al. (2012)	Walking	Rigid	1	1	1	Revolute Motor	0.6	8.1	0.59
Berenguer and Monasterio-Huelin (2008)	Walking	Rigid	1	1	1	Revolute Motor	0.7	0.05	0.15
Simon et al. (2018)	Hopping	Flexible	4	2	4	Revolute Motor[3]	0.047	N/A	0.21
Casarez et al. (2013)	Walking	Rigid	1	1	1	Revolute Motor	0.005	0.0395	0.12
Chang-Siu et al. (2013)	None	Rigid	1	3	2	Revolute Motor	0.07	0.105	0.73
Jusufi et al. (2010)	None	Rigid	1	1	1	Spring	0.048	0.204	N/A
Guan-Horng et al. (2014)	Hopping	Rigid	1	1	1	Revolute Motor	0.371	0.423	0.177
Aiello and Crespo (2013)	Wheeled	Rigid	1	1	1	Revolute Motor	0.089	0.862	0.27
Sato et al. (2016)	Hopping	Pseudo-Flexible	6	2	6[2]	Revolute Motor	0.1	1.045	0.235
Santiago et al. (2016)	None	Flexible	2	4	4	Revolute Motor[3]	N/A	N/A	0.41
Kim and Shell (2018, 2017)	Wheeled	Flexible	∞	4	∞	Unactuated	0.035	0.7	0.7
Jovanova et al. (2018)	None	Flexible	3	4	7	Revolute Motor	N/A	N/A	N/A

[1](Static) [2](1 Active, 5 Passive) [3](Cable Driven)

Table 4. Table of all the papers that did not use physical robots. "Tail DoF" refers to the total number of DoF, including passive joints. Papers with passive DoF are denoted with a superscript. Number of decimal places reflects the precision found in the references.

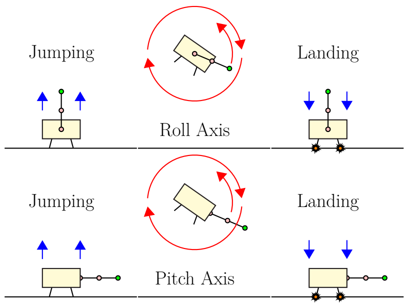
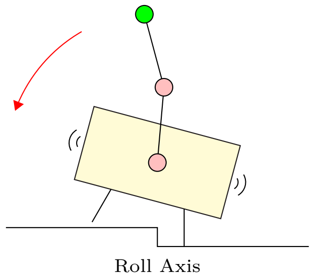
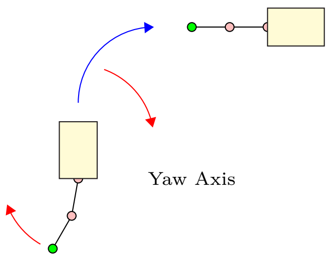
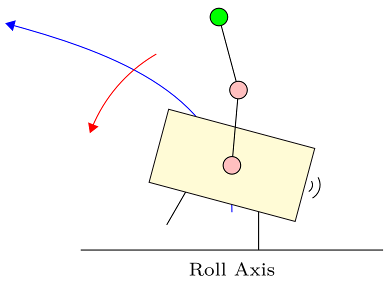
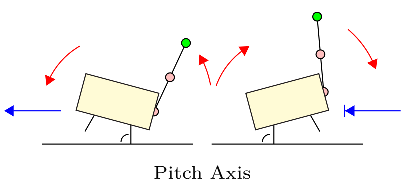
Reference	Locomotion	Tail Structure	Number of Segments	Tail Dimension Class	Tail DoF	Actuator	Tail Mass (kg)	Body Mass (kg)	Tail Len (m)
Berenguer and Monasterio-Huelin (2007)	Walking	Rigid	1	1	1	N/A	0.7	N/A	0.15
Graichen and Hentzelt (2015)	Hopping	Rigid	1	1	1	N/A	N/A	N/A	N/A
Iwamoto and Yamamoto (2015)	Wheeled	Pseudo-Flexible	10	1	10[1]	Revolute Motor	0.307	N/A	0.2
Iwamoto and Nishikawa (2018)	N/A	Rigid	N/A	N/A	Multiple	N/A	N/A	N/A	N/A
Karakasiliotis et al. (2012)	Walking	Flexible	10	2	12[2]	N/A	1327.96 (mg)	N/A	0.26
Xiaoyun et al. (2015)	Wheeled	Rigid	1	1	1	N/A	N/A	N/A	N/A
Liu and Ben-Tzvi (2018)	Walking	Rigid	1	3	2	N/A	1.4347	26.9078	0.6
Machairas and Papadopoulos (2015b)	Walking	N/A	1	1	1	N/A	N/A	N/A	N/A
Machairas and Papadopoulos (2015a)	N/A	Rigid	1	1	1	N/A	0.5-4	N/A	0.2-0.5
Mutka et al. (2013)	Walking, Hopping	Rigid	1	3	2	N/A	N/A	N/A	0.1
Patel and Braae (2015)	Walking	Rigid	1	1	1	N/A	1	N/A	0.8
Rone and Ben-Tzvi (2014)	None	Flexible	1/2	3/4	2/4	Linear Screw Motor[3]	2.25	N/A	0.5
Rone and Ben-Tzvi (2015)	None	Pseudo-Flexible	2	3	4	Cable Driven	0.33585	N/A	0.44
Rone and Ben-Tzvi (2016)	Walking	Rigid	6	2	1–6	N/A	2.4	N/A	0.5
Rone et al. (2017)	None	Rigid	6	4	6	N/A	0.449	N/A	0.48
Rone and Ben-Tzvi (2017)	Walking	Pseudo-Flexible	12	4	3	Revolute Motor	3.96	N/A	0.51
Saab and Ben-Tzvi (2016)	Walking	Pseudo-Flexible	2	3	2	Revolute Motor[3]	1	N/A	0.12
Saab and Ben-Tzvi (2017)	Walking	Rigid	1	1	1	N/A	0.3	15	1.0
Sadati and Meghdari (2017)	Walking	Rigid	1	3	2	N/A	0.026, 0.00753	N/A	0.10 0.11
Shin et al. (2011)	Wheeled	Rigid	1	1	1	N/A	N/A	N/A	N/A
Yu et al. (2017)	Hopping	Rigid	1	1	1	N/A	1.5	28.5	0.4

[1](1 Active, 9 Stiffness Adjustment) [2](10 Active, 2 Passive) [3](Cable Driven)

Table 5. Comparison of control system classification with paper structure. In addition to the classifications specified in figure 5, 0 indicates a tail with no control system.

	Control System Classification					
Paper Category	N/A	0	1	2	3	4
Abstract Model	Machairas and Papadopoulos (2015b); Rone and Ben-Tzvi (2014, 2016); Saab and Ben-Tzvi (2017)	Ren and Hong (2010)	Berenguer and Monasterio-Huelin (2007)	-	Graichen and Hentzelt (2015); Iwamoto and Yamamoto (2015); Mutka et al. (2013); Patel and Braae (2015); Xiaoyun et al. (2015); Yu et al. (2017)	Machairas and Papadopoulos (2015a); Sadati and Meghdari (2017); Shamsah et al. (2018); De and Koditschek (2018)
Modelling & Simulation	Rone and Ben-Tzvi (2015); Saab and Ben-Tzvi (2016); Shin et al. (2011); Rone et al. (2017); Iwamoto and Nishikawa (2018)	-	Karakasiliotis et al. (2012)	-	Rone and Ben-Tzvi (2017); Liu and Ben-Tzvi (2018)	-
Experimental	Brill et al. (2015)	Jusufi et al. (2010); Kim and Shell (2017); Ren et al. (2009); Kim and Shell (2018)	Casarez et al. (2013); Kohut et al. (2012); Patel and Boje (2015); Simon et al. (2018); Jovanova et al. (2018)	Berenguer and Monasterio-Huelin (2008); Heim et al. (2016); Kessens and Dotterweich (2017); Kwak and Bae (2015); McInroe et al. (2016); Santiago et al. (2016); Sato et al. (2016); Takita et al. (2002a,b, 2003); Xiuli et al. (2016); Rone et al. (2018); Saab et al. (2018a); Casarez and Fearing (2017, 2018)	Briggs et al. (2012); Chang-Siu et al. (2011); Jianguo et al. (2013, 2015b,a); Johnson et al. (2012); Kohut et al. (2013); Libby et al. (2012, 2016); Patel and Braae (2013)	Aiello and Crespo (2013); Chang-Siu et al. (2013); De and Koditschek (2015); Guarnieri et al. (2009); Guan-Horng et al. (2014); Patel and Braae (2014); Pullin et al. (2012); Wenger et al. (2016); Saab et al. (2018c)

Table 6. Non-unique tail functions operating in free space.

Function	Diagram	Description	Papers
Aerial Reorientation		The robot either jumps or moves off an edge. Whilst airborne, the tail is used to correct any torques induced on the robot so it lands with the correct orientation. This can be in the pitch or roll axis.	Briggs et al. (2012); Chang-Siu et al. (2011); De and Koditschek (2015); Jianguo et al. (2013, 2015b,a); Johnson et al. (2012); Libby et al. (2012, 2016); Guan-Horng et al. (2014); Wenger et al. (2016); Shamsah et al. (2018); De and Koditschek (2018); Yu et al. (2017) (Total: 14)
Locomotion Stability		The robot walks or drives over a rough surface. The tail is used to correct torques induced by the unstable motion of the robot to prevent it falling over.	Berenguer and Monasterio-Huelin (2008); Heim et al. (2016); Takita et al. (2002a,b, 2003); Xiuli et al. (2016); Rone et al. (2017, 2018); Simon et al. (2018); Saab et al. (2018a); Iwamoto and Nishikawa (2018); Liu and Ben-Tzvi (2018); Saab et al. (2018c) (Total: 13)
Induced Turning		The tail is used to initiate a yaw torque on the robot, enabling it to have a smaller turning circle.	Casarez et al. (2013); Kohut et al. (2012); Pullin et al. (2012); Saab and Ben-Tzvi (2017) (Total: 4)
Turning Stability		When a fast moving robot makes a turn, the tail is used to minimise roll torques to prevent it falling over.	Aiello and Crespo (2013); Kohut et al. (2013); Patel and Braae (2013); Patel and Boje (2015) (Total: 4)
Velocity Change Stability		When a fast moving robot undergoes acceleration (or deceleration) the tail is used to minimise pitch torques to prevent it falling over.	Kwak and Bae (2015); Patel and Braae (2014) (Total: 2)