

A Novel Triad Twisted String Actuator for Controlling a Two Degrees of Freedom Joint: Design and Experimental Validation

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Abstract—Actuated universal joints, or equivalent joint systems, are found in a number of robotic applications, in particular mobile snake robots, snake-arm robots and robotic tails. Such joints use a variety of actuation methods, including direct drive motors, linear screw drives, cable based systems, and hydraulics/pneumatics. In this paper the authors present a novel design for a mechanism that utilises the twisted string actuator in an antagonistic triad to actuate the universal joint, using orientation sensors and load cells to create a robust cascading closed loop control system. This design realises a light, compact, high-performance actuation system that avoids the extra mass and hardware complexity that alternative actuation methods present, with the additional challenge of nonlinearity. The authors were able to develop a closed loop control system that could track a universal joint orientation setpoint to within $\pm 1.8^\circ$ within a $\pm 11^\circ$ range for a single axis and $\pm 6^\circ$ range for dual axis.

Index Terms—???

I. INTRODUCTION

Actuated universal joint (AUJ) mechanisms are found in a wide range of robotic applications, such as confined space inspection using continuum robots [1], highly manoeuvrable mobile snake robots [2], and biomimetic robot tails for stability [3]. Mobile snake robots must usually incorporate electric actuators inline with their joints. This results in an AUJ having to shift the mass of the follower segments and all the actuators inside the follower segments, which results in high torque requirements. Continuum robots and robotic tails can reduce the mass and size of the AUJ by moving their actuators away from the AUJs and use cables to transfer the force to the joints, or use hydraulic or pneumatic actuators which tend to be lighter than equivalent electric motors. This comes at the expense of increased mass and bulk at the base of the arm or tail.

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First developed by Würtz *et al.* [4] in 2010, the TSA uses two or more strings between two fixtures as a linear actuator. When one fixture is rotated (typically by an electric motor), the looped string twists into a helix, decreasing the distance between them. TSA actuators have been used for a hand orthosis [5], elbow joint [6] and foldable robot arm [7] among other functions.

The primary advantage of TSA over similar linear actuators such as a leadscrew is the reduction (lower velocity, higher torque) the TSA provides is not proportional to the mass of the actuator, in fact it is slightly inversely proportional. Generally, to increase the reduction in an actuator requires the addition of a gearbox which increases mass, but in the case of the TSA, by decreasing the string cross-section radius, the reduction increases given a constant unwound length and motor angle, resulting in a greater reduction with no increase, or even a slight decrease, in actuator mass.

While the reduction in a leadscrew can be increased by decreasing the lead on the thread, which also has no increase in mass, this has a limited range and can quickly run up against manufacturing tolerances or material strength requirements. In order to achieve greater or more robust reductions, the screw radius has to be increased, or the driving motor has to have a larger reduction before driving the screw, both of which usually result in more material (typically steel) and therefore more mass.

However, TSA does have some disadvantages, the most significant of which is a nonlinear reduction equation, which is also dependent on the motor angle (and therefore actuator position). The reduction decreases in a nonlinear fashion as the angle increases, with the derivative decreasing as the angle increases. There is also the compliance of the strings to consider, depending on the thickness and material chosen, which becomes a significant factor under high forces. Both of these issues can be addressed with accurate modelling [8] and/or a robust control strategy, as demonstrated in [4]. What is more of an issue is the unidirectional force of the TSA, which can only impart force in tension. This means that for an AUJ, which is a 2 degree of freedom (DOF) joint, a minimum of three TSA are required, unless spring return mechanisms are used, which would impart additional force on the TSA and therefore reduce performance. However, the potential high force ratio of the TSA due to the non-proportional

reduction may adequately compensate for the additional actuator requirement.

The focus of this research is to investigate if the TSA is a suitable candidate for control of an AUJ considering both the benefits and drawbacks. To this end, the objective is to simulate a model and then construct a physical experimental prototype to validate the proposed control system.

The use of TSA as an actuator for an AUJ is an under-studied area of research. **bombara2021physics** have proposed a similar design using a flexible core with continuous curvature as opposed to a rigid universal joint, however currently this research has not demonstrated control of both axes of motion with multiple TSA. For the first time the authors demonstrate a robust closed loop control of an AUJ in both axes of motion using three TSA in an “antagonistic triad” configuration. The result is a light, compact AUJ design that has the potential to significantly improve upon exiting inline actuation options.

This publication will first give an outline of the TSA based on the existing literature, and the concept of an antagonistic triad. Then a detailed explanation of the control system is given, followed by results from the simulation and experimental system. Finally, a theoretical analysis of the TSA AUJ compared to a similar AUJ using leadscrews is conducted, followed by a discussion and conclusion.

This work was previously published as a conference proceeding at the International Conference on Robotics and Automation (ICRA) 2022 [9]. This publication extends that research by...

A. A Twisted string actuator

First developed by Würtz, May, Holz, *et al.* [4] in 2010, TSA uses two or more strings between two fixtures as a 1 DOF linear actuator. When one fixture is rotated (typically by an electric motor), the strings twist into a helix, decreasing the distance between the fixtures, as shown in figure 1. Given the unwound length l_u , and the cross-section radius of the string r_s (or $r_s + r_c$ when there are more than two strings) as shown in figure 2, the actuator length is given by

$$l_s(\theta_s) = \sqrt{l_u^2 - \theta_s^2 r_s^2} \quad (1)$$

where θ_s is the motor angle, as shown in figure 3.

This equation assumes an infinite string stiffness, so is only reasonably accurate under low tension. Although theoretically the stroke of the TSA can be the entire domain of $[0, l_u]$, in reality the thickness of the string prevents a geometric helix from forming once the helix pitch $q < 4r_s$ (or $q < 2nr_s$ for n strings). This limits the lower bound of the stroke as follows,

$$l_{min} = \frac{l_u}{\sqrt{\frac{\pi^2}{2} + 1}} \approx 0.46 l_u \quad (2)$$

or approximately 46% of l_u for a two string TSA.

$$\theta_s = 0 \quad \theta_s = 2\pi \quad \theta_s = 20\pi$$

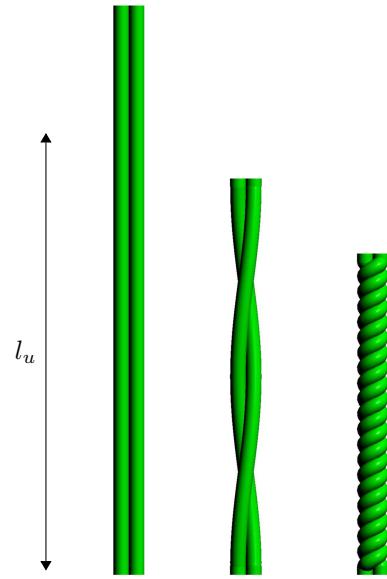


Fig. 1: The value of θ_s increases the number of twists in a string bundle with a string length l_u .

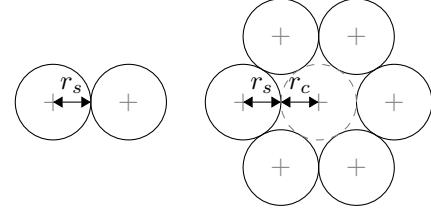


Fig. 2: The location of r_s and optionally r_c in a string bundle.

B. Antagonistic Triad

As mentioned in the introduction, because the TSA provides only tensile force, a minimum of three actuators are required for a 2 DOF actuation system. These can be arranged in a triangular configuration to create an “antagonistic triad”, akin to the antagonistic pairs of muscles found in animals. Where a revolute joint would be found between the connecting ends of the actuator, a universal joint is found instead. The geometric structure of the system as shown in figure 4 can be described with two equilateral triangles of exradius r on two planes separated in the z axis. The centroids are then connected via a universal joint from each plane normal to an intersecting point, let the vector $\boldsymbol{\theta} = [\theta_1 \ \theta_2]$ denote the rotation of the second plane relative to the first, in the x and y axes around the intersecting point, and let l_1 and l_2 denote the normal distance from the intersection to the first and second plane centroids respectively. When $\boldsymbol{\theta} = [0 \ 0]$ the triangles are parallel to each other. The distance between the vertex pairs of each triangle is then denoted as $[\lambda_1 \ \lambda_2 \ \lambda_3]$ for the “top”, “left” and “right” vertices of the triangles. When $\boldsymbol{\theta}$ is changed, this will change λ_1 ,

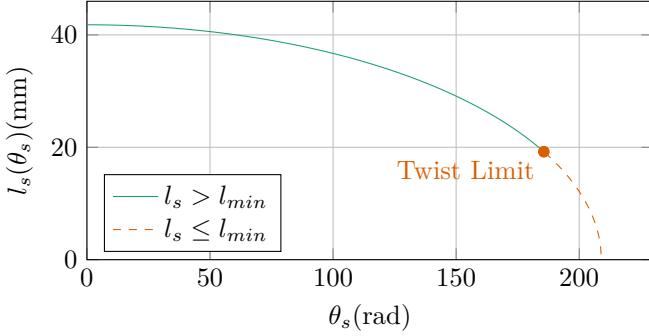


Fig. 3: TSA string length against motor angle with coefficients from table I.

λ_2 and λ_3 respectively.

To calculate the lengths of the strings for a given θ of the universal joint, a transformation matrix for each string can be computed, following a path from one vertex to the other via the plane normals. This incorporates the universal joint rotations $R_1(\theta_1)$ and $R_2(\theta_2)$, the normal distance translations $P_l(l_1)$ and $P_l(l_2)$, the triangle exradius translation P_r and triangle exradius rotation $R_r(\phi)$, where $\phi = [0, \frac{2\pi}{3}, \frac{4\pi}{3}]$ to represent the three vertices of the triangle, hereafter referred to as *top*, *left* and *right*.

$$\begin{aligned} R_1(\theta_1) &= \begin{bmatrix} \cos \theta_1 & 0 & \sin \theta_1 & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \theta_1 & 0 & \cos \theta_1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ R_2(\theta_2) &= \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & \cos \theta_2 & -\sin \theta_2 & 0 \\ 0 & \sin \theta_2 & \cos \theta_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ P_l(l) &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ P_r &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & r \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ R_r(\phi) &= \begin{bmatrix} \cos \phi & -\sin \phi & 0 & 0 \\ \sin \phi & \cos \phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned} \quad (3)$$

$$T_n(\theta) = P_l(l_1) R_1(\theta_1) R_2(\theta_2) P_l(l_2)$$

$$T_1(\theta) = P_r^{-1} T_n(\theta) P_r$$

$$T_2(\theta) = R_r\left(\frac{2\pi}{3}\right) P_r^{-1} R_r\left(-\frac{2\pi}{3}\right) T_n(\theta) R_r\left(\frac{2\pi}{3}\right) P_r$$

$$T_3(\theta) = R_r\left(-\frac{2\pi}{3}\right) P_r^{-1} R_r\left(\frac{2\pi}{3}\right) T_n(\theta) R_r\left(-\frac{2\pi}{3}\right) P_r$$

Then, to get the distances between the vertex pairs, the euclidean norm for the position components of each transformation matrix T can be calculated. These distances can then be combined into a vector function $\Lambda(\theta) = [\lambda_1(\theta) \ \lambda_2(\theta) \ \lambda_3(\theta)]$.

$$\begin{aligned} \ell(T) &= \| [t_{14} \ t_{24} \ t_{34}] \|_2 \\ \Lambda(\theta) &= [\ell(T_1(\theta)) \ \ell(T_2(\theta)) \ \ell(T_3(\theta))] \\ &= [\lambda_1(\theta) \ \lambda_2(\theta) \ \lambda_3(\theta)] \end{aligned}$$

where:

$$\lambda_1(\theta) = \sqrt{l_1^2 + 2l_1l_2 \cos(\theta_1) \cos(\theta_2) + 2l_1r \sin(\theta_1) \cos(\theta_2) + l_2^2 + 2l_2r \sin(\theta_2) - 2r^2 \cos(\theta_2) + 2r^2}$$

$$\lambda_2(\theta) = \sqrt{a + b + c - d}$$

$$\lambda_3(\theta) = \sqrt{a - b - c + d}$$

where:

$$\begin{aligned} a &= l_1^2 + 2l_1l_2 \cos(\theta_1) \cos(\theta_2) \\ b &= \sqrt{3}l_1r \sin(\theta_1) - l_1r \sin(\theta_2) \cos(\theta_1) + l_2^2 \\ c &= \sqrt{3}l_2r \sin(\theta_1) \cos(\theta_2) - l_2r \sin(\theta_2) \\ d &= \frac{\sqrt{3}r^2 \sin(\theta_1) \sin(\theta_2)}{2} - \frac{3r^2 \cos(\theta_1)}{2} - \frac{r^2 \cos(\theta_2)}{2} + 2r^2 \end{aligned} \quad (4)$$

The output of this is shown in figure 5 for a domain of $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

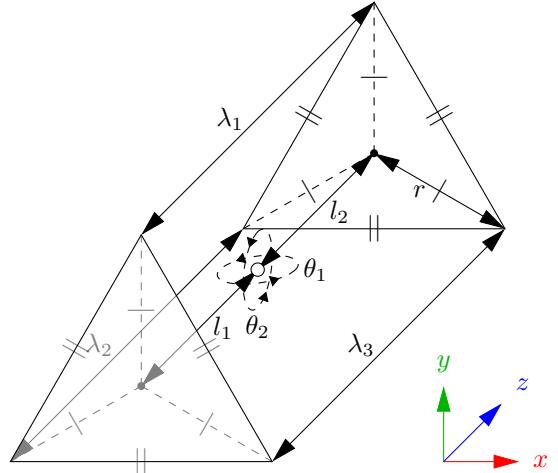


Fig. 4: Kinematic diagram of the antagonistic triad, where the universal joint rotation is defined by $\theta_{1,2}$ on the *y* and *x* axes respectively, and the actuator lengths are defined by $\lambda_{1,2,3}$ for the *top*, *left* and *right* strings. r and $l_{1,2}$ define the anchor points of the strings.

II. CONTROL SYSTEM

The control system is a four layer cascade design, joining an inverse dynamic control system [10], to the triad force controller in [11], to a proportional controller for each TSA. It uses feedback signals of the joint position from the accelerometers and TSA force from the load cells. A second order setpoint trajectory q is used as the input, which can either be pre-defined or generated dynamically from user input. Feedback is provided by the AUJ angular position θ as shown in figure 4, angular velocity $\dot{\theta}$, and TSA tension force \hat{f} . Figure 7 shows a complete block diagram of the control system.



Fig. 5: Surface plots of each element of the vector function $\Lambda(\theta)$, assuming coefficient values from table I. Note that λ_2 and λ_3 are symmetric.

- 1) C_1 AUJ Position PID Controller
- 2) C_2 Inverse Dynamics
- 3) C_3 TSA Force Optimisation Algorithm
- 4) C_4 TSA Force P Controller

Functions $C_{1\dots 4}$ are then combined into a cascade function $C_4(C_3(C_2(C_1(\dots),\dots),\dots),\dots)$.

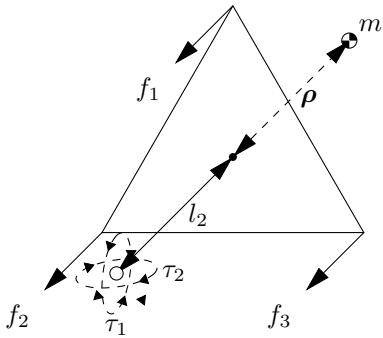


Fig. 6: Dynamics of an antagonistic triad based on figure 4, where the follower mass is m , the follower center of mass (COM) relative to the universal joint pivot is ρ , the forces on each anchor point are $f_{1,2,3}$ and the universal joint torque is $\tau_{1,2}$.

A. Actuated universal joint Position PID Controller with Acceleration Feedforward

Firstly, a PID controller is used to generate a control signal \mathbf{u} with the input \mathbf{q} as the setpoint, and the AUJ angular position θ and velocity $\dot{\theta}$ as feedback, plus the addition of a feedforward term for the input acceleration $\ddot{\mathbf{q}}$, i.e.

$$\mathbf{u} = \mathbf{k}_p(\mathbf{q} - \theta) + \mathbf{k}_i \left(\int_0^t (\mathbf{q} - \theta) dt \right) + \mathbf{k}_d (\dot{\mathbf{q}} - \dot{\theta}) + \ddot{\mathbf{q}}. \quad (5)$$

In the discrete implementation used for fixed step simulation and experimental system control, the integral term is replaced by the trapezoidal rule.

$$K_I \int_0^t \epsilon \approx \sum_{i=0}^N \frac{\epsilon(t_i) + \epsilon(t_{i-1})}{2} \Delta t \quad (6)$$

B. Inverse Dynamics

The control signal \mathbf{u} from the PID controller is then converted to the desired AUJ torque $\boldsymbol{\tau}$. Firstly the affine transformation matrix $T(\theta)$ for a coordinate frame between the AUJ pivot and the COM of the follower segment can be defined, with the rotation matrices $R_1(\theta_1)$ and $R_2(\theta_2)$ defining the orientation of the universal joint, and P defining the offset between the center of the universal joint and the follower mass. The order of $R_1(\theta_1)$ and $R_2(\theta_2)$ can be reversed, but this requires other terms to be reversed as well.

$$P = \begin{bmatrix} 1 & 0 & 0 & \rho_1 \\ 0 & 1 & 0 & \rho_2 \\ 0 & 0 & 1 & l_2 + \rho_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (7)$$

$$T(\theta) = R_1(\theta_1) R_2(\theta_2) P$$

Then the linear velocity jacobian J_v is simply the jacobian of the translation vector of T .

$$J_v(\theta) = \begin{bmatrix} \frac{\partial t_{14}}{\partial \theta_1} & \frac{\partial t_{24}}{\partial \theta_1} & \frac{\partial t_{34}}{\partial \theta_1} \\ \frac{\partial t_{14}}{\partial \theta_2} & \frac{\partial t_{24}}{\partial \theta_2} & \frac{\partial t_{34}}{\partial \theta_2} \end{bmatrix} \quad (8)$$

The angular velocity jacobian J_ω is calculated using the first joint angle relative to the base frame, and the second in the frame of the first. If R_x is the first rotation in T then the first column of the jacobian is $[100]^\top$ (to represent the pitch angle) and the second column is $R_x [010]^\top$.

$$J_\omega(\theta) = \begin{bmatrix} 1 & r_{x_{11}} \\ 0 & r_{x_{21}} \\ 0 & r_{x_{31}} \end{bmatrix} \quad (9)$$

Then the mass matrix D can be created from the jacobian, the mass for the follower segment m and its inertia tensor $I \in \mathbb{R}^{3 \times 3}$, and R_x and R_y to express the inertia in the correct frame.

$$D(\theta) = m J_v^\top J_v + J_\omega^\top (R_x R_y) I (R_x R_y)^\top J_\omega^\top \quad (10)$$

The centrifugal/coriolis matrix C is created from the christoffel symbols of D , along with the AUJ velocity vector $\dot{\theta}$.



Fig. 7: Block diagram of the complete experimental control system, excluding the hardware velocity controllers for the motors.

$$C(\theta, \dot{\theta})_{k,j} = \sum_{i=1}^N \frac{1}{2} \frac{\partial d_{kj}}{\partial \theta_i} + \frac{\partial d_{ki}}{\partial \theta_j} - \frac{\partial d_{ij}}{\partial \theta_k} \dot{\theta}_i \quad (11)$$

Then the gravity term G . As the gravity vector direction is the same as the z axis as in figure 4, the height is equal to $-l_2 \cos \theta_1 \cos \theta_2$, therefore the potential energy is $mg(-l_2 \cos \theta_1 \cos \theta_2)$. The jacobian of this then becomes the gravity term.

$$G(\theta) = \begin{bmatrix} \frac{\partial mg - l_2 \cos \theta_1 \cos \theta_2}{\partial \theta_1} \\ \frac{\partial mg - l_2 \cos \theta_1 \cos \theta_2}{\partial \theta_2} \end{bmatrix} \quad (12)$$

D , C and G are then combined to form the dynamics equation C_2 , along with the AUJ position and velocity vectors. C_1 is used as the acceleration term. This results in a setpoint joint torque which can be used in the optimisation algorithm.

$$\tau = D(\theta) u + C(\theta, \dot{\theta}) \dot{\theta} + G(\theta). \quad (13)$$

C. Twisted string actuator Force Optimisation Algorithm

This uses the *inverse force transformation* algorithm from [11] with the jacobian of equation 4 to select an optimal force vector from the desired joint torque. Here it is presented in an unexpanded and more general form. A force matrix F is created from the torque input τ , jacobian J_Λ from the vector function Λ as defined in equation 4, and minimum force constant f_{min} . f_{ii} is equal to f_{min} , while the other elements in the column are based on a calculation using $J_{\Lambda_{-i,*}}$ where $-i$ is a row removed from the matrix.

$$\begin{aligned} J_\Lambda &= \begin{bmatrix} \frac{\partial \lambda_1}{\partial \theta_1} & \frac{\partial \lambda_2}{\partial \theta_1} & \frac{\partial \lambda_3}{\partial \theta_1} \\ \frac{\partial \lambda_1}{\partial \theta_2} & \frac{\partial \lambda_2}{\partial \theta_2} & \frac{\partial \lambda_3}{\partial \theta_2} \end{bmatrix} \\ \gamma(i) &= -J_{\Lambda_{-i,*}}^{-T} (J_{\Lambda_{i,*}}^T f_{min} + \tau) \quad (14) \\ F(\tau, \theta) &= \begin{bmatrix} f_{min} & \gamma(2)_1 & \gamma(3)_1 \\ \gamma(1)_1 & f_{min} & \gamma(3)_2 \\ \gamma(1)_2 & \gamma(2)_2 & f_{min} \end{bmatrix} \end{aligned}$$

The following algorithm then selects one column of F to be the output force vector f , where \top and \perp are boolean *true* and *false* respectively, and $f_{*,i}$ is the i th column of F . The solution to this algorithm minimises the net force on all TSA while producing the desired output torque on the universal joint.

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1:  $s \leftarrow [\top \top \top]$ 
2: if  $f_{23} > f_{min}$  then  $s_2 \leftarrow \perp$  else  $s_3 \leftarrow \perp$  end if
3: if  $f_{31} > f_{min}$  then  $s_3 \leftarrow \perp$  else  $s_1 \leftarrow \perp$  end if
4: if  $f_{12} \geq f_{min}$  then  $s_1 \leftarrow \perp$  else  $s_2 \leftarrow \perp$  end if
5: for  $i = 1$  to  $3$  do
6:   if  $s_i \rightarrow \top$  then  $f \leftarrow f_{*,i}$  end if
7: end for

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D. Twisted string actuator Force Proportional Controller

The selected forces are then used as an input to a P controller with gain k_{ps} using the measured load cell forces \hat{f} as feedback. The output from this can then be used to control the top, left and right TSA motors, corresponding to the actuators in figure 4.

1) *Simulation Current Control*: In the simulation, each TSA was modelled as a state-space system which takes motor current u as an input and outputs y as the TSA tension force. [4] defines it as such, where J is the motor inertia, C is the motor coulomb friction (modified from viscous friction as the motor only has dry friction), K_t is the motor torque constant, and K_L is the load stiffness. As the original definition is for a fixed load l_u distance from the motor a modified model is required which takes into account the varying length between the motor and load defined by $\Lambda(\theta)$. A saturation function, with the compact notation $\text{sat}_x^y z = \max(x, \min(y, z))$ is used to prevent incorrect compression forces when the string is slack. All of the motor coefficients were taken from the Faulhaber 1724TSR datasheet [12] as this is the motor used in the experimental model. An estimated value is used for the load stiffness, this was chosen to be a high number as the model is expected to be very stiff.

$$\begin{aligned}
h(\theta_s) &= \frac{\theta r_s^2}{\sqrt{l_u^2 - \theta_s^2 r_s^2}} \\
k(\theta_s, \boldsymbol{\theta}) &= \lambda_n(\boldsymbol{\theta}) - \sqrt{l_u^2 - \theta_s^2 r_s^2} \\
\dot{\boldsymbol{x}} &= \begin{bmatrix} x_2 \\ -\frac{K_L}{J} h(x_1) k(x_1, \boldsymbol{\theta}) - \frac{C}{J} \operatorname{sgn}(x_2) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{K_t}{J} \end{bmatrix} u \\
y &= K_L \operatorname{sat}_0^\infty k(x_1, \boldsymbol{\theta})
\end{aligned} \tag{15}$$

The state space model was then adapted to include constraints on motor velocity and acceleration set by the motor controller in order to keep the motor within design limits, by replacing $\dot{\boldsymbol{x}}$ with $\dot{\boldsymbol{x}}'$ which contains saturation functions for maximum motor velocity v_s and acceleration α_s .

$$\dot{\boldsymbol{x}}' = \begin{bmatrix} \operatorname{sat}_{-\omega_s}^{\omega_s} \dot{x}_1 \\ \operatorname{sat}_{-\alpha_s}^{\alpha_s} \dot{x}_2 \end{bmatrix} \tag{16}$$

2) Experimental Velocity Control with Deadband Compensation: Due to a controller deadband within $\pm 10 \text{ min}^{-1}$, an adjustable deadband compensator is used,

$$\phi_i(\omega_i) = \begin{cases} 10 & 10 \leq \omega_i < h \\ -10 & -h < \omega_i \leq -10 \\ 0 & h \leq \omega_i \leq -h \\ \omega_i & \text{otherwise} \end{cases} \tag{17}$$

where ϕ_i is the compensator for the controller i . An adjustment value $h \in [0, 10]$ changes the threshold at which the compensator switches on and off, allowing for a small deadband to remain.

The result from the TSA is then a compressive force acting between each of the three TSA and its corresponding endpoint on the Antagonistic Triad, imparting a torque on the axes of the universal joint.

III. SIMULATION & EXPERIMENTAL RESULTS

To design and refine the parameters of the control system, a Simscape Multibody™ model of the antagonistic triad and control system was created in MATLAB®/Simulink™. This allowed for model design coefficients $l_1, 2$ and controller gains k_p, k_i, k_d, k_{ps} to be modified in order to have the most stable control within design limits.

A. Experimental Setup

For the experimental validation, a physical prototype of the mechanism was constructed with coefficients from table I as design parameters. This was mounted vertically, in order for the inertial measurement unit (IMU) to measure the orientation of the universal joint. The TSA mechanisms consist of a compact high torque motor attached to the base segment, with a string clamp attached to the motor shaft. On the follower segment, a load cell is mounted on top of a universal joint to ensure a purely axial load, with a capstan bolt attached to the load cell. The string itself is attached to the clamp at both ends using

TABLE I: Model coefficients.

Coefficient	Value	Coefficient	Value
l_1	41.8 mm	J	$1 \times 10^{-6} \text{ kg m}^{-2}$
l_2	0 mm	K_L	1000 N m^{-1}
r	13 mm	f_{min}	3 N
l_u	41.8 mm	ω_s	441.9 rad s^{-1}
r_s	200 μm	I_s	0.19 A
m	72.619 13 g	K_t	$0.0263 \text{ N m A}^{-1}$
C	0.1315 N mm	τ_s	4.5 mN m
α_s	$1 \times 10^5 \text{ rad s}^{-2}$	ρ	[0 0 0] mm
Coefficient	Value		
I	$\begin{bmatrix} 3 \times 10^{-5} & 0 & 0 \\ 0 & 3.2 \times 10^{-5} & 0 \\ 0 & 0 & 1.4 \times 10^{-5} \end{bmatrix} \text{ kg m}^{-2}$		

TABLE II: PID gains in the simulation and experiment.

Gain	Value	
	Simulation	Experiment*
k_p	800	3×10^4
k_i	3000	350
k_d	50	50
k_{ps}	19	100

* Tracking mode, see section III-B.

two grub screws for extra security and easy adjustment, and looped through the hole in the capstan bolt. The total mass of the prototype, excluding the mount, is $\approx 176 \text{ g}$. Figures 8 and 18 detail the construction of the experiment with all the constituent parts.

The motors were controlled by Fauhaber MCDC3002 motor controllers, which could interface with a National Instruments MyRIO via the USB port, using a USB to serial converter. The load cells were Futek LCM100 miniature load cells, selected for their small size. The signals from these were amplified using Flyde FE-359-TA instrumentation amplifiers and decoded using an external AD7606 ADC before being fed into the MyRIO using SPI. The orientation of the AUJ was measured using a Bosch Sensortec BNO080 IMU using the accelerometer data.

B. Windup & Tracking States

When the mechanism is started with the TSA in a completely unwound state, before it can begin tracking a motion trajectory, the TSA strings must “wind up” to closely match the initial state of \boldsymbol{f} . During this phase, the outer PID gains k_p, k_i are unsuitable and can result in damage to the mechanism. To mitigate this, two sets of PID gains are chosen, one for the windup state ($k_p = 800$, $k_i = 3000$), and another for the tracking state ($k_p = 3 \times 10^4$, $k_i = 350$), which the windup state transitions to once suitable stability is achieved. This transition trigger is defined on a per-axis basis, as the first zero crossing of the angle error (as $\boldsymbol{q} = 0$ this is effectively $\boldsymbol{\theta}$). A graph showing the difference this state change makes to the AUJ orientation is shown in figure 10.

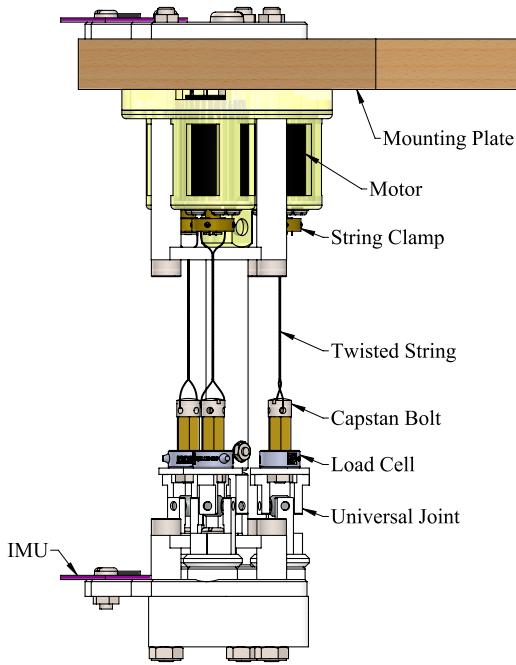


Fig. 8: Schematic of the experimental system with labelled components.

C. Results

1) *Force Proportional Controller & Motor Characterisation:* Before verifying robust control of the AUJ, the control of each individual TSA needed to be tested in order to characterise the performance of the motors and to ensure the inner loop control system was robust. This would involve selecting the control strategy that gave the best performance. A test trajectory consisting of a smooth ramp followed by a sine wave was fed into the inner loop of the cascade function as $[f(t) \ f(t) \ f(t)]$. Each control strategy was tested, and in the end the velocity control strategy proved most optimal, as is shown in figure 11. The results of a sinusoidal force trajectory for the velocity control strategy is shown in figure 12. As can be seen in the inset, there is a stick-slip friction effect which causes a sawtooth like effect on the measured force. This means the motor angle has to be constantly adjusted even with a constant force input, in order to maintain the setpoint force.

2) *Actuated universal joint Angle Tracking:* Figure 13 plots the tracking response of both the simulation and experiment. Three trajectories were created to test the capabilities of the mechanism. Two were only in one axis of the Universal Joint, and the third was in both axes.

D. The Effect of f_{\min} on AUJ Roll Range Limits

As the TSA use a closed loop force controller, the pitch and roll angle range is determined by the “opposing” TSA motor angle at f_{\min} when $\theta = [0 \ 0]$. This is because

Experiments were able to achieve modest increases of $\approx 4.004^\circ N^{-1}$ for the positive (upper) limit of the universal joint roll θ_1 , and $\approx -6.062^\circ N^{-1}$ for the negative

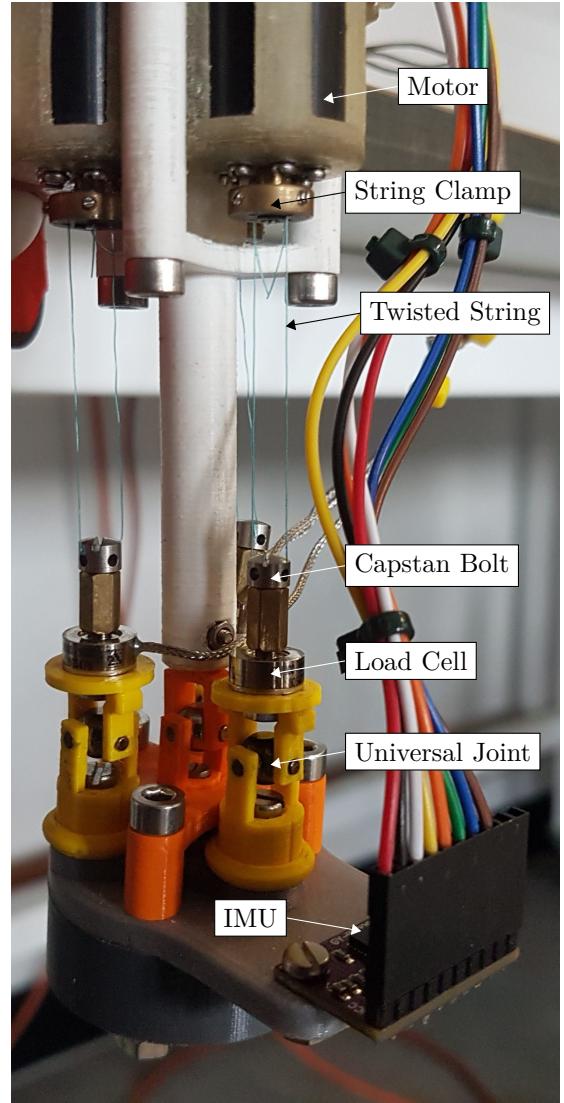


Fig. 9: Annotated photograph of the experimental system.

(lower) limit, within the f_{\min} interval [3, 3.5], as shown in figure ???. Attempts to increase the upper bound of this interval resulted in the opposing TSA exceeding torsion limits, causing string failure.

E. The Effect of AUJ Angular Velocity on AUJ Angle Tracking

To verify the performance of the AUJ at higher angular velocities, the single axis experiments from section III-C2 were repeated with a trapezoidal “chirp” signal, with the maximum angular velocity increasing by $\omega_0 + (2(n-1)\omega_0)$ each cycle, where n is the cycle number, and ω_0 is the initial maximum angular velocity, and the angular acceleration increasing by $\alpha_0 + (16(n-1)\alpha_0)$ each cycle, where α_0 is the initial angular acceleration. A total of four cycles were performed, with the maximum velocity and acceleration values shown in table III. Figure 15 shows the results of these experiments, which shows the tracking error increasing marginally as maximum angular velocity and acceleration increases.

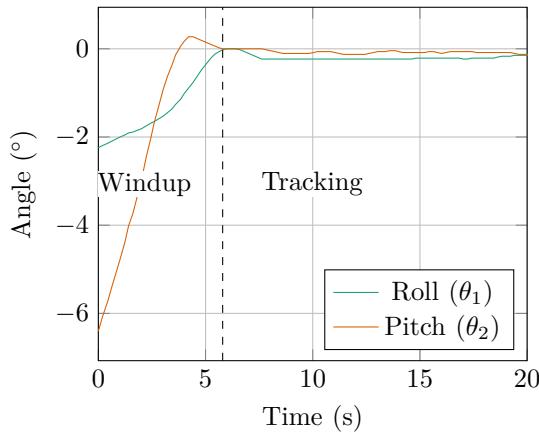


Fig. 10: The initial startup of the mechanism showing the transition between “windup” and “tracking” states.

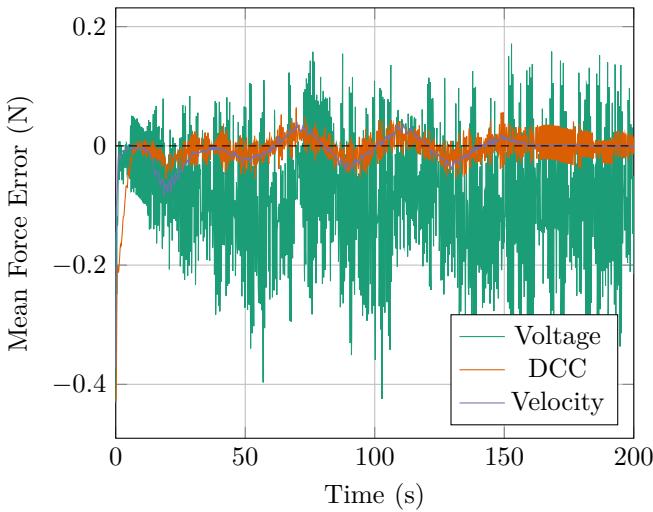


Fig. 11: Mean tracking error for each control strategy.

F. The Effect of Follower Mass on AUJ Angle Tracking

In order to demonstrate the feasibility of a multiple joint system, additional mass was added to the follower segment and the *Pitch & Roll* trajectory was performed to analyze the effect of the additional mass on the control system. This was performed by the modification of the end of the follower segment to allow multiple standard disc weights, each with a mass of 100 g, to be attached. Four experiments were performed, one with no disc weights attached, and three with one, two, and three disc weights attached, adding approximately 100 g each time. 3D printed custom length spacers were used to maintain an approximately

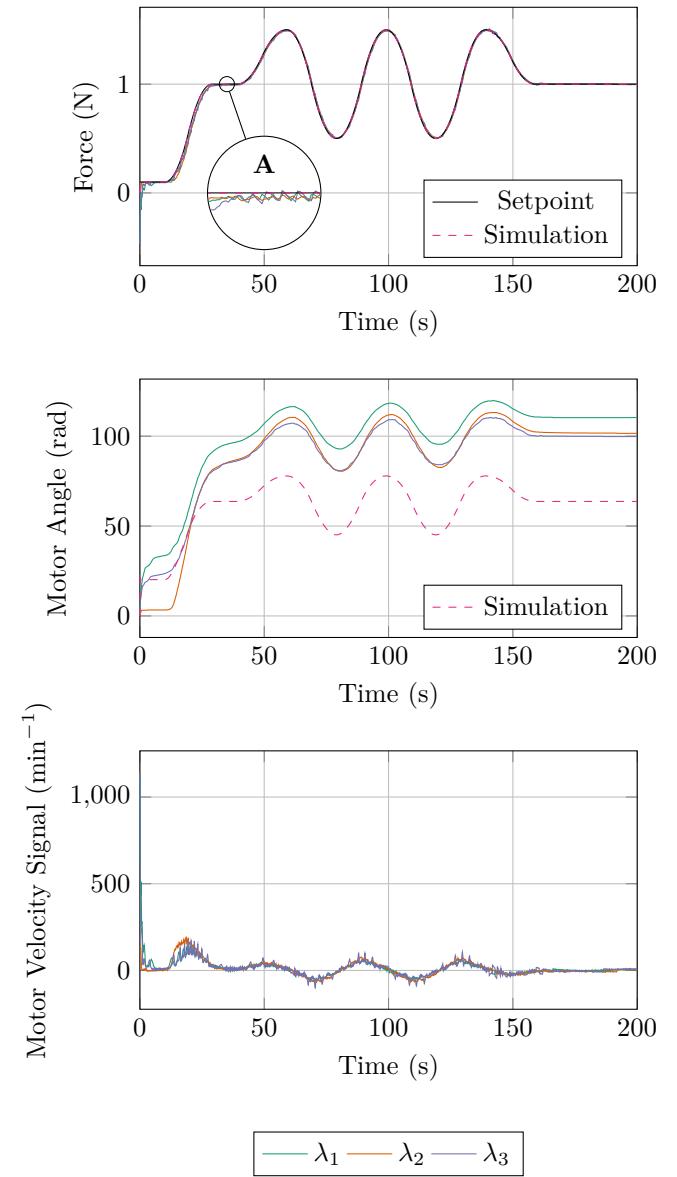


Fig. 12: Load cell, motor angle, and motor velocity signal plot from each string using the velocity control strategy for an experimental trajectory. Inset A shows a zoomed in portion of the graph, illustrating the stick-slip phenomenon noticed in the load cell measurements.

equal COM for each of the experiments with disc weights. Figure ?? shows each configuration of the follower segment for the experiments. Table IV lists the four follower mass configurations that were tested, with follower mass and inertia matrix m and I , and follower COM z component ρ_3 . The follower COM x and y components were set to 0 for these experiments, as they did not vary significantly between experiments, and had a negligible contribution to the dynamics. These figures are based on estimates from a computer aided design (CAD) model developed in SolidWorks®, with known weights of 100 g. Any inertial terms less than 1×10^{-5} were rounded down to 0, which resulted in a diagonal inertia matrix. For each experiment,

Cycle	Max./Min. Angle [rad]	Max. Velocity [rad s⁻¹]	Acceleration [rad s⁻²]
1	0.2	0.01	0.00125
2	0.2	0.03	0.02125
3	0.2	0.05	0.04125
4	0.2	0.07	0.06125

TABLE III

the inverse dynamics parameters m and ρ were updated so the correct setpoint torque would be calculated.

Figure 16 shows the setpoint and response trajectory for each experiment. Initially, k_p was set to the same value as in table I, however the system was unable to reach a steady state in any configuration other than “No Mass”. Reducing k_p to 1000 in the weighted configurations solved the steady state issue and resulted in an average maximum tracking error of 0° over all configurations with added mass, similar to the result from the initial experiments. However, for the “No Mass” configuration, $k_p = 1000$ resulted in a very poor tracking response, whereas k_p from table I resulted in a maximum tracking error of 0° , once again similar to the result from the initial experiments. In future implementations gain scheduling can be employed to select the most optimal k_p for a given follower mass, that allows for the smallest tracking error while being able to reach a steady state.

IV. PERFORMANCE

A. TSA Comparison with Leadscrew

To compare the performance of a TSA AUJ against alternatives, we can measure two metrics, the maximum tension force f_{max} and maximum stroke velocity \dot{p}_{max} . This corresponds to the equivalent of maximum torque and maximum velocity in a rotary motor, a larger f_{max} would be able to actuate a larger follower mass, and a larger \dot{p}_{max} would be able to rotate the AUJ more quickly. The alternatives chosen for comparison are leadscrews of various rod diameters d_m and pitches λ , and a “direct drive” where the motor is rotating the universal joint directly without any reduction or motion transformation.

1) *Twisted string actuator*: For the TSA metrics, the equations from [4], in particular $h(\theta)$ and $k(\theta)$ as used for the state space, which can be used to determine f_{max} and \dot{p}_{max} . By extracting coefficient r_s as an input to make $f(p, r_s)$ and $\dot{p}(\dot{\theta}, p, r_s)$ the performance of different string thicknesses can be compared for a given unwound length l_u and $\tau_{k(\theta)}$, $\dot{\theta}_{max}$ over the range of the contraction length p .

$$\begin{aligned} k^{-1}(p) &= \pm \frac{\sqrt{p(2l_u - p)}}{r_s} \\ h^{-1}(\theta) &= \frac{\sqrt{l_n^2 - r_s^2\theta^2}}{r_s^2\theta} \end{aligned} \quad (18)$$

$$\begin{aligned} f(p) &= h^{-1}(k^{-1}(p)) = \pm \frac{\sqrt{(l_u - p)^2}}{r_s \sqrt{p(2l_u - p)}} \\ f_{max} &= f(p)\tau_{max} \end{aligned}$$

$$\begin{aligned} k(\dot{\theta}, \theta) &= \frac{\dot{\theta}r_s^2\theta}{\sqrt{l_n^2 - r_s^2\theta^2}} \\ \dot{p}(\dot{\theta}, p) &= \dot{k}(\dot{\theta}, k^{-1}(p)) = \pm \frac{\dot{\theta}r_s \sqrt{p(2l_n - p)}}{\sqrt{(l_n - p)^2}} \\ \dot{p}_{max} &= \dot{p}(\dot{\theta}_{max}, p) \end{aligned} \quad (19)$$

2) *Leadscrew*: For the leadscrew metrics, the raising torque calculation [13] can be used as the absolute value of f_{max} , since the TSA only operates in tension, which can be used to determine the same metrics. The performance of different screw diameters d_m and leads λ can then be compared for a given τ_{max} and coefficient of friction μ . \dot{p}_{max} is then calculated by multiplying λ with $\dot{\theta}_{max}$. The performance of different λ can then be compared for a given $\dot{\theta}_{max}$.

$$\begin{aligned} |\tau(f)| &= \frac{d_m f (\lambda + \pi d_m \mu)}{2(\pi d_m - \lambda \mu)} \\ |f(\tau)| &= \frac{2\tau(\pi d_m - \lambda \mu)}{d_m (\lambda + \pi d_m \mu)} \\ f_{max} &= |f(\tau_{max})| \end{aligned} \quad (20)$$

$$\begin{aligned} \dot{p}(\dot{\theta}) &= \lambda \frac{\dot{\theta}}{2\pi} \\ \dot{p}_{max} &= \dot{p}(\dot{\theta}_{max}) \end{aligned} \quad (21)$$

3) *Direct Drive*: The direct drive metrics are trivially calculated using the lever force and tangential velocity that would be generated at the endpoint of a linear actuator able to impart equivalent angular velocity and torque on the universal joint.

$$f_{max} = \frac{\tau_{max}}{\sqrt{l_2^2 + r^2}} \quad (22)$$

$$\dot{p}_{max} = \dot{\theta}_{max} \sqrt{l_2^2 + r^2} \quad (23)$$

4) *Comparison between TSA and Leadscrew*: As the values for τ_{max} and $\dot{\theta}_{max}$ for the TSA depend on p , but remain constant for the leadscrew, the performance of the TSA is going to be better or worse than a given leadscrew depending on the value p . Figure 17 compares the TSA configuration using the coefficients from table I against a number of common leadscrew configurations that are practical for the dimensions of the AUJ. The TSA outperforms or underperforms different leadscrew configurations depending on p . In simpler terms, the performance of the TSA is dependent on the contraction length. The maximum linear velocity increases with the contraction length, and the maximum tension force decreases with contraction length, both in a nonlinear fashion.

V. DISCUSSION & CONCLUSION

A. Discussion

1) *Monofilament Torsion Fatigue & String Breakage*: TSA string breakage was common during experiments, this was often due to a loss of control stability during tuning, or operation beyond previously explored motor angle or load limits. However, failure rates were observed to increase with the number of working cycles (winding and unwinding) of each TSA string, particularly when taken to high motor angles or loads. Nylon monofilament, as was used for the TSA string, is susceptible to torsion fatigue [14], [15] which reduces tensile strength [16]. This

Configuration	m [g]	ρ_3 [mm]	I [kg m ⁻²]	Image
No Mass	67	6.7	$\begin{bmatrix} 3.3 \times 10^{-5} & 0 & 0 \\ 0 & 3.1 \times 10^{-5} & 0 \\ 0 & 0 & 1.4 \times 10^{-5} \end{bmatrix}$	
+100 g	170	49	$\begin{bmatrix} 1.3 \times 10^{-4} & 0 & 0 \\ 0 & 1.3 \times 10^{-4} & 0 \\ 0 & 0 & 4.6 \times 10^{-5} \end{bmatrix}$	
+200 g	220	50	$\begin{bmatrix} 1.7 \times 10^{-4} & 0 & 0 \\ 0 & 1.7 \times 10^{-4} & 0 \\ 0 & 0 & 7.7 \times 10^{-5} \end{bmatrix}$	
+300 g	320	50	$\begin{bmatrix} 2.0 \times 10^{-4} & 0 & 0 \\ 0 & 2.0 \times 10^{-4} & 0 \\ 0 & 0 & 1.1 \times 10^{-4} \end{bmatrix}$	

TABLE IV: Table of all the follower mass configurations, with the parameters for follower mass m and follower COM z offset ρ_3 .

decrease in strength could be a potential explanation for the increase in failure rate. There was also the route and securing method of the string itself, which used four grub screws to clamp onto the string to hold both ends in place, and also passed close to six potentially sharp edges, as shown in figure 19. These locations could weaken the string, by either an edge biting into it when under high tension, or when being clamped by the grub screws.

The cause of the string break could be determined by the location of the break. If it was located at or near one of the grub screws or biting edges, then it was likely to be caused by those. If it was located on the “active” part of the string that twists, then torsion fatigue is likely to be the culprit. To somewhat mitigate the biting and clamping issue, polytetrafluoroethylene (PTFE) sleeving was added around the string near those locations, and this did result in a significant reduction in string breakages due to edge biting or clamping. This could be improved upon by redesigning the string clamp and capstan to round off sharp edges, and to consider alternatives to clamping such as end knotting or loop wrapping.

The experiments in [17] used 0.2 mm Dyneema® and Fast Flight® Plus polyethylene (PE) polyfilament, and were able to achieve over 1000 working cycles before failure at a load of ≤ 20 N at a motor angle of nearly 300 rad. Use of nylon monofilament was purely for convenience, assembly of polyfilament was difficult as slicing the ends usually resulted in individual fibres splaying, which made it impossible thread through the string clamp. A redesign of the string clamp could make it easier to use polyfilament, which based on the results from [17] could greatly

improve string longevity.

2) Improving AUJ Angle Range & Enabling Active Transmission Adjustment by Reducing Triad Radius: As can be seen in figure 20, decreasing the triad radius r decreases the stroke range of each TSA for a given AUJ angle range. This increases the AUJ angle range limits without having to increase f_{min} as was done in section III-D.

If r or the AUJ angle range is small enough, this allows for active transmission adjustment, where increasing or decreasing f_{min} shifts the active TSA stroke “window” to the right or left along the maximum force and maximum velocity curves from figure 17, as shown in figure 21. This would allow the performance of the AUJ to be adjusted during operation, which could be useful in some applications, such as a mobile snake robot which could increase AUJ angular velocity when all segments are on the ground, and sacrifice angular velocity in some segments for increased AUJ torque when they are off the ground and therefore subject to gravitational forces.

An issue with a small triad radius is the difficulty of accommodating the central shaft with the universal joint. One solution to this is to use an Another alternative is to use

3) Hardware Improvements: Using the BNO080 as the AUJ orientation sensor proved to be challenging. The magnetometer was unusable within the indoor experimental environment even after several calibration attempts, so the universal joint angle would not be able to be calculated when the gravity vector is not orthogonal to the universal joint DOF, hence the experiments had the AUJ in a

vertical orientation. An alternative method for sensing the AUJ orientation could be used, such as linear variable differential transformers (LVDTs), hall effect sensors or potentiometers. If the “hollow spider” universal joint design in section V-A2 is used, the larger spider pins would be able to accommodate potentiometers. The 1724TSR motors also had poor performance at low speeds, necessitating the deadband compensator. The use of similar size and torque brushless motors with sinusoidal commutation for stable low speed control could eliminate the need for the compensator, and allow for smoother AUJ motion.

B. Alternative Control Algorithm

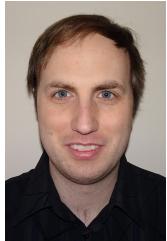
Alternative control algorithms that are less or not dependant on the dynamic properties of the follower segments (mass and inertia) will also be considered, in order to improve the robustness of the control system to external forces.

1) *Multiple Segments:* Development of a system comprised of multiple segments is the eventual goal of the research, to demonstrate its suitability for applications such as mobile snake robots or snake-arm robots. This can use a embedded controller for each segment, controlled by a primary controller for individual joint control or inverse kinematics, as in figure 22.

C. Conclusion

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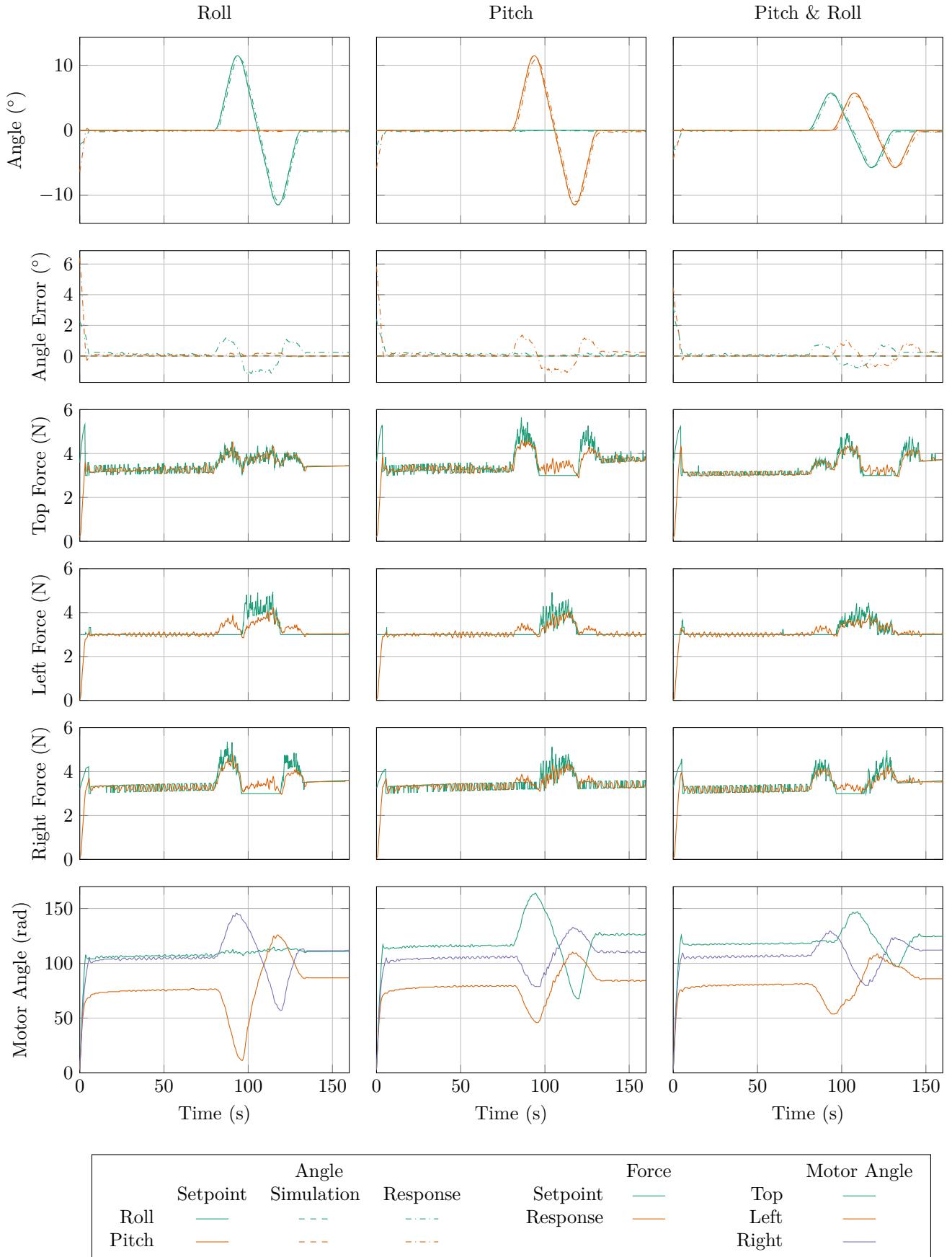


Fig. 13: Plots of the response for three different trajectories, one on only the roll axis θ_1 (column 1), one on only the pitch axis θ_2 (column 2), and one on both axes θ_1 and θ_2 (column 3). Plots include AUJ orientation, forces at the top, left and right TSA, and the motor positions. Note the simulation error is very small, so the plot cannot be seen on the graph.

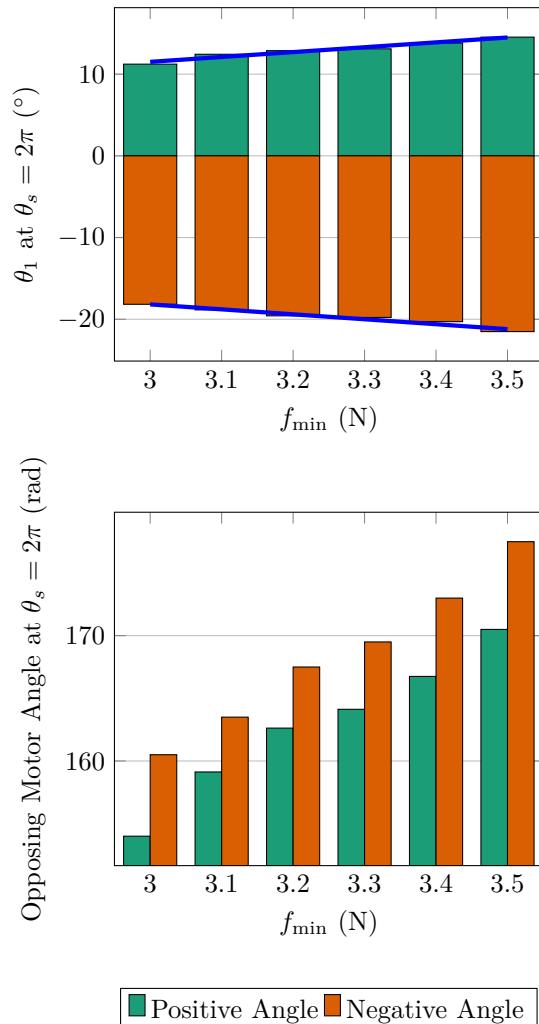


Fig. 14

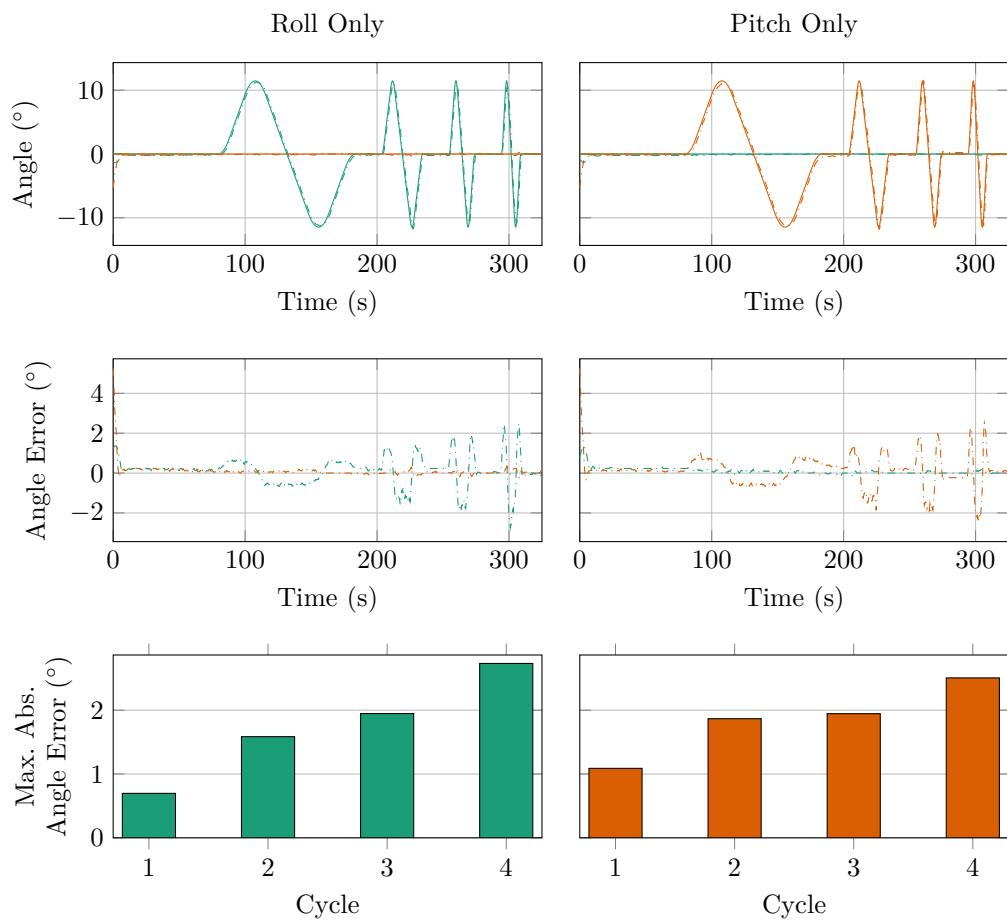


Fig. 15

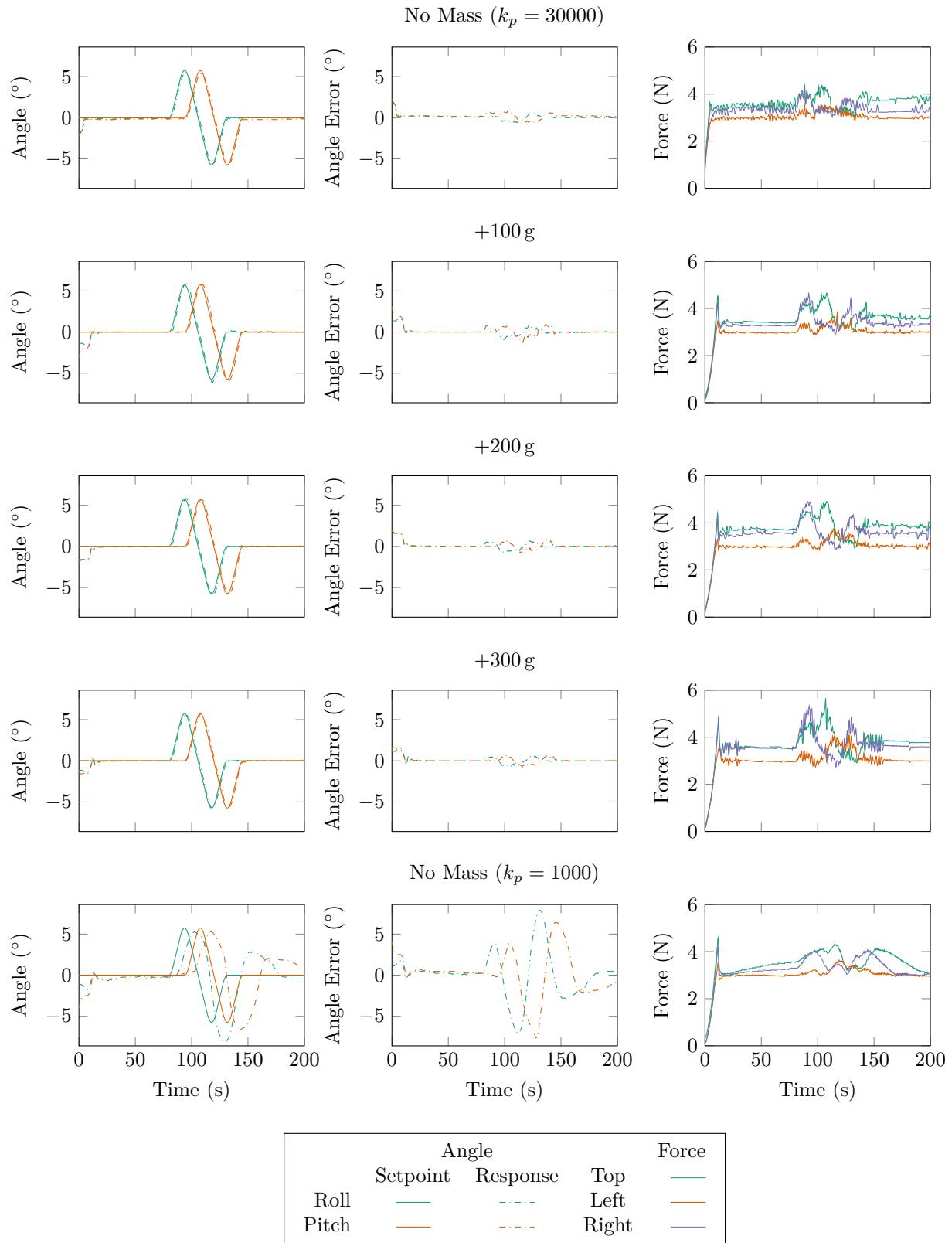


Fig. 16

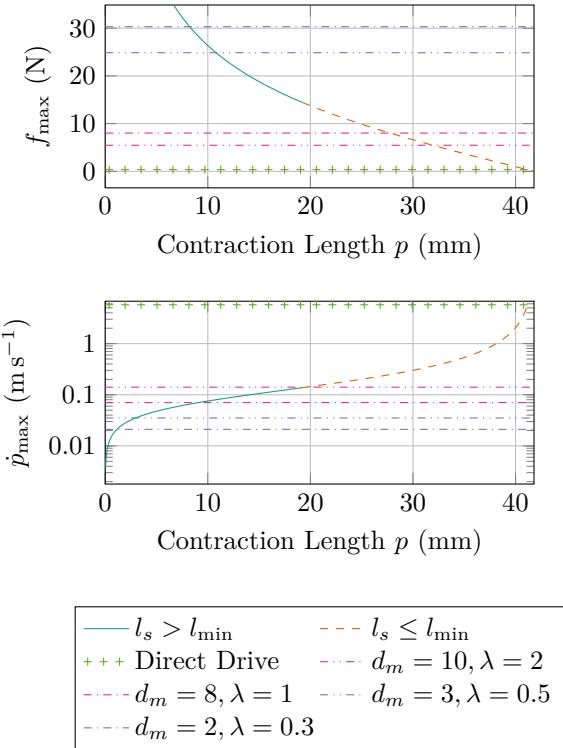


Fig. 17: Performance comparison of the TSA configuration using coefficients from table I to various leadscrew configurations with different d_m and λ , and the direct drive, where $\mu = 0.1$ for the leadscrews.

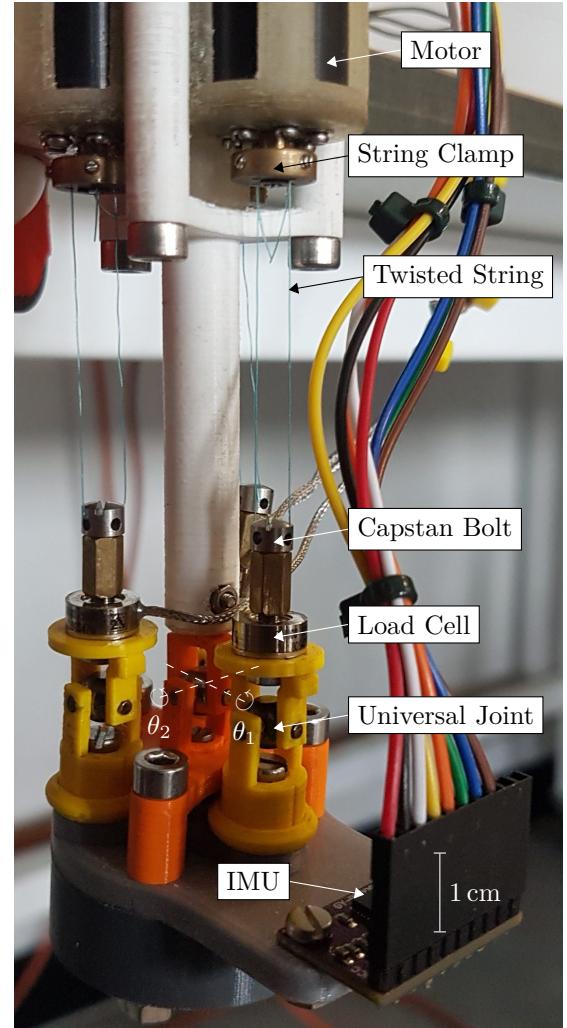


Fig. 18: Annotated photograph of the experimental model, with the roll θ_1 and pitch θ_2 axes marked.

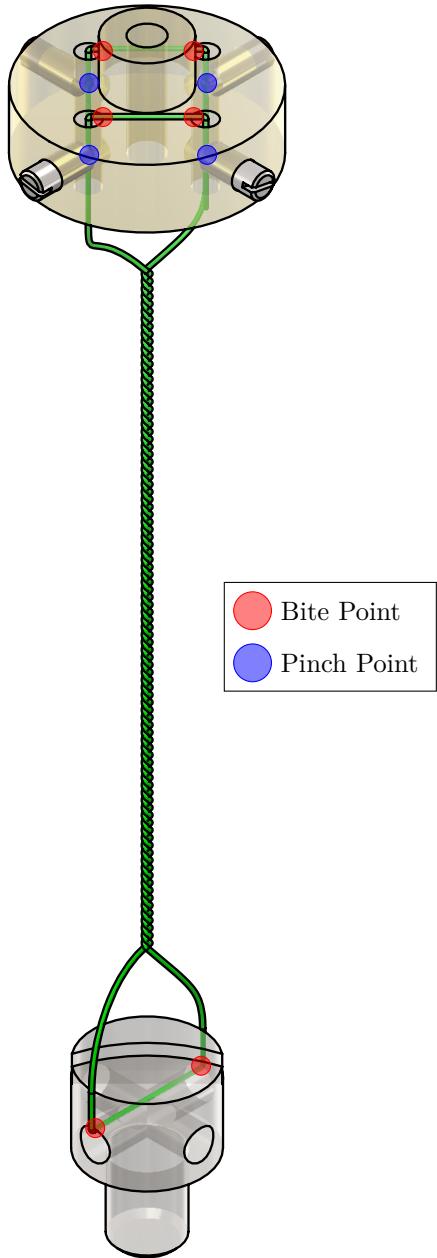


Fig. 19: Cutaway of a single TSA string assembly, including the capstan and string mount, highlighting the “pinch” points and “bite” points that could cause weaknesses in the string material, which may lead to premature failure.

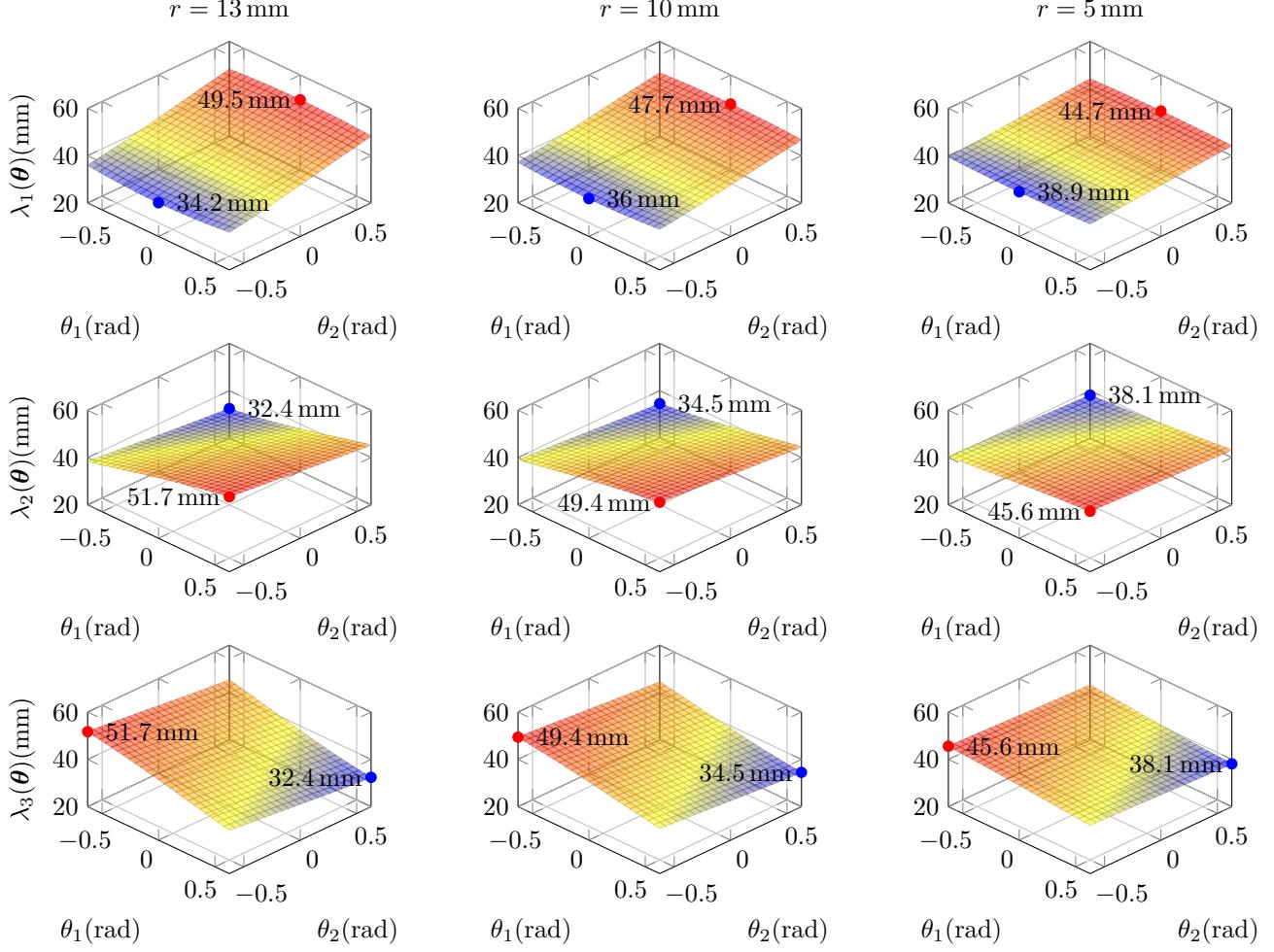


Fig. 20: Surface plots, minima and maxima for the lengths of $\lambda_{1,2,3}$ at $r = 13 \text{ mm}$, $r = 10 \text{ mm}$ and $r = 5 \text{ mm}$ in the range $[-\frac{\pi}{5}, \frac{\pi}{5}]$. As r decreases, the difference between the minima and maxima also decreases. This means that a lower value of r requires a smaller stroke range of the TSA for a given AUJ angle range.

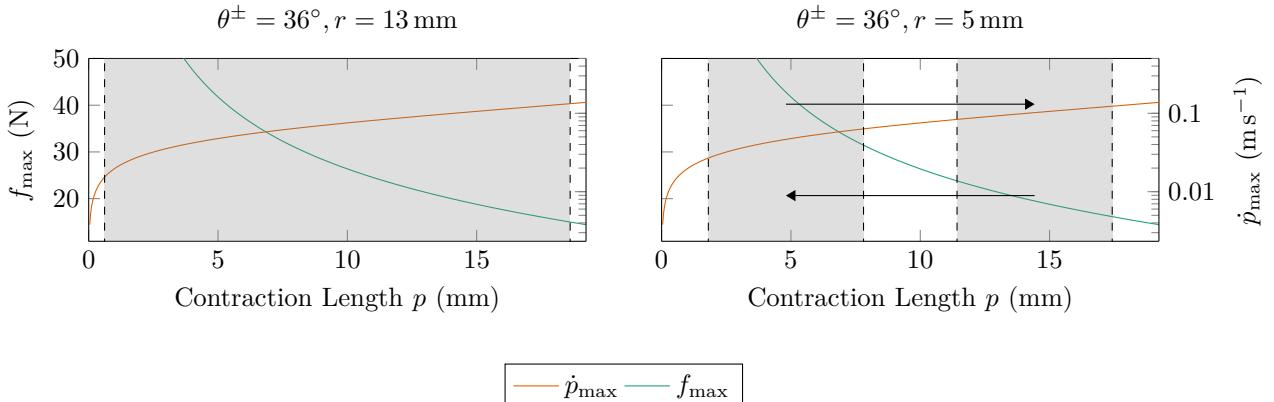


Fig. 21: By decreasing r , not only can the AUJ angle range be increased by reducing the TSA stroke range, which is marked in grey, but by adjusting f_{\min} , the transmission ratio of the TSA can be altered. Reducing f_{\min} increases the maximum TSA force f_{\max} while reducing the maximum TSA stroke velocity \dot{p}_{\max} . Conversely, increasing f_{\min} reduces f_{\max} and increases \dot{p}_{\max} . This can be used to actively modify the dynamic properties of the AUJ during operation.

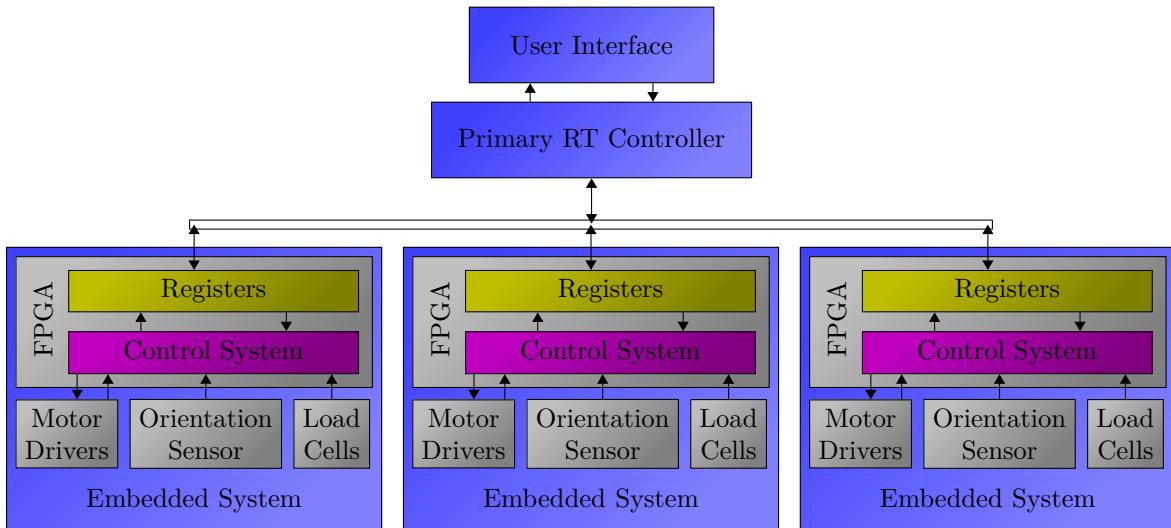


Fig. 22: Proposed system architecture for a future multi-segment system. Each segment has an embedded FPGA controller programmed with the cascaded control loop in section II. The FPGA interfaces with the load cells for each TSA, the orientation sensor for the AUJ, and the drivers for the TSA motors (the FPGA may include some motor control for a more compact design). A primary real-time controller then uses a common control bus to interface with the embedded controllers, reading and writing FPGA registers to issue motion commands and get status updates.