Introduction & Literature Review

November 11, 2020

Stability is a significant issue for mobile robot design. Loss of stability can mean the robot is unable to move and must be reorientated or retrieved, which maybe difficult or impossible in some extreme environments, such as in outer space or a nuclear fuel pool. In the worst case it can result in severe damage or destruction of the robot, and any objects it is carrying. This has become more of an issue as mobile robots have become increasingly fast and agile, often running [], jumping [] and hopping [] around less controlled environments.

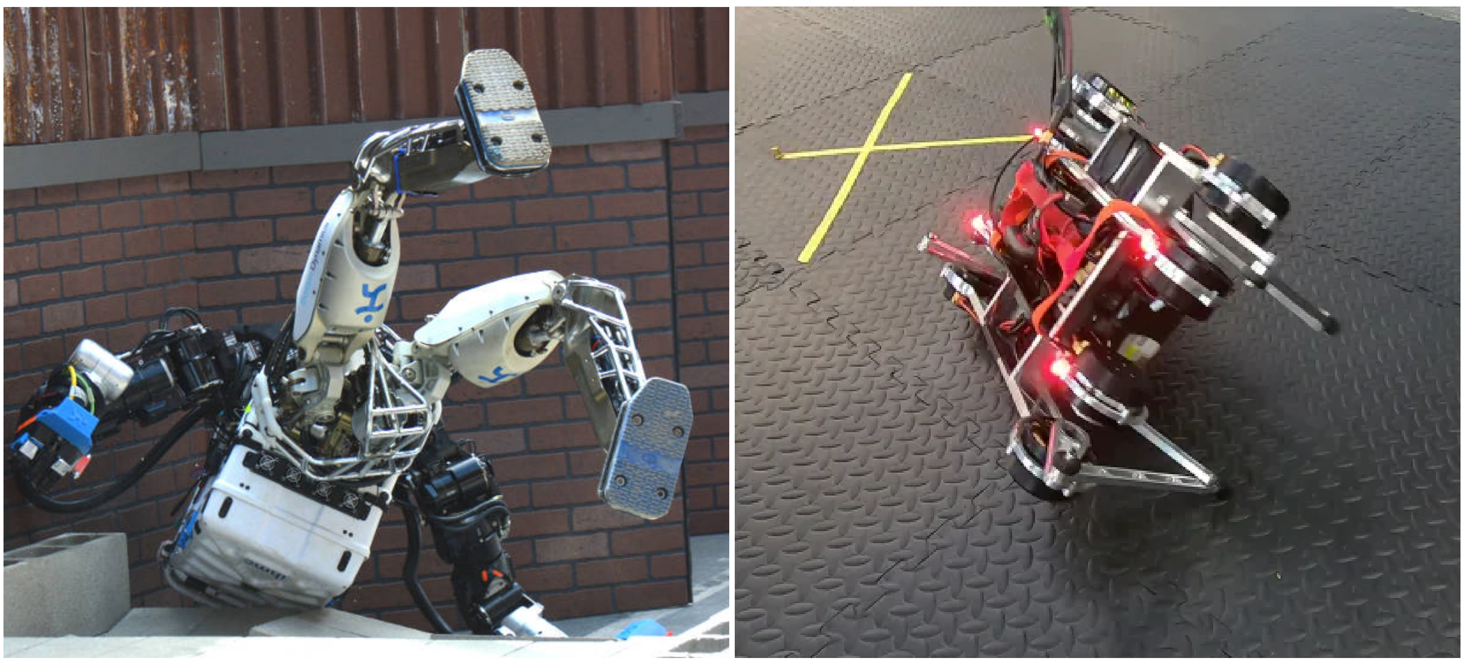


Figure 1: Examples of loss of stability in bipedal and quadrupedal robots.

In many ways the consequences of stability loss in mobile robots are analogous to other situations. A human that falls over has to pick themselves up before continuing on, or if they are infirm they may require assistance. Likewise they could also suffer injury, or if walking along the edge of a long drop, fall to cause severe injury or death. A forklift truck or other piece of heavy plant can topple, injuring the driver and causing the damage or destruction of vehicles and materials.

In general, stability from a biomechanical perspective can be divided into two different types, *static* and *dynamic*. While there are numerous ways to define the difference between the two, such as the maximum lyupanov exponent for dynamic stability [], the following definitions will be used:

- **Static stability** only considers the uniform force of gravity and assumes no other forces are acting on the object.

- **Dynamic stability** considers other forces and torques on the object, both internal and external, as well as gravity.

A stationary object that has no external forces or torques being applied needs to be statically stable, a moving object, or an object that is having a force or torque applied to it other than gravity, needs to be dynamically stable.

Figure 2: Examples of loss of stability (static or dynamic) in a human and forklift truck.

1 Static Stability

To determine if the robot is statically stable, the gravity axis projection of the center of gravity needs to fall within a defined "support polygon" on the plane perpendicular to the gravity axis plane, as in figure 3. If gravity is treated as a constant force along vector g, then the center of mass and center of gravity are equivalent. The center of mass can be calculated for using equation 1 for n bodies of masses $m_{1\ldots n}$ and COM positions $\boldsymbol{p_{1\ldots n}}$. If the gravity vector is parallel to any of the basis vectors, then the perpendicular components of the COM/COG can be used to determine the static stability.

$$\mathrm{COG} = \mathrm{COM} = \frac{\sum_{i=1}^{n} m_i \boldsymbol{p_i}}{\sum_{i=1}^{n} m_i} \tag{1}$$

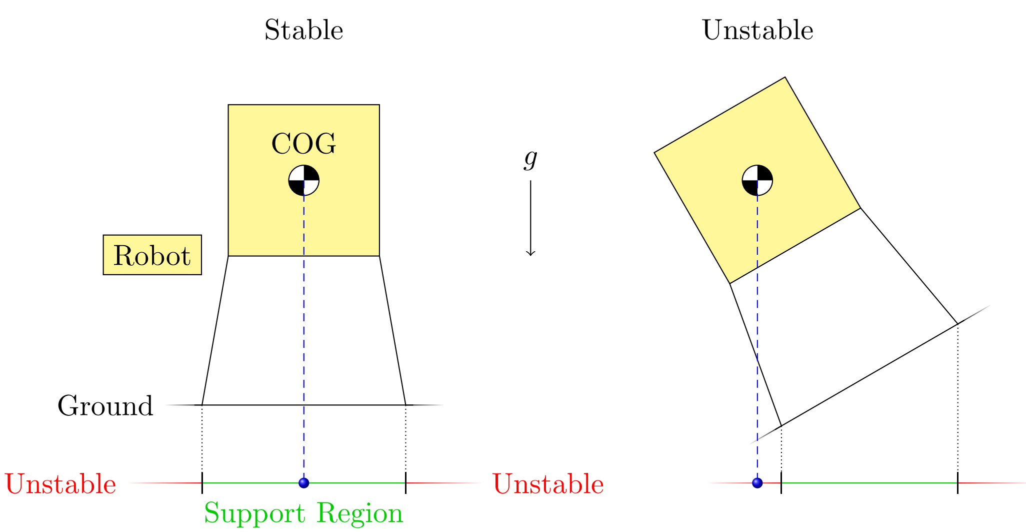


Figure 3: 2D representation of the static stability of a legged robot, with the support region defined by the contact points of the legs with the ground. Notice how the orientation of the robot with respect to the gravity axis can move it in and out of the static stability region.

Fundamentally, there are two different methods to maintain static stability:

- Change the region or shape of the support polygon so the gravity axis projection of the center of gravity remains within the bounds of the support polygon.

- Move the gravity axis projection of the center of gravity so it remains within the bounds of the support polygon.

The former method can be considered equivalent to using a walking pole to steady yourself on uneven ground when hiking, the leg of the pole acts as a new vertex that is used to calculate the support polygon, expanding it sufficiently, or placing your foot in front of you when walking. The latter method can be considered equivalent to leaning back to remain upright when falling forward. Leaning back moves the centre of gravity to keep it within the support polygon.

However, just because the center of gravity falls outside of the support polygon does not mean that loss of stability is inevitable, as long as a force or torque applied to the body counteracts the forces and torques induced by the force of gravity in order to maintain stability. This is similar to applying a torque to your ankle when standing on one leg, or a strong wind keeping you upright when leaning forward. If this is the case, then *dynamic* stability is maintained while *static* stability is lost. If the force or torque is removed, then stability is lost. Conversely, forces and torques can also cause loss of stability even if the center of gravity does not fall outside of the support polygon. This is similar to being pushed over, or stopping too quickly and falling forward. So dynamic stability can maintain stability even if static stability is lost, but static stability cannot maintain stability if dynamic stability is lost.

2 Dynamic Stability

To determine if the robot is dynamically stable,

The concept of the Zero Moment Point, first defined in [], is useful for mobile robots to check if dynamic stability will be maintained. It extends the calculation used for static stability by including inertial forces caused by accelerations of the bodies, as shown in figure 4. Though it has mostly been utilised for bipedal robots to ensure stability while walking, it is also applicable to quadruped robots [], and has even been investigated for the development of a stability warning system in road vehicles [**?**]. The ZMP is formally defined as the point at which the point where the total of horizontal inertia and gravity forces equals zero. It can be thought of as a *dynamically augmented* version of the gravity axis projection of the center of mass.

Equation 2 defines the position of the ZMP for a robot or vehicle with n bodies of masses $m_{1\ldots n}$, COM positions $\boldsymbol{p_{1\ldots n}}$ and COM accelerations $\boldsymbol{\ddot{p}_{1\ldots n}}$, in contact with a planar surface of normal vector $\boldsymbol{n}$ (ZMP cannot be calculated for non-planar surfaces). $\boldsymbol{\tau_{i\ldots n}}$ defines the torque acting on each COM, which can be calculated from .

$$\begin{aligned} \boldsymbol{\tau_i} &= \boldsymbol{R_i}\left(\boldsymbol{I_i}\ddot{\theta}_i - \left(\boldsymbol{I_i}\dot{\theta}_i\right) \times \dot{\theta}_i\right) \\ \text{ZMP} &= \frac{\boldsymbol{n} \times \sum_{i=1}^{n}\left(\boldsymbol{p_i} \times m_i\boldsymbol{g} - \boldsymbol{p_i} \times m_i\boldsymbol{\ddot{p}_i} - \boldsymbol{\tau_i}\right)}{\boldsymbol{n} \cdot \left(\left(\sum_{i=1}^{n} m_i\boldsymbol{g}\right) - \left(\sum_{i=1}^{n} m_i\boldsymbol{\ddot{p}_i}\right)\right)} \end{aligned} \tag{2}$$

3 Payload

When a payload is added to the robot, it is equivalent to instantaneously adding an extra body to the robot of mass m_p and position p_p thus changing the COM. Initially the payload will also create an extra contact point with the ground, changing the support polygon so the robot remains statically stable. However, as soon as contact between the payload and ground is severed, the robot can become statically unstable. This does not mean the robot will immediately lose stability, the dynamic forces created picking up the payload may keep the robot dynamically stable, but once the robot is stationary without other forces acting upon it, it may lose stability. The best way to compensate for this is to change the position of the other bodies so the COM remains within the support polygon. This is akin to leaning back when carrying something heavy. But leaning generally has limited range, so can only compensate for lighter payloads. An alternative is needed for heavy payloads.

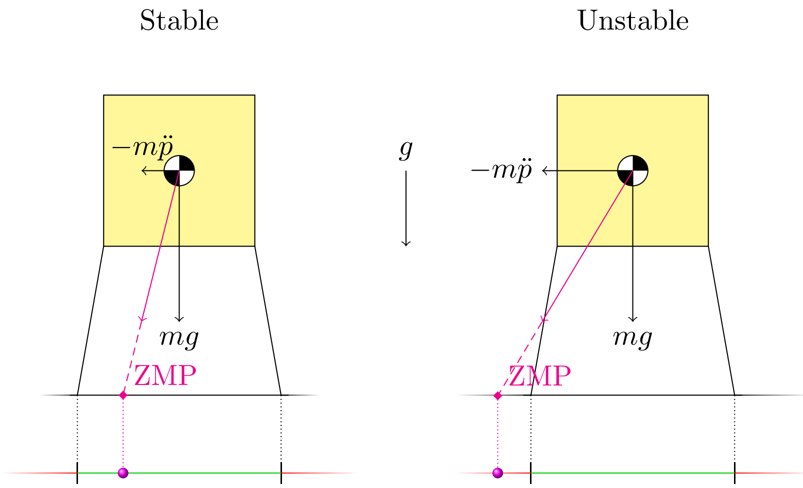


Figure 4: 2D representation of the ZMP of a legged robot under horizontal acceleration, with the support region defined by the contact points of the legs with the ground. Notice how increasing the horizontal acceleration of the robot can make it dynamically unstable.

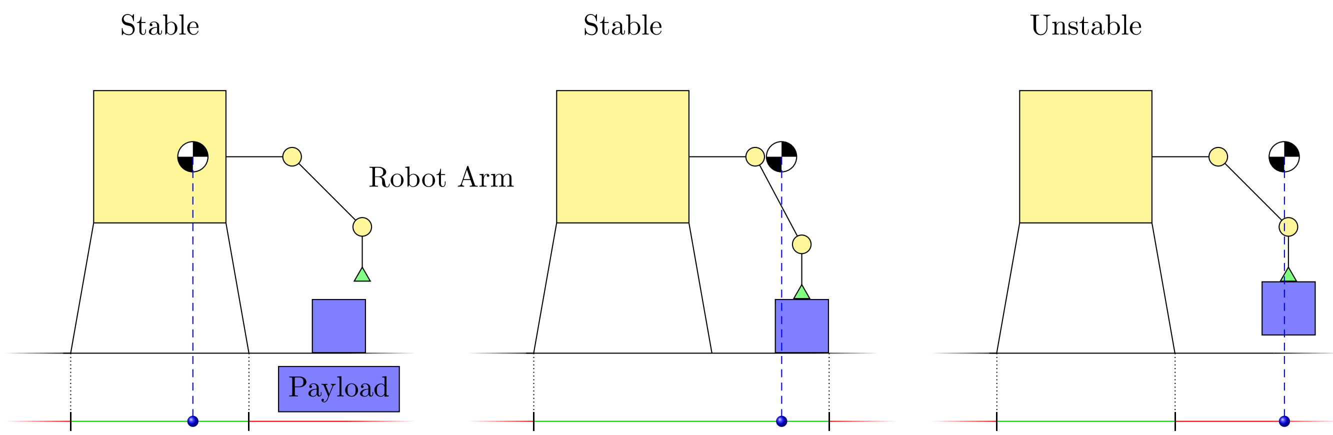


Figure 5: 2D representation of the static stability of a legged robot picking up a payload, with the support region defined by the contact points of the arm and legs with the ground. Notice how the payload acts as a contact point until it is lifted off the ground, preventing loss of stability even as the COM/COG is translated due to the mass of the payload.

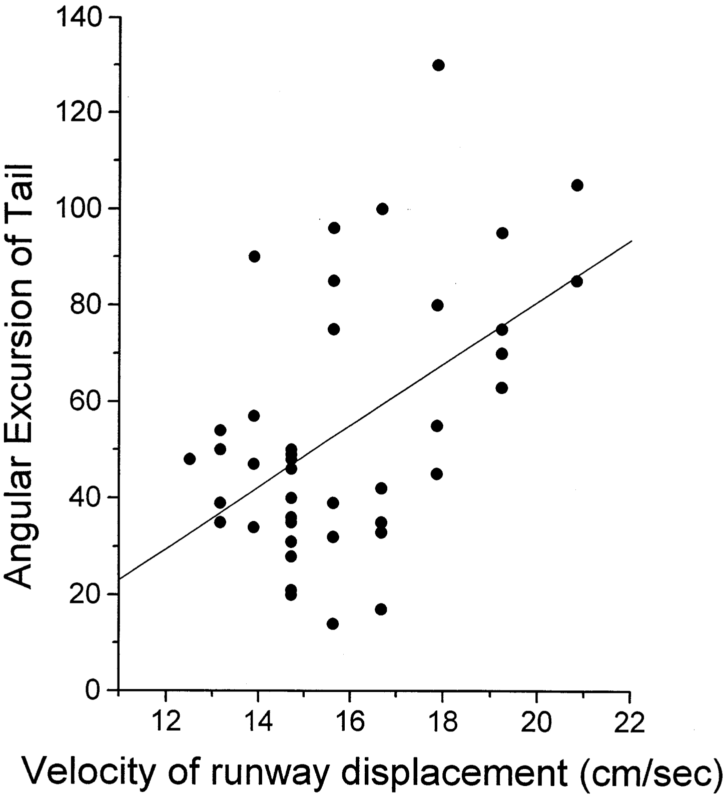
4 Tails

4.1 Tails for Stability in the Animal Kingdom

Tails are a common sight in vertebrate animals, a natural extension of the spinal column. While some tails are used purely for grasping, locomotion, communication or decoration, many have some function in maintaining stability. For example, [5] demonstrates how the domestic cat uses its tail for balance when walking along a narrow beam, which was shifted laterally at a certain velocity by 2.5 cm or 5 cm while they are traversing it. Four cats were trained to walk across the beam, before and after a surgical procedure that severed their spinal cord just above their tail, severely affecting its function. As can be seen from figure 6, this procedure caused the cats to fall from the beam far more often than before surgery.

Other animals use a tail to compensate for inertial forces and torques induced during a change in velocity, either in magnitude or direction. [3, 4] examines how the cheetah uses its tail to counteract centrifugal force when turning at high speed, and acceleration and braking forces when speeding up and slowing down. They then applied this to a robotic vehicle, which is discussed in section 4.2.

Finally, some animals use their tail to remain upright while airborne. [1] examines the aerial stability of the arboreal lizard with an intact tail and with their tail removed. Lizards with the tail removed are unable to maintain



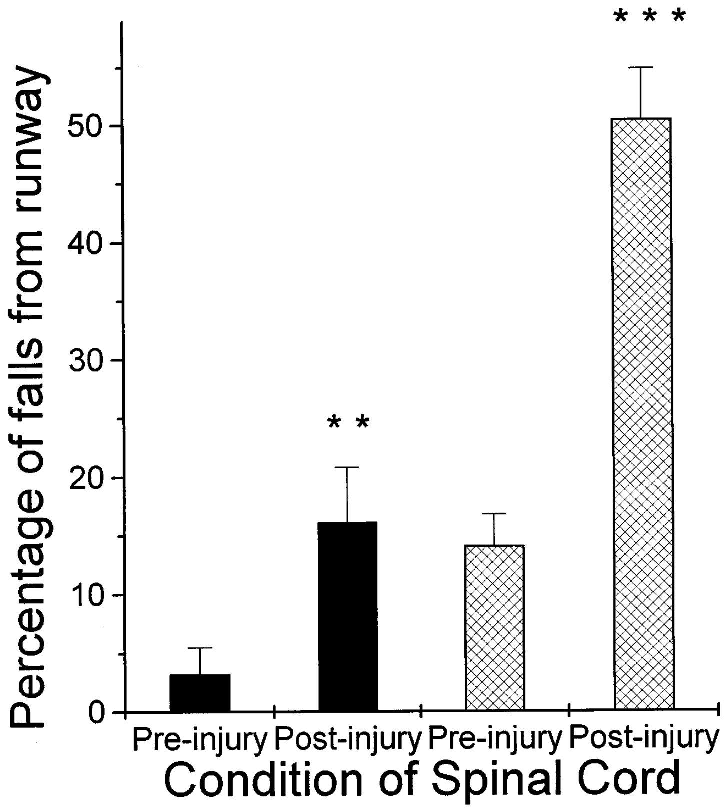


Figure 6: Charts from [5] showing how the cat's tail is used to maintain balance on the beam when it is shifted, and how impairing it causes a major loss of stability. Dark bars are a 2.5 cm displacement, cross-hatched bars are a 5 cm displacement.

their body orientation and do not land cleanly, as can bee seen in figure 8. [2] then applies this to a robotic vehicle, discussed in section 4.2.

4.2 State of the Art Mobile Robots

As many mobile robots have looked at animals for inspiration when it comes to solving various problems with dynamics and locomotion, tails have been used in the development of a significant selection of mobile robots.

5 Literature Review

5.1

References

[1] G. B. Gillis, L. A. Bonvini, and D. J. Irschick. Losing stability: tail loss and jumping in the arboreal lizard anolis carolinensis. *Journal of Experimental Biology*, 212(5):604–609, 2009.

[2] T. Libby, T. Moore, E. Chang-Siu, D. Li, J. Jusufi, D. Cohen, and R. Full. Tail assisted pitch control in a lizard, robot, and dinosaur. In *Integrative and Comparative Biology*, volume 52, pages E106–E106. Oxford univ press inc journals dept, 2001 Evans rd, Cary, NC 27513 USA, 2012.

[3] A. Patel and M. Braae. Rapid turning at high-speed: Inspirations from the cheetah's tail. In *Intelligent Robots and Systems (IROS), 2013 IEEE/RSJ International Conference on*, pages 5506–5511. IEEE, 2013.

[4] A. Patel and M. Braae. Rapid acceleration and braking: Inspirations from the cheetah's tail. In *Robotics and Automation (ICRA), 2014 IEEE International Conference on*, pages 793–799. IEEE, 2014.

Figure 7: Images from [3, 4] of a cheetah using its tail during a turn and braking while chasing a lure.

[5] C. Walker, C. J. Vierck Jr, and L. A. Ritz. Balance in the cat: role of the tail and effects of sacrocaudal transection. *Behavioural brain research*, 91(1-2):41–47, 1998.

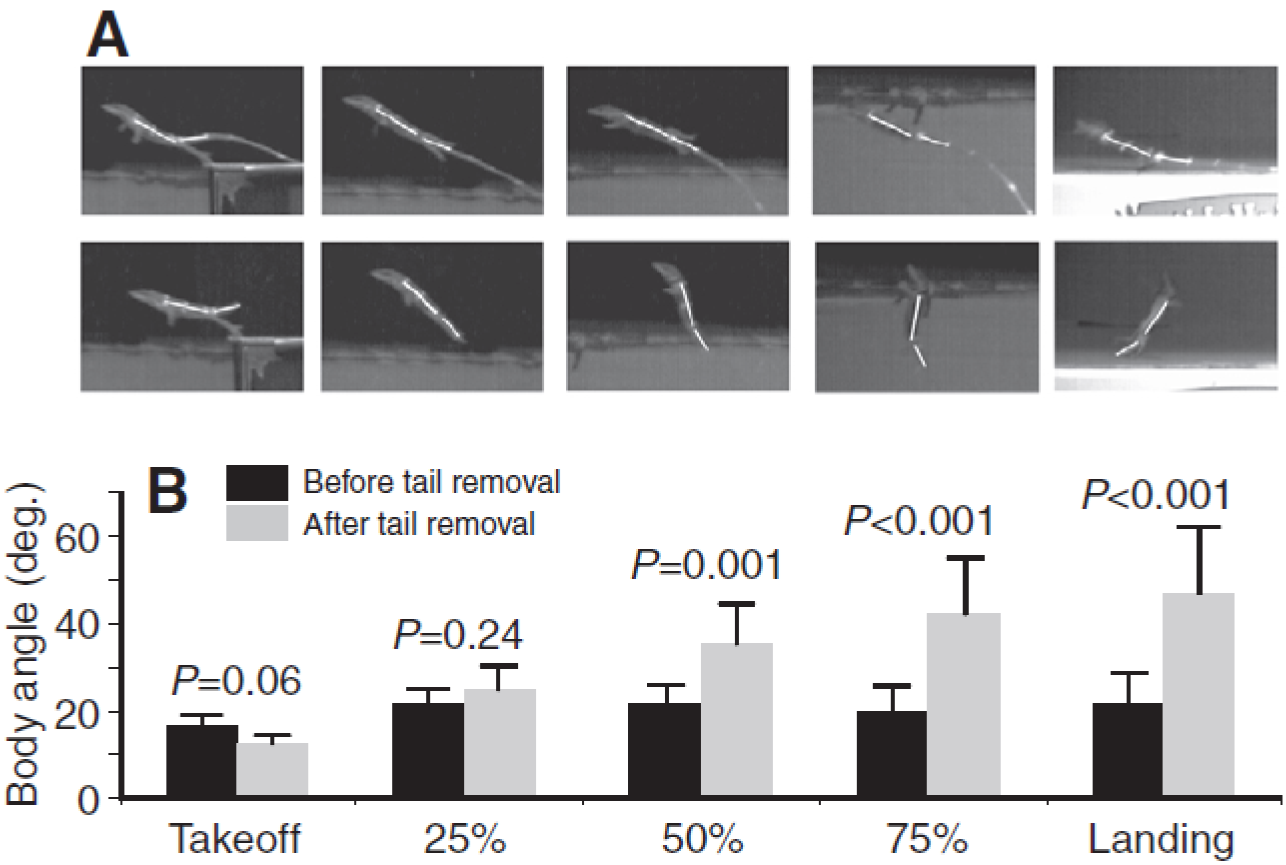


Figure 8: Image from [1], showing the body angle of a lizard during a jump, before and after tail removal.