Fall 2024 - 2025.

· INDE 504 - Discrete Event Simulation.

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Assignment 4 - Input Analysis

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Check python file for 9.7 - 9.11 and other computations.

Check Excel file for 9.14 computations.

Check Arena Input Analyzer for last ex.

Ex 9.12: MLE for Gamma distribution.

The pdf for Gamma is:

$$f(x) = \begin{cases} \frac{\beta\theta}{\Gamma(\beta)} (\beta\theta x)^{\beta-1} e^{-\beta\theta x} & , x > 0 \\ 0, & \text{otherwise} \end{cases}$$

$\Gamma(\beta) = \int^{\infty} x^{\beta-1} e^{-x} dx = (\beta-1)\Gamma(\beta-1)$ being gamma fct.

We need to find $\hat{\beta}$ and $\hat{\theta}$ that maximize likelihood of sample data.

· For $\hat{\beta}$: $M = \ln(\bar{x}) - \frac{1}{n} \ln(x_i)$.

$\bar{x} = \sum_{i=1}^{n} x_i / n = 3.5106$. $\ln(\bar{x}) = 1.2558$.

$\frac{1}{n}\sum_{i=1}^{n} \ln(x_i) = \frac{1}{20} \cdot 21.356 = 1.0678$.

$\Rightarrow M = 1.2558 - 1.0678$

$= 0.188$. $\Rightarrow 1/M = 5.3191$.

From Table A.9, $\hat{\beta} = \frac{5.3191 + 2.855}{5.4} = 2.8123$.

· For $\hat{\theta}$:

$\frac{1}{\bar{x}} = 1/3.5106 = 0.2849$.

Formulas of $\hat{\beta}$ and $\hat{\theta}$ were taken from Chap 9 of BCNN book.

Ex 9.14: Kolmogorov - Smirnov goodness of fit test.

Public safety officers say occurence of accidents ~ uniform.

KS test: Compare $F_n(x)$ and $\hat{F}(x)$ (here, $U(0,100)$).

Null Hypothesis: H_0: X_i's IID with cdf $\hat{F}(x)$.

Alternative Hyp: H_a: X_i's not IID with cdf $\hat{F}(x)$

$\hat{F}(x)$: $X_i \sim U(0,100)$,

$$\Rightarrow \hat{F}(x) = \begin{cases} 0, & x < 0 \\ \frac{x}{100}, & 0 \leq x < 100 \\ 1, & \text{otherwise} \end{cases}$$

$F_n(x_i) = i/n$ where $x_1 \leq x_2 \leq \cdots \leq x_i \ldots \leq x_n$.

$\hat{F}(x_i) = x_i/n$. check excel file for computations.

$$D_n^- = \max_{i=1,\ldots,n} \left\{ \hat{F}(x_i) - \frac{i-1}{n} \right\} = 0.172.$$

$$D_n^+ = \max_{i=1,\ldots,n} \left\{ \frac{i}{n} - \hat{F}(x_i) \right\} = 0.047$$

$$\Rightarrow D_n = \max\{D_n^-; D_n^+\} = \max\{0.172; 0.047\}$$
$$= 0.172.$$

$$AD_n = 0.956 = \left(\sqrt{n} + 0.12 + \frac{0.11}{\sqrt{n}}\right) D_n.$$

Ex 9.17: Chi square Goodness of fit test - (for exponential distribution).

We have 50 samples/observations of times for 50 employees.

$k = 6$ intervals; $\alpha = 0.05$.

$[a_0, a_1)$, $[a_1, a_2)$, $[a_2, a_3)$, $[a_3, a_4)$, $[a_4, a_5)$, $[a_5, a_6)$.

We set $a_0 = 0$ and $a_6 = \infty$.

$\hat{F}(a_j) = j/6$. $\Rightarrow p_j = \hat{F}(a_j) - \hat{F}(a_{j-1}) = \frac{j}{6} - \frac{j-1}{6} = 1/6$.

To find a_j: $\hat{F}(a_j) = j/6 \Rightarrow a_j = -\frac{1}{\lambda} \ln(1 - j/6)$

$(a_j = F^{-1}(j/6))$.

Let's get $\hat{\lambda}$: using MLE:

$\hat{\lambda} = 1/\bar{x} = 1/1.206 = 0.829$.

$\Rightarrow a_j = -1.206 \ln\left(1 - \frac{j}{6}\right)$

$$\begin{cases} a_0 = 0. \\ a_1 = 0.22 \\ a_2 = 0.489 \\ a_3 = 0.836 \\ a_4 = 1.325 \\ a_5 = 2.161 \\ a_6 = \infty \end{cases}$$

H_0: Exp. dist.

H_a: Not exp dist.

$\hat{F}(x) = 1 - e^{-0.829x}$

Exp. Nb of observations in each interval:

$E_j = mp_j = 50 \times 1/6 \simeq 8.33.$

Nb. of observations: N_j in $[a_{j-1} ; a_j)$.

$N_1 = 8 ; N_2 = 11 ; N_3 = 9 ; N_4 = 5 ; N_5 = 10 ; N_6 = 7.$

$$\chi^2 = \sum_{j=1}^{k} \frac{(N_j - mp_j)^2}{np_j} = 0.0133 + 0.8533 + 0.0533 + 1.333 + 0.333 + 0.2133 = 2.8$$

$\chi^2_{5,0.95} = 1.145.$

$\Rightarrow \chi^2 > \chi^2_{5,0.95} \Rightarrow$ Reject H_0.

Ex 9.19 : Check Input Analyzer file.

Observations:

Our data follows a Normal Dist $\sim N(99.2, 10.1)$ since this distribution incurs least MSE. $= 0.00453$.

This distribution passed goodness of fit tests:

→ KS = 0.0648.
P-value > 0.15.

→ χ^2 square = 0.815.
P-value = 0.676.