Fall 2024-2025.

- INDE 504: Discrete Event Simulation.
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Assignment 5: Random Number Generator.

Ex. 7.2:

Applications for Pseudo Random Number Generators:
- Cryptography: Generate keys for encrypting & decrypting data;
- Finance: use Monte Carlo methods for option pricing.
- Machine learning: Initialize weights randomly while training.

Ex 7.4:

$X_0 = 27$; $a = 8$; $c = 47$; $m = 100$.

Using linear congruential generators (LCG):

$X_1 = (8X_0 + 47) \pmod{100}$
$= (8 \times 27 + 47) \pmod{100}$
$= 263 \bmod 100$
$= 63.$

$U_1 = X_1/m = 63/100 = 0.63$;

$X_2 = (8 \times 63 + 47) \pmod{100}$
$= 551 \bmod 100$
$= 51$

$U_2 = 51/100 = 0.51$.

$X_3 = (8 \times 51 + 47) \pmod{100}$
$= 455 \bmod 100$
$= 55.$

$U_3 = 55/100 = 0.55.$

Ex 7.5:

If we set $X_0 = 0$ in the previous exercise, we should not get a problem with our RNG as we are not changing the important parameters such as a, c, m.

$a = 8$; $c = 47$; $m = 100$.

→ c & m are relatively prime (common div = 1).
→ 2 & 5 don't divide $a-1 = 7$.
→ 4 doesn't divide 7.

This RNG doesn't already ~~satisfy~~ full period theorem, but changing X_0 doesn't play a role in it.

Ex 7.6:

Multiplicative Linear Congruential Generator:

LCG with $c = 0$; $X_0 = 117$; $a = 43$; $m = 1000$.

$X_1 = (43 X_0) \bmod 1000$

$= (43 \times 117) \pmod{1000}$

$= 5031 \bmod 1000 = 31.$

$U_1 = 31/1000 = 0.031.$

$X_2 = 43 \times 31 \pmod{1000}$

$= 1333 \bmod 1000 = 333.$

$U_2 = 333/1000 = 0.333.$

$X_3 = (43 \times 333) \bmod 1000$

$= 14319 \bmod 1000$

$= 319.$

$U_3 = 319/1000 = 0.319.$

$X_4 = (43 \times 319) \bmod 1000$

$= 13717 \bmod 1000$

$= 717.$

$U_4 = 717/1000$

$= 0.717.$

Supplement Problems:

31) Check EXCEL Files for RNG Tests;

$O_{11} = 20$; $O_{12} = 22$; $O_{13} = 17$;

$O_{21} = 18$; $O_{22} = 22$; $O_{23} = 23$;

$O_{31} = 21$; $O_{32} = 29$; $O_{33} = 28$.

H_0: U_i's are iid $U(0,1) \times U(0,1)$;

H_a: U_i's aren't iid $U(0,1) \times U(0,1)$.

$$\chi^2 = \sum_{j_1=1}^{3}\sum_{j_2=1}^{3} \frac{[O_{j_1 j_2} - 200/3^2]^2}{(200/3^2)} = 5.92.$$

$\alpha = 0.05;$

$\chi^2_{8, 0.05} = 15.057.$

$\chi^2 < \chi^2_{8, 0.05}$

$\Rightarrow$ We fail to reject H_0;

Hence, U_i's are iid $U(0,1) \times U(0,1)$.

5.2) Runs up and down test;

Check excel for scatter plot. $n = 60$.

H_0: u_i's are indep;

H_a: u_i's are not indep.

$r = 40.$

$$E[R] = \frac{2n-1}{3} = \frac{2 \times 60 - 1}{3} = \frac{119}{3}.$$

$$\sigma_R^2 = \frac{16n - 29}{90} = \frac{16 \times 60 - 29}{90} = \frac{931}{90}.$$

TS: $$Z = \frac{r - E[R]}{\sigma_R} = \frac{40 - 119/3}{\sqrt{\frac{931}{90}}} = \frac{1/3}{3.22} = 0.1035.$$

$\alpha = 0.05.$

$Z_{\alpha/2} = Z_{0.025} = 1.96.$

$Z < Z_{\alpha/2}$

$\Rightarrow$ Don't reject H_0.

$\Rightarrow$ u_i's are independent.