Fall 2024 - 2025.

- INDE 504: Discrete Event Simulation.
- Instructor: Dr. Bacel Maddah.

Assignment 6 - Generating Random Variates and performing output Analysis.

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Ex 8.4)

Triang (1, 10, 4) requires the following algorithm to generate random variates.

1) set $m_i = \frac{m-a}{b-a} = 1/3$

2) Generate $U \sim U(0,1)$ using LCG.

3) If $u < 1/3$: $y = \sqrt{1/3\, u}$.
   If $u \geq 1/3$: $y = \sqrt{(1-u)^2/3}$.

4) Set $X = 1 + 9Y$.

Check python file for generated nbs.

Ex 8.5)

$$F(x) = \begin{cases} 0, & x \leq -3 \\ 1/2 + x/6 & -3 \leq x \leq 0 \\ 1/2 + x^2/32, & 0 \leq x \leq 4 \\ 1, & x > 4 \end{cases}$$

We'll apply inverse transform method: $u = F(x)$.

- $x \leq -3 \Rightarrow u = 0$;
  skip.

- $-3 < x \leq 0$: $F(x) = 1/2 + x/6 = u$.

$\Rightarrow x/6 = u - \frac{1}{2}$

$x = 6u - 3$.

$-3 < x \leq 0 \Rightarrow -3 < 6u - 3 \leq 0$.

$0 < 6u \leq 3$

$0 < u \leq 1/2$

- $0 \leq x \leq 4$: $F(x) = 1/2 + x^2/32$.

$\Rightarrow 1/2 + x^2/32 = u \Rightarrow x^2/32 = u - 1/2$.

$x^2 = 32u - 16$

$x = \sqrt{16(2u-1)} \Rightarrow x = 4\sqrt{2u-1}$ as $x \geq 0$.

$0 < x \leq 4 \Rightarrow 0 < 4\sqrt{2u-1} \leq 4.$

$\Rightarrow 0 < 32u - 16 \leq 16.$

$16 < 32u \leq 32.$

$1/2 \leq u \leq 1.$

. $x > 4$: $F(x) = 1 \Rightarrow u = 1.$

$\Rightarrow$ when $u \geq 1$; set $x = 4$. (this is max value of x).

$\Rightarrow$ use following algorithm:

1) Generate $u \sim U(0,1)$.

2) If $u = 0 \Rightarrow x = -3$.

. If $0 < u \leq 1/2 \Rightarrow x = 6u - 3$.

If $1/2 < u \leq 1 \Rightarrow x = 4\sqrt{2u-1}$

otherwise $\Rightarrow x = 4$.

Ex 8.9)

we have the following CDF: (discrete distribution).

$$F(x) = \frac{x(x+1)(2x+1)}{n(n+1)(2n+1)} \; ; \; x = 1, 2, \cdots, n.$$

Here we cannot invert the CDF as our function is not smooth.

The method used to generate random variates from discrete distrib:

- Generate $u \sim U(0,1)$;
- Find smallest x_i s.t: $u \leq F(x_i)$.
- set $X = x_i$.

For $n = 4$:

$$F(x) = \frac{x(x+1)(2x+1)}{180} \; ; \; x = 1, 2, 3, 4.$$

$F(1) = \frac{6}{180} = 0.0333$. ; $F(2) = \frac{30}{180} = 0.16667$. ; $F(3) = \frac{84}{180} = 0.46667$. ; $F(4) = \frac{180}{180} = 1.0$

we generated the following:

$R_1 = 0.83 \in [F_3; F_4] \Rightarrow x_1 = 3.$

$R_2 = 0.24 \in (F_2; F_3) \Rightarrow x_2 = 2.$

$R_3 = 0.57 \in (F_3; F_4) \Rightarrow x_3 = 3.$

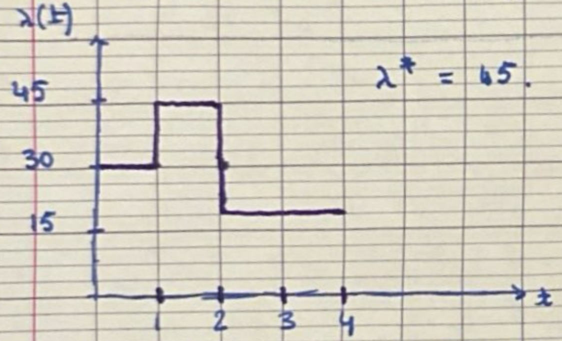
Ex 8.29)

At $t = 0$, 6:00 AM. Arrival rate (arr/hr) of customers to a breakfast restaurant opening from 6-9:00 AM is:

$$\lambda(t) = \begin{cases} 30, & 0 \leq t \leq 1; \\ 45, & 1 \leq t < 2; \\ 20, & 2 \leq t \leq 4. \end{cases}$$

Goal: derive a thinning algorithm to generate 100 arrival times.

Here, we can draw $\lambda(t)$ wrt "t".



$\lambda^* = 45.$

We'll generate random variates based on λ^* & Thinning approach.

1) Set $t_0 = 0$: (or 6:00 AM); $i = 1$;

2) $t = t_{i-1}$;

3) Generate $U_1, U_2 \sim \mathcal{U}(0,1)$;

4) $t = t - 1/\lambda^* \ln(U_1)$;

5) If $U_2 \leq \frac{\lambda(t)}{\lambda^*} \Rightarrow t_i = t$; $i = i+1$ & go to step 2.

otherwise

↳ step 3.

I'll repeat this until I have 100 observations.

EX 11.1)

Terminating	Steady state
a) If its shift ends at a certain time	a) If it's operating 24/7.
b) Simulate for fixed duration?	b) Simul. for a cont. time.
c) No consec. shifts	c) Shifts continuous.
d) Like others	d) Like others.

EX 11.4) $\alpha = 95\%$. $\varepsilon = 0.05$ hrs (3 min);

Need to determine m^*;

$\bar{X}(10) = 46.86;$

$S^2(10) = 0.2.$

$\Rightarrow m^* = 330$ (Check Excel File)...