Exercise 4 10/26/22

1A. In an adjacency list Adj of a directed graph the out degree of a vertex has length Adj[v] and the sum of all adj-lists are $|E|$. So the time to compute one vertex is $\Theta(Adj[v])$ and for all its $\Theta(V+E)$. In the in-degree if we search all the lists, for each vertex, the time complexity is $\Theta(VE)$

1B.

Adjacency Matrix

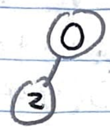
$$
\begin{matrix}
0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0
\end{matrix}
$$

Adjacency List

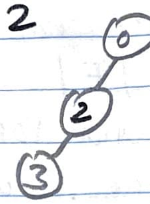
0	$\rightarrow 2 \rightarrow 5$
1	$\rightarrow 4$
2	$\rightarrow 0 \rightarrow 3$
3	$\rightarrow 1 \rightarrow 4$
4	
5	$\rightarrow 6 \rightarrow 7$
6	$\rightarrow 7$
7	$\rightarrow 5$

Step-by-Step DFS

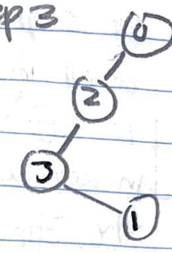
Step 1



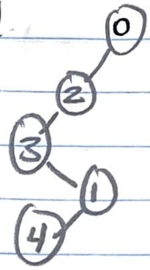
Step 2



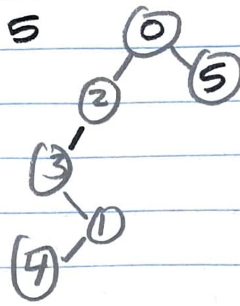
Step 3



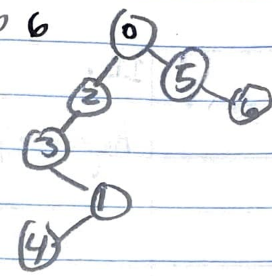
Step 4



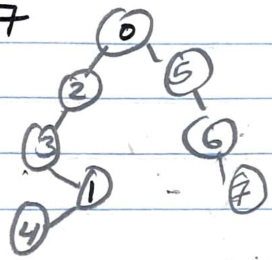
Step 5



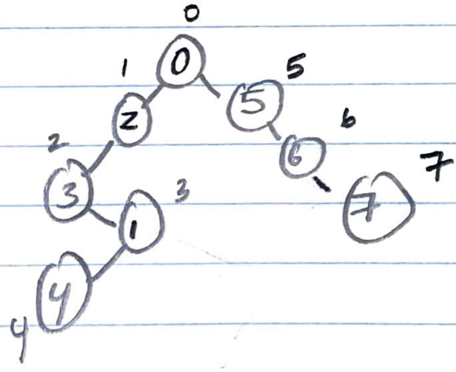
Step 6



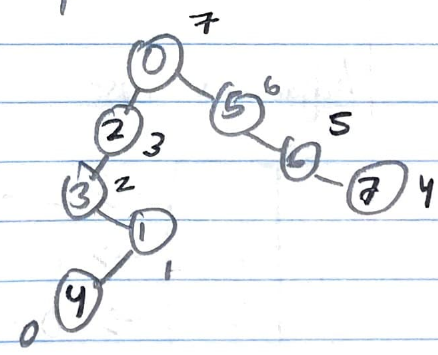
Step 7



Visit - Order

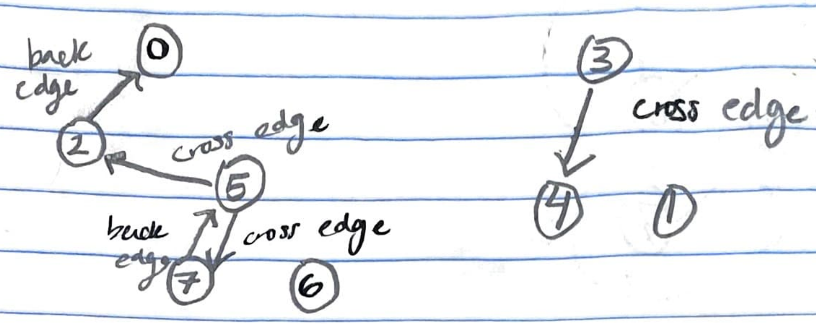


Completion - Order

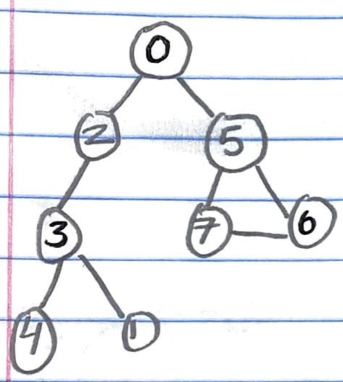


The strong components in the graph are 0, 2, 5 and 5, 6, 7

Classify non-tree edges



Undirected Graph



BFS Queue

0
2, 5
5, 3, 5
3, 5, 6, 7
~~5~~, 6, 7, 1, 4
6, 7, 1, 4
7, 1, 4, 7
1, 4, 7
4, 7, 4
~~7~~, 4
~~4~~

1C.

Suppose we have a directed graph on vertices {1, 2, 3} and edges on (1, 2) and (2, 3). Then 2 has both incoming and outgoing edges. If 3 is the root then that will be its own DFS, then our second root would be 2 so we won't explor it. Then, picking 1 as the root we don't mess up the fact that 2 is along in a DFS tree even though it has both an incoming and outgoing edge in the graph