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AERSP 304 Project 3

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Abstract

This report investigates the design of a pitch motion controller for an aircraft. First, a state space is defined and used to make both open and closed loop transfer functions. After analyzing these functions in the laplace and time domains, the root locus method is used to implement a lead compensator. This lead compensator makes the otherwise unstable system stable via addition of a pole and zero, along with a gain value K. This project is a comprehensive demonstration of the material taught in AERSP 304.

Question 1 Analysis

See appendix A

Question 3 Output and Discussion

Open Loop Poles: s = 0, -0.3695+/-0.8860i

Open Loop Zeroes: s = -0.1514

Shown in *Figure 1* is the step response of the open loop transfer function for our system, derived from the state space defined in Question 1.The system is unstable, which makes sense since we have not yet implemented a control system.

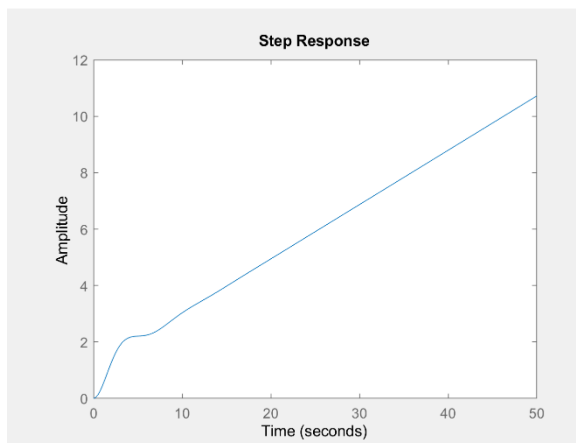


Figure 1 (Step Response of Open Loop System)

Questions 4 & 5 Output and Discussion

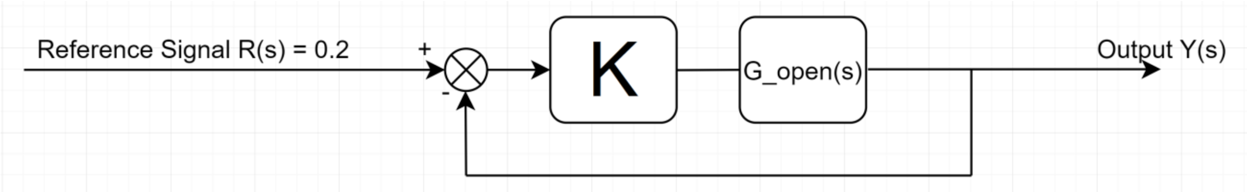
Closed Loop Poles: s = -0.3255 +/- 1.3187i, -0.0880

Closed Loop Zeroes: s = -0.1541

Shown here is the step response of our closed loop system, in both the laplace domain (*Figure 2*) and time domain (*Figure 3*). The closed loop system was directly derived from the open loop system defined in Question 3 using the formula below:

$$G_{closed}(s) = \frac{G_{open}(s)}{1 + G_{open}(s)}$$

The block diagram for $G_{closed}(s)$ is shown below:



As expected, the response of closed loop system approaches our desired value of 0.2 as t approaches infinity. Furthermore, this behavior is also exhibited in the time domain. For the derivation of $\theta(t)$ from $G(s)$, see Appendix B. The response characteristics of the closed loop system are provided in Appendix C.

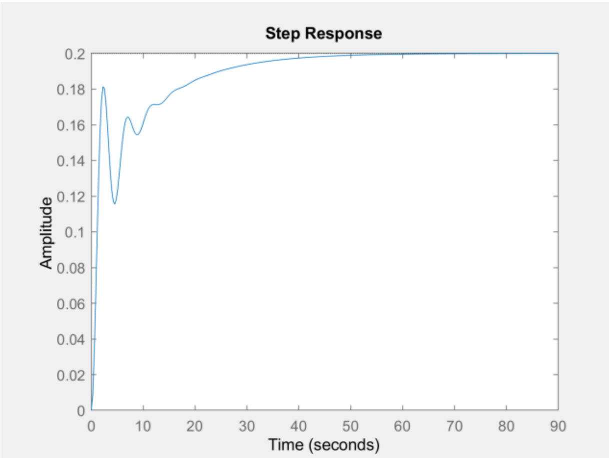


Figure 2 (Step Response of Closed Loop System)

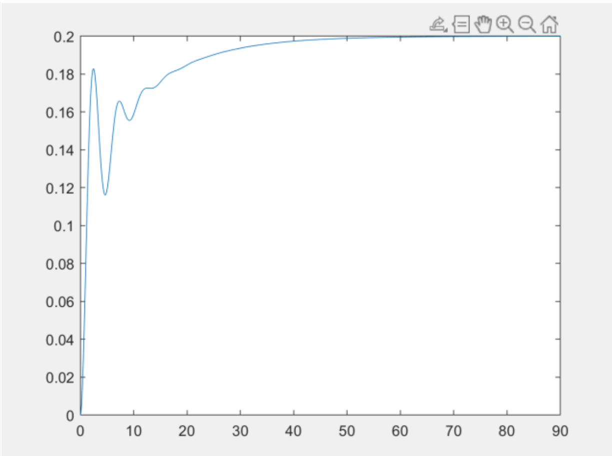


Figure 3 (apologies for the lack of a title. This graph is of $\theta(t)$ vs t)

Questions 6 & 7 Output and Discussion

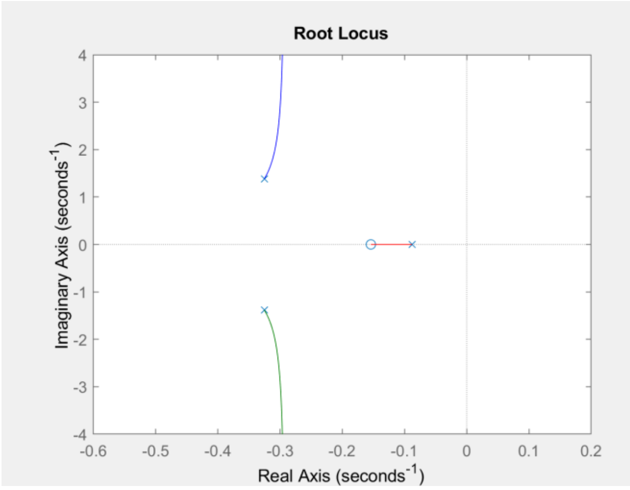
Questions 6 and 7 involved the implementation of a lead compensator in order to have our closed loop response meet the following performance criteria:

$Settling\ Time < 10s$

$Overshoot < 10\%$

$Natural\ Frequency\ \omega_n > 0.9\ rad/s$

The root locus method is used to display the implications of these performance requirements on the system in the imaginary plane. Firstly, *Figure 4* shows the root locus of the closed loop system pre-compensator implementation. Secondly, *Figure 5* shows the root locus of the system with the compensator adding a pole at -3 and zero at -0.9. In addition, the performance requirements are represented by the shaded areas— for the system to meet said requirements, the root locus must enter the *white* region as K approaches infinity. Figures 6,7, & 8 show the step response of the system for K = 2,50, and 200 respectively. The performance characteristics for these plots are also shown in Appendix C.



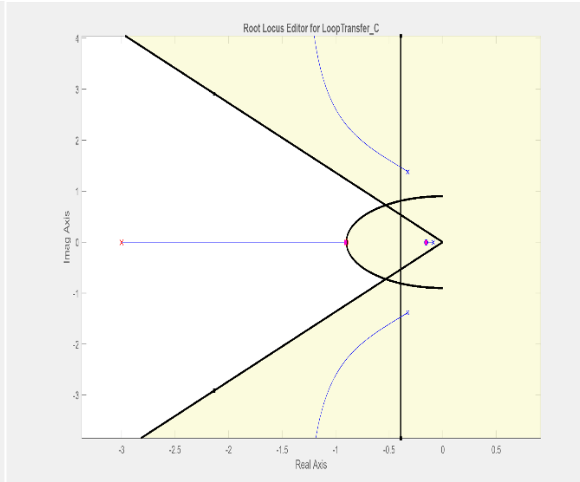
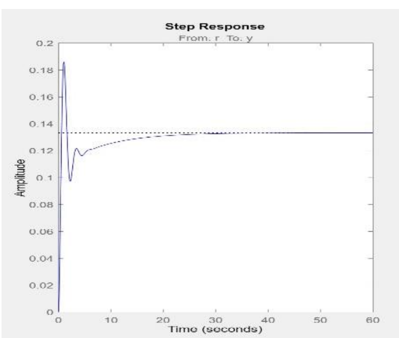


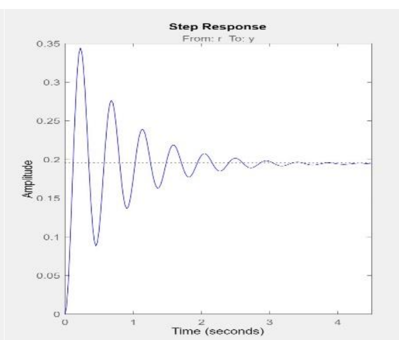
Figure 4

Figure 5 (Root Locus with compensator and requirements)

Without the compensator, the root locus never exists in the white region and the system can therefore never meet the performance criteria. However, by implementing a compensator *some* of the performance requirements can be met for certain values of K. The natural frequency* requirement was met for all values of K, the settling time requirement was met for K = 50 & 200, and the overshoot requirement was not met for any values of K. This demonstrates the influence of the compensator on system response: by manually placing poles and zeroes in the root locus the behavior of the system was changed.

**bode plots provided by the ControlSystemDesigner MATLAB command were used to determine the natural frequency at different values of K. While this was not directly covered in class I do not know any way to calculate it from only class material.*





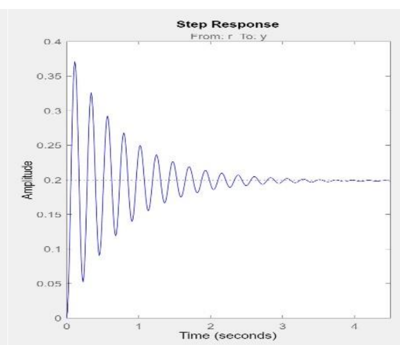


Figure 6 (K = 2)

Figure 7 (K = 50)

Figure 8 (K = 200)

Appendix A

1) $\dot{\vec{x}} = \begin{bmatrix} \alpha \\ q \\ \theta \end{bmatrix}$; $\dot{\vec{x}} = \begin{bmatrix} -0.313\alpha + 56.7q \\ -0.0139\alpha - 0.426q \\ 56.7q \end{bmatrix} + \begin{bmatrix} 0.232 \\ 0.203 \\ 0 \end{bmatrix} \delta_e = \begin{bmatrix} \dot{\alpha} \\ \dot{q} \\ \dot{\theta} \end{bmatrix}$

Using the following state space general formula:

$$\dot{\vec{x}} = A\vec{x} + Bu$$
$$\dot{\vec{y}} = C\vec{y} + Du$$

And the following eqns:

$$\vec{y} = [0 \quad 0 \quad \theta]^T$$
$$u = \delta_e$$

We obtain the following state space model:

$\vec{s} = \vec{x} = \begin{bmatrix} \alpha \\ q \\ \theta \end{bmatrix}$; $\dot{\vec{x}} = \begin{bmatrix} -0.313 & 56.7 & \emptyset \\ -0.0139 & -0.426 & \emptyset \\ \emptyset & 56.7 & \emptyset \end{bmatrix} \begin{bmatrix} \alpha \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} 0.232 \\ 0.203 \\ \emptyset \end{bmatrix} \delta_e$

$= \begin{bmatrix} \dot{\alpha} \\ \dot{q} \\ \dot{\theta} \end{bmatrix}$

$$\dot{\vec{y}} = \begin{bmatrix} 0 \\ 0 \\ \theta \end{bmatrix} \quad \left.\begin{matrix} {}^*C = [0 \quad 0 \quad 1] \\ D = \emptyset \end{matrix}\right\} \text{From } y = \theta$$

Appendix B

Calculating $y(t)$ for G_{closed} & $\theta_{des} = 0.2$

$$G_{closed} = \frac{1.151s + 0.1774}{s^3 + 0.739s^2 + 2.072s + 0.1774}$$

$$\frac{Y}{R} = G_{closed} \; ; \; R = 0.2$$

$$\therefore Y = G_{closed}(0.2) = \frac{0.2302 + 0.03548}{s^3 + 0.739s^2 + 2.072s + 0.1774}$$

using residue,

$$Y = \frac{0.2302(s+0.1541)}{s(s+0.08804)(s^2+0.651s+2.015)}$$

using residue again,

$$Y = \frac{0.2}{s} - \frac{0.088}{s+0.088} - \frac{0.1121s+0.081}{s^2+0.651s+2.015}$$

$$y(t) = \mathcal{L}^{-1}(Y(s)) = 0.2 - 0.088e^{(-0.088t)} - e^{(-0.3255t)}(0.1121\cos(1.3817t) + 0.032\sin(1.3817t))$$

this function of t is hard-coded into MATLAB on line 31.

Appendix C

1. Response Characteristics of Closed Loop System

```
      RiseTime: 1.7893
 TransientTime: 35.0977
  SettlingTime: 35.0977
   SettlingMin: 0.5776
   SettlingMax: 0.9999
     Overshoot: 0
    Undershoot: 0
          Peak: 0.9999
      PeakTime: 98.7594
```

K=2 Response Characteristics

Characteristic	Value
Settling Time	18.9
% Overshoot	39.4
Natural Frequency	2.67

K = 50 Response Characteristics

Characteristic	Value
Settling Time	3.23
% Overshoot	75.8
Natural Frequency	13.8

K = 200 Response Characteristics

Characteristic	Value
Settling Time	2.96
% Overshoot	86.5
Natural Frequency	27.7

**green font indicates that the performance requirement is met. Red indicates that it is not.*

I have completed this work with integrity

Ralph Quartiano