ASTR 562 - High-Energy Astrophysics

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Homework 2

Problem 1
Data Analysis: Describe in detail what is meant by analyzing the: a) energy, b) arrival time, and c) spatial position of photons. Mention the complications to building a perfect telescope in each case. Then describe how one might perform data analysis using all measurements of a photon together (spatially-resolved time-dependent spectroscopy).

Solution. Analyzing the energy, arrival time, and spatial position of photons is crucial in various fields such as astronomy, quantum mechanics, and medical imaging. Each aspect provides valuable information about the source of the photons and the environment through which they have traveled. Here's a detailed explanation of each aspect and the complications associated with building a perfect telescope for each:

(a) **Energy Analysis:** The energy of a photon is determined by its frequency or wavelength. Analyzing the energy spectrum of photons allows us to identify the chemical composition, temperature, and physical processes occurring in the source.

Complications:

- Building a perfect telescope for energy analysis requires high spectral resolution to differentiate between closely spaced energy levels.
- Detectors must be sensitive across a wide range of wavelengths to capture the entire energy spectrum.
- Calibration is critical to accurately determine the energy of detected photons, accounting for instrumental effects and atmospheric absorption.

(b) **Arrival Time Analysis:** Arrival time analysis involves determining the precise time when photons reach the detector. This information can reveal dynamic processes such as pulsations, explosions, or interactions with matter.

Complications:

- Achieving nanosecond or even femtosecond timing resolution requires advanced timing electronics and fast detectors.
- Background noise and timing jitter can obscure the true arrival time of photons, necessitating sophisticated data processing algorithms.
- Atmospheric effects and dispersion introduce delays, which must be corrected to accurately determine arrival times.

(c) **Spatial Position Analysis:** Spatial position analysis involves determining the origin or direction of incoming photons. This is crucial for imaging objects in space, locating sources of radiation, and studying the distribution of matter.

Complications:

- Achieving high spatial resolution requires large aperture telescopes with precise optics to minimize aberrations.
- Atmospheric turbulence causes image distortion, limiting the achievable spatial resolution.
- Detector imperfections and electronic noise can introduce errors in determining the spatial position of photons.

Performing data analysis using spatially-resolved time-dependent spectroscopy: Spatially-resolved time-dependent spectroscopy combines information from all three aspects (energy, arrival time, and spatial position) to gain a comprehensive understanding of photon emissions. Sophisticated data analysis techniques such as Fourier transforms, deconvolution, and maximum likelihood estimation are employed to extract meaningful information from the data. By analyzing photons together, we can study how energy, timing, and spatial distribution correlate with each other, providing insights into the underlying physical processes. Data from different detectors or instrument channels are correlated to reconstruct the complete spatiotemporal spectrum of the source. However, this approach requires careful calibration, synchronization, and computational resources to handle the large volumes of data generated.

In summary, analyzing the energy, arrival time, and spatial position of photons is essential for understanding various phenomena in astronomy and other fields. Building a perfect telescope for each aspect is challenging due to technical limitations and environmental factors. Spatially-resolved time-dependent spectroscopy offers a powerful approach to extracting comprehensive information from photon data, but it requires sophisticated instrumentation and data analysis techniques. ■

Problem 2
Photons scatter many times in the interior of the sun. The cross-section for that scattering is approximately given by the Thomson Cross-Section ($\sigma_T = 6.65 \times 10^{-25}\,\mathrm{cm}^2$). Calculate the mean free path of a photon in the Sun assuming that the density is about $150\,\mathrm{g\,cm}^{-3}$ and the plasma is fully ionized and contains 25% Helium and 75% Hydrogen.

Solution. The mean free path is the typical distance a particle will travel between interactions. The mean free path for electron scattering in the sun is

$$\ell_{es} = \frac{1}{n_e \sigma_T},$$

where n_e is the number density of electrons and σ_T is the Thomson cross-section.

Assuming that the plasma is fully ionized, containing 25% Helium and 75% Hydrogen, and that there is one electron per atom of mass m_H and two electrons per atom of mass m_He, then

$$n_e = \frac{\rho}{0.75 m_\mathrm{H} + 0.25 m_\mathrm{He}} = \frac{0.15}{0.75(1.67 \times 10^{-27}) + 0.25(2)(1.67 \times 10^{-27})} = 7.185 \times 10^{25}\,\mathrm{cm}^{-3}.$$

The mean free path for electron scattering in the sun is then

$$\ell_{es} = \frac{1}{n_e \sigma_T} = \frac{1}{(7.185 \times 10^{25})\,(6.65 \times 10^{-25})} = 2.1 \times 10^{-2}\,\mathrm{cm}.$$

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Problem 3
The spatial energy density per unit frequency interval of blackbody radiation is given by

$$u_\nu = \frac{8\pi h}{c^3} \frac{\nu^3}{\mathrm{e}^{\frac{h\nu}{k_B T}} - 1}$$

(a) Write down an expression for the total energy of a photon gas of volume V (Hint: this will be an integral over frequency).

(b) Approximate the denominator by $\mathrm{e}^{\frac{h\nu}{k_B T}} - 1 \approx \mathrm{e}^{\frac{h\nu}{k_B T}}$ and evaluate the integral in part (a).

Solution. (a) The total energy of a photon gas of volume V is given by

$$E = \int_0^\infty u_\nu V \,\mathrm{d}\nu = \frac{8\pi V}{c^3} \int_0^\infty \frac{h\nu^3}{\mathrm{e}^{\frac{h\nu}{k_B T}} - 1} \,\mathrm{d}\nu.$$

(b) Approximating the denominator by $\mathrm{e}^{\frac{h\nu}{k_B T}} - 1 \approx \mathrm{e}^{\frac{h\nu}{k_B T}}$, we have

$$E = \frac{8\pi V}{c^3} \int_0^\infty \frac{h\nu^3}{\mathrm{e}^{\frac{h\nu}{k_B T}}} \,\mathrm{d}\nu$$

Letting $x = \frac{h\nu}{k_B T}$, $\mathrm{d}x = \frac{h}{k_B T}\,\mathrm{d}\nu$, we have

$$\begin{aligned}
E &= \frac{8\pi V k_B T}{c^3} \int_0^\infty \frac{\left(\frac{k_B T}{h} x\right)^3}{\mathrm{e}^x} \,\mathrm{d}x \\
&= \frac{8\pi V (k_B T)^4}{(hc)^3} \int_0^\infty x^3 \mathrm{e}^{-x} \,\mathrm{d}x \\
&= \frac{8\pi V (k_B T)^4}{(hc)^3} \left[- \left(x^3 + 3x^2 + 6x + 6\right) \mathrm{e}^{-x}\right]_0^\infty \\
&= \frac{48\pi V (k_B T)^4}{(hc)^3}.
\end{aligned}$$

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