PHYS630: Advanced Theory Of Electricity And Magnetism

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PHYS 313

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Area of research: theoretical astrophysics, plasma physics.

Second most important physics theory, after mechanics

$$\nabla \cdot \mathbf{E} = 4\pi \rho$$

$$\nabla \cdot \mathbf{B} = 0$$

$$abla extbf{x} extbf{E} = -rac{1}{c} rac{\partial extbf{B}}{\partial t}$$

$$abla extbf{X} extbf{X} extbf{B} = rac{1}{c} \left(4\pi extbf{J} + rac{\partial extbf{E}}{\partial t}
ight)$$

- Physics: differential form of Maxwell's eqns (integral form of little use - only a few special cases)
- cgs units...

Prerequisites

- This is graduate-level physics E&M course
- not introductory General Physics
- Expected that you took a basic undergraduate E&M course
- not a "hands-on" eg, lots of math problems like "find a electric potential in some complicated cavity"
 - we'll do "simple" examples
 - will not do too many "practical" problems
- Not much numerics/computational
 - those are different, often problem-specific "arts"

Homeworks

- About once a week, 2-3 problems.
- Must turn upload to Brightspace before the class starts we'll be discussing solutions in class
- No real option for late homework

Our TAs

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Grading

- Homeworks 50%
- Midterm 20%
- Final 30%
- No relative grading (everybody can get A+).

Collaboration policy

- Try it first yourself.
- Then talk to others: collaboration is encouraged (but not copying!) Talk to each other - that's how progress in science is made!
- If a problem has been done in close collaboration, please state that
- Please uphold "academic integrity"
- ChatGPT...? We'll see... Use it as a tool to learn, not to cheat

Attendance policy

- "<u>University Academic Regulations regarding class</u>
 <u>attendance</u> state that students are expected to be
 present for every meeting of the classes in which they are
 enrolled.
- No track of attendance will be kept, but let me know ahead of time if you have to miss the class

- office hours Monday 10am-11pm, PHYS 313
- Also we can chat via Zoom etc
- Communication
 - homeworks: Brightspace,
 - quick question: email

- Ask questions at any moment!
- If I am not asked at least three question per class, then I failed...

Textbooks...

Textbooks (recommended)

- Landau & Lifshitz, (top notch, but a bit too much at first run...)
 - vol 2, Field Theory
 - vol 8, Electrodynamics of continuous media
- Jackson, Classical electrodynamics more of detailed explanation
 - L&L: top down, Jackson: bottom-up
 - Somewhat different choices of what is more important, and applications I'll make mine.
 - I will not follow exactly, will be jumping in-between a bit
 - Will give you the chapters #s for each lecture
- Griffiths, Introduction to Electrodynamics somewhat slower, more like undergraduate, but explains things clearly. Especially vector analysis.
- I will not be distributing lecture notes (sometimes...)

Special relativity

Landau&Lifshitz

CHAPTER 1. THE PRINCIPLE OF RELATIVITY

- 1 Velocity of propagation of interaction
- 2 Intervals
- 3 Proper time
- 4 The Lorentz transformation
- 5 Transformation of velocities
- 6 Four-vectors
- 7 Four-dimensional velocity

Griffiths

12	Ele	ctrodynamics and Relativity	47
	12.1	The Special Theory of Relativity	47
		12.1.1 Einstein's Postulates	47
		12.1.2 The Geometry of Relativity	48
		12.1.3 The Lorentz Transformations	49
		12.1.4 The Structure of Spacetime	50
	12.2	Relativistic Mechanics	50
		12.2.1 Proper Time and Proper Velocity	50
		12.2.2 Relativistic Energy and Momentum	50
		12.2.3 Relativistic Kinematics	51
		12.2.4 Relativistic Dynamics	51
	12.3	Relativistic Electrodynamics	52
		12.3.1 Magnetism as a Relativistic Phenomenon	52
		12.3.2 How the Fields Transform	52
		12.3.3 The Field Tensor	53
		12.3.4 Electrodynamics in Tensor Notation	53
		12.3.5 Relativistic Potentials	54
A		tor Calculus in Curvilinear Coordinates	54
	A.1	Introduction	54
	A.2	Notation	54
	A.3	Gradient	54
	A.4	Divergence	54
	A.5	Curl	55
	A.6	Laplacian	55

Jackson

Chapter

11 / 5	Special Theory of Relativity	514		
11.1	The Situation Before 1900, Einstein's Two Postulates 515			
11.2	Some Recent Experiments 518			
11.3	1.3 Lorentz Transformations and Basic Kinematic Results of Spe			
	Relativity 524			
11.4	Addition of Velocities; 4-Velocity 530			
11.5				
11.6 Mathematical Properties of the Space-Time of Special				
	Relativity 539			
11.7	Matrix Representation of Lorentz Transformations, Infinitesimal			
	Generators 543			
11.8	Thomas Precession 548			
11.9				
11.10				
11.11	Relativistic Equation of Motion for Spin in Uniform or Slowly Var	ving		
	External Fields 561	, ,		
11.12	Note on Notation and Units in Relativistic Kinematics 565			
	References and Suggested Reading 566			
	Problems 568			

We are physicists here - understanding principles! But Landau is way too fast in the beginning

EM is relativistic tensor theory

$$\partial_{\alpha}F^{\alpha\beta} = \frac{4\pi}{c}J^{\beta}$$

$$\left[\partial_{\alpha}F_{\beta\gamma} + \partial_{\beta}F_{\gamma\alpha} + \partial_{\gamma}F_{\alpha\beta} = 0\right]$$

$$F_{\alpha\beta} = \begin{pmatrix} 0 & E^{x} & E^{y} & E^{z} \\ -E^{x} & 0 & -B^{z} & B^{y} \\ -E^{y} & B^{z} & 0 & -B^{x} \\ -E^{z} & -B^{y} & B^{x} & 0 \end{pmatrix}.$$

There is no separate E-field or B-field: one EM field Separation in E & B depends on the observer Still, can be practical

I'll try to balance

- We will be jumping between the books/chapters, no continuous reading of a given book
- Reading assignments will be given ahead off time (today)
- Many results will be given and explained, not derived from first principles
- but some basics will be derived
- Often: order-of-magnitude derivation of principal results
- There are many practical problems (eg., given some shape of a charged conductor, find fields) - will try to avoid details, will discuss principles, few simple examples, a "physicsy course"
- Mathematics is/can be interesting, but can quickly become super-messy

Course outline

- We'll start with what you know, but at higher mathematical level: charges create fields,
 - Fields and potentials
 - Electrostatics
 - Magnetostatics
 - conductors in EM fields (EM with boundary conditions)
- Charge in EM fields:
 - relativistic mechanics
 - 4-vectors
 - EM tensor, EM stress-energy tensor
 - gauges
 - Lorentz transformation of the fields, field invariants
- Maxwell's equations
- EM waves
 - spectrum
 - polarization

- EM emission (addition of phases of retarded potentials)
 - retarded potentials
 - multipole emission
 - emission during collisions
 - cyclotron emission
 - emission by relativistic particles
 - radiation reaction
- EM scattering by charged particles (Thomson cross-section, and the like)
- EM propagation
 - ray tracing
 - diffraction
- Advanced topics:
 - EM phenomena in dielectrics
 - Plasmas, dispersion
 - laser-plasma interaction
 - Cherenkov/transition radiation
 - quantization of EM fields (where to put "hat"s)
 - Some quantum EM (eg, Casimir effect)

Reading assignments

(see file in Brightspace)

Mathematics will be given as we go

- Lagrangian vs Hamiltonian approaches in EM
- EM is a tensor theory. Vectors still useful
- vector identities and derivatives, curvilinear coords
- the *nabla* operator ∇
- Fourier
- spherical harmonics (Fourier on a sphere)
- special functions
- Mathematically, results quickly become super-complicated (even for linear problems)
 - some results will be given without derivation
 - intuition is needed

Vector analysis in EM

- It's complicated...
- Vectors, tensors, ∇ operators, curvilinear coordinates will be give as we go

Vector Formulas

$$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \mathbf{b} \cdot (\mathbf{c} \times \mathbf{a}) = \mathbf{c} \cdot (\mathbf{a} \times \mathbf{b})$$

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$$

$$(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d}) = (\mathbf{a} \cdot \mathbf{c})(\mathbf{b} \cdot \mathbf{d}) - (\mathbf{a} \cdot \mathbf{d})(\mathbf{b} \cdot \mathbf{c})$$

$$\nabla \times \nabla \psi = 0$$

$$\nabla \cdot (\nabla \times \mathbf{a}) = 0$$

$$\nabla \times (\nabla \times \mathbf{a}) = \nabla(\nabla \cdot \mathbf{a}) - \nabla^2 \mathbf{a}$$

$$\nabla \cdot (\psi \mathbf{a}) = \mathbf{a} \cdot \nabla \psi + \psi \nabla \cdot \mathbf{a}$$

$$\nabla \cdot (\psi \mathbf{a}) = \mathbf{a} \cdot \nabla \psi + \psi \nabla \times \mathbf{a}$$

$$\nabla \times (\psi \mathbf{a}) = \nabla \psi \times \mathbf{a} + \psi \nabla \times \mathbf{a}$$

$$\nabla (\mathbf{a} \cdot \mathbf{b}) = (\mathbf{a} \cdot \nabla)\mathbf{b} + (\mathbf{b} \cdot \nabla)\mathbf{a} + \mathbf{a} \times (\nabla \times \mathbf{b}) + \mathbf{b} \times (\nabla \times \mathbf{a})$$

$$\nabla \cdot (\mathbf{a} \times \mathbf{b}) = \mathbf{b} \cdot (\nabla \times \mathbf{a}) - \mathbf{a} \cdot (\nabla \times \mathbf{b})$$

$$\nabla \times (\mathbf{a} \times \mathbf{b}) = \mathbf{a}(\nabla \cdot \mathbf{b}) - \mathbf{b}(\nabla \cdot \mathbf{a}) + (\mathbf{b} \cdot \nabla)\mathbf{a} - (\mathbf{a} \cdot \nabla)\mathbf{b}$$

NRL_FORMULARY

$$igg|
abla \cdot {f E} = rac{
ho}{arepsilon_0}$$

$$abla \cdot \mathbf{B} = 0$$

$$abla extbf{x} extbf{E} = -rac{\partial extbf{B}}{\partial t}$$

$$oxed{
abla imes \mathbf{B} = \mu_0 \left(\mathbf{J} + arepsilon_0 rac{\partial \mathbf{E}}{\partial t}
ight)}$$

SI - cgs

$$abla \cdot \mathbf{E} = 4\pi
ho$$

$$\nabla \cdot \mathbf{B} = 0$$

$$abla extbf{x} extbf{E} = -rac{1}{c} rac{\partial extbf{B}}{\partial t}$$

$$abla imes \mathbf{B} = rac{1}{c} \left(4\pi \mathbf{J} + rac{\partial \mathbf{E}}{\partial t}
ight)$$

- $\mathbf{D} = \epsilon_0 \mathbf{E}$, $\mathbf{B} = \mu_0 \mathbf{H}$
- D,E,B,H all different dimension

• E, B - same dimension

$$abla \cdot {f E} = rac{
ho}{arepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$abla imes \mathbf{E} = -rac{\partial \mathbf{B}}{\partial t}$$

$$abla imes \mathbf{B} = \mu_0 \left(\mathbf{J} + arepsilon_0 rac{\partial \mathbf{E}}{\partial t}
ight)$$

SI - cgs

$$\nabla \cdot \mathbf{E} = 4\pi \rho$$

$$\nabla \cdot \mathbf{B} = 0$$

$$abla imes \mathbf{E} = -rac{1}{c}rac{\partial \mathbf{B}}{\partial t}$$

$$abla imes \mathbf{B} = rac{1}{c} \left(4\pi \mathbf{J} + rac{\partial \mathbf{E}}{\partial t}
ight)$$

•
$$F = \left(\frac{1}{4\pi\epsilon_0}\right) \frac{e^2}{r}$$
 (SI) $\rightarrow F = \frac{e^2}{r}$ (cgs)

$$\bullet \quad \left(\frac{1}{4\pi\epsilon_0}\right) \to 1$$

•
$$\left(\frac{1}{4\pi\epsilon_0}\right) \to 1$$

• $\epsilon_0 \mu_0 = \frac{1}{c^2}$, so $\mu_0 \to \frac{4\pi}{c^2}$

$$\bullet \quad B = \frac{\mu_0 I}{2\pi r} \to \frac{2I}{rc}$$

Sorry - I am just the messenger here

Maxwell's equations

$$\nabla \cdot \mathbf{E} = 4\pi \rho$$

$$\nabla \cdot \mathbf{B} = 0$$

$$abla extbf{x} extbf{E} = -rac{1}{c}rac{\partial extbf{B}}{\partial t}$$

$$abla extbf{X} extbf{X} extbf{B} = rac{1}{c} igg(4\pi extbf{J} + rac{\partial extbf{E}}{\partial t} igg)$$

cgs:

- E & B same dimension
- differential

EM is relativistic theory

There is no way to "avoid" relativistic kinematics in dealing with EM field

• plus dynamics
$$\partial_t \mathbf{p} = e \left(\mathbf{E} + (\mathbf{v}/c) \times \mathbf{B} \right)$$

$$\mathbf{p} = \frac{\mathbf{v}}{\sqrt{1 - (v/c)^2}} m$$

Differential and integral forms

$$\begin{cases}
\operatorname{rot} \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{H}}{\partial t}, & \oint \mathbf{E} d\mathbf{l} = -\frac{1}{c} \frac{\partial}{\partial t} \int \mathbf{H} d\mathbf{S}; \\
\operatorname{div} \mathbf{H} = 0, & \oint \mathbf{H} d\mathbf{S} = 0.
\end{cases}$$

$$\begin{cases}
\operatorname{rot} \mathbf{H} = \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi \mathbf{j}}{c}, & \oint \mathbf{H} d\mathbf{l} = \frac{4\pi}{c} \mathbf{I} + \frac{1}{c} \frac{\partial}{\partial t} \int \mathbf{E} d\mathbf{S}; \\
\operatorname{div} \mathbf{E} = 4\pi \rho, & \oint \mathbf{E} d\mathbf{S} = 4\pi e.
\end{cases}$$

- Gauss theorem: $\oint a \, dS = \int \operatorname{div} a \, dV$
- Stokes theorem: $\oint a \, dl = \int \operatorname{rot} a \, dS$

Classical Maxwell's are linear

$$\nabla \cdot \mathbf{E} = 4\pi \rho$$

$$\nabla \cdot \mathbf{B} = 0$$

Can depend on fields: non-linearity

$$abla extbf{x} extbf{E} = -rac{1}{c} rac{\partial extbf{B}}{\partial t}$$

$$abla extbf{X} extbf{B} = rac{1}{c} \left(4 \sqrt[4]{\mathbf{J}} + rac{\partial \mathbf{E}}{\partial t}
ight)$$

Quantum E&M is nonlinear

EM in matter

- Conductors are easy: E_{inside}=0+ boundary conditions
- Dielectrics are trickier:
 - ρ , **j** are functions of external E&B

$\mathbf{D}^{\mathrm{G}} = \mathbf{E}^{\mathrm{G}} + 4\pi \mathbf{P}^{\mathrm{G}}$	$\mathbf{B}^{\mathrm{G}} = \mathbf{H}^{\mathrm{G}} + 4\pi \mathbf{M}^{\mathrm{G}}$
$\mathbf{P}^{\mathrm{G}} = \chi_{\mathrm{e}}^{\mathrm{G}} \mathbf{E}^{\mathrm{G}}$	$\mathbf{M}^{\mathrm{G}} = \chi_{\mathrm{m}}^{\mathrm{G}} \mathbf{H}^{\mathrm{G}}$
$\mathbf{D}^{\mathrm{G}} = arepsilon^{\mathrm{G}} \mathbf{E}^{\mathrm{G}}$	$\mathbf{B}^{\mathrm{G}} = \mu^{\mathrm{G}}\mathbf{H}^{\mathrm{G}}$
$arepsilon^{ m G} = 1 + 4\pi \chi_{ m e}^{ m G}$	$\mu^{ m G}=1+4\pi\chi_{ m m}^{ m G}$

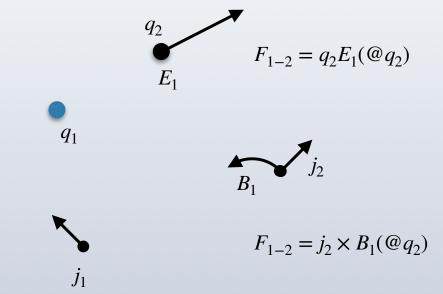
- tensorial functions, eg induced $E_{\alpha}^{ind}=\epsilon_{\alpha\beta}E_{\beta}^{ex}$
- often $\epsilon(\omega)$ dispersion
- Also, non-linear $E^{ind} = \alpha_1 E^{ex} + \alpha_2 (E^{ind})^2$
- Typically applications are so wide and different we'll discuss just the basic principles
- Collective effects (eg. plasma)

Brightspace

• Questions?

"Action at a distance"

- How two bodies that are not in contact interact with each other?
 - what is "in contact"?
 - Source -> field -> force
 - (and from 2nd on 1st)
- No need for medium to carry the field



"Action at a distance"

- No need for "medium" to carry the field
- Maxwell's ϵ_0 , μ_0 were designed to quantify the vacuum as medium not needed.
- $\epsilon_0 \mu_0 = \frac{1}{c^2}$, no need for ϵ_0, μ_0