# PHYSICS 601

## Homework Assignment 2

1. **Fermat's Principle**: If the velocity of $u$ of light is given by the continuous function $u = u(y)$, the actual light path connecting $(x_1, y_1)$ and $(x_2, y_2)$ in a plane is the one which extremizes the time integral

$$I = \int_{x_1,y_1}^{x_2,y_2} \frac{ds}{u(y)}$$

   (a) Derive Snell's Law from Fermat's Principle: Prove that $\dfrac{\sin\phi}{u}$ is a constant, where $\phi$ is the angle between the tangent of the light path and a vertical line at that point.

   (b) Suppose that light travels in the $x$-$y$ plane in such a way that its speed is proportional to $y$. Prove that the light rays emitted from any point are circles with their centers on the $x$-axis.

2. The Lagrangian density $\tilde{L}$ which generates a given set of Euler-Lagrange equations is not unique. Prove this result by showing that adding a divergence to $\tilde{L}$ does not alter the Euler-Lagrange equations. Specifically let

$$\tilde{L} = \tilde{L}\left(x_k, w_j, \frac{\partial w_j}{\partial x_k}\right) ; \tilde{L}' = \tilde{L} + \sum_k \frac{\partial f_k}{\partial x_k},$$

where $f_k = f_k(w_j)$, and $j = 1, \cdots, m$; $k = 1, \cdots, n$. Then show that $\tilde{L}'$ and $\tilde{L}$ lead to the same Euler-Lagrange equations.

3. Show that, if $\psi(x)$ and $\bar{\psi}(x)$ are taken as two independent functions, the Lagrangian density ($\bar{\psi} = \psi^*$ and $\dot{\psi} = \partial\psi/\partial t$)

$$\tilde{L} = \frac{\hbar^2}{2m}\nabla\psi\cdot\nabla\bar{\psi} + V\psi\bar{\psi} - \frac{i\hbar}{2}(\bar{\psi}\dot{\psi} - \psi\dot{\bar{\psi}})$$

leads to the time-dependent Schroedinger equation

$$H\psi = \left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\psi = i\hbar\frac{\partial\psi}{\partial t}$$

and the complex conjugate of this equation.