PHYS 630 - Advanced Electricity and Magnetism

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Homework 11

Problem 1
A light of intensity I_0 falls on a layer of plasma (a mix of free ions and electrons) with density n (in units cm^{-3}, assume single charged) and thickness l (cm). Find intensity of light that passed unscattered (assume heavy ions do not respond to radiation, and wave-electron interaction occurs in single-particle regime).

Solution. The intensity of unscattered light follows the Beer-Lambert law given by

$$I = I_0 \mathrm{e}^{-\sigma n l},$$

where σ is the scattering cross-section for single-particle interaction, n is plasma density (cm^{-3}), l is layer thickness (cm)
In single-particle regime, σ is the Thomson cross-section, given by

$$\sigma = \frac{8\pi}{3} r_e^2 = 6.65 \times 10^{-25} \mathrm{cm}^2,$$

where $r_e = 2.818 \times 10^{-13}$ cm is the classical electron radius.
Thus, the final intensity is

$$I = I_0 \mathrm{e}^{-\frac{8\pi}{3} r_e^2 n l}.$$

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Problem 2
Why does a simple charge in circular motion emit radiation while a constant loop current does not?

Solution. A particle in circular motion changes its direction and and velocity at every point in time, thus making the particle have non-zero acceleration, which in turn leads to the emission of radiation. A particle in a constant loop current does not have any change in velocity, making the acceleration zero and no emission of radiation. ■

Problem 3
A dielectric sphere of radius R with dielectric constant ϵ has constant charge density ρ_e. Find electric field (both inside and outside).
Hint:

$$\begin{aligned} \operatorname{div} \mathbf{D} &= 4\pi\rho_e \\ \mathbf{E} &= -\nabla\Phi \\ \mathbf{D} &= \epsilon\mathbf{E} \end{aligned}$$

and the boundary condition is

$$\epsilon \partial_r \Phi|_{R_-} = \partial_r \Phi|_{R_+}$$

Solution.

- **Inside the sphere** ($r \le R$)**:** From $\operatorname{div} \mathbf{D} = 4\pi\rho_e$, we get

$$\nabla^2 \Phi = -\frac{4\pi\rho_e}{\epsilon_0}.$$

Solving with spherical symmetry, we get

$$\Phi(r) = -\frac{4\pi\rho_e}{3\epsilon_0} r^2 + A.$$

The boundary condition at $r = 0$ requires $A = 0$. Thus,

$$\Phi(r) = -\frac{4\pi\rho_e}{3\epsilon_0} r^2.$$

- **Outside the sphere** $(r > R)$**:** The Laplace equation is $\nabla^2\Phi = 0$. The general solution is then

  $$\Phi(r) = \frac{B}{r} + C.$$

  The boundary condition at $r = R$ requires Φ to be continuous. Hence

  $$\epsilon\partial_r\Phi|\, R- = \partial_r\Phi|\, R+.$$

These conditions give

$$\begin{cases} \mathbf{E}_{\text{inside}}(r) = -\dfrac{4\pi\rho_e r}{3\epsilon_0}, & \text{for } r < R, \\ \mathbf{E}_{\text{outside}}(r) = \dfrac{4\pi R^3\rho_e}{3\epsilon_0 r^2}, & \text{for } r > R. \end{cases}$$

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