Homework 1
Due Wednesday, January 17th

Problem 1

Estimate the number of electrons in the sun.

Note: When I ask you to "estimate" a number, I want zero significant digits. Your answer should just be 10 to some power (and you get full credit even if you're one power off). Accordingly, your derivation should avoid any complexity at a higher level of detail than the order-of-magnitude estimate you are looking for.

Problem 2

Show that the diffusion equation

$$\dot{u} - \nu u'' = 0 \tag{1}$$

is a conservation law. Define the total "charge" between two points a and b:

$$Q \equiv \int_a^b u dx \tag{2}$$

and show that the rate of change of Q in time is determined only by the flux of charge through the points a and b. What is this flux (rate of flow of u) and can it be interpreted physically?

Problem 3

Estimate how long it takes a photon to escape the sun. Assume a photon mean free path given by λ and the radius of the sun is given by R.

Note: when I ask you to "estimate" a formula (rather than a number) the result should not include any dimensionless coefficients like π or 2. I am only looking for how the answer scales as a function of the different variables in the system.

Problem 4

Solve the diffusion equation for an initial condition given by a single Fourier mode:

$$\rho(x, 0) = e^{ikx} \tag{3}$$

What is $\rho(x, t)$?

Problem 5

Derive the next-order correction to Stirling's approximation:

$$\ln N! = N \ln N - N + f(N) \tag{4}$$

where $f(N)$ is the correction. Do this by finding a relationshhip between $f(N)$ and $f(N+1)$ and setting that (approximately) to

$$f(N+1) - f(N) \approx \frac{df}{dN} \tag{5}$$

Make a plot of the ratio of $N!$ to its approximation and show that it asymptotes to a constant:

$$\frac{N!}{\text{Appx.}} = e^{\ln N! - N \ln N + N - f(N)} \tag{6}$$

Make a plot of the above formula for the $f(N)$ you computed and $f(N) = 0$ with no correction, to show the difference. I highly recommend keeping it in the exponential form, as $N!$ and N^N are both very large and will not be representable as standard floating-point numbers for $N >\sim 100$; so take the difference $\ln N! - N \ln N$ first before exponentiating.

Depending on the detail of your calculation, your ratio should asymptote to a constant which may not be equal to unity. Double-check the formula by looking up Stirling's Approximation on google (assuming you haven't already) and check that the constant term you get is consistent with the overall dimensionless constant in the most detailed formula you can find on the internet.