PHYSICS 601

Homework Assignment 8

1. Show that the Green's function for the 2-dimensional Laplace over the entire 2-dimensional space is

$$G(\mathbf{r}, \mathbf{r}') = -\frac{1}{2\pi}\ln[(x - x')^2 + (y - y')^2]^{1/2},$$

where, $\mathbf{r} = (x, y)$ and $\mathbf{r}' = (x', y')$.

2. Write down the solution to the equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x, y)$$

in terms of $G(\mathbf{r}, \mathbf{r}')$.

3. $\psi(x, t)$ satisfies the 1-dimensional Schroedinger equation

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} = i\hbar\frac{\partial \psi}{\partial t},$$

with initial condition $\psi(x, 0) = \delta(x)$, and boundary condition

$$\frac{\partial \psi}{\partial x}\left(-\frac{L}{2}, t\right) = \frac{\partial \psi}{\partial x}\left(\frac{L}{2}, t\right) = 0.$$

Show by the method of separation of variables that

$$\psi(x, t) = \frac{1}{L}\sum_{n=-\infty}^{\infty}\cos\frac{2n\pi x}{L}\exp\left[-i\frac{\hbar}{2m}\left(\frac{2n\pi}{L}\right)^2 t\right].$$