PHYSICS 601

Homework Assignment 1

1. Find the shortest distance between two points using polar coordinates, i.e., using the line element $ds^2 = dr^2 + r^2 d\theta^2$.

2. **Inverse Isoperimetric Problem** ("Isoareametric" Problem): Prove that of all simple closed curves enclosing a given area, the least perimeter is possessed by the circle.

3. In all our discussions so far on finding the function f for which

$$I = \int_{x_1}^{x_2} f dx$$

is an extremum, it has been assumed that f depends on x, y, and y', that is, $f = f(x, y, y')$. Show that, if f also depends on $y'' = d^2y/dx^2$, and for fixed end points at which y and y' are prescribed, the Euler-Lagrange equation is

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) + \frac{d^2}{dx^2}\left(\frac{\partial f}{\partial y''}\right) = 0 .$$

4. We proved in class that the curve which encloses the most area for a given perimeter was a circle. To do this, we demonstrated that this curve was characterized by a constant curvature $1/\lambda$ everywhere. Obtain the same result by using the Euler-Lagrange equation to solve for $r = r(\theta)$ or $\theta = \theta(r)$ directly. [Hint: Use the fact that $f(\theta, r, r')$ and $g(\theta, r, r')$ are independent of θ.]

5. In connection with Problem 4 above, show that the curvature K in polar coordinate system is

$$K = \left| \frac{r^2 + 2r'^2 - rr''}{(r^2 + r'^2)^{3/2}} \right| ,$$

where $r' = dr/d\theta$ and $r'' = d^2r/d\theta^2$.