PHYSICS 601

Homework Assignment 5

1. In class we have derived Stirling's Formula for real S,

$$S\,! \approx \sqrt{2\pi S}\, S^{S} e^{-S} \; .$$

Show that the same result holds for arbitrary complex z as well, namely

$$\Gamma(z+1) = z\,! \approx \sqrt{2\pi} z^{z+1/2} e^{-z} \; .$$

2. Repeat the derivation given in class of the result

$$I = \Gamma(a)\Gamma(1-a) = \int_0^\infty du \frac{u^{a-1}}{1+u} = \frac{\pi}{\sin \pi a} \; .$$

Use the substitution $u = e^{x}$ to write I in the form

$$I = \int_{-\infty}^{+\infty} dx \frac{e^{ax}}{1+e^{x}} \; .$$

Integrate around the rectangular contour with the four corners at $+R$, $+R + i\,2\pi$, $-R + i\,2\pi$, and $-R$. Determine that the residue at $z = i\,\pi$ is $-e^{ia\pi}$.

PHYSICS 601

Homework Assignment 6

1. Consider the error function

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \ .$$

a) Develop an asymptotic series for $\mathrm{erf}(x)$. [Hint: look at $1 - \mathrm{erf}(x)$ and develop this in a series by partial integration]

b) Show that this series is not uniformly convergent.

2. In class we developed asymptotic expansions for $C_i(x)$ and $S_i(x)$. Rederive the same expansions by a series of partial integrations. [Hint: Recall that

$$C_i(x) + iS_i(x) = -\int_x^\infty \frac{e^{it}}{t} dt \ .$$