Homework 3
Due Wednesday, Feb 7th

Problem 1

a) Show that the ratio of specific heats in a gas can be determined by the slope of an isentropic path through (P, V) space:

$$\frac{C_P}{C_V} = -\left(\frac{\partial \ln P}{\partial \ln V}\right)_S \tag{1}$$

You may assume a multiplicity factor $\Omega(E, V) \propto E^{\alpha N} V^N$.

b) Show generally that a gas with an adiabatic equation of state

$$P = (\gamma - 1)E/V \tag{2}$$

obeys a polytropic equation of state under adiabatic transformations (i.e. when entropy is kept fixed):

$$P = K n^\gamma \tag{3}$$

where $n = N/V$ is the number density. Again you may assume the system has multiplicity $\Omega \propto E^{\alpha N} V^N$, as above.

Problem 2

Let's think about the atmosphere. Atoms are bound by gravity, but they still seem to float some distance above the Earth (rather than all staying on the ground, at a minimum energy). We can figure out the distribution of atoms with height, using the magic of estimation.

a) First, assume a given mean free path λ for molecules (assuming a single species with mass m) and assume the atmosphere is at some fixed temperature T . Estimate the diffusion constant ν in the atmosphere.

b) Atoms are always trying to fall in earth's gravity but they keep having collisions. Estimate the time Δt between collisions.

c) In the time Δt, gravity can give molecules a net drift velocity $v_d \sim -g\Delta t$, before the molecule is scattered and its velocity is totally random again. So, there is a downward flux of atoms given by $F = n v_d$, where n is the number density. This is cancelled by an equal and opposite diffusive flux (computed from homework 1, remember?). In equilibrium, these fluxes cancel (so that $\dot{n} = 0$); set the fluxes to cancel and use this fact to compute $n(z)$, the number density in the atmosphere, as a function of z. (call n_0 the number density at the base of the Earth). Assume g is a constant (not dependent on z).

Problem 3

a) Show that you can attain the results of problem (2c) by assuming that the chemical potential gets an external component

$$\mu = \mu_0(n, T) + \mu_{\text{ext}}(z) \tag{4}$$

where

$$\mu_{\text{ext}}(z) = mgz \tag{5}$$

and assuming μ = constant in the atmosphere. Why would we add such a term to the chemical potential and why would we assume that μ is a constant throughout the atmosphere?

b) The atmosphere is roughly a 4-to-1 mixture of Nitrogen and Oxygen. By what factor does the ratio n_{N_2}/n_{O_2} change when I go 10 km above the ground? You can assume the atmosphere has $T = 300$ Kelvin.