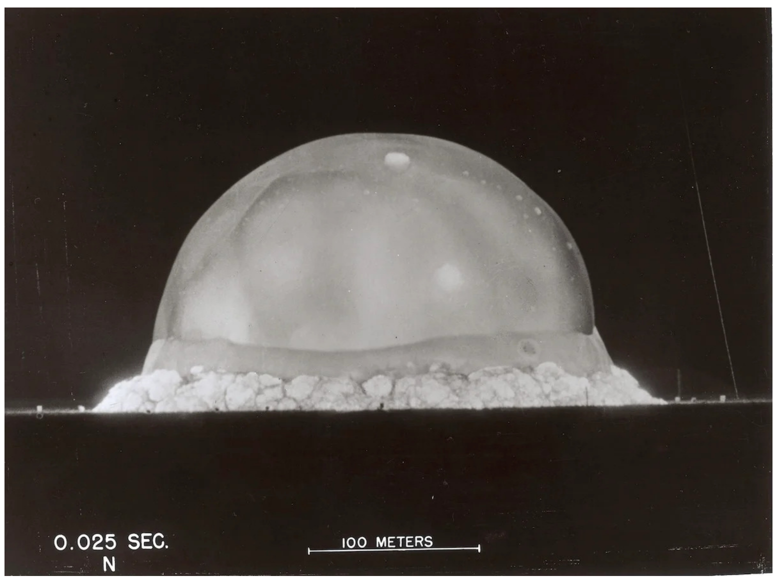
Homework 5
Due Wednesday, February 21st

Problem 1

The figure below was published in Time magazine in September of 1945. It is an image of an atomic test. At the time of publication, this was extremely new military technology and the explosion energy was classified by the US government.

Estimate the explosion energy. Express your answer in tons of TNT, as (for some reason) those are the units the military preferred. 1 ton of TNT is about 4×10^9 Joules or 4×10^{16} erg.



Problem 2

a) In class, we discussed the condition for convective instability, being that the entropy has a negative vertical gradient. Please explain this reasoning again in your own words.

b) Assuming hydrostatic equilibrium, show that this implies a maximally steep temperature gradient in the atmosphere, beyond which instability occurs. What is the steepest negative temperature gradient allowed in Earth's atmosphere, in °C/km? Don't forget the Earth's atmosphere is a diatomic gas, made mostly out of nitrogen molecules, with $m = 28m_p$.

Problem 3

Imagine a black hole with mass M embedded in a uniform gas. Far from this mass, as $r \to \infty$, the density $\rho \to \rho_0$ and sound speed $c_s \to c_0$ are constants. Near the hole, gas will accrete until it reaches a steady-state, spherically symmetric solution as a function of r. For this problem, consider a "black hole" just to be an accreting point mass; interestingly, relativity never enters into this calculation.

a) Estimate the location of the "sonic point"; i.e. the distance from the black hole where the accretion flow becomes supersonic, and sound waves can no longer travel upstream.

b) Estimate the accretion rate $\dot{M}$ by determining its value at the sonic point.

Problem 4

2D turbulence behaves fundamentally differently from 3D turbulence. The reason is that in 2D the "vorticity" $\omega = \nabla \times v$ obeys a conservation law (maybe I can show this at some point; it's not too hard to derive.). Formulate an analogous vorticity cascade argument (analogous to the energy cascade we derived in class) and derive the following:

a) How does velocity v of an eddy scale with its size l? (in 3D we saw $v \propto l^{1/3}$)

b) Given this, what is the slope of the power spectrum dE/dk of 2D turbulence (instead of the $k^{-5/3}$ law we found in 3D)?

Note: the most familiar example you might have of 2D turbulence is weather patterns on the surface of the earth (on scales much larger than the height of the atmosphere, so the flow is effectively 2D). Another example is convective zones on planets like Jupiter. The steeper power-law in 2D turbulence puts more energy in long-lived large-scale structures like hurricanes or Jupiter's big red dot.