ASTR 562 - High-Energy Astrophysics

High Energy Astrophysics by *Malcolm S. Longair*
Student: **Ralph Razzouk**

Homework 3

Problem 1
Bremsstrahlung: The total power emitted from thermal bremsstrahlung we said was

$$\frac{\mathrm{d}E}{\mathrm{d}V\,\mathrm{d}t} = \frac{32\pi e^6}{3hmc^3}\left(\frac{2\pi k_B}{3m}\right)^{\frac{1}{2}} Z^2 n_e n_{ion} T^{\frac{1}{2}},$$

ignoring the Gaunt factor. Now, the energy per unit volume of a thermal plasma is nk_BT. What is the cooling time of a plasma due to bremsstrahlung (i.e. the time it takes the plasma to lose its energy due to radiation)? How long does it take a $10^8\,\mathrm{K}$ plasma with number density $10^{-2}\,\mathrm{cm}^{-3}$ to cool down (typical of plasma in a cluster of galaxies)?

Solution. A typical astrophysical plasma is made up of Hydrogen ($Z = 1$, ions and free electrons). In that case, we can assume that the number of free electrons and the number of ions distributed among the shared volume is the same, *i.e.* $n_e = n_{ion}$. Given that the energy per unit volume of a thermal plasma is nk_BT, where $n = n_e + n_{ion}$, we have

$$\frac{\mathrm{d}E}{\mathrm{d}V\,\mathrm{d}t} = P = \frac{E}{t} \implies t = \frac{E}{P} = \frac{(n_e + n_{ion})k_BT}{P}.$$

Computing the value of P, we have

$$P = \frac{32\pi e^6}{3hmc^3}\left(\frac{2\pi k_B}{3m}\right)^{\frac{1}{2}} Z^2 n_e n_{ion} T^{\frac{1}{2}} = 2.4\times 10^{-27} n_e n_{ion} T^{\frac{1}{2}} \quad \left[\mathrm{erg\,s^{-1}\,cm^{-3}}\right]$$

The time needed to cool down is then

$$\begin{aligned} t &= \frac{(n_e + n_{ion})k_BT}{2.4\times 10^{-27}\times n_e n_{ion} T^{\frac{1}{2}}} \\ &= \frac{2nk_BT^{\frac{1}{2}}}{2.4\times 10^{-27}\times n^2} \\ &= \frac{2k_BT^{\frac{1}{2}}}{2.4\times 10^{-27}\times n} \\ &= 6\times 10^3 \frac{T^{\frac{1}{2}}}{n} \quad \text{[years]}. \end{aligned}$$

For a plasma with temperature $10^8\,\mathrm{K}$ and number density $10^{-2}\,\mathrm{cm}^{-3}$ to cool down, the time needed is

$$t = 6\times 10^3 \frac{(10^8)^{\frac{1}{2}}}{10^{-2}} = 6\times 10^9 \text{ years}.$$

■

Problem 2
Synchrotron Radiation: Given that the synchrotron power for a single electron in a magnetic field is

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{4}{3}\sigma_T c\beta^2\gamma^2 u_B,$$

calculate the time it takes an electron to cool from synchrotron emission. How does it depend on energy? How long does it take a 1 keV electron to cool down in a 1 μG magnetic field (typical of our own galaxy)?

Solution. The time needed for an electron to cool down is given by

$$t(\gamma) = \frac{E}{\mathrm{d}E/\,\mathrm{d}t} = \frac{\gamma m_e c^2}{\frac{4}{3}\sigma_T c\beta^2\gamma^2 u_B} = \frac{3m_e c}{4\sigma_T\beta^2\gamma u_B} = \frac{6m_e c\pi}{\sigma_T\beta^2\gamma B^2}.$$

For $\beta \to 1$, we have

$$t(\gamma) = 2.5\times 10^3\frac{1}{\gamma B^2}.$$

An electron with energy 1 keV is moving at a speed

$$E = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} \implies v = c\sqrt{1-\left(\frac{mc^2}{E}\right)^2} = 1.87\times 10^7 \text{ m/s}.$$

Then, we have

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \approx 1$$

Thus, the time needed is

$$t = 2.5\times 10^3(10^6)^2 = 2.5\times 10^{15}\text{ s}.$$

■