ASTR 562 - High-Energy Astrophysics

High Energy Astrophysics by *Malcolm S. Longair*
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Homework 5

Problem 1
(Gamma Ray Bursts) Assume a gamma ray burst has a flux of $2.6 \times 10^5\,\mathrm{ergs\,s^{-1}\,cm^{-2}}$ for 1 second and is located at a distance of 1 Mpc. First, estimate the total energy of the burst if the emission occurred isotropically. Then, estimate the effective energy if all the emission occurred in two cones with opening angle of 10 degrees.

Solution. We have a gamma ray burst with a flux density of $\Phi = 2.6 \times 10^5\,\mathrm{ergs\,cm^{-2}}$. The total energy of the burst if the emission occurred isotropically is given by

$$\begin{aligned} E_{\mathrm{iso}} &= 4\pi D^2 \Phi \\ &= 4\pi (1\,\mathrm{Mpc})^2 (2.6 \times 10^5) \\ &= 4\pi (3.086 \times 10^{24})^2 (2.6 \times 10^5) \\ &= 3.11 \times 10^{55}\ \mathrm{ergs}. \end{aligned}$$

The effective energy if all the emission occurred in two cones of an opening angle is

$$E_{\mathrm{eff}} = \frac{\Delta\Omega}{4\pi} E_{\mathrm{iso}},$$

where $\Delta\Omega$ is the range of the solid angle, which, for small angles, can be estimated by

$$\begin{aligned} \Delta\Omega &= 2 \times \text{Cone Opening} \\ &= 2 \int_0^{2\pi} \mathrm{d}\phi \int_0^{\alpha} \sin(\theta)\,\mathrm{d}\theta \\ &\approx 2 \int_0^{2\pi} \mathrm{d}\phi \int_0^{\alpha} \theta\,\mathrm{d}\theta \\ &= 2(2\pi)\left(\frac{\alpha^2}{2}\right) \\ &= 2\pi\alpha^2. \end{aligned}$$

Then, the effective energy for an opening angle of $10^\circ = 0.175\,\mathrm{rad}$ is

$$\begin{aligned} E_{\mathrm{eff}} &= \frac{\Delta\Omega}{4\pi} E_{\mathrm{iso}} \\ &= \frac{2\pi\alpha^2}{4\pi} E_{\mathrm{iso}} \\ &= \frac{\alpha^2}{2} E_{\mathrm{iso}} \\ &= \frac{(0.175)^2}{2} (3.11 \times 10^{55}) \\ &= 4.762 \times 10^{53}\ \mathrm{ergs}. \end{aligned}$$

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Problem 2
(Clusters of Galaxies) Consider the intracluster medium where there is some plasma with a density of 10^{-2} particles per sq. cm, and a temperature of 10^7 degrees. If the plasma is distributed in a sphere uniformly of 100 kpc, what would be the mass enclosed to achieve that temperature? Compare that to the mass of the plasma itself.

Solution. The ideal gas law states that

$$PV = Nk_BT \implies P = nk_BT.$$

Since the particles are distributed uniformly, *i.e.* in hydrostatic equilibrium, the pressure gradient balances the gravitational force and we can relate them by

$$\begin{aligned}
\frac{1}{\rho}\frac{\mathrm{d}P}{\mathrm{d}r} &= -\frac{GM}{r^2} \\
\frac{\mathrm{d}P}{\mathrm{d}r} &= -\frac{GM}{r^2}\rho \\
\frac{\mathrm{d}P}{\mathrm{d}r} &= -\frac{GM}{r^2}\frac{M}{\frac{4}{3}\pi r^3} \\
\frac{\mathrm{d}P}{\mathrm{d}r} &= -\frac{3GM^2}{4\pi r^5} \\
\mathrm{d}P &= -\frac{3GM^2}{4\pi r^5}\,\mathrm{d}r \\
P &= \frac{3GM^2}{16\pi r^4}.
\end{aligned}$$

Then, by equating them, we have

$$\frac{3GM^2}{16\pi r^4} = nk_BT \implies M = \sqrt{\frac{16\pi nk_BTr^4}{3G}}$$

Plugging in the values, we get the mass enclosed to achieve the given temperature, which is

$$\begin{aligned}
M &= \sqrt{\frac{16\pi nk_BTr^4}{3G}} \\
&= \sqrt{\frac{16\pi(10^{-2})(1.38\times10^{-23})(10^7)(3.09\times10^{21})^4}{3(6.67\times10^{-11})}} \\
&= \sqrt{\frac{6.324\times10^{69}}{2\times10^{-10}}} \\
&= \sqrt{3.162\times10^{79}} \\
&= 5.62\times10^{39}\,\mathrm{kg}.
\end{aligned}$$

Now to find the mass of the plasma itself, we will assume it is completely hydrogen. The mass of a hydrogen atom is essentially the mass of a proton m_p. The volume enclosed by a sphere with a radius of $r = 100$ kpc is

$$\begin{aligned}
V &= \frac{4}{3}\pi r^3 \\
&= \frac{4}{3}\pi(3.09\times10^{23})^3 \\
&= \frac{4}{3}\pi(2.95\times10^{69}) \\
&= 1.24\times10^{70}\,\mathrm{cm}^2.
\end{aligned}$$

The mass of a plasma itself is

$$\begin{aligned} m_{\text{plasma}} &= n m_p V \\ &= (10^{-2})(1.67 \times 10^{-27})(1.24 \times 10^{70}) \\ &= 2.07 \times 10^{41}\,\text{kg}. \end{aligned}$$

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