Homework 6
Due Thursday, February 29th

Problem 1

a) Estimate the kinematic viscosity ν of a plasma made out of electrons and protons. You may assume the plasma has mass density ρ and temperature T.

b) Show for a plasma with characteristic mass M, size L, temperature T and velocity V, the Reynolds number is given by

$$Re \sim LV/\nu = C\left(\frac{M}{M_\odot}\right)\left(\frac{L}{R_\odot}\right)^{-2}\left(\frac{T}{1\mathrm{K}}\right)^{-5/2}\left(\frac{V}{1\mathrm{km/s}}\right) \tag{1}$$

What is your estimate for the coefficient C? (this might help to explain why astrophysical flows are often treated as having zero viscosity).

Problem 2

Compute the electric susceptibility χ_E of water vapor. Assume that a water molecule has a given electric dipole moment p and it can have any random orientation in the presence of an electric field. The susceptibility should be some function of number density n and temperature T. You may find it useful to know the following expansion of $\coth(x)$ for small x:

$$\coth(x) \approx \frac{1}{x} + \frac{1}{3}x \tag{2}$$

If it's been awhile since your last E&M course, the electric susceptibility of a material is determined by its affinity for inducing polarization in the presence of an electric field:

$$\vec{P} = \epsilon_0 \chi_E \vec{E} \tag{3}$$

Where $\vec{P}$ is the dipole moment per unit volume. Don't worry about the elecric field produced by the dipoles themselves; $\vec{E}$ is meant to represent the total electric field, so it is already taken into account.

Problem 3

a) Compute the single-particle partition function

$$z = \int \frac{d^3x d^3p}{h^3} e^{-\beta \epsilon_p} \tag{4}$$

for a single particle in a classical ideal monatomic gas.

b) Let's look at the quantum version. Imagine an infinite square well with sides of length L. Show that the partition function for this system can be expressed as

$$z = \left(\sum_{n=1}^{\infty} e^{-\beta \epsilon_0 n^2} \right)^3 \tag{5}$$

for some energy scale ϵ_0.

Write a computer program to compute this explicitly as a function of $\beta\epsilon_0$ by approximating the infinite sum with a finite sum up to very large N. Make a plot of $z(\beta\epsilon_0)$, where

$$z(x) = \left(\sum_{n=1}^{\infty} e^{-xn^2} \right)^3 \tag{6}$$

and examine the limiting cases $\beta\epsilon_0 \ll 1$ and $\beta\epsilon_0 \gg 1$. Does your solution agree with the classical result in the appropriate limit? Plot the classical result alongside the quantum result. Make sure the plot is clear enough that I can actually see where these curves agree and where they disagree.