Homework 2
Due Wednesday, January 31st

Problem 1

Derive the multiplicity factor $\Omega(E, V)$ and the equation of state for a relativistic gas of particles. Recall the formula relating energy and momentum

$$\epsilon^2 = p^2c^2 + m^2c^4 \tag{1}$$

And assume the system is ultra-relativistic, so that $p^2c^2 \gg m^2c^4$.

What is the total energy E of the system as a function of the temperature T?

What is the pressure P as a function of the temperature?

What is the adiabatic index γ describing this system? (i.e. $P = Kn^\gamma$ for adiabatic expansion or compression, where n is the number density).

Problem 2

Van Der Waals came up with a method for describing a system of hard spheres with a physical size R_0. The only difference to the multiplicity factor is that each hard sphere removes some volume $V_0 \sim R_0^3$ from the system, such that the total volume that the particles have available to them is actually $V - NV_0$, instead of V. Derive the multiplicity factor and equation of state for this system (assuming a monotonic, nonrelatvistic gas, and answering the same questions as in problem 1) except show that it is not really consistent with a simple γ-law; derive a relationship between pressure and density for this system while undergoing adiabatic changes in density and pressure. There should be some unusual behavior when the number density n is close to some n_{crit}. How do we interpret this behavior and what is the value of n_{crit}?

Problem 3

Imagine you have a room filled with N gas particles. What is the probability that if you observe this room at some instant, all of the particles will be in the left half of the room, with none in the right half (suffocating half the class)?

If N is Avogadro's number, how small is this probability? Are we safe from spontaneous death?

Problem 4

Estimate the temperature below which you do not excite a single quantum rotational mode in a diatomic gas (as a function of the molecular moment of inertia I).

For molecular Hydrogen where the hydrogen atoms are separated by about 3 Angstroms, what is this temperature in Kelvin?

Problem 5

Estimate the temperature of the interior of the sun, knowing the mass $M_\odot \sim 10^{33}$ g and radius $R_\odot \sim 10^{11}$ cm. Assume the pressure in the sun is sufficient to keep it supported against gravitational collapse.

Note: Be careful when googling this number. It might give you a much smaller answer than your estimate! Don't trust google.

Problem 6

Pathria & Beale derive the multiplicity for an ideal gas using quantum mechanics. Imagine a 3D infinite square well with sides of length L. As we are all experts on quantum mechanics, we know the infinite square well admits energy eigenvalues of

$$\epsilon(n_x, n_y, n_z) = \frac{\pi^2\hbar^2}{2mL^2}(n_x^2 + n_y^2 + n_z^2). \tag{2}$$

Given the constraint that the total energy

$$E = \sum_{i=1}^{N} \epsilon_i \tag{3}$$

for a fixed total number of particles N estimate the multiplicity factor $\Omega(E, V)$, i.e. for a given energy, how many possible quantum states of N particles can have that energy? You can assume large N and large E (large enough total energy that the average energy per particle is way above the ground state, that will help a lot). You should only worry about dependence on E and V, don't worry about terms that look like N^N unless you really want to. It should be equivalent to what we derived in class.

Note: the full formula they give is:

$$\ln\Omega = N \ \ln\left(\frac{V}{Nh^3}\left(\frac{4\pi mE}{3N}\right)^{3/2}\right) + \frac{5}{2}N \tag{4}$$