# ASTR 562 - High-Energy Astrophysics

**High Energy Astrophysics** by *Malcolm S. Longair*
Student: **Ralph Razzouk**

## Homework 1

**Problem 1**
**Radiation:** Make a table having a row for each of the electromagnetic wavebands (radio, sub-mm, IR, Optical, UV, X-ray, and Gamma ray). Then add columns describing:

1. whether light from those wavebands can be detected from the ground or space,
2. what technology is used to detect this radiation, and
3. what complications there are in doing astronomy from this waveband.

*Proof.* The table is as follows

- **Radio**:
  1. Ground
  2. Single radio antennae and large interferometer arrays
  3. Reflection of low frequency radio waves by the plasma of the ionosphere and by the interplanetary and interstellar plasma for frequencies less than 1 MHz
- **Sub-mm**:
  1. Space
  2. Single element detectors (heterodyne receivers or bolometers)
  3. $H_2O$ and $CO_2$ in Earth's atmosphere
- **Infrared**:
  1. Space
  2. Infrared detector arrays
  3. Strong thermal emitters of infrared radiation in Earth's atmosphere
- **Optical**:
  1. Ground
  2. CCD detector arrays
  3. Short wavelength absorption by the ozone in the upper atmosphere
- **Ultraviolet**:
  1. Space
  2. Ultraviolet spectrographs
  3. Ozone and molecular absorption
- **X-ray**:
  1. Space
  2. Proportional counters and scintillation detectors are used as well as other devices such as CCDs
  3. Photoelectric absorption by the atoms which make up the molecular gases of the atmosphere

- **Gamma ray**:
  1. Space
  2. Scintillation detectors
  3. Photoelectric absorption (between 100 keV and 1 MeV) and Compton scattering and electron–positron pair production at higher energies

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**Problem 2**
**Relativity:** Cosmic rays that hit the Earth's atmosphere produce muons as secondary particles (rest mass $mc^2$ of 105 MeV). They are produced typically at a height of 6 km and have a lifetime of 2200 ns before they decay into electrons and neutrinos. How is it possible that they are detected on the ground? Give a lower limit on their energy. (See Appendix for help).

*Proof.* Muons are very energetic as they are produced in the Earth's atmosphere at are moving at a velocity very close to the speed of light $c$ with $\beta \approx 0.9999$. Classically, to find the distance covered by muons after being produced, we use

$$d = vt = (299,792,458)(2.2 \times 10^{-6}) = 659.5\,\mathrm{m}$$

which is much less than the distance between their point of production and the ground. Since they are travelling at relativistic speeds, we must account for the time dilation they experience. In fact, the classical answer mixes up reference frames

- $\tau = 2.2 \times 10^{-6}$ s is the time in the muon's frame of reference.
- $d = 6$ km is the length of the atmosphere in Earth's frame of reference.

Thus, relativistically, the time them muon experiences is

$$t = \frac{\tau}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{2.2 \times 10^{-6}}{\sqrt{1-0.9999^2}} = 70.712(2.2 \times 10^{-6}) = 1.55 \times 10^{-4}\,\mathrm{s}$$

which gives

$$d = vt = (299,792,458)(1.55 \times 10^{-4}) = 46.5\,\mathrm{km} > 6\,\mathrm{km},$$

and, hence, them muon does reach the ground. Equivalently, the same approach can be done using length contraction to explain this.

In the ultra-relativistic case, we find that the total energy ≈ kinetic energy = $\gamma m_0 c^2 = 70.712(105) =$ 7.424 GeV. ■

**Problem 3**
**Plasmas:** What is the Debye length for the plasma in the intracluster medium ($kT = 10^8$ K, $n = 10^{-3}\,\mathrm{cm}^{-3}$)? What is the Debye length for the plasma in the solar coronae ($kT = 10^7$ K, $n = 10^{15}\,\mathrm{cm}^{-3}$)? How about a molecular cloud ($kT = 10$ K, $n = 10^7\,\mathrm{cm}^{-3}$)?

*Proof.* The equation for the Debye length is given by

$$\lambda_D = 69 \left(\frac{T}{n_e}\right)^{\frac{1}{2}}\,\mathrm{m}.$$

- For plasma in the intracluster medium:

$$\lambda_D = 69 \left(\frac{10^8}{10^{-3} \cdot 10^{-6}}\right)^{\frac{1}{2}} = 69\left(10^{8.5}\right) = 2.181 \times 10^{10}\,\mathrm{m}.$$

- For plasma in the solar coronae:

$$\lambda_D = 69 \left( \frac{10^7}{10^{15} \cdot 10^{-6}} \right)^{\frac{1}{2}} = 69 \left(10^{-1}\right) = 6.9\,\mathrm{m}.$$

- For plasma in the molecular cloud:

$$\lambda_D = 69 \left( \frac{10}{10^7 \cdot 10^{-6}} \right)^{\frac{1}{2}} = 69 \left(10^{8.5}\right) = 69\,\mathrm{m}.$$

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**Problem 4**
**Observing Photons:** The light from a faint star has an energy flux of $10^{-7}\,\mathrm{ergs\,cm^{-2}\,s^{-1}}$. Assuming that the light has a wavelength of $5 \times 10^{-5}$ cm estimate the number of photons from this star that enter a human eye in one second.

*Proof.* The radius of the average pupil opening of the human eye is around $r_p = 1\,\mathrm{mm} = 10^{-3}\,\mathrm{m}$. Thus, the surface area of the pupil is $A_p = \pi r_p^2 = \pi \times 10^{-6}\,\mathrm{m}^2$. The duration for which we are measuring for is $t = 1\,\mathrm{s}$.

The energy of each photon with a wavelength of $\lambda = 5 \times 10^{-5}\,\mathrm{cm}$ is given by

$$E = \frac{hc}{\lambda} = 3.972 \times 10^{-21}\,\mathrm{J}.$$

The number of photons $N$ entering the pupil of the human eye is then

$$N = \frac{\Phi A_p t}{E} = \frac{\left(1\,\mathrm{J\,m^{-2}\,s^{-1}}\right)\left(\pi \times 10^{-6}\,\mathrm{m}^2\right)(1\,\mathrm{s})}{3.972 \times 10^{-21}\,\mathrm{J}} = 7.909 \times 10^{14} \text{ photons}.$$

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