PHYS630: Advanced Theory Of Electricity And Magnetism

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Area of research: theoretical astrophysics, plasma physics.

Second most important physics theory, after mechanics

$\nabla \cdot \mathbf{E} = 4\pi\rho$
$\nabla \cdot \mathbf{B} = 0$
$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$
$\nabla \times \mathbf{B} = \frac{1}{c}\left(4\pi\mathbf{J} + \frac{\partial \mathbf{E}}{\partial t}\right)$

- Physics: differential form of Maxwell's eqns (integral form of little use - only a few special cases)
- cgs units…

Prerequisites

- This is graduate-level physics E&M course
- not introductory General Physics
- Expected that you took a basic undergraduate E&M course
- not a “hands-on” - eg, lots of math problems like “find a electric potential in some complicated cavity”
  - we’ll do “simple” examples
  - will not do too many “practical” problems
- Not much numerics/computational
  - those are different, often problem-specific “arts”

Homeworks

- About once a week, 2-3 problems.
- Must turn upload to Brightspace before the class starts - we'll be discussing solutions in class
- No real option for late homework

Our TAs

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Grading

- Homeworks 50%
- Midterm 20%
- Final 30%
- No relative grading (everybody can get A+).

Collaboration policy

- Try it first yourself.
- Then talk to others: collaboration is encouraged (but not copying!) Talk to each other - that's how progress in science is made!
- If a problem has been done in close collaboration, please state that
- Please uphold "academic integrity"
- ChatGPT…? We'll see… Use it as a tool to learn, not to cheat

Attendance policy

- "University Academic Regulations regarding class attendance state that students are expected to be present for every meeting of the classes in which they are enrolled.
- No track of attendance will be kept, but let me know ahead of time if you have to miss the class

- office hours - Monday 10am-11pm, PHYS 313
- Also we can chat via Zoom etc
- Communication
  - homeworks: Brightspace,
  - quick question: email

- Ask questions at any moment!
- If I am not asked at least three question per class, then I failed…

Textbooks…

Textbooks (recommended)

- Landau & Lifshitz, (top notch, but a bit too much at first run…)
    - vol 2, *Field Theory*
    - vol 8, Electrodynamics of continuous media
- Jackson, *Classical electrodynamics* - more of detailed explanation
    - L&L: top down, Jackson: bottom-up
    - Somewhat different choices of what is more important, and applications - I'll make mine.
    - I will not follow exactly, will be jumping in-between a bit
    - Will give you the chapters #s for each lecture
- Griffiths, *Introduction to Electrodynamics* - somewhat slower, more like undergraduate, but explains things clearly. Especially vector analysis.
- I will not be distributing lecture notes (sometimes…)

Special relativity

- Landau&Lifshitz

CHAPTER 1. THE PRINCIPLE OF RELATIVITY

- Griffiths

- Jackson

We are physicists here - understanding principles!
But Landau is way too fast in the beginning

EM is relativistic tensor theory

$$\boxed{\partial_\alpha F^{\alpha\beta} = \frac{4\pi}{c} J^\beta}$$

$$\boxed{\partial_\alpha F_{\beta\gamma} + \partial_\beta F_{\gamma\alpha} + \partial_\gamma F_{\alpha\beta} = 0}$$

$$F_{\alpha\beta} = \begin{pmatrix} 0 & E^x & E^y & E^z \\ -E^x & 0 & -B^z & B^y \\ -E^y & B^z & 0 & -B^x \\ -E^z & -B^y & B^x & 0 \end{pmatrix}.$$

There is no separate E-field or B-field: one EM field
Separation in E & B depends on the observer
Still, can be practical

I'll try to balance

- We will be jumping between the books/chapters, no continuous reading of a given book
- Reading assignments will be given ahead off time (today)
- Many results will be given and explained, not derived from first principles
- but some basics will be derived
- Often: order-of-magnitude derivation of principal results
- There are many practical problems (eg., given some shape of a charged conductor, find fields) - will try to avoid details, will discuss principles, few simple examples, a "physicsy course"
- Mathematics is/can be interesting, but can quickly become super-messy

Course outline

- We'll start with what you know, but at higher mathematical level: charges create fields,
    - Fields and potentials
    - Electrostatics
    - Magnetostatics
    - conductors in EM fields (EM with boundary conditions)
- Charge in EM fields:
    - relativistic mechanics
    - 4-vectors
    - EM tensor, EM stress-energy tensor
    - gauges
    - Lorentz transformation of the fields, field invariants
- **Maxwell's equations**
- EM waves
    - spectrum
    - polarization

- EM emission (addition of phases of retarded potentials)
    - retarded potentials
    - multipole emission
    - emission during collisions
    - cyclotron emission
    - emission by relativistic particles
    - radiation reaction
- EM scattering by charged particles (Thomson cross-section, and the like)
- EM propagation
    - ray tracing
    - diffraction
- Advanced topics:
    - EM phenomena in dielectrics
    - Plasmas, dispersion
    - laser-plasma interaction
    - Cherenkov/transition radiation
    - quantization of EM fields (where to put "hat"s)
    - Some quantum EM (eg, Casimir effect)

Reading assignments

(see file in Brightspace)

Mathematics will be given as we go

- Lagrangian vs Hamiltonian approaches in EM
- EM is a tensor theory. Vectors still useful
- vector identities and derivatives, curvilinear coords
- the *nabla* operator ∇
- Fourier
- spherical harmonics (Fourier on a sphere)
- special functions
- Mathematically, results quickly become super-complicated (even for linear problems)
  - some results will be given without derivation
  - intuition is needed

Vector analysis in EM

- It's complicated…
- Vectors, tensors, ∇ operators, curvilinear coordinates - will be give as we go

Vector Formulas

$$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \mathbf{b} \cdot (\mathbf{c} \times \mathbf{a}) = \mathbf{c} \cdot (\mathbf{a} \times \mathbf{b})$$

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$$

$$(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d}) = (\mathbf{a} \cdot \mathbf{c})(\mathbf{b} \cdot \mathbf{d}) - (\mathbf{a} \cdot \mathbf{d})(\mathbf{b} \cdot \mathbf{c})$$

$$\nabla \times \nabla\psi = 0$$

$$\nabla \cdot (\nabla \times \mathbf{a}) = 0$$

$$\nabla \times (\nabla \times \mathbf{a}) = \nabla(\nabla \cdot \mathbf{a}) - \nabla^2\mathbf{a}$$

$$\nabla \cdot (\psi\mathbf{a}) = \mathbf{a} \cdot \nabla\psi + \psi\nabla \cdot \mathbf{a}$$

$$\nabla \times (\psi\mathbf{a}) = \nabla\psi \times \mathbf{a} + \psi\nabla \times \mathbf{a}$$

$$\nabla(\mathbf{a} \cdot \mathbf{b}) = (\mathbf{a} \cdot \nabla)\mathbf{b} + (\mathbf{b} \cdot \nabla)\mathbf{a} + \mathbf{a} \times (\nabla \times \mathbf{b}) + \mathbf{b} \times (\nabla \times \mathbf{a})$$

$$\nabla \cdot (\mathbf{a} \times \mathbf{b}) = \mathbf{b} \cdot (\nabla \times \mathbf{a}) - \mathbf{a} \cdot (\nabla \times \mathbf{b})$$

$$\nabla \times (\mathbf{a} \times \mathbf{b}) = \mathbf{a}(\nabla \cdot \mathbf{b}) - \mathbf{b}(\nabla \cdot \mathbf{a}) + (\mathbf{b} \cdot \nabla)\mathbf{a} - (\mathbf{a} \cdot \nabla)\mathbf{b}$$

NRL_FORMULARY

SI - cgs

SI
$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$
$\nabla \cdot \mathbf{B} = 0$
$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$

cgs
$\nabla \cdot \mathbf{E} = 4\pi\rho$
$\nabla \cdot \mathbf{B} = 0$
$\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}$
$\nabla \times \mathbf{B} = \frac{1}{c} \left(4\pi \mathbf{J} + \frac{\partial \mathbf{E}}{\partial t} \right)$

- $\mathbf{D} = \epsilon_0 \mathbf{E}$, $\mathbf{B} = \mu_0 \mathbf{H}$
- D,E,B,H - all different dimension

- E, B - same dimension

SI - cgs

SI	cgs
$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$	$\nabla \cdot \mathbf{E} = 4\pi\rho$
$\nabla \cdot \mathbf{B} = 0$	$\nabla \cdot \mathbf{B} = 0$
$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$
$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}\right)$	$\nabla \times \mathbf{B} = \frac{1}{c}\left(4\pi\mathbf{J} + \frac{\partial \mathbf{E}}{\partial t}\right)$

- $F = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{e^2}{r}$ (SI) -> $F = \frac{e^2}{r}$ (cgs)
- $\left(\frac{1}{4\pi\epsilon_0}\right) \to 1$
- $\epsilon_0\mu_0 = \frac{1}{c^2}$, so $\mu_0 \to \frac{4\pi}{c^2}$
- $B = \frac{\mu_0 I}{2\pi r} \to \frac{2I}{rc}$

Sorry - I am just the messenger here

Maxwell's equations

$\nabla \cdot \mathbf{E} = 4\pi\rho$
$\nabla \cdot \mathbf{B} = 0$
$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$
$\nabla \times \mathbf{B} = \frac{1}{c}\left(4\pi\mathbf{J} + \frac{\partial \mathbf{E}}{\partial t}\right)$

cgs:
- E & B same dimension
- differential

EM is relativistic theory

There is no way to " avoid" relativistic kinematics in dealing with EM field

- plus dynamics $\partial_t \mathbf{p} = e\left(\mathbf{E} + (\mathbf{v}/c) \times \mathbf{B}\right)$
- $\mathbf{p} = \frac{\mathbf{v}}{\sqrt{1-(v/c)^2}} m$

Differential and integral forms

$$\begin{cases} \operatorname{rot} \boldsymbol{E} = -\dfrac{1}{c}\dfrac{\partial \boldsymbol{H}}{\partial t}, & \displaystyle\oint \boldsymbol{E}\, d\boldsymbol{l} = -\frac{1}{c}\frac{\partial}{\partial t}\int \boldsymbol{H}\, d\boldsymbol{S}; \\ \operatorname{div} \boldsymbol{H} = 0, & \displaystyle\oint \boldsymbol{H}\, d\boldsymbol{S} = 0. \end{cases}$$

$$\begin{cases} \operatorname{rot} \boldsymbol{H} = \dfrac{1}{c}\dfrac{\partial \boldsymbol{E}}{\partial t} + \dfrac{4\pi \boldsymbol{j}}{c}, & \displaystyle\oint \boldsymbol{H}\, d\boldsymbol{l} = \frac{4\pi}{c} I + \frac{1}{c}\frac{\partial}{\partial t}\int \boldsymbol{E}\, d\boldsymbol{S}; \\ \operatorname{div} \boldsymbol{E} = 4\pi\rho, & \displaystyle\oint \boldsymbol{E}\, d\boldsymbol{S} = 4\pi e. \end{cases}$$

- Gauss theorem: $\displaystyle\oint \boldsymbol{a}\, d\boldsymbol{S} = \int \operatorname{div} \boldsymbol{a}\, dV$
- Stokes theorem: $\displaystyle\oint \boldsymbol{a}\, d\boldsymbol{l} = \int \operatorname{rot} \boldsymbol{a}\, d\boldsymbol{S}$

Classical Maxwell's are linear

$$\nabla \cdot \mathbf{E} = 4\pi\rho$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \frac{1}{c}\left(4\pi\mathbf{J} + \frac{\partial \mathbf{E}}{\partial t}\right)$$

Can depend on fields:
non-linearity

Quantum E&M is nonlinear

EM in matter

- Conductors are easy: $\mathbf{E}_{\text{inside}}$=0+ boundary conditions
- Dielectrics are trickier:
  - $\rho, \mathbf{j}$ are functions of external E&B

$\mathbf{D}^{\mathrm{G}} = \mathbf{E}^{\mathrm{G}} + 4\pi\mathbf{P}^{\mathrm{G}}$	$\mathbf{B}^{\mathrm{G}} = \mathbf{H}^{\mathrm{G}} + 4\pi\mathbf{M}^{\mathrm{G}}$
$\mathbf{P}^{\mathrm{G}} = \chi_{\mathrm{e}}^{\mathrm{G}}\mathbf{E}^{\mathrm{G}}$	$\mathbf{M}^{\mathrm{G}} = \chi_{\mathrm{m}}^{\mathrm{G}}\mathbf{H}^{\mathrm{G}}$
$\mathbf{D}^{\mathrm{G}} = \varepsilon^{\mathrm{G}}\mathbf{E}^{\mathrm{G}}$	$\mathbf{B}^{\mathrm{G}} = \mu^{\mathrm{G}}\mathbf{H}^{\mathrm{G}}$
$\varepsilon^{\mathrm{G}} = 1 + 4\pi\chi_{\mathrm{e}}^{\mathrm{G}}$	$\mu^{\mathrm{G}} = 1 + 4\pi\chi_{\mathrm{m}}^{\mathrm{G}}$

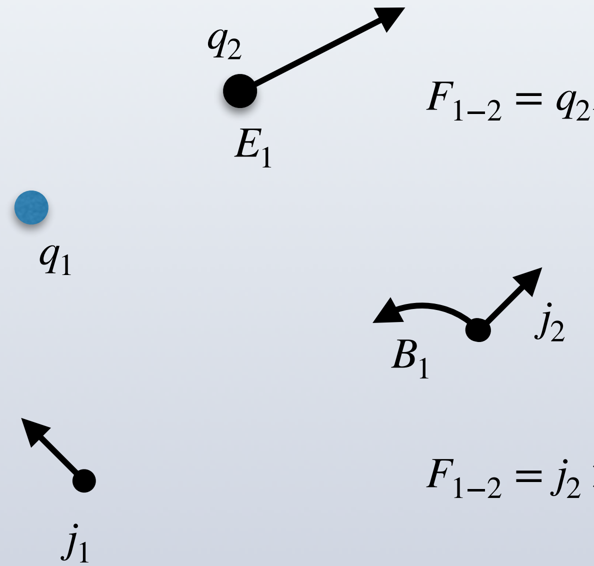
  - tensorial functions, eg induced $E_{\alpha}^{ind} = \epsilon_{\alpha\beta}E_{\beta}^{ex}$
  - often $\epsilon(\omega)$ - dispersion
  - Also, non-linear $E^{ind} = \alpha_1 E^{ex} + \alpha_2 (E^{ind})^2$
- Typically applications are so wide and different - we'll discuss just the basic principles
- Collective effects (eg. plasma)

Brightspace

- Questions?

“Action at a distance”

- How two bodies that are not in contact interact with each other?
  - what is “in contact”?
  - Source -> field -> force
  - (and from 2nd on 1st)
- No need for medium to carry the field



$$F_{1-2} = q_2 E_1(@q_2)$$

$$F_{1-2} = j_2 \times B_1(@q_2)$$

“Action at a distance”

- No need for “medium” to carry the field
- Maxwell’s ϵ_0, μ_0 were designed to quantify the vacuum as medium - not needed.
- $\epsilon_0\mu_0 = \dfrac{1}{c^2}$, no need for ϵ_0, μ_0