Fall 2024 MA 562
Introduction to Differential Geometry and Topology

- **Schedule and location:** MWF 8:30-9:20AM in Hampton Hall of Civil Engineering 2101
- **Course webpage and syllabus:** https://www.math.purdue.edu/~esampert/IQC/
- **Detailed course calendar:** https://www.math.purdue.edu/~esampert/IQC/cal
- **Gradebook and announcements are handled via Brightspace (Purdue login required):** https://purdue.brightspace.com/d2l/home/1102972
- **Video recording archive and link to live-stream:** Purdue Mediaspace Channel
- **Instructor:** Eric Samperton
  - **Email:** my first name followed by AT purdue.edu
  - **Office hour:** Fridays after class, 9:30am-11:30am in my office MATH 706, and by appointment
- **Grader:** Homework is graded by an assigned course grader, but all questions about grading should be directed to Eric

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Content

This course will focus mainly on differential topology, which is the study of smooth manifolds and smooth functions between them. At the end of the semester we will learn some of the basics of Riemannian geometry. The overarching goal is to introduce enough perspective to help orient students toward further studies in subjects such as algebraic and geometric topology, differential equations, or differential geometry.

For the bulk of the semester we will be concerned with developing the basic definitions, structures and examples of smooth manifolds. Highlights will include the Whitney embedding theorem, Stoke's theorem, the hairy ball theorem, and the construction of de Rham cohomology groups. Towards the end of the semester we will develop the basic definitions of Riemannian geometry, especially as applied to curves and surfaces, culminating in the Gauss-Bonnet theorem. Time permitting, and based on the class's preferences, we may discuss some of the following additional topics: the Poincare-Hopf

index theorem, the classification of surfaces, the Frobenius integrability theorem, or the Thurston-Perelman geometrization theorem.

Prerequisites

Other than graduate standing, there are no official prequisites for the class. However, we will assume comfort with multivariable calculus and linear algebra at the level of a first or second year graduate student in mathematics, as well as some basic familiarity with differential equations. Familiarity with the definitions of point-set topology will be helpful but will not be assumed. Please inquire with Eric if you have any questions about whether this course is appropriate for you.

Reading

The required course textbook is available for free online through the Purdue library:

- Introduction to Smooth Manifolds by John M. Lee

You are expected to keep up with the assigned course reading, which you can find on the detailed course calendar page. For supplemental reading, you may also find the following two references helpful:

- Differential Topology by Victor Guillemin and Alan Pollack
- Topology from the Differentiable Viewpoint by John W. Milnor

Grading

Grades will be based on several homework assignments (60%), 4 in-class quizzes (21%) and 1 final exam (19%).

- **Homework:** I will aim to assign homework every week, for a total of roughly 15 assignments. Homework will be posted on the detailed course calendar page. You should submit your assignments as a single scanned PDF to our Brightspace site. It will generally be due Monday mornings by the beginning of class. Depending on the size of the class, I may have some flexibility with late homework submissions, but don't count on it unless you ask me in advance, are sick, or have some kind of emergency.
- **In-class quizzes:** there will be 4 in-class quizzes throughout the semester. I will announce them well in advance, and provide a list of problems to study for them. I plan to have each quiz about 25 minutes long. Your lowest 1 score will be dropped. Depending on how large course enrollment is, make-up quizzes may not be possible.
- **In-class final exam:** the comprehensive in-class final will be 2 hours long. I will send more info after the registrar finishes scheduling finals for the semester.

Final course letter grades will be computed using a scale that is at least as generous as the usual 10 point scale.

Attendance

Regular attendance is expected but not mandatory except on the days of quizzes and exams. If you are sick, then for the consideration of your peers, please either attend class virtually (a live-stream option is available via Brightspace, and recordings can always be watched later) or else mask.

Academic integrity

Homework. When it comes to solving homework problems, an information source is generally allowed if it is legal, free, public, and passive. Legal means that your access to the information does not involve the violation of any law (e.g. copyright law); free means you do not pay for the information; public means the information is available to any Purdue student without having to create an account or register for a service that is not provided by Purdue; and passive means that the information is prerecorded and not created in response to your input.

The use of almost any calculator or computer program is disallowed a priori, as it fails to meet the "passive" requirement. However, common programs such as a TI-87 calculator, MatLab or GeoGebra that do not use AI are exempted from this.

Similarly, the use of other people is generally disallowed when it comes to getting help on homework problems, since asking another person a question fails to generate an answer in the form of passive information. However, any individual that is officially associated with this course (that is, your instructor and your classmates) is exempt from this.

In fact, *you are encouraged to collaborate with your classmates when solving homework problems*, and you are generally welcome to use any textbooks, research papers, or other notes to solve homework problems, as long as the sources are legal, free, public and passive as explained above. However, you must use your own words and understanding when writing up your solutions (you can not just copy or quote), and you must cite any sources you use.

Examples of allowed resources: textbooks from the Purdue library; answers to questions that you yourself did not post on public online discussion forums such as Math Stack Exchange; publicly available YouTube videos. Of course, the use of any such source must be accompanied with a proper citation.

Examples of disallowed resources: typing a homework question directly into an online search engine or artificial intelligence program such as ChatGPT or WolframAlpha; posting a homework question on a forum.

Violation of these expectations will typically result in a 0 for the offending assignment and the filing of an Academic Dishonesty Report with the Office of Student Rights and Responsibilities.

Finally, the use of "homework help services" (that is, professional cheating companies) such as Chegg, CourseHero, etc, is disallowed for help on any homework problems unless I give you my explicit permission. Any un-authorized use I discover will result in an F for the course and the filing of an Academic Dishonesty Report.

General studying. All of the above also applies when it comes to studying the recommended practice problems for the course quizzes and exams. However, for other studying, you are free to use whatever technologies or people you want as long as you do not violate the other policies. For example, if you

want to use an AI to create a summary of your textbook and self-written homework solutions, then have at it. Of course, you should also feel free to talk to whoever you want about smooth manifolds!

Quizzes and exam. The in-class quizzes and final exam are of course not to be done in collaboration with anyone. They will be closed-notes and closed-book.

Communication

My preferred modes of communication, in order of preference: talk to me before or after class > talk to me in office hours > send me an email > call my office phone at 41937. Don't send me messages on Brightspace, as I won't see those.

Brightspace

Most course materials will be hosted on the detailed course calendar page. However Brightspace will be used to send class announcements, host the Boilercast livestream, submit homework, handle the gradebook, and host links to a large amount of boilerplate that I am supposed to include in this syllabus per university policy. Consider this your notification that the policies you can find there are considered to be part of this syllabus.

Privacy

All of the regularly scheduled course meetings will be recorded and made publicly available on the course Media Space channel. If you have privacy concerns, please let me know.

Reasonable accommodations

Any students with disabilities that require reasonable accommodations must request accommodations via DRC before any accommodations can be made. These students are encouraged to discuss briefly with the instructor well in advance of any necessary accommodations.

Other boilerplate

In extraordinary circumstances, this syllabus may be amended or changed as necessary in order to facilitate fair learning and grading.