ASTR 562 - High-Energy Astrophysics

High Energy Astrophysics by *Malcolm S. Longair*
Student: **Ralph Razzouk**

Homework 4

Problem 1
Supernovae: A supernovae explodes with kinetic energy of 2×10^{51} ergs s^{-1}. It ejects 2 Solar masses of material. What is the velocity of the ejecta right after the explosion? If the density of the ISM is 2×10^{-24} g cm^{-3}, how long does it take to sweep up another 2 Solar masses of plasma?

Solution. The energy from a supernova explosion is given by

$$E_{SN} = \frac{1}{2} v^2.$$

Rewriting to find the velocity right after the explosion, we have

$$\begin{aligned} v &= \sqrt{\frac{2E_{SN}}{m_{ej}}} \\ &= \sqrt{\frac{2\left(2 \times 10^{51}\right) \times 10^{-7}}{2\left(2 \times 10^{30}\right)}} \\ &= \sqrt{\frac{10^{44}}{10^{30}}} \\ &= \sqrt{10^{14}} \\ &= 10^7 \text{ m/s}. \end{aligned}$$

The mass swept up from the interstellar material by the ejecta is given by

$$M_{sweep} \sim \frac{4\pi}{3} \rho_{ISM} r^3.$$

The time taken for the ejecta to sweep up another 2 solar masses is given by $t \sim \frac{r}{v}$. To find the radius at which that happens, we have

$$\begin{aligned} r &\sim \left(\frac{3M_{sweep}}{4\pi\rho_{ISM}}\right)^{\frac{1}{3}} \\ &\sim \left(\frac{3\left(2 \times 10^{30}\right)}{4\pi\left(2 \times 10^{-21}\right)}\right)^{\frac{1}{3}} \\ &\sim \left(\frac{3 \times 10^{51}}{4\pi}\right)^{\frac{1}{3}} \\ &\sim 10^{17} \text{ m}. \end{aligned}$$

Thus, the time taken is

$$t \sim \frac{r}{v} \sim \frac{10^{17} \text{ m}}{10^7 \text{ m/s}} \sim 10^{10} \text{ s} \sim 317 \text{ years}.$$

∎

Problem 2
Neutron Stars: Imagine a star like the sun with mass of 2×10^{30} kg and radius of 7×10^{8} m shrinks and becomes a neutron star with radius of 10 km. If the Sun normally rotates with a period of 26 days, what will be its period afterwords assuming conservation of momentum. If the sun has a magnetic field of about 40 Gauss what will be the magnetic field intensity afterwords assuming conservation of magnetic flux $\left(B4\pi R^2\right)$.

Solution. Converting the period to angular frequency, we have

$$\omega_i = \frac{2\pi}{T_i} = \frac{2\pi}{26 \times 60 \times 60 \times 24} = \frac{\pi}{1,123,200} = 2.797 \times 10^{-6} \text{ rad/s.}$$

Since there is no external torque being applied, we can apply the law of conservation of angular momentum. Clearly the star before and the neutron star are both spherical, then

$$\begin{aligned} L_i = L_f \implies \omega_f &= \frac{I_i \omega_i}{I_f} \\ &= \frac{\left(\frac{2}{5} M r_i^2\right) \omega_i}{\left(\frac{2}{5} M r_f^2\right)} \\ &= \frac{r_i^2 \omega_i}{r_f^2} \\ &= \frac{\left(7 \times 10^8\right)^2 \left(2.797 \times 10^{-6}\right)}{\left(10^4\right)^2} \\ &= \frac{1.37 \times 10^{12}}{10^8} \\ &= 1.37 \times 10^4 \text{ rad/s.} \end{aligned}$$

Then the period afterwards will be

$$T_f = \frac{2\pi}{\omega_f} = \frac{2\pi}{1.37 \times 10^4} = 4.586 \times 10^{-4} \text{ s.}$$

■

Problem 3
Black Holes and Accretion: A ten Solar mass black hole swallows up a cloud of rock the mass of the Earth. Assume the rock falls from infinity and then fall directly reaches the event horizon at $\frac{2GM}{c^2}$. How much energy will be produced during this accretion event? How does this compare to the energy liberated if 0.7% of the rest mass of the rock was released during Hydrogen fusion?

Solution. The energy produced in an accretion event is given by

$$E_{\text{accretion}} = \frac{GMm}{R}, \quad \text{(from infinity to the surface).}$$

We have that $M = 10 M_\odot = 2 \times 10^{31}$ kg, $m = 6 \times 10^{24}$ kg, and $r = \frac{2GM}{c^2}$, we have

$$\begin{aligned} E_{\text{accretion}} &= \frac{GMm}{\frac{2GM}{c^2}} \\ &= \frac{mc^2}{2} \\ &= \frac{\left(6 \times 10^{24}\right)(299792459)^2}{2} \\ &= 2.7 \times 10^{41} \text{ J.} \end{aligned}$$

The energy liberated if 0.7% of the rest mass was released during Hydrogen fusion is

$$\begin{aligned} E_{\text{fusion}} &= 0.007mc^2 \\ &= 0.007\left(6 \times 10^{24}\right)(299792459)^2 \\ &= 3.7 \times 10^{39} \text{ J.} \end{aligned}$$

Thus, the energy produced from the accretion of Earth's mass into a black hole's event horizon is 100 times larger than the energy produced by 0.7% of Earth's rest mass undergoing Hydrogen fusion. ■