

# Practice Final Exam

## Monday, April 29th, 8am

Closed book, no notes. If you need a formula, please just ask for it. I am happy to provide a range of formulas within reason.

Your grade will be based on your process to the solution, not on the answer itself. If you just write down an answer with no explanation, that gets zero credit.

### Problem 1

Estimate the critical temperature  $T_{\text{BE}}$  of transition to a Bose-Einstein condensate, as a function of the number density  $n$  and the particle mass  $m$  (assume massive bosons, like  $^4\text{He}$ ).

### Problem 2

Imagine a set of non-interacting particles that can each be in one of two quantum states, separated by energy  $\Delta E$  (if you want, you can imagine them as particles with spins in a magnetic field). Compute the total energy of the system as a function of the number of particles and the temperature  $E(N, T)$ .

### Problem 3

Recall for Fermions we could write the total number of particles as a sum over eigenstates:

$$N = \sum_{\text{eigenstates}} (\text{occupation\#}) \quad (1)$$

Use this fact to derive the Fermi energy for a cold (zero temperature) gas of electrons as a function of the number density  $n_e$  and electron mass  $m_e$ . Actually, if you have some other method of deriving the Fermi energy, that's fine too. If you're having trouble you may estimate the Fermi energy instead for half credit.

### Problem 4

Recall in class we discussed the heat capacity of solids using (at least) three different models. Derive the energy  $E(T)$  for a solid using any one of these models (in principle you could take the derivative to get the heat capacity but I won't make you do that). When you are done, you must also write out in words the next correction you would need to make to improve the model.

### Problem 5

Derive the ideal gas law  $PV = NkT$  by any means.