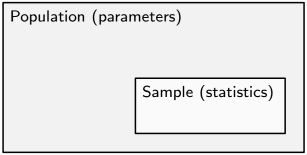
# Math 207: Statistics

## Chapter 21: The Accuracy of Percentages

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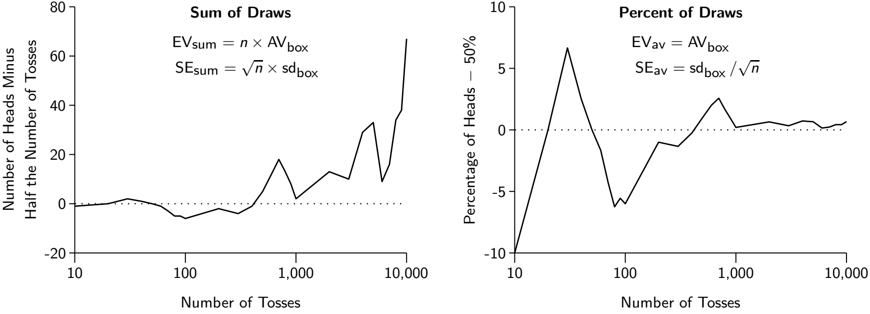
# Expected Value, Standard Error, Central Limit Theorem

- Many statistics problems are modeled as samples from a box of numbered tickets.
- Solution procedure:
  - Formulate a box model.
  - Compute the average and SD of the contents of the box.
  - Determine if you are computing a sum or average (% in a 0/1 box is an average).
  - Use the appropriate formulas to compute the EV and SE for the sample.
  - Use the normal curve to compute chances of the sample being in a specified range.



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- $\text{AV}_{\text{box}} = \frac{433{,}211}{433{,}211+253{,}785} \approx 0.631$ and $\text{SD}_{\text{box}} = (1-0)\sqrt{0.631\,(1-0.631)} \approx 0.483$

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- Compute the $z$ scores for 61% and 65%:
  $z_1 = (61 - 63.1)/4.83 = -0.435$
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- Area under the normal curve: $\texttt{pnorm}(0.393) - \texttt{pnorm}(-0.435) = 0.321 = 32.1\%$

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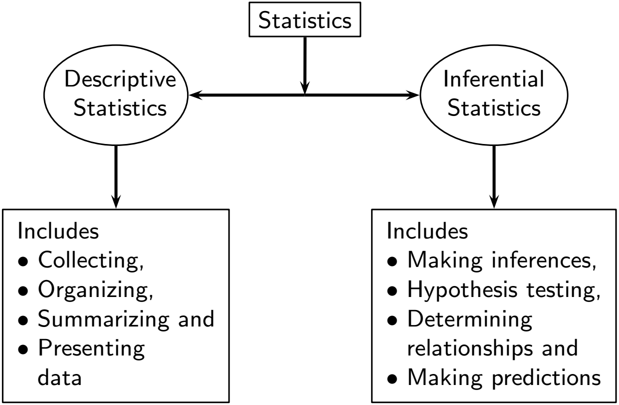
- $\text{SE}_{\%} = \dfrac{\text{SD}_{\text{box}}}{\sqrt{n}} \approx \frac{0.483}{\sqrt{100}} = 0.0483 = 4.83\%$.
- If we assume that the samples follow a normal curve, then

$$63\% \ \pm\ 2\cdot(4.83\%) = 63\% \ \pm 9.7\%$$

is 95% confidence interval on the fraction of $\boxed{1}$s.

# Inferential Statistics

- In real situations, we typically don't know the contents of the box!
- That is where inferential statistics is used.



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- Prior to taking the sample, we are 95% confident that this procedure will give an interval that contains the true average.

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- Estimated fraction of $\boxed{1}$s in the box: $317/400 = 79.3\%$.
- Now compute a $\pm$ on this estimate:
  - $\text{SD}_{\text{box}} \approx \sqrt{0.793 \cdot (1 - 0.793)} = 0.41.$
  - $\text{SE}_{\text{av}} = 0.41/\sqrt{400} \approx 0.020 = 2.0\%$

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- 95% confidence interval:

$$79.3\% \ \pm \ 2 \cdot (2.0\%) = 79.3\% \ \pm \ 4\%$$