Math 207
Intro to Statistics

Worksheet: Hypothesis Testing

1. **Areas Under the Normal Curve**. Sketch the following regions under the normal curve and estimate their areas.

a) to the left of $z = 0$

b) between $z = -1$ and $z = 2$

c) between $z = -2$ and $z = 1$

d) to the left of $z = 3$

e) to the right of $z = 2$

f) $z > 5$

2. **Standard Units**. A list of numbers has mean $= 4$ and SD $= 2$.

a) Convert $x = 6$ to standard units.

b) What value x as value -2 in standard units.

3. **Central Limit Theorem**. Would taking the average of 25 measurements likely divide the error by a factor of 5, 10 or 25?

4. ***p*-Values**.

a) Which of the following p-values is best for the null hypothesis? Explain.

0.1%   3%   17%   32%

b) Repeat a) for the alternative hypothesis.

c) True or false: The p-value depends on the data.

d) True or false: If the p-value is 5%, there is only a 5% chance that the null hypothesis is correct.

5. **Hypothesis Tests**. For each question, (1) state the test you would use, (2) state the null and alternative hypotheses, (3) compute the test statistic and p-value (approximately) and (4) make a conclusion.

a) You flip a coin 100 times and it lands heads 51% of the time. Is that too many heads?

b) Repeat a) if $n = 10,000$.

c) A coin lands heads four times in a row. Is that too many heads?

d) A local pizza shop advertises that "75% of Shenandoah students enjoy our pizza every week!" You take a random sample of 100 SU students and find that 50 of them ordered pizza from that shop last week. Do the data support the advertisement?

e) In 1970, 59% of college freshmen thought that capital punishment should be abolished; by 2005, the percentage had dropped to 35%. Is the difference real, or can it be explained by chance? You may assume that the percentages are based on two independent simple random samples, each of size 1,000.

Solutions

1. a) 50% b) $(95\%/2) + (68\%/2) = 81.5\%$ c) same as b) d) $(99.7\% + (0.3\%/2) = 99.85\%$ e) about 0%

2. a) $z = (6-4)/2 = 2/2 = 1$ b) Solve $-2 = (x-4)/2$ to get $x = 0$.

3. $\text{SE}_{\text{av}} = \text{SD}/\sqrt{n} = \text{SD}/\sqrt{25} = \text{SD}/5$. So, the answer is 5.

4. a) 32%. If the null hypothesis is true, there is a 32% chance of seeing data as extreme or more extreme than was observed. b) 0.1%. If the null were true then there would be very little chance of seeing such extreme data. c) True. d) False. We assume that the null is true in order to compute the p-value.

5. a) z-test. Null hypothesis: the difference between expected (50%) and observed (51%) is due to chance. Alternative hypothesis: No, it's not. The box model has a 0 and 1 in it. It's mean is 0.5 and SD is 0.5. The SE is $0.5/\sqrt{100} = 5\%$. So, the z-score is $(51-50)/5 \approx 0.2$ So, the p-value is close to 50% (it's 42.1%). This supports the null.

b) z-test. Null, alt and box are the same. SE is 0.5% and $z = (51-50)/0.5 = 2$. That gives a p-value of about 2.5% which supports the alternative.

c) t-test since n is small. Null, alt and box are the same as above. SE is $0.5/\sqrt{4} = 0.25 = 25\%$. So, $z = (0-50)/25 \approx -2$. If we use a t-distribution with 3 degrees of freedom, we get a p-value of about 7% which supports the null. Note that if we had used the z-test, the p-value would be 2.5% (which would support the alternative).

d) z-test. Null hypothesis: The difference between expected (75%) and observed (50%) is due to chance. Alternative hypothesis: No, it's not. Box: 3 ones and a zero. $\text{AV}_{\text{box}} = 0.75$, $\text{SD}_{\text{box}} = 0.43$, $\text{SE} = 0.43/\sqrt{100} = 0.043 = 4.3\%$. The test statistic is $z = (50-75)/4.3 < -5$ so the p-value is about 0%. This supports the alternative.

e) Two-sample z-test. Null hypothesis: The observed difference (50% and 55%) is due to chance. Alternative hypothesis: No, it's not. For the 1970s box, the SD is $\sqrt{0.59 * 0.41} \approx 0.49$ so the SE is $0.49/\sqrt{1000} \approx 1.5\%$. For the 2005 box, the SD is $\sqrt{0.35 * 0.65} \approx 0.48$ so the SE is $0.48/\sqrt{1000} \approx 1.5\%$. The SE for the difference is $\sqrt{1.5^2 + 1.5^2} \approx 2.1\%$. So, the test statistic is $z = (59-35)/2.1 = 11.4$. This gives a p-value close to 0 which supports the alternative hypothesis.

Solutions

1. a) 50% b) $(95\%/2) + (68\%/2) = 81.5\%$ c) same as b) d) $(99.7\% + (0.3\%/2) = 99.85\%$ e) about 0%

2. a) $z = (6-4)/2 = 2/2 = 1$ b) Solve $-2 = (x-4)/2$ to get $x = 0$.

3. $\text{SE}_{\text{av}} = \text{SD}/\sqrt{n} = \text{SD}/\sqrt{25} = \text{SD}/5$. So, the answer is 5.

4. a) 32%. If the null hypothesis is true, there is a 32% chance of seeing data as extreme or more extreme than was observed. b) 0.1%. If the null were true then there would be very little chance of seeing such extreme data. c) True. d) False. We assume that the null is true in order to compute the p-value.

5. a) z-test. Null hypothesis: the difference between expected (50%) and observed (51%) is due to chance. Alternative hypothesis: No, it's not. The box model has a 0 and 1 in it. It's mean is 0.5 and SD is 0.5. The SE is $0.5/\sqrt{100} = 5\%$. So, the z-score is $(51-50)/5 \approx 0.2$ So, the p-value is close to 50% (it's 42.1%). This supports the null.

b) z-test. Null, alt and box are the same. SE is 0.5% and $z = (51-50)/0.5 = 2$. That gives a p-value of about 2.5% which supports the alternative.

c) t-test since n is small. Null, alt and box are the same as above. SE is $0.5/\sqrt{4} = 0.25 = 25\%$. So, $z = (0-50)/25 \approx -2$. If we use a t-distribution with 3 degrees of freedom, we get a p-value of about 7% which supports the null. Note that if we had used the z-test, the p-value would be 2.5% (which would support the alternative).

d) z-test. Null hypothesis: The difference between expected (75%) and observed (50%) is due to chance. Alternative hypothesis: No, it's not. Box: 3 ones and a zero. $\text{AV}_{\text{box}} = 0.75$, $\text{SD}_{\text{box}} = 0.43$, $\text{SE} = 0.43/\sqrt{100} = 0.043 = 4.3\%$. The test statistic is $z = (50-75)/4.3 < -5$ so the p-value is about 0%. This supports the alternative.

e) Two-sample z-test. Null hypothesis: The observed difference (50% and 55%) is due to chance. Alternative hypothesis: No, it's not. For the 1970s box, the SD is $\sqrt{0.59 * 0.41} \approx 0.49$ so the SE is $0.49/\sqrt{1000} \approx 1.5\%$. For the 2005 box, the SD is $\sqrt{0.35 * 0.65} \approx 0.48$ so the SE is $0.48/\sqrt{1000} \approx 1.5\%$. The SE for the difference is $\sqrt{1.5^2 + 1.5^2} \approx 2.1\%$. So, the test statistic is $z = (59-35)/2.1 = 11.4$. This gives a p-value close to 0 which supports the alternative hypothesis.