Independent Events

Multiplication Rule:

- $P(A \text{ and } B) = P(A) P(B \mid A)$.
- If A and B are independent, then $P(A \text{ and } B) = P(A) P(B)$.

In particular, A and B are independent if and only if $P(A|B) = P(A)$. This is true if and only if $P(B|A) = P(B)$. In other words, knowing A doesn't change the chance of B; knowing B doesn't change the chance of A.

1. A six-sided die has the sides with 1, 2 and 3 painted blue. The sides with 4, 5, and 6 are painted red.

   a) I roll the die and the top side shows blue. What is the chance that the top side is a 2?

   b) I roll the die and the top side shows blue. What is the chance that the top side is a 5?

   c) I roll the die and the top side shows red. What is the chance that the top side is a 2?

   d) Are the color shown and the number shown independent or dependent?

   e) The top side shows blue. What is the chance that the top side shows an even number?

   f) The top side shows an even number. What is the chance that the top side is blue?

   g) The top side shows an blue. What is the chance that the top side shows an odd number?

   h) The top side shows an odd number. What is the chance that the top side is blue?

2. In the box shown below, each ticket should have two numbers:

   | 1 | | | 1 | 2 | | 1 | 2 | | 1 | 3 | | 3 | 1 | | 3 | 2 | | 3 | | | 3 | |

   A ticket will be drawn at random. Can you fill in the blanks so that the two numbers are independent?

3. In the box shown below, each ticket should have two numbers:

   | 5 | | | 5 | 2 | | 5 | 2 | | 5 | 3 | | 3 | 1 | | 3 | 2 | | 3 | | | 3 | |

   A ticket will be drawn at random. Can you fill in the blanks so that the two numbers are independent?