The Binomial Formula

The chance that an event will occur exactly k times out of n is given by the binomial formula:

$$\frac{n!}{k!\,(n-k)!}\,p^k\,(1-p)^{n-k}$$

In this formula, n is the number of trials, k is the number of times the event is to occur, and p is the probability that the event will occur on any particular trial. The assumptions:

- The value of n must be fixed in advance.
- p must be the same from trial to trial.
- The trials must be independent.

1. A coin is to be flipped 5 times. Use the binomial formula to calculate the following probabilities.

   a) Exactly 3 heads

   b) Exactly 4 heads

   c) Exactly 2 tails

   d) 3 or 4 tails

   e) At least 3 heads

2. Find the number of different ways of arranging one R and three Gs in a row. Write out all the patterns.

3. Find the number of different ways of arranging two Rs and two Gs in a row. Write out all the patterns.

4. A die is rolled four times. Find the chance that—

   a) an ace (one dot) never appears

   b) an ace appears exactly once

   c) an ace appears exactly twice.

5. A box contains one red marble and five green ones. Four draws are made at random with replacement. Find the chance that—

a) a red marble is never drawn

b) a red marble appears exactly once

c) a red marble appears exactly twice

d) a red marble appears exactly three times

e) a red marble appears on all the draws

f) a red marble appears at least twice

5. (Review) An investigator wants to use a straight line to predict IQ from lead levels in the blood, for a representative group of children aged 5–9. There is a weak positive association in the data. True or false, and explain—

a) The investigator can use many different lines.

b) The investigator has to use the regression line.

c) Only the regression line has an r.m.s. error.

d) Any line the investigator uses will have an r.m.s. error.

e) Among all lines, the regression line has the smallest r.m.s. error.

f) The r.m.s. error of his line will be $SD_y \sqrt{1 - r^2}$.