Computer Project 2: Probability

Your Name 2020-06-18

Flipping Coins

For a fair coin, the chance of getting tails is 1/2.

Counting the number of tails

Let's experiment with the total number of tails in n trials.

Total number of tails in a small number of coin flips

Suppose we flip 10 coins (with 1 = Tails and 0 = Heads). Here are six samples of size 10.

```
n = 10
trials = 6
for (i in 1:trials) {
    print(sample(c(0, 1), n, replace=TRUE))
}

## [1] 0 1 1 0 1 1 1 1 1 0 1
## [1] 1 0 1 1 0 0 1 0 1 1
## [1] 1 0 0 0 0 1 0 1 0 0
## [1] 1 1 0 0 1 1 1 1 0 0 1
## [1] 0 1 0 0 1 1 1 1 1 1
## [1] 0 1 0 0 0 1 1 0 0 1
```

Let's count the number of tails then subtract the expected number of tails. The expected number of tails is half the number of coin flips.

```
for (i in 1:trials) {
    print(sum(sample(c(0, 1), n, replace=TRUE)) - n/2)
}
```

```
## [1] 3
## [1] 0
## [1] 3
## [1] -2
## [1] 1
## [1] -1
```

Based on the calculations, approximately what is the \pm on the difference between the total number of tails and the expected total? (Remember: 95% of the area under a normal curve is \pm 2SD.) In other words, approximately what is the SD of the list of numbers shown above?

Total number of tails in a large number of coin flips

Let's repeat the experiment with 10,000 coins.

```
n = 10000
for (i in 1:trials) {
    print(sum(sample(c(0, 1), n, replace=TRUE)) - n/2)
}

## [1] -6
## [1] 28
## [1] -92
## [1] 34
## [1] 84
## [1] 90
```

Based on the calculations, approximately what is the \pm on the difference between the total number of tails and expected total?

Include your answer here.

Does the \pm on the number of tails increase or decrease with the number of trails? Briefly justify your conclusion.

Fraction of tails

Now let's look at the fraction of tails in n trials.

Fraction of tails in a small number of coin flips

Suppose we flip 10 coins and compute the fraction of tails (with 1 = Tails and 0 = Heads).

```
n = 10
trials = 6
for (i in 1:trials) {
   print(sum(sample(c(0, 1), n, replace=TRUE)) / n)
}
## [1] 0.4
## [1] 0.5
```

[1] 0.5 ## [1] 0.3 ## [1] 0.3 ## [1] 0.8 ## [1] 0.5

[1] 0.2

The expected fraction of tails is 0.5. Let's compute the difference between the observed fraction and the expected fraction:

```
n = 10
trials = 6
for (i in 1:trials) {
    print(sum(sample(c(0, 1), n, replace=TRUE)) / n - 0.5)
}
## [1] -0.1
## [1] 0.2
## [1] -0.3
## [1] 0.2
## [1] -0.3
```

Based on the calculations, approximately what is the \pm on the difference between the observed fraction of tails and the expected fraction (0.5)? In other words, approximately what is the SD of the list of numbers shown above?

Fraction of tails in a large number of coin flips

The expected fraction of tails is 0.5. Let's repeat the experiment with a larger number of coins.

```
n = 10000
for (i in 1:trials) {
    print(sum(sample(c(0, 1), n, replace=TRUE)) / n - 0.5)
}

## [1] -0.0149
## [1] 0.0059
## [1] 0.0097
## [1] 0.0047
## [1] 5e-04
## [1] -0.0095
```

Based on the calculations, approximately what is the \pm on the difference between the observed fraction of tails and 0.5

Include your answer here.

Does the \pm on the difference between the observed fraction of tails and 0.5 increase or decrease with the number of trails? Briefly justify your conclusion.

Binomial Formula

If we roll a die, the chance of getting a 4 is 1/6. If we roll 60 dice we expect to get about ten 4s. The number of ways of getting exactly ten 4s out of 60 rolls is:

```
n = 60
k = 10
choose(n, k)
```

[1] 75394027566

The chance of getting exactly ten 4s is given by the binomial formula:

```
p = 1/6
dbinom(k, size=n, prob=p)
```

```
## [1] 0.1370131
```

The chance of getting ten or fewer 4s is:

```
pbinom(k, size=n, prob=p)
```

```
## [1] 0.5833866
```

Why do the two probabilities differ so much?