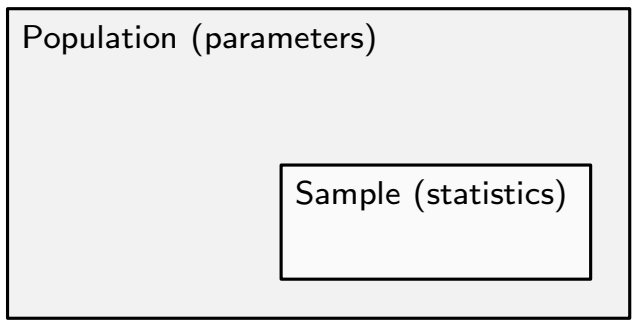
Math 207: Statistics

Chapter 24: A Model for Measurement Error



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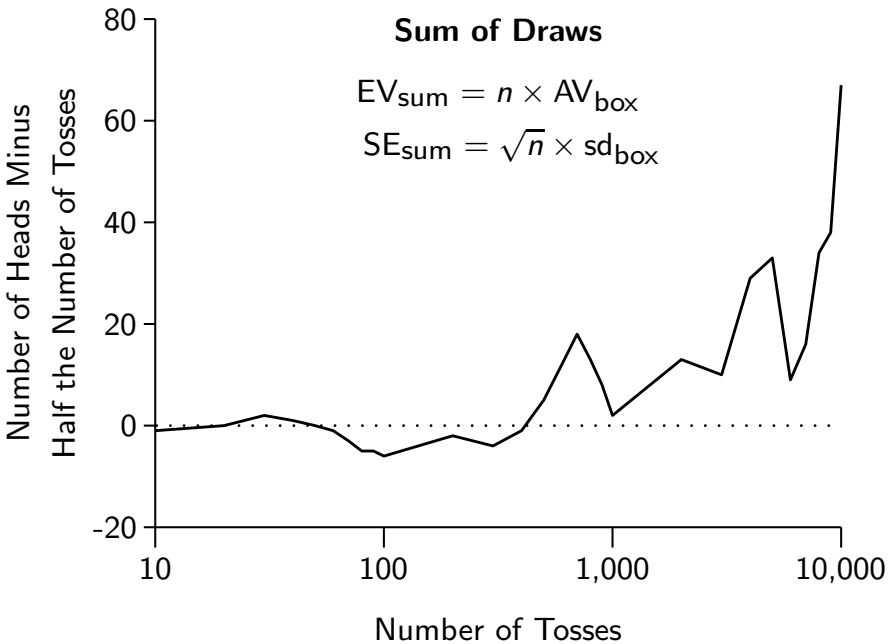
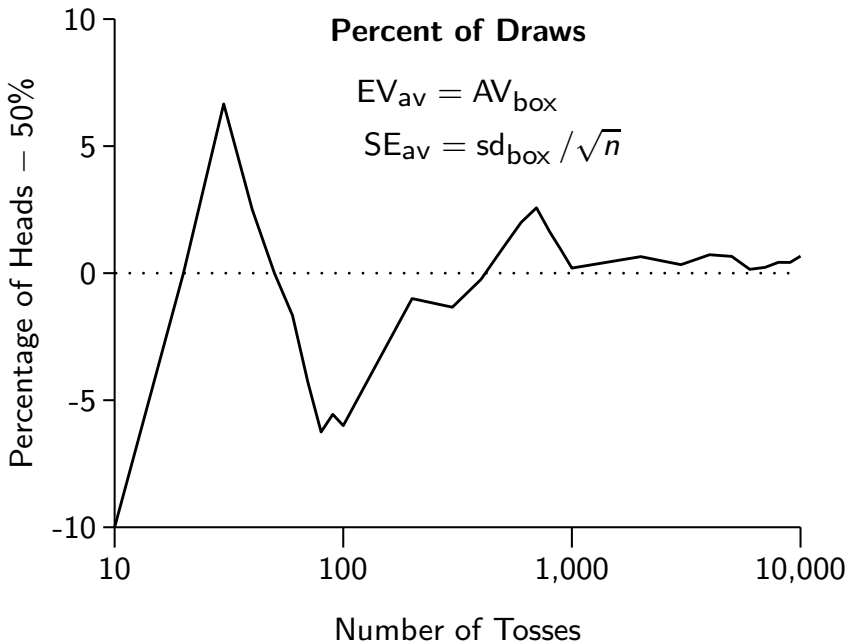
Expected Value, Standard Error, Central Limit Theorem

- Many statistics problems are modeled as samples from a box of numbered tickets.
- Solution procedure:
  - Formulate a box model.
  - Compute the average and SD of the contents of the box.
  - Determine if you are computing a sum or average (% in a 0/1 box is an average).
  - Use the appropriate formulas to compute the EV and SE for the sample.
  - Use the normal curve to compute chances of the sample being in a specified range.





Chance Error

- In Chapter 6 we saw:

$$\text{measured value} = \text{true value} + \text{bias} + \text{chance error}$$

- The bias is a systematic error in the measurement process
  - tape measure has stretched over time and is now 3'4" instead of 3'
  - scale is not callibrated to 0
  - Bias can be corrected!
- The chance error is due to random fluctuations (variables that we don't measure).
- Chance error can't be eliminated.
- We control chance error by taking the average of repeated measurements:

$$\text{SE}_{\text{av}} = \frac{\text{SD}_{\text{box}}}{\sqrt{n}}$$

where the box is the 'box of error measurements' which we typically assume follows a normal curve.

Example: NB10 Measurements

- Chapter 6 gave a table of 100 chance error measurements of the mass of NB10: a 10 kg standard weight.
- The SD of this list is 60 micrograms.
- So, for the careful measurement process, the SD of the error box is 60 micrograms.
- If we take the average of 100 measurements, we expect the average to be off by:

$$\text{SE}_{\text{av}} = \frac{60}{\sqrt{100}} = \frac{60}{10} = 6 \text{ micrograms}$$

Exercise 7 page 454

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- d) A 95%-confidence interval for the speed of light is 300,007 $\pm$ 4. True

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- c) Each measurement is off 300,007 by 10 or so. True
- d) A 95%-confidence interval for the speed of light is 300,007 $\pm$ 4. True
- e) A 95%-confidence interval for the average of the 25 measurements is 300,007 $\pm$ 4. False
- f) If a 26th measurement were made, there is a 95% chance that it would be off the exact value for the speed of light by less than 4. False