



## Warmup Exercises: Independence

Multiplication Rule:

- $P(A \text{ and } B) = P(A)P(B|A)$ .
- If  $A$  and  $B$  are independent, then  $P(A \text{ and } B) = P(A)P(B)$ .

Events  $A$  and  $B$  are independent if and only if  $P(B|A) = P(B)$ . This is true if and only if  $P(B|A) = P(B)$ . In other words, knowing  $A$  doesn't change the chance of  $B$ ; knowing  $B$  doesn't change the chance of  $A$ .

1. Sets of paper tickets are shown below. Each ticket has a number on the left and a shape on the right. For each set of tickets, Determine if number and shape are independent or dependent.

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2. Referring to the polio vaccine trials, are the events  $A$  and  $B$  independent or dependent?

- $A$  = a randomly selected subject is from a high-income family  
 $B$  = a randomly selected subject will get polio
- $A$  = a randomly selected subject is from a high-income family  
 $B$  = the parents of the randomly selected subject will consent to their child getting the vaccine
- $A$  = a randomly selected subject will be diagnosed with polio  
 $B$  = the medical examiner will know that the patient received the vaccine



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3. Are the two events independent or dependent.

- a)  $A$  = it will snow on 21 December  
 $B$  = the high temp on 21 December will be  $31^{\circ}$  F
- b)  $A$  = it will snow on 21 December  
 $B$  = 21 December will be a Saturday
- c)  $A$  = it will snow on 21 December  
 $B$  = the next coin flip will be tails

4. Are the two events independent or dependent.

- a)  $A$  = first card drawn will be a  $\heartsuit$   
 $B$  = first card drawn will be a 10
- b)  $A$  = first card drawn will be a face card  
 $B$  = first card drawn will be a king
- c)  $A$  = first card drawn will be a  $\clubsuit$   
 $B$  = second card drawn will be a  $\clubsuit$
- d)  $A$  = first card drawn will be a  $\clubsuit$   
 $B$  = second card drawn will be a  $\diamondsuit$

**Solutions:** 1.a) independent; b) dependent; c) independent; d) dependent.      2. all are dependent.

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