Math 207: Statistics

Chapter 6: Measurement Error

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Introduction

- In the real world, if we measure something several times, we observe different values each time.
- Each result is thrown off by chance error.
- How do these errors arise?
- How big are they likely to be? (Chapter 6)
- How much is likely to cancel out in the average? (Chapters 20–21, 23–24) Standard error: SE (for sum, average or percent)

Chance Error

- Standards weights are maintained at local, state, national and international levels for commercial, scientific and other purposes.
- The International Bureau of Weights and Measures near Paris maintains the International Prototype Kilogram.
- The National Bureau of Standards in Washington, D.C. maintains a national prototype kilogram (Kilogram #20) that is calibrated against the international standard.
- The Bureau maintains several other standard weights that are calibrated against Kilogram #20.
- NB 10 is one such standard weight. It weighs very nearly 10 grams.
- Our text has a table of 100 measurements of NB 10.

NB 10

- NB 10 is a 10 gram weight maintained by the National Bureau of Standards.
- The first five NB 10 measurements (in grams) from Table 1 on page 99 are:

9.999591	9.999600	9.999594	9.999601	9.999598

- Measurements are in terms of micrograms below 10 grams:

409	400	406	399	402

- For the measurements in Table 1, mean ≈ 405 and SD ≈ 6 in micrograms.

NB 10 Data Exercises

- Use the following to load the data into R:

```
> nb10 <- read.table("http://www.adjoint-functors.net/su/web/314/R/NB10")
```

- Use R to compute the mean, SD, max, min, and quantiles (using `summary(nb10)`).
- Compare the median (use the quantiles or `median(nb10$V1)`) to the mean.
- Plot a histogram of the data. Does it look like the normal curve?

```
> hist(nb10$V1, probability=TRUE, breaks=40)
```

- Calculate the fraction of the data within 1 SD of the mean.

```
> m <- mean(nb10$V1)
> s <- SD(nb10$V1)
> length(nb10$V1[m-s < nb10$V1 & nb10$V1 < m+s])
```

- What fraction of the data is within 2 SDs of the mean? 3 SDs? 4? 5? 6? How do these percentages compare to those of the normal curve?
- Compare `SD(nb10)` to `sd(nb10)`. Why are the values so close?

Outliers

- The NB 10 data does not fit the normal curve very well. The data is a bit more crowded around the mean than it should be. Value #36 is 3 SDs from the mean and #86 and #94 are 5 SDs away.
  - Run the following test: `shapiro.test(nb10$V1)`.
    The *p*-value reported by the Shapiro test answers the following question: If the measurement process has a normal distribution, what is the probability of getting the observed data? What was the *p*-value that your test calculated and what does it mean?
  - Try the following: `shapiro.test(rnorm(100, mean=0, sd=5))`.
    Repeat this several times. What do the results tell you?
- Repeat the exercises from the previous slide using the file
  `http://www.adjoint-functors.net/su/web/314/R/NB10_noOutliers`
- This data fits the normal curve better.
- Should the outliers be discarded? No. See page 103.

Bias

- Bias affects all measurements the same way, pushing them in the same direction.
- Chance errors change from measurement to measurement, sometimes up and sometimes down.

$$(\text{individual measurement}) = (\text{exact value}) + \text{bias} + (\text{chance error})$$

- If there is no bias, the long-run average of repeated measurements should approach the exact value. Chance errors should cancel out in the average.