Multiplication and Sum Rule Exercises

Multiplication Rule:
- $P(A \text{ and } B) = P(A) P(B \mid A)$.
- If A and B are independent, then $P(A \text{ and } B) = P(A) P(B)$.

Sum Rule:
- $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
- If A ane B are mutually exclusive, then $P(A \text{ or } B) = P(A) + P(B)$

1. Cards are drawn at random without replacement from a well-shuffled deck. Calculate the following probabilities.

a) First card is a $\heartsuit$

b) First card is a K

c) First card is a $K\heartsuit$

d) First card is a $K\heartsuit$ and the second is $Q\heartsuit$

e) Both the first card and the second card are $10\clubsuit$

f) The second card is a 10 given that the first card was.

g) The first and second cards are both 10s

h) First card is a $\heartsuit$ or a 10

i) First card is a $\clubsuit$ or a face card

j) First card is a 10 or a 3

k) First card is $Q\spadesuit$ or J$\clubsuit$

ℓ) First card is $\diamondsuit$ or 3$\clubsuit$

m) First card is an 5 or $\clubsuit$

n) At least one of the first two cards is a $\diamondsuit$

2. Two dice are rolled. Calculate the following probabilities.

    a) Sum is 12

    b) Sum is 12 or 2

    c) Sum is even and at least 10

    d) Sum is even

    e) Sum is even or at least 10

    f) Sum is six or even

    g) Sum is six given that it is even

    h) At least one die shows an even number

    i) Exactly one die shows an even number

3. A box contains 6 red marbles and 4 green ones. Calculate the following probabilities assuming that two marbles are drawn with replacement.

    a) Exactly one of the marbles is red

    b) Exactly two of the marbles are red

    c) None of the marbles are red.

4. Repeat exercise 3 assuming that two marbles are drawn without replacement.

5. Repeat exercise 4 assuming that three marbles are drawn without replacement.

6. **Independent** / **Mutually Exclusive Events**. For each event A described below, describe events B for which A and B are i) independent, ii) dependent, iii) mutually exclusive, iv) not mutually exlusive.

a) The first card drawn from a deck is a 7.

b) The first card drawn from a deck is a $\heartsuit$.

c) It will snow in Winchester on February 21 of next year.

d) Candidate X will win the Michigan primary in the next presidential election.