Math 207
Introduction to Statistics

Exam III

1. Tickets are to be drawn at random with replacement from the box shown below.

$$\boxed{7} \quad \boxed{13}$$

a) (10 points) Find the average and SD of the box.

b) (5 points) If you draw 100 tickets, what is the minimum value of the sum? What is the maximum?

c) (10 points) The sum of 100 draws will be around ______________ give or take ______________ or so.

d) (5 points) If you draw 100 tickets, what is the minimum value of the average? What is the maximum?

e) (10 points) The average of 100 draws will be around ______________ give or take ______________ or so.

2. A box has 8 blue marbles and 2 green ones.

a) (10 points) If you draw seven marbles with replacement, what is the chance of getting exactly four blue ones?

b) (5 points) If you draw seven marbles with replacement, what is the chance of getting three or four blue ones?

c) (5 points) If you draw seven *without* replacement, what is the chance of of getting exactly three blue ones?

3. (10 points) You plan to roll two dice. Let A be the event: The sum of the dice will be an even number. Describe events B for which A and B are

a) independent

b) dependent,

c) mutually exclusive,

d) not mutually exlusive.

4. Compute the following probabilities.

a) (10 points) Cards are drawn without replacement from a deck.

i) What is the chance that the first card is a ♣ or a 3?

ii) What is the chance that the second card is a ♣ given that the first card was a ♣?

iii) What is the chance that the first three cards are face cards (J, Q, K)?

b) (10 points) A coin is flipped five times.

i) What is the chance that tails appears all five times?

ii) What is the chance that tails appears on the last flip given that the first four were heads?

c) (10 points) One ticket will be drawn at random from each of the two boxes shown below.

(A) $\boxed{1}\quad\boxed{2}\quad\boxed{3}\quad\boxed{3}$ (B) $\boxed{1}\quad\boxed{2}\quad\boxed{3}$

Find the chance that:

i) The number drawn from (A) equals the one from (B).

ii) The number drawn from (A) is larger than the one from (B).

iii) The number drawn from (A) is smaller than the one from (B).

A standard deck of cards has the following properties.

- 52 cards
- 4 suits: hearts ♡, spades ♠, clubs ♣, and diamonds ♢
- Hearts and diamonds are red. Spades and clubs are black
- 13 cards of each suit: 2 – 10, J, Q, K, and ace
- 4 cards of each rank: 2 – 10, J, Q, K, and ace
- J, Q, and K are face cards

Multiplication Rule for Probabilities:

- $P(A \text{ and } B) = P(A)\,P(B \mid A)$.
- If A and B are independent, then $P(A \text{ and } B) = P(A)\,P(B)$.

Sum Rule for Probabilities:

- $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
- If A and B are mutually exclusive, then $P(A \text{ or } B) = P(A) + P(B)$

Binomial Formula: The chance that an event will occur exactly k times out of n is given by the binomial formula:

$$\frac{n!}{k!\,(n-k)!}\,p^k\,(1-p)^{n-k}$$

In this formula, n is the number of trials, k is the number of times the event is to occur, and p is the probability that the event will occur on any particular trial. The assumptions:

- The value of n must be fixed in advance.
- p must be the same from trial to trial.
- The trials must be independent.

The **mean** of a list of numbers $x_1, \ldots, x_n$ is

$$\text{mean} = \frac{1}{n}\,(x_1 + x_2 + \cdots + x_n) = \frac{1}{n}\sum_{i=1}^{n} x_i.$$

The **population standard deviation** is

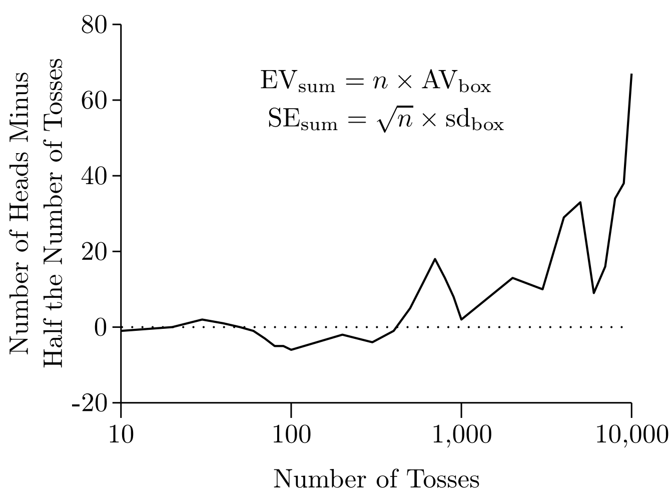
$$\text{SD} = \sqrt{\frac{1}{n}\left((x_1 - \text{mean})^2 + \cdots + (x_n - \text{mean})^2\right)}$$

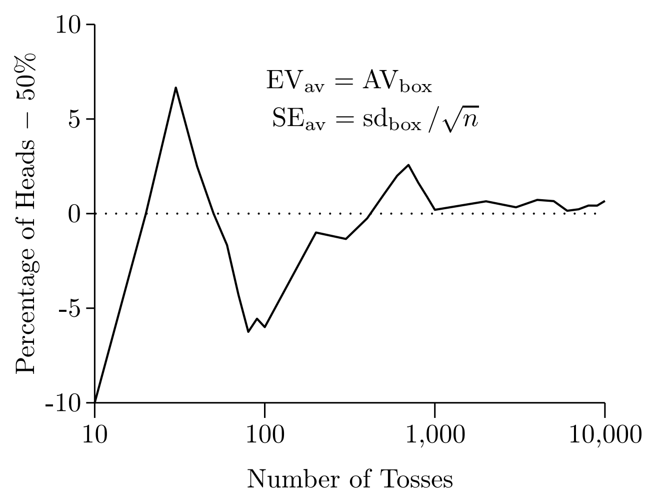
If we know the contents of a box of numbered tickets, we use SD.

The **sample standard deviation** is

$$\text{sd} = \sqrt{\frac{1}{n-1}\left((x_1 - \text{mean})^2 + \cdots + (x_n - \text{mean})^2\right)}$$

The figures below show how the standard error changes as the sample size increases. The figure on the right illustrates the Law of Averages.





Math 207
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1. Compute the following probabilities.

   a) (10 points) Cards are drawn without replacement from a deck.

      i) What is the chance that the first card is a face card (J, Q, K) or a ♡?

      ii) What is the chance that the second card is a 5 given that the first card was a 5?

      iii) What is the chance that the first three cards are aces?

   b) (10 points) A coin is flipped four times.

      i) What is the chance that tails appears on the last flip given that the first three were heads?

      ii) What is the chance that tails appears all four times?

   c) (10 points) One ticket will be drawn at random from each of the two boxes shown below.

(A) $\boxed{1}$ $\boxed{2}$ $\boxed{2}$ $\boxed{3}$ (B) $\boxed{1}$ $\boxed{2}$ $\boxed{3}$

Find the chance that:

   i) The number drawn from (A) equals the one from (B).

   ii) The number drawn from (A) is larger than the one from (B).

   iii) The number drawn from (A) is smaller than the one from (B).

2. Tickets are to be drawn at random with replacement from the box shown below.

$$\boxed{-2}\quad\boxed{2}$$

a) (10 points) Find the average and SD of the box.

b) (5 points) If you draw 400 tickets, what is the minimum value of the sum? What is the maximum?

c) (10 points) The sum of 400 draws will be around ______________ give or take ______________ or so.

d) (5 points) If you draw 400 tickets, what is the minimum value of the average? What is the maximum?

e) (10 points) The average of 400 draws will be around ______________ give or take ______________ or so.

3. (10 points) You plan to draw a card from a deck. Let A be the event: The card will be a face card (J, Q, K). Describe events B for which A and B are

a) independent

b) dependent,

c) mutually exclusive,

d) not mutually exlusive.

4. A box has 5 blue marbles and 9 green ones.

a) (10 points) If you draw six marbles with replacement, what is the chance of getting exactly five blue ones?

b) (5 points) If you draw six marbles with replacement, what is the chance of getting five or six blue ones?

c) (5 points) If you draw six *without* replacement, what is the chance of of getting exactly five blue ones?

A standard deck of cards has the following properties.

- 52 cards
- 4 suits: hearts ♡, spades ♠, clubs ♣, and diamonds ♢
- Hearts and diamonds are red. Spades and clubs are black
- 13 cards of each suit: 2 – 10, J, Q, K, and ace
- 4 cards of each rank: 2 – 10, J, Q, K, and ace
- J, Q, and K are face cards

Multiplication Rule for Probabilities:

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The **mean** of a list of numbers $x_1, \ldots, x_n$ is

$$\text{mean} = \frac{1}{n}\,(x_1 + x_2 + \cdots + x_n) = \frac{1}{n}\sum_{i=1}^{n} x_i.$$

The **population standard deviation** is

$$\text{SD} = \sqrt{\frac{1}{n}\left((x_1 - \text{mean})^2 + \cdots + (x_n - \text{mean})^2\right)}$$

If we know the contents of a box of numbered tickets, we use SD.

The **sample standard deviation** is

$$\text{sd} = \sqrt{\frac{1}{n-1}\left((x_1 - \text{mean})^2 + \cdots + (x_n - \text{mean})^2\right)}$$

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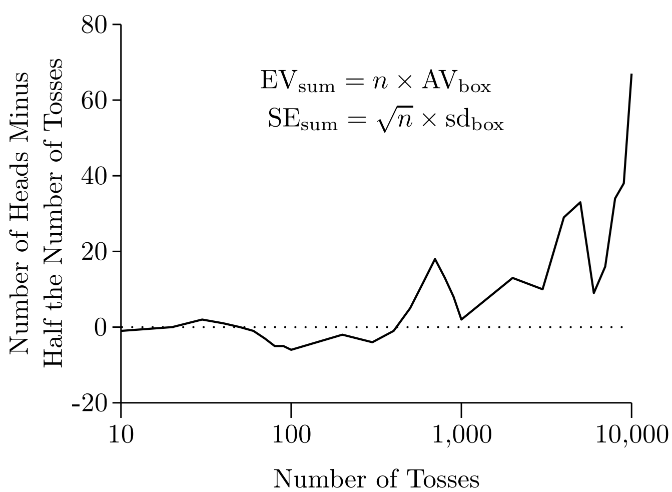

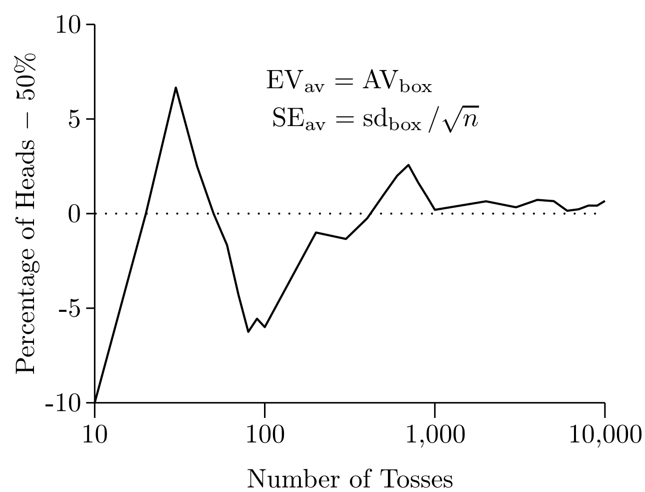

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1. Compute the following probabilities.

   a) (10 points) Cards are drawn without replacement from a deck.

      i) What is the chance that the first card is a 10 or a $\diamondsuit$?

      ii) What is the chance that the second card is a face card (J, Q, K) given that the first card was a face card.

      iii) What is the chance that the first four cards are $\clubsuit$?

   b) (10 points) A coin is flipped three times.

      i) What is the chance that tails appears on the last flip given that the first two were heads?

      ii) What is the chance that tails appears all three times?

   c) (10 points) One ticket will be drawn at random from each of the two boxes shown below.

(A) $\boxed{1}\ \boxed{2}\ \boxed{2}$ (B) $\boxed{1}\ \boxed{2}\ \boxed{2}\ \boxed{3}$

Find the chance that:

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   iii) The number drawn from (A) is smaller than the one from (B).

2. Tickets are to be drawn at random with replacement from the box shown below.

$$\boxed{\boxed{0}\quad\boxed{8}}$$

a) (10 points) Find the average and SD of the box.

b) (5 points) If you draw 25 tickets, what is the minimum value of the sum? What is the maximum?

c) (10 points) The sum of 25 draws will be around ______________ give or take ______________ or so.

d) (5 points) If you draw 25 tickets, what is the minimum value of the average? What is the maximum?

e) (10 points) The average of 25 draws will be around ______________ give or take ______________ or so.

3. (10 points) You plan to roll one die. Let A be the event: The die shows an even number. Describe events B for which A and B are

a) independent

b) dependent,

c) mutually exclusive,

d) not mutually exlusive.

4. A box has 10 blue marbles and 3 green ones.

a) (10 points) If you draw five marbles with replacement, what is the chance of getting exactly three blue ones?

b) (5 points) If you draw five marbles with replacement, what is the chance of getting two or three blue ones?

c) (5 points) If you draw five *without* replacement, what is the chance of of getting exactly two blue ones?

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$$\text{SD} = \sqrt{\frac{1}{n}\left((x_1 - \text{mean})^2 + \cdots + (x_n - \text{mean})^2\right)}$$

If we know the contents of a box of numbered tickets, we use SD.

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