Math 207
Introduction to Statistics

Exam IV

1. (25 points) A survey is conducted of 100 freshmen at public universities. It is found that these students work 11.5 hours a week for pay, on average. The SD of the data is 5.

a) Find a 95% confidence interval on the hours a week worked by freshmen at public universities.

b) True or false and explain: There is a 95% chance that if a second sample of 100 freshmen were taken the average of the sample would be in the range you found in a).

c) True or false and explain: There is a 95% chance that the average hours a week worked for pay among *all* freshmen at public universities is in the range you found in a).

d) True or false and explain: There is a 95% chance that the average hours a week worked for pay among all 18–20 year olds is in the range you found in a).

2. (15 points) Based on the survey data in #1, a news organization reports that the freshmen at public universities work 12 hours a week for pay, on average. Use a z-test to comment on this assertion. State the null and alternative hypotheses. Calculate the test statistic and estimate the P value.

3. (10 points) Would taking the average of 100 measurements decrease the likely size of the chance error by a factor of 5, 10 or 25? Justify your answer.

4. (10 points) A surveyor is measuring the distances between five points V, W, X, Y, and Z along a straight line. She finds that each of the four distances measures one mile, give or take two inches or so. The distance from V to Z is about four miles; but the estimate is likely to be off by around: 4 inches, 2 inches, 1 inch, 1/2 inch or 1/4 inch. Justify your answer.

$$V \text{ —— } W \text{ —— } X \text{ —— } Y \text{ —— } Z$$

5. (20 points) Five hundred draws are made at random from the box

$$\boxed{60{,}000\ \boxed{0}\text{s} \quad 20{,}000\ \boxed{1}\text{s}}$$

Note that $\sqrt{0.25 \times 0.75}\,/\sqrt{500} \approx 0.02$. True or false? Explain your conclusions.

i. The expected value for the percentage of 1s among the draws is around 25%, give or take 2% or so.

ii. The expected value for the percentage of 1s among the draws is exactly 25%.

iii. The percentage of 1s among the draws will be around 25%, give or take 2% or so.

iv. The percentage of 1s in the box is exactly 25%.

v. The percentage of 1s among the draws will be exactly 25%.

vi. The percentage of 1s in the box is around 25%, give or take 2% or so.

6. (10 points) Find the specified area under the normal curve. Write down the `R` command that you use.

a) $z > -1.15$

b) $z < -1.75$

c) $-0.5 < z < 2.3$

7. (10 points) Fill in the blanks:

The _____ hypothesis says that the difference is due to chance but the _____ says that the difference is real.

Formula Sheet

$$\text{mean} = \frac{1}{n}\left(x_1 + \cdots + x_n\right)$$

$$\text{SD} = \sqrt{\frac{1}{n}\left((x_1 - \text{mean})^2 + \cdots + (x_n - \text{mean})^2\right)}$$

$$\text{sd} = \sqrt{\frac{1}{n-1}\left((x_1 - \text{mean})^2 + \cdots + (x_n - \text{mean})^2\right)}$$

$$\begin{aligned} \text{EV}_{\text{sum}} &= n\,\text{AV}_{\text{box}} \\ \text{SE}_{\text{sum}} &= \sqrt{n}\ \text{SD}_{\text{box}} \end{aligned}$$

$$\begin{aligned} \text{EV}_{\text{av}} &= \text{AV}_{\text{box}} \\ \text{SE}_{\text{av}} &= \text{SD}_{\text{box}} \,/\sqrt{n} \end{aligned}$$

Shortcut formula (if there are only two different kinds of numbers in the box):

$$\text{SD}_{\text{box}} = (\text{big } \# - \text{small } \#)\sqrt{\left(\begin{array}{c}\text{fraction of}\\ \text{tickets with the}\\ \text{big number}\end{array}\right)\left(\begin{array}{c}\text{fraction of}\\ \text{tickets with the}\\ \text{small number}\end{array}\right)}$$

$$95\%\text{ confidence interval} = \text{observed } \pm\ 2\,\text{SE}$$

$$z = \frac{\text{observed} - \text{expected}}{\text{standard error}}$$