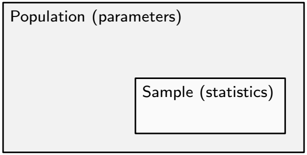
Math 207: Statistics

Chapter 20: Chance Error in Sampling



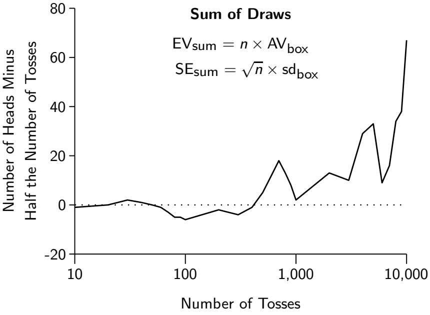
Dr. Ralph Wojtowicz

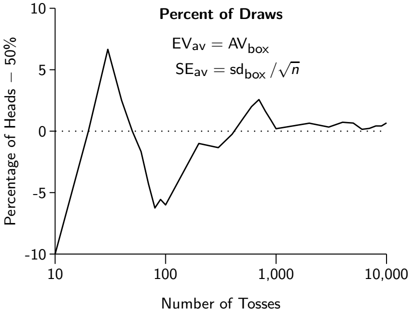
Mathematics Department

Shenandoah University

Expected Value, Standard Error, Central Limit Theorem

- Many statistics problems are modeled as samples from a box of numbered tickets.
- Solution procedure:
  - Formulate a box model.
  - Compute the average and SD of the contents of the box.
  - Determine if you are computing a sum or average (% in a 0/1 box is an average).
  - Use the appropriate formulas to compute the EV and SE for the sample.
  - Use the normal curve to compute chances of the sample being in a specified range.





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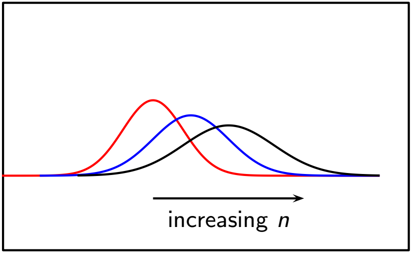

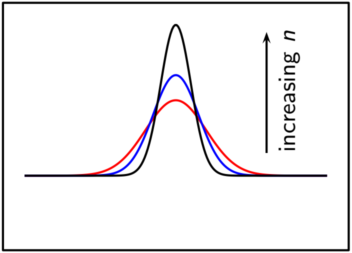

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- Compute the z scores for 375:
  $z = (375 - 350)/17.1 = 1.46$
- Area under the normal curve: $1 - \texttt{pnorm}(1.46) \approx 0.0721 = 7.21\%$