Math 207
Introduction to Statistics

Exam IV

1. (25 points) A survey is conducted of 100 freshmen at public universities. It is found that these students work 11.5 hours a week for pay, on average. The SD of the data is 5.

a) Find a 95% confidence interval on the hours a week worked by freshmen at public universities.

b) True or false and explain: There is a 95% chance that if a second sample of 100 freshmen were taken the average of the sample would be in the range you found in a).

c) True or false and explain: There is a 95% chance that the average hours a week worked for pay among *all* freshmen at public universities is in the range you found in a).

d) True or false and explain: There is a 95% chance that the average hours a week worked for pay among all 18–20 year olds is in the range you found in a).

2. (15 points) Would taking the average of 25 measurements divide the likely size of the chance error by a factor of 5, 10 or 25? Justify your answer.

3. (15 points) A surveyor is measuring the distances between five points A, B, C, D, and E along a straight line. She finds that each of the four distances measures one mile, give or take an inch or so. The distance from A to E is about four miles; but the estimate is likely to be off by around: 4 inches, 2 inches, 1 inch, 1/2 inch or 1/4 inch. Justify your answer.

4. (20 points) Five hundred draws are made at random from the box

$$\underline{|\, 60{,}000\ \boxed{0}\,\text{s}\quad 20{,}000\ \boxed{1}\,\text{s}\,|}$$

Note that $\sqrt{0.25 \times 0.75}\,/\sqrt{500} \approx 0.02$. True or false?

- The expected value for the percentage of 1s among the draws is exactly 25%.
- The expected value for the percentage of 1s among the draws is around 25%, give or take 2% or so.
- The percentage of 1s among the draws will be around 25%, give or take 2% or so.
- The percentage of 1s among the draws will be exactly 25%.
- The percentage of 1s in the box is exactly 25%.
- The percentage of 1s in the box is around 25%, give or take 2% or so.

5. (20 points) Based on the survey data in #1, a news organization reports that the freshmen at public universities work 12 hours a week for pay, on average. Use a z-test to comment on this assertion. State the null and alternative hypotheses. Calculate the test statistic and estimate the P value.

6. (10 points) Fill in the blanks:

The ____________ hypothesis says that the difference is due to chance but the ____________ says that the difference is real.

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1. (25 points) A survey is conducted of 100 freshmen at public universities. Among the surveyed students, 20% (or 1/5) work at least 15 hours per week for pay.

a) Find a 95% confidence interval on the percent of freshmen at public universities who work at least 15 hours per week for pay.

b) True or false and explain: There is a 95% chance that the percent among *all* freshmen at public universities is in the range you found in a).

c) True or false and explain: There is a 95% chance that if a second sample of 100 freshmen were taken, the sample percent would be in the range you found in a).

d) True or false and explain: There is a 95% chance that the average hours a week worked for pay among all 18–20 year olds is in the range you found in a).

2. (10 points) Would taking the sum of 25 measurements multiply the likely size of the chance error by a factor of 5, 10 or 25? Justify your answer.

3. (10 points) A surveyor is measuring the distances between five points A, B, C, D, and E along a straight line. She finds that each of the four distances measures one mile, give or take an inch or so. The distance from A to E is about four miles; but the estimate is likely to be off by around: 4 inches, 2 inches, 1 inch, 1/2 inch or 1/4 inch. Justify your answer.

$$A \text{ —— } B \text{ —— } C \text{ —— } D \text{ —— } E$$

4. (15 points) The Zorro News Organization reports that 50% of freshmen at public universities work 15 hours a week for pay. If that is report is accurate, approximately what is the chance of getting a sample percent as low as 20% (as we did in #1) or lower? Hint: Use a z-test.

5. (20 points) Five hundred draws are made at random from the box

$$\underline{\left| \; 60{,}000 \; \boxed{0}\text{s} \quad 20{,}000 \; \boxed{1}\text{s} \; \right|}$$

Note that $\sqrt{0.25 \times 0.75}\,/\sqrt{500} \approx 0.02$. True or false? Explain your conclusions.

i. The expected value for the percentage of 1s among the draws is exactly 25%.

ii. The expected value for the percentage of 1s among the draws is around 25%, give or take 2% or so.

iii. The percentage of 1s among the draws will be around 25%, give or take 2% or so.

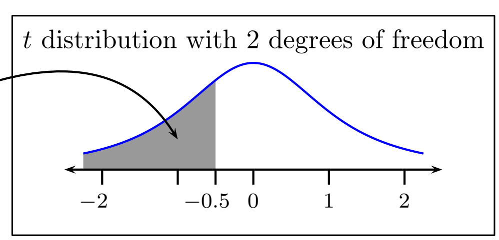
iv. The percentage of 1s among the draws will be exactly 25%.

v. The percentage of 1s in the box is exactly 25%.

vi. The percentage of 1s in the box is around 25%, give or take 2% or so.

6. (20 points) If a particular scientific instrument is working correctly, it's averge measurement should be 9. It is tested twice and the results are 9 and 11. Run a t test to see if the difference between the observed and expected averages is significant. State the null and alternative hypotheses.

Degrees of Freedom	t = -2.0	-1.5	-1.0	-0.5
1	0.148	0.187	0.250	0.352
2	0.092	0.136	0.211	0.333
3	0.070	0.115	0.196	0.326



Formula Sheet

$$\text{mean} = \frac{1}{n}\,(x_1 + \cdots + x_n)$$

$$\text{SD} = \sqrt{\frac{1}{n}\,\left((x_1 - \text{mean})^2 + \cdots + (x_n - \text{mean})^2\right)}$$

$$\text{sd} = \sqrt{\frac{1}{n-1}\,\left((x_1 - \text{mean})^2 + \cdots + (x_n - \text{mean})^2\right)}$$

$$\text{EV}_{\text{sum}} = n\,\text{AV}_{\text{box}}$$
$$\text{SE}_{\text{sum}} = \sqrt{n}\;\;\text{SD}_{\text{box}}$$

$$\text{EV}_{\text{av}} = \text{AV}_{\text{box}}$$
$$\text{SE}_{\text{av}} = \text{SD}_{\text{box}}\,\big/\sqrt{n}$$

Shortcut formula (if there are only two different kinds of numbers in the box):

$$\text{SD}_{\text{box}} = (\text{big \# } - \text{ small \#})\;\sqrt{\begin{pmatrix}\text{fraction of}\\ \text{tickets with the}\\ \text{big number}\end{pmatrix}\begin{pmatrix}\text{fraction of}\\ \text{tickets with the}\\ \text{small number}\end{pmatrix}}$$

$$\text{95\% confidence interval} = \text{observed } \pm\ 2\,\text{SE}$$

$$z = \frac{\text{observed} - \text{expected}}{\text{standard error}}$$

Formula Sheet

$$\text{mean} = \frac{1}{n}\,(x_1 + \cdots + x_n)$$

$$\text{SD} = \sqrt{\frac{1}{n}\,\left((x_1 - \text{mean})^2 + \cdots + (x_n - \text{mean})^2\right)}$$

$$\text{sd} = \sqrt{\frac{1}{n-1}\,\left((x_1 - \text{mean})^2 + \cdots + (x_n - \text{mean})^2\right)}$$

$$\text{EV}_{\text{sum}} = n\,\text{AV}_{\text{box}}$$
$$\text{SE}_{\text{sum}} = \sqrt{n}\;\;\text{SD}_{\text{box}}$$

$$\text{EV}_{\text{av}} = \text{AV}_{\text{box}}$$
$$\text{SE}_{\text{av}} = \text{SD}_{\text{box}}\,\big/\sqrt{n}$$

Shortcut formula (if there are only two different kinds of numbers in the box):

$$\text{SD}_{\text{box}} = (\text{big \# } - \text{ small \#})\;\sqrt{\begin{pmatrix}\text{fraction of}\\ \text{tickets with the}\\ \text{big number}\end{pmatrix}\begin{pmatrix}\text{fraction of}\\ \text{tickets with the}\\ \text{small number}\end{pmatrix}}$$

$$\text{95\% confidence interval} = \text{observed } \pm\ 2\,\text{SE}$$

$$z = \frac{\text{observed} - \text{expected}}{\text{standard error}}$$

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Exam IV

1. (25 points) A survey is conducted of 100 freshmen at public universities. Among the surveyed students, 20% (or 1/5) work at least 15 hours per week for pay.

a) Find a 95% confidence interval on the percent of freshmen at public universities who work at least 15 hours per week for pay.

b) True or false and explain: There is a 95% chance that the percent among *all* freshmen at public universities is in the range you found in a).

c) True or false and explain: There is a 95% chance that if a second sample of 100 freshmen were taken, the sample percent would be in the range you found in a).

d) True or false and explain: There is a 95% chance that among all 18–20 year olds, the percent who work at least 15 hours per week for pay is in the range you found in a).

2. (15 points) Would taking the average of 25 measurements decrease the likely size of the chance error by a factor of 5, 10 or 25? Justify your answer.

3. (15 points) A surveyor is measuring the distances between five points A, B, C, D, and E along a straight line. She finds that each of the four distances measures one mile, give or take an inch or so. The distance from A to E is about four miles; but the estimate is likely to be off by around: 4 inches, 2 inches, 1 inch, 1/2 inch or 1/4 inch. Justify your answer.

$$A \text{ —— } B \text{ —— } C \text{ —— } D \text{ —— } E$$

4. (20 points) The Zorro News Organization reports that 50% of freshmen at public universities work 15 hours a week for pay. If that is report is accurate, approximately what is the chance of getting a sample percent as low as or lower than 20% (as we did in #1)? Hint: calculate z then a p-value (area under the normal curve).

5. (25 points) Five hundred draws are made at random from the box

$$\boxed{60{,}000\ \boxed{0}\text{s}\quad 20{,}000\ \boxed{1}\text{s}}$$

Note that $\sqrt{0.25 \times 0.75}\,/\sqrt{500} \approx 0.02$. True or false? Explain your conclusions.

i. The expected value for the percentage of 1s among the draws is exactly 25%.

ii. The expected value for the percentage of 1s among the draws is around 25%, give or take 2% or so.

iii. The percentage of 1s among the draws will be around 25%, give or take 2% or so.

iv. The percentage of 1s among the draws will be exactly 25%.

v. The percentage of 1s in the box is exactly 25%.

vi. The percentage of 1s in the box is around 25%, give or take 2% or so.

Formula Sheet

$$\text{mean} = \frac{1}{n}\left(x_1 + \cdots + x_n\right)$$

$$\text{SD} = \sqrt{\frac{1}{n}\left((x_1 - \text{mean})^2 + \cdots + (x_n - \text{mean})^2\right)}$$

$$\text{sd} = \sqrt{\frac{1}{n-1}\left((x_1 - \text{mean})^2 + \cdots + (x_n - \text{mean})^2\right)}$$

$$\begin{aligned} \text{EV}_{\text{sum}} &= n\,\text{AV}_{\text{box}} \\ \text{SE}_{\text{sum}} &= \sqrt{n}\;\;\text{SD}_{\text{box}} \end{aligned}$$

$$\begin{aligned} \text{EV}_{\text{av}} &= \text{AV}_{\text{box}} \\ \text{SE}_{\text{av}} &= \text{SD}_{\text{box}} \big/ \sqrt{n} \end{aligned}$$

$$\begin{aligned} \text{EV}_{\%} &= \text{AV}_{\text{box}} \\ \text{SE}_{\%} &= \text{SD}_{\text{box}} \big/ \sqrt{n} \end{aligned}$$

Shortcut formula (if there are only two different kinds of numbers in the box):

$$\text{SD}_{\text{box}} = (\text{big }\# - \text{small }\#)\sqrt{\left(\begin{array}{c}\text{fraction of}\\ \text{tickets with the}\\ \text{big number}\end{array}\right)\left(\begin{array}{c}\text{fraction of}\\ \text{tickets with the}\\ \text{small number}\end{array}\right)}$$

$$95\%\text{ confidence interval} = \text{observed } \pm\ 2\,\text{SE}$$

$$z = \frac{\text{observed} - \text{expected}}{\text{standard error}}$$

Math 207
Introduction to Statistics

Exam IV

1. (25 points) A survey is conducted of 100 freshmen at public universities. Among the surveyed students, 80% (or 4/5) work at least 5 hours per week for pay.

a) Find a 95% confidence interval on the percent of freshmen at public universities who work at least 5 hours per week for pay.

b) True or false and explain: There is a 95% chance that the percent among *all* freshmen at public universities is in the range you found in a).

c) True or false and explain: There is a 95% chance that if a second sample of 100 freshmen were taken, the sample percent would be in the range you found in a).

d) True or false and explain: There is a 95% chance that among all 18–20 year olds, the percent who work at least 5 hours per week for pay is in the range you found in a).

2. (10 points) Would taking the average of 25 measurements decrease the likely size of the chance error by a factor of 5, 10 or 25? Justify your answer.

3. (15 points) A surveyor is measuring the distances between five points A, B, C, D, and E along a straight line. She finds that each of the four distances measures one mile, give or take an inch or so. The distance from A to E is about four miles; but the estimate is likely to be off by around: 4 inches, 2 inches, 1 inch, 1/2 inch or 1/4 inch. Justify your answer.

$$A \text{ —— } B \text{ —— } C \text{ —— } D \text{ —— } E$$

4. (20 points) The Zorro News Organization reports that 85% of freshmen at public universities work 5 hours a week for pay. If that is report is accurate, approximately what is the chance of getting a sample percent as low as or lower than 80% (as we did in #1)? Hint: calculate z then a p-value (area under the normal curve).

5. (20 points) Five hundred draws are made at random from the box

$$\underline{\left|\ 20{,}000\ \boxed{0}\text{s}\quad 60{,}000\ \boxed{1}\text{s}\ \right|}$$

Note that $\sqrt{0.25 \times 0.75}\,/\sqrt{500} \approx 0.02$. True or false? Explain your conclusions.

i. The expected value for the percentage of 1s among the draws is exactly 75%.

ii. The expected value for the percentage of 1s among the draws is around 75%, give or take 2% or so.

iii. The percentage of 1s among the draws will be around 75%, give or take 2% or so.

iv. The percentage of 1s among the draws will be exactly 75%.

v. The percentage of 1s in the box is exactly 75%.

vi. The percentage of 1s in the box is around 75%, give or take 2% or so.

6. (10 points) Find the specified area under the normal curve. Write down the `R` command that you use.

a) $z > 1.35$

b) $z < -2$

c) $-1.5 < z < 3.5$

Formula Sheet

$$\text{mean} = \frac{1}{n}(x_1 + \cdots + x_n)$$

$$\text{SD} = \sqrt{\frac{1}{n}\left((x_1 - \text{mean})^2 + \cdots + (x_n - \text{mean})^2\right)}$$

$$\text{sd} = \sqrt{\frac{1}{n-1}\left((x_1 - \text{mean})^2 + \cdots + (x_n - \text{mean})^2\right)}$$

$$\text{EV}_{\text{sum}} = n\,\text{AV}_{\text{box}}$$
$$\text{SE}_{\text{sum}} = \sqrt{n}\ \text{SD}_{\text{box}}$$

$$\text{EV}_{\text{av}} = \text{AV}_{\text{box}}$$
$$\text{SE}_{\text{av}} = \text{SD}_{\text{box}} / \sqrt{n}$$

Shortcut formula (if there are only two different kinds of numbers in the box):

$$\text{SD}_{\text{box}} = (\text{big \#} - \text{small \#})\sqrt{\begin{pmatrix}\text{fraction of} \\ \text{tickets with the} \\ \text{big number}\end{pmatrix}\begin{pmatrix}\text{fraction of} \\ \text{tickets with the} \\ \text{small number}\end{pmatrix}}$$

$$95\%\ \text{confidence interval} = \text{observed} \pm 2\,\text{SE}$$

$$z = \frac{\text{observed} - \text{expected}}{\text{standard error}}$$

6. (10 points) Find the specified area under the normal curve. Write down the `R` command that you use.

a) $z < 2.15$

b) $z > -1.75$

c) $-1.25 < z < 2.5$

Formula Sheet

$$\text{mean} = \frac{1}{n}(x_1 + \cdots + x_n)$$

$$\text{SD} = \sqrt{\frac{1}{n}\left((x_1 - \text{mean})^2 + \cdots + (x_n - \text{mean})^2\right)}$$

$$\text{sd} = \sqrt{\frac{1}{n-1}\left((x_1 - \text{mean})^2 + \cdots + (x_n - \text{mean})^2\right)}$$

$$\begin{aligned} \text{EV}_{\text{sum}} &= n\,\text{AV}_{\text{box}} \\ \text{SE}_{\text{sum}} &= \sqrt{n}\ \text{SD}_{\text{box}} \end{aligned}$$

$$\begin{aligned} \text{EV}_{\text{av}} &= \text{AV}_{\text{box}} \\ \text{SE}_{\text{av}} &= \text{SD}_{\text{box}} \big/ \sqrt{n} \end{aligned}$$

$$\begin{aligned} \text{EV}_{\%} &= \text{AV}_{\text{box}} \\ \text{SE}_{\%} &= \text{SD}_{\text{box}} \big/ \sqrt{n} \end{aligned}$$

Shortcut formula (if there are only two different kinds of numbers in the box):

$$\text{SD}_{\text{box}} = (\text{big \#} - \text{small \#})\sqrt{\begin{pmatrix}\text{fraction of} \\ \text{tickets with the} \\ \text{big number}\end{pmatrix}\begin{pmatrix}\text{fraction of} \\ \text{tickets with the} \\ \text{small number}\end{pmatrix}}$$

$$95\%\ \text{confidence interval} = \text{observed} \pm 2\,\text{SE}$$

$$z = \frac{\text{observed} - \text{expected}}{\text{standard error}}$$

5. (25 points) One hundred draws will be made at random with replacement from the box shown below.

$$\boxed{\boxed{0}\ \boxed{0}\ \boxed{1}}$$

Note that $\text{SD}_{\text{box}} = \sqrt{2}\,/\,3 = 0.47$.

a) The number of 1s among the draws will be around ________ give or take ________ or so.

b) The percentage of 1s among the draws will be around ________ give or take ________ or so.

c) Roughly what is the probability that the percentage of 1s will be over 38%.