Math 207
Introduction to Statistics

Exam III Review Exercises (B)

1. **Probability Rules**:

   a) **Coins**: Suppose that you flip a fair coin twice.

      i) What is the chance that both flips show heads?

      ii) What is the chance that the second flip shows tails given that the first showed tails?

   b) **Cards**: Cards are drawn without replacement from a deck.

      i) What is the chance that the first two cards are ♣?

      ii) What is the chance that the first card is a ♡ or a J?

      iii) What is the chance that the second card is a ♡ given that the first card was?

      ii) What is the chance that the 10th card is a 5 (given no information about the first nine cards)?

   c) **Dice**: Two fair dice are rolled.

      i) What is the chance that the sum is at least 8?

      ii) What is the chance that the sum is at least 8 or at most 4?

      iii) What is the chance that the sum is at least 8 given that one die shows a 5?

   d) **Normal Curve**: A computer program randomly generates a z-score from a normal distribution.

      i) What is the chance that the value is between -2 and 2?

      ii) What is the chance that the value is greater than 1?

2. **Mutually Exclusive Events**. Describe an event B for which A and B are mutually exclusive.

   a) $A =$ the first two cards drawn from a deck are $8\heartsuit$ and $J\spadesuit$.

   b) $A =$ in flipping three coins, you will get all tails

   c) $A =$ in rolling two dice, the sum will be at most 4

3. **Independent Events**

   a) $A =$ the first card drawn from a deck is a $\heartsuit$

   b) $A =$ the first two coin flips will both show heads

   c) $A =$ in rolling a blue die and a red die, the blue die will show an even number.

   d) Sets of paper tickets are shown below. Each ticket has a number on the left and a shape on the right. For each set of tickets, Determine if number and shape are independent or dependent.

   i) $\boxed{3}\boxed{\triangle}$ $\boxed{3}\boxed{\square}$ $\boxed{3}\boxed{\star}$ $\boxed{3}\boxed{\star}$ $\boxed{7}\boxed{\triangle}$ $\boxed{7}\boxed{\square}$ $\boxed{7}\boxed{\square}$ $\boxed{7}\boxed{\star}$

   ii) $\boxed{3}\boxed{\triangle}$ $\boxed{3}\boxed{\star}$ $\boxed{3}\boxed{\square}$ $\boxed{3}\boxed{\star}$ $\boxed{7}\boxed{\triangle}$ $\boxed{7}\boxed{\square}$ $\boxed{7}\boxed{\star}$ $\boxed{7}\boxed{\star}$

4. **Binomial Formula**. A box contains 6 blue marbles and 7 red marbles.

   a) If six marbles are drawn with replacement, what is the chance that two are blue?

   b) If six are drawn with replacement, what is the chance that at most two are blue?

   c) If six are drawn without replacement, what is the chance that two are blue?

5. **Law of Averages**. A coin is flipped repeatedly and the percent of tails recorded.

a) You win \$1 if there are at least 45% tails. Which is better 10 tosses or 1000?

b) You win \$1 if there are between 49.999% and 50.001% tails. Which is better 10 tosses or 1000?

c) You win \$1 if there are at least 55% tails. Which is better 10 tosses or 1000?

d) You win \$1 if there are exactly 50% tails. Which is better 10 tosses or 1000?

6. **Expected Value and Standard Error for Sums**. 100 draws will be made at random with replacement from the box shown below.

$$\boxed{\boxed{3}\quad\boxed{7}}$$

a) How small can the sum be? How large?

b) How many times do you expect the ticket $\boxed{3}$ to turn up?

c) About how much do you expect the sum to be?

d) Put a plus or minus on your answer to c).

7. **Expected Value and Standard Error for Averages**. 100 draws will be made at random with replacement from the box shown below.

$$\boxed{\boxed{3}\quad\boxed{7}}$$

a) How small can the average be? How large?

b) About how much do you expect the average to be?

c) Put a plus or minus on your answer to b).

A standard deck of cards has the following properties.

- 52 cards
- 4 suits: hearts $\heartsuit$, spades $\spadesuit$, clubs $\clubsuit$, and diamonds $\diamondsuit$
- Hearts and diamonds are red. Spades and clubs are black
- 13 cards of each suit: 2 – 10, J, Q, K, and ace
- 4 cards of each rank: 2 – 10, J, Q, K, and ace
- J, Q, and K are face cards

Multiplication Rule for Probabilities:

- $P(A \text{ and } B) = P(A)\,P(B \mid A)$.
- If A and B are independent, then $P(A \text{ and } B) = P(A)\,P(B)$.

Sum Rule for Probabilities:

- $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
- If A and B are mutually exclusive, then $P(A \text{ or } B) = P(A) + P(B)$

Binomial Formula: The chance that an event will occur exactly k times out of n is given by the binomial formula:

$$\frac{n!}{k!\,(n-k)!}\,p^k\,(1-p)^{n-k}$$

In this formula, n is the number of trials, k is the number of times the event is to occur, and p is the probability that the event will occur on any particular trial. The assumptions:

- The value of n must be fixed in advance.
- p must be the same from trial to trial.
- The trials must be independent.

The **mean** of a list of numbers $x_1, \ldots, x_n$ is

$$\text{mean} = \frac{1}{n}\,(x_1 + x_2 + \cdots + x_n) = \frac{1}{n}\sum_{i=1}^{n} x_i.$$

The **population standard deviation** is

$$\text{SD} = \sqrt{\frac{1}{n}\left((x_1 - \text{mean})^2 + \cdots + (x_n - \text{mean})^2\right)}$$

If we know the contents of a box of numbered tickets, we use SD.

The **sample standard deviation** is

$$\text{sd} = \sqrt{\frac{1}{n-1}\left((x_1 - \text{mean})^2 + \cdots + (x_n - \text{mean})^2\right)}$$

The figures below show how the standard error changes as the sample size increases. The figure on the right illustrates the Law of Averages.