Auction Algorithms

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Assignment Problem

- N persons and M objects and benefits a_{ij}
- Match one-to-one basis
- Maximize total benefit $\sum_{i=1}^{n} a_{ij_i}$

Economic equilibrium Problem

- Match n persons and objects and maximize the profit
- Person i would want object j_i, such that

$$a_{ij_i} - p_{ji} = \max_{j=1,\ldots,n} a_{ij} - p_j$$

- Equilibrium if all persons are happy

Basic auction algorithm

- Choose object j_i with maximal value

$$j_i \in \arg \max_{j=1,\ldots,n} a_{ij} - p_j$$

- Get object, increment its price

$$p_{ji} = p_{ji} + \gamma_i \qquad \gamma_i = v_i - w_i$$

$$v_i = \max_j a_{ij} - p_j, \qquad w_i = \max_{j \neq j_i} a_{ij} - p_j$$

D.P. Bertsekas, *A distributed algorithm for the assignment problem.* Lab. for Information and Decision System Working Paper, M.I.T., March 1997.

Points to note

- γ_i is always non decreasing
- The object becomes less desirable as it is bid more
- What if v_i equals w_i

Modified Auction Algorithm

- Person is almost happy if

$$a_{ij_i} - p_{ji} = \max_{j=1,\ldots,n} a_{ij} - p_j - \epsilon$$

$$\gamma_i = v_i - w_i + \epsilon$$

- almost at equilibrium

Optimality

- Total benefit within nε of the optimal value
- When weights are integers and

$$\epsilon < \frac{1}{n}$$

  - optimal assignment

ε-scaling

- Convergence depends on ε and maximum object value (C)
- Low initial prices
  - number of rounds α C/ε
- Fasten process with ε-scaling

Centralized Algorithm

- Centralized Controller
- Bidding process in synchronized rounds
- Communication between controller and person threads through RMI calls

Controller

- Asks the current person to bid
- If the person returns with no assignment, terminate the person
- If the person returns with an assignment, assign next round to displaced person, else go on with next person in queue

Person

- Person finds the object offering the best value
- If this exists, assigns itself to that and returns the assignment
- Otherwise returns terminated

Points to Note

- Overdependence on controller
- Not really distributed
- Bidding and selection can be parallelized

Asynchronous-Parallel Algorithm

- Controller starts all person and object threads
- Responsible for termination detection

Person

- If currently not assigned to an object, sends a bid to the object having max benefit
- If there are no objects with positive benefit, it reports terminated to the controller
- On receiving accept from the object, updates the price of the object and reports the new assignment to controller

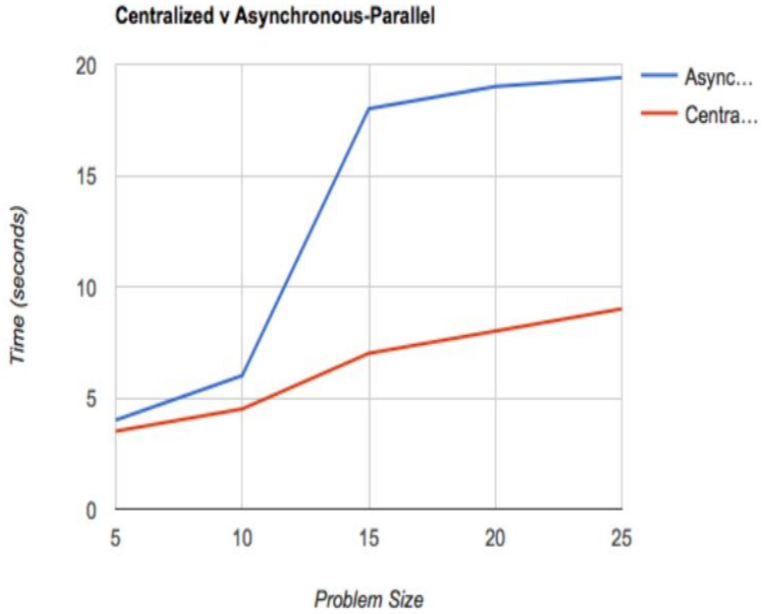
Object

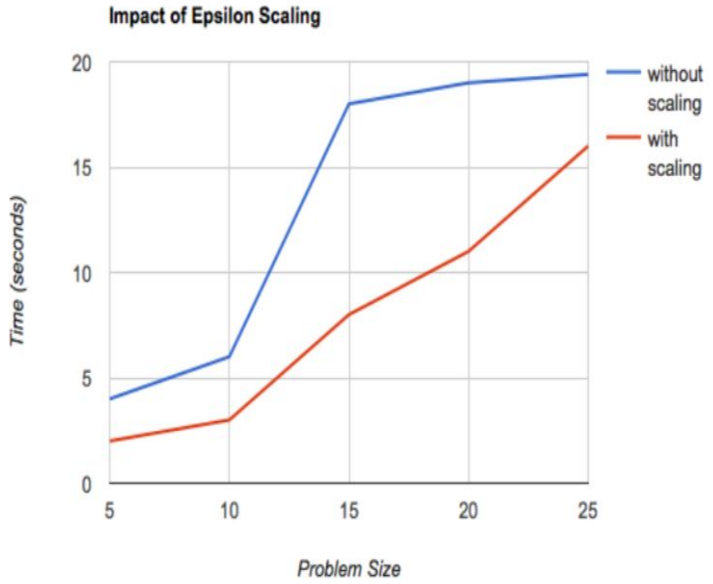
- On receiving bid from object, if it is higher than current price, sends accept, else reject
- If bid is accepted, then informs all other objects of its new price

Termination Detection - Controller

- Keeps track of the persons assigned and their respective objects.
- Process terminated when
  - All persons have an object assigned or are terminated
  - The number of assigned objects are $\min(N_{person}, N_{object})$

Experiments





Demo