Examining the Dynamics of Systemic Risk in the Context of Asset Bubbles: An Inquiry Utilizing Stochastic Dominance

# Methodology

## Definition of the Financial Network

To model the evolution of financial bubbles and their impact on systemic risk, the Romanian stock market is represented as a dynamic, directed, and weighted financial network. This network evolves over time, capturing firm-level dependencies based on bubble co-occurrence patterns. Formally, at each time step , the financial network is denoted as:

where represents the set of firms active in the market during period , is the set of directed edges encoding the interactions between firms based on financial bubble overlap, and denotes the matrix of edge weights, which quantify the strength of these interactions. The construction of this network allows for the analysis of how speculative bubbles propagate through the financial system and the extent to which firms contribute to systemic instability.

Each node in the network corresponds to a publicly traded firm that exhibits characteristics of a financial bubble during the given time period. A firm is considered to be in a bubble phase if its price dynamics satisfy specific statistical criteria defined by bubble detection algorithms. The set of firms is therefore not static but evolves over time as new bubbles emerge and existing bubbles collapse. The inclusion of firms in the network is determined by the presence of financial bubbles rather than by simple market participation, ensuring that the constructed network focuses explicitly on systemic risk arising from speculative episodes.

The formation of edges within the network is governed by the temporal overlap of financial bubbles between firms. A directed edge from firm to firm is established if firm enters a bubble while firm is already experiencing one. This directional relationship reflects the hypothesis that earlier bubbles may contribute to the development of subsequent bubbles, suggesting the presence of financial contagion. Mathematically, an edge exists if:

where and represent the start and end dates of the bubble period for firm , while and denote the corresponding dates for firm . The additional condition ensures that edges respect the chronological order of bubble formation, thereby maintaining the causal structure of the network.

The strength of an edge is determined by the duration of bubble overlap, capturing the extent to which firms experience simultaneous speculative episodes. The weight of an edge is formally defined as:

This formulation ensures that the network does not merely reflect binary relationships but encodes the intensity of co-bubbling phenomena. A longer overlap period suggests a more prolonged exposure to shared market exuberance, potentially reinforcing systemic risk through interconnected speculation.

Unlike traditional financial networks based on static correlation matrices, the proposed network dynamically evolves as bubbles appear and disappear over time. The construction of a time series of networks enables the study of systemic risk propagation through the financial system, providing insight into how market exuberance transitions from localized speculative episodes to broader financial instability.

By representing the financial system as a dynamic graph, this framework facilitates the identification of firms that act as major risk amplifiers. Firms with a high number of incoming edges may be particularly susceptible to contagion, as they tend to enter bubbles following the onset of speculative activity in other firms. Conversely, firms with a high number of outgoing edges may play a central role in initiating or transmitting financial instability. The structure of the network, particularly the distribution of edge weights and node connectivity patterns, offers a rich source of information for assessing systemic risk.

The advantages of this network-based approach lie in its ability to capture market interdependencies in a way that traditional econometric models fail to do. While classical systemic risk measures such as CoVaR and marginal expected shortfall assess risk at the individual firm level based on historical price dynamics, they do not explicitly account for how bubbles propagate through the system. By contrast, the financial network model directly incorporates information about the interconnectedness of speculative episodes, providing a more holistic understanding of systemic instability.

The construction of this network serves as the foundation for the subsequent analysis, where systemic risk measures such as delta CoVaR are integrated into the model to quantify firm-level contributions to market-wide financial distress. Furthermore, the time-evolving nature of the network allows for the implementation of machine learning techniques, including Graph Neural Networks, to extract patterns of financial contagion and improve the prediction of systemic events.

## Edge Formation: Defining Bubble Co-Occurrence

The formation of edges within the financial network is driven by the concept of bubble co-occurrence, which captures the extent to which speculative episodes across firms overlap in time. The underlying assumption is that the simultaneity of financial bubbles reflects systemic market exuberance, where speculative activity in one firm can spill over to another. By encoding these relationships as directed edges, the network provides a framework for analyzing the dynamics of financial contagion.

To formally define edge formation, let each firm have an associated bubble period , where represents the date on which the bubble begins, and denotes the date on which the bubble ends. Similarly, for another firm , the bubble period is given by . The fundamental criterion for establishing a directed edge from firm to firm is that the bubble of firm begins while firm is still in a bubble state. Mathematically, this is expressed as:

The first condition ensures that firm enters a bubble at a time when firm is already experiencing speculative activity, indicating potential financial contagion. The second condition imposes a chronological constraint, requiring that firm enters a bubble before firm , ensuring that the direction of the edge reflects the order of bubble formations rather than arbitrary connections.

Unlike static correlation-based financial networks, where edges typically represent long-term associations between firms, the proposed methodology establishes edges based on dynamic temporal dependencies. This approach recognizes that systemic risk does not arise merely from firms having historically similar return patterns but from the contemporaneous nature of financial bubbles, where shared speculative dynamics may amplify systemic instability.

Once an edge is established, its weight is determined by the degree of bubble overlap, which quantifies the strength of the speculative connection between firms. The weight of the edge is given by:

This expression calculates the number of days during which both firms simultaneously experience a financial bubble. A longer overlap period implies a stronger connection, as it suggests a prolonged period of shared speculative behavior. If the overlap duration is zero or negative, no edge is formed, as this would indicate that firm entered a bubble only after firm had already exited its speculative phase, thereby lacking a meaningful temporal connection.

The introduction of weighted edges provides a richer representation of systemic risk than a simple binary adjacency matrix. The edge weights allow for the differentiation between firms that experience short-lived coincidental bubbles and those that remain interconnected in speculative phases for extended periods. This distinction is crucial for assessing the stability of the financial system, as firms with numerous high-weighted edges may act as amplifiers of systemic risk.

Furthermore, the structure of edge formation has important implications for identifying key players in financial contagion. Firms that frequently initiate connections (i.e., those with high out-degree) can be interpreted as *bubble leaders*, whose speculative activity may precede broader market instability. Conversely, firms with a high in-degree are more likely to enter bubbles after others, potentially acting as *risk absorbers* that accumulate market-wide speculative pressure. The network’s directed nature allows for a more nuanced analysis of these roles compared to undirected financial networks, where causality in bubble propagation cannot be explicitly inferred.

Another key feature of this approach is its flexibility in capturing market dynamics across different time scales. By adjusting the temporal resolution of bubble detection, the model can be adapted to study short-term speculative surges as well as prolonged financial instability. This adaptability is particularly useful for analyzing systemic risk in emerging markets, where market conditions evolve rapidly and speculative activity may be highly clustered.

By incorporating edge weights that reflect the duration of bubble overlaps and by structuring edges according to the temporal sequence of bubble formations, the network effectively encodes the propagation of speculative dynamics within the financial system. This foundation enables further analysis of systemic risk transmission, where the network topology is examined to identify periods of heightened instability and the key firms driving financial contagion.

## Node Features: Systemic Risk Contribution via Delta CoVaR

The effectiveness of the financial network in capturing systemic risk dynamics depends not only on the structure of its edges but also on the attributes assigned to its nodes. In this framework, each firm in the network is characterized by a set of financial risk indicators that quantify its contribution to overall market instability. Among these, one of the most relevant measures is *delta Conditional Value at Risk* (CoVaR), which provides a firm-specific assessment of systemic risk. By integrating this measure as a node feature, the network representation extends beyond purely structural relationships to incorporate firm-level financial distress.

### Systemic Risk Measurement: Delta CoVaR

Systemic risk is traditionally assessed through risk measures that estimate potential losses under extreme market conditions. One such measure is the *Conditional Value at Risk* (CoVaR), which quantifies the risk of the financial system given that a specific firm is in distress. Mathematically, the CoVaR of the financial system conditional on firm is defined as:

where represents the return of the financial system, is the return of firm , is the Value at Risk (VaR) of firm at confidence level , and is the systemic risk measure capturing the impact of firm ’s distress on the overall market.

To isolate the specific contribution of firm to systemic risk, the *delta CoVaR* (CoVaR) is computed as the difference between the system’s CoVaR when firm is in distress and when firm is in its median state:

A higher CoVaR value for a firm indicates that its distress significantly increases systemic risk, identifying it as a potential source of financial instability.

### Integration of Delta CoVaR into the Financial Network

In the constructed financial network, delta CoVaR is assigned as a node attribute, ensuring that the model captures not only structural relationships but also firm-level risk contributions. At each time step , the feature vector for a firm is given by:

This formulation ensures that systemic risk information is embedded directly into the network representation, allowing machine learning models, such as Graph Neural Networks (GNNs), to leverage both topological and financial data in predicting future instability.

While delta CoVaR provides a firm-level measure of systemic importance, it is subject to data availability constraints. In cases where delta CoVaR is not directly observable for a firm at a given time step, an imputation strategy is applied to maintain the completeness of the dataset. The missing values may be replaced using sector-wide averages, interpolation from historical values, or set to zero in cases where systemic risk exposure is deemed negligible. This ensures that the network remains well-defined across all time steps, preventing missing data from introducing inconsistencies in model training and evaluation.

### Economic Interpretation and Implications

The incorporation of delta CoVaR into the network model has important implications for systemic risk analysis. Firms with consistently high CoVaR values are identified as major contributors to financial instability, implying that their distress could have widespread repercussions on the market. Furthermore, by analyzing the temporal evolution of delta CoVaR within the network, it is possible to detect patterns of systemic risk accumulation and dissipation over time.

Another key advantage of integrating delta CoVaR into the network is the ability to identify clusters of firms with correlated systemic risk. In a traditional systemic risk framework, firms are often analyzed in isolation, without fully considering their interdependencies. However, when CoVaR values are embedded into a financial network, firms with similar risk profiles can be identified as forming tightly connected subgraphs, providing regulators and policymakers with more targeted intervention strategies.

The combination of delta CoVaR with network topological measures further enhances the predictive power of systemic risk models. Firms that exhibit high CoVaR values while also occupying central positions in the financial network are particularly concerning, as they serve as both systemic risk sources and amplifiers. Conversely, firms with lower CoVaR values but high connectivity may act as risk transmission channels, facilitating the spread of financial contagion.

By incorporating firm-level systemic risk attributes into the network representation, this methodology provides a comprehensive framework for analyzing financial contagion. This approach enables a more accurate identification of firms that play a crucial role in systemic instability, improving the effectiveness of early warning systems and risk mitigation strategies.

## Temporal Evolution of the Network

Financial markets are inherently dynamic, with systemic risk and speculative interactions evolving continuously over time. The structure of financial relationships is not static; firms enter and exit bubble phases, their interdependencies fluctuate, and risk propagation patterns shift in response to macroeconomic conditions. To accurately model these evolving dynamics, the proposed financial network is constructed as a time-dependent system, where each time step corresponds to a snapshot of the market’s systemic risk structure.

### Time-Indexed Network Representation

To formally capture the temporal nature of financial contagion, the financial system is modeled as a sequence of time-evolving graphs:

where each financial network represents the market at time . At each time step, the set of nodes consists of firms that are actively experiencing a financial bubble, while the edges capture the relationships between firms based on their bubble co-occurrence. The weight matrix quantifies the strength of these relationships by measuring the duration of bubble overlaps.

Because financial bubbles appear and dissipate over time, the set of firms participating in the network fluctuates across time steps. Unlike static financial networks, which assume a fixed set of entities and persistent relationships, this dynamic formulation allows for the study of how speculative episodes propagate across firms over time. Firms may emerge as systemic risk sources at certain time steps and lose their importance as market conditions change. Similarly, connections between firms may strengthen or weaken as bubble overlap periods expand or contract.

### Rolling Window Approach for Network Construction

To ensure that the network accurately reflects recent market conditions while maintaining a degree of historical context, a rolling window approach is employed. Instead of constructing networks based on isolated daily observations, each graph is built using data from a fixed historical period of length , where:

The choice of determines the balance between short-term and long-term financial dependencies. A shorter window captures immediate market fluctuations but may introduce excessive noise, while a longer window smooths out short-term volatility but risks overlooking sudden shifts in systemic risk. In empirical applications, typical choices for range from 30 to 90 trading days, depending on the desired granularity of systemic risk analysis.

### Graph Transition and Market Dynamics

The temporal evolution of the financial network is characterized by continuous changes in firm participation, edge formation, and weight distribution. At each time step , the network undergoes structural modifications as follows:

1. **Node Dynamics:** Firms that exit their bubble phases are removed from , while new firms entering bubbles are added.
2. **Edge Evolution:** New edges are created as fresh bubble co-occurrences emerge, while edges corresponding to past bubble interactions that have expired are removed.
3. **Weight Updates:** The duration of bubble overlaps is recomputed, leading to real-time adjustments in edge weights.
4. **Feature Updates:** Firm-specific attributes, such as delta CoVaR, are updated to reflect current systemic risk conditions.

This dynamic updating process ensures that the network remains an accurate representation of financial contagion at each time step, allowing for a granular examination of how systemic risk propagates in real time.

### Graph Similarity and Network Stability

To quantify the extent of structural changes in the financial network over time, a similarity measure between consecutive graph snapshots is introduced. Given two consecutive networks and , their similarity is computed using the Jaccard index for edge sets:

A high similarity score indicates that the financial network remains relatively stable over time, with persistent systemic risk relationships. Conversely, a sharp decline in similarity suggests a structural shift in market conditions, potentially signaling the onset or collapse of speculative phases. By tracking this similarity metric, one can assess whether systemic risk is gradually evolving or undergoing abrupt regime shifts.

### Time-Varying Graph-Theoretic Measures

The evolving nature of the financial network necessitates a dynamic analysis of graph-theoretic measures to capture systemic risk patterns over time. Key metrics that are monitored across time steps include:

* **Network Density:** Measures the proportion of observed edges relative to the total possible edges, reflecting how interconnected firms are during speculative episodes.
* **Degree Centrality Variation:** Tracks how the importance of specific firms in bubble propagation changes over time.
* **Eigenvector Centrality Dynamics:** Identifies firms that gain or lose systemic significance, providing early indicators of shifting risk concentrations.

The temporal trajectories of these measures offer insights into the development of systemic risk clusters, helping identify periods of market overheating and potential crises.

### Implications for Systemic Risk Monitoring

The temporal evolution of the financial network provides a powerful tool for systemic risk assessment. By analyzing how firms transition in and out of speculative phases and how their interdependencies change over time, the model offers a dynamic perspective on financial stability. This approach is particularly valuable for regulators and policymakers seeking to implement preemptive interventions to mitigate systemic risk before it escalates into a financial crisis.

A key advantage of this methodology is its ability to distinguish between *transient* and *persistent* risk episodes. Short-term fluctuations in network structure may indicate temporary speculative bursts, whereas prolonged periods of high network density and persistent systemic risk hubs suggest deeper structural vulnerabilities in the financial system. The ability to make this distinction is critical for designing effective risk mitigation strategies.

Furthermore, by integrating this temporal network framework with machine learning approaches, it becomes possible to develop predictive models that forecast future systemic risk based on historical contagion patterns. The learned temporal dependencies can be used to anticipate which firms are most likely to become systemic risk amplifiers, enabling proactive regulatory measures.

### Conclusion

The dynamic nature of financial contagion necessitates a time-evolving network framework that accurately reflects the shifting relationships between firms. By constructing a sequence of financial networks and continuously updating their structure, this approach provides a real-time view of systemic risk propagation. The use of rolling window construction, graph similarity metrics, and dynamic centrality measures allows for a comprehensive analysis of how speculative episodes develop and dissipate. This foundation supports further advancements in systemic risk forecasting, where machine learning techniques can be employed to extract predictive insights from the evolving network topology.

## Summary of Network Properties

The financial network constructed in this study provides a comprehensive representation of how financial bubbles propagate through the system and how firms contribute to systemic risk. By integrating both structural dependencies and firm-specific risk attributes, the network serves as a valuable tool for understanding financial contagion mechanisms. The key properties of the network, which distinguish it from traditional financial risk models, can be categorized into its topological characteristics, dynamic nature, and systemic risk implications.

### Structural Properties of the Financial Network

The financial network is inherently *directed* and *weighted*, capturing the chronological sequencing and intensity of financial bubble interactions. Unlike static correlation-based networks, which only quantify long-term statistical associations between firms, this network explicitly encodes the *temporal dependencies* of speculative episodes. The directed nature of the network ensures that financial contagion is modeled with causality, where speculative activity in one firm may lead to subsequent bubbles in other firms.

The weighting scheme, based on the duration of bubble overlaps, further enhances the network’s ability to quantify systemic interactions. Instead of binary connections, which only indicate whether a relationship exists, edge weights reflect the *strength* of bubble co-occurrence. A firm that remains in a speculative phase for an extended period while overlapping with multiple others will exhibit stronger systemic influence than a firm experiencing short-lived, isolated bubbles.

From a graph-theoretic perspective, the network can be analyzed using measures that quantify systemic importance. Degree centrality, eigenvector centrality, and clustering coefficients provide insight into the firms that act as systemic risk amplifiers. High-degree nodes correspond to firms that are frequently involved in speculative interactions, whereas firms with high eigenvector centrality play a more global role in financial contagion. The clustering coefficient, which measures the tendency of firms to form speculative clusters, offers additional information about localized risk concentration.

### Time-Varying Nature and Systemic Risk Evolution

A defining feature of this network is its ability to evolve over time. Unlike static systemic risk models, which assume fixed relationships between firms, the proposed framework continuously updates its structure as financial bubbles emerge and dissipate. This dynamic formulation provides a more accurate representation of systemic risk propagation, as firms may transition between speculative and stable phases, influencing or being influenced by market conditions.

The use of rolling windows in constructing the network ensures that systemic risk is analyzed within a recent and relevant timeframe. By capturing short- and long-term dependencies, the model balances responsiveness to market fluctuations with stability in risk assessment. The ability to track changes in network topology over time allows for the identification of *systemic regime shifts*, where market instability transitions between speculative booms and crises.

To assess the stability of the network structure, similarity metrics such as the Jaccard index are used to quantify changes in firm interconnections over time. A persistent network structure suggests prolonged systemic vulnerabilities, whereas a rapidly shifting network may indicate transient speculative phases. These insights are crucial for understanding whether financial instability is systemic or temporary in nature.

### Graph-Theoretic Measures for Systemic Risk Assessment

The financial network provides a rigorous mathematical framework for systemic risk analysis, where key graph-theoretic measures serve as indicators of financial instability. Among these, the most relevant are:

which quantifies the overall level of interconnectedness among firms. A higher density suggests more widespread financial contagion, whereas a lower density indicates localized speculative activity.

Another important measure is degree centrality, which assesses how influential a firm is within the bubble network:

Firms with high degree centrality are more likely to play a significant role in spreading financial instability, either as initiators of speculation or as entities highly susceptible to market contagion.

Eigenvector centrality extends this concept by considering not just the number of connections a firm has, but also the importance of the firms it is connected to:

This measure is particularly useful for identifying firms that may not be directly involved in many speculative interactions but nonetheless exert systemic influence due to their position in the financial network.

To assess the persistence of financial contagion, the similarity of consecutive network structures is analyzed using:

This metric captures the extent to which financial instability propagates over time, providing insight into whether systemic risk is escalating or dissipating.

### Economic and Regulatory Implications

The insights derived from this network-based approach have significant implications for financial stability monitoring and policymaking. By identifying firms that act as systemic risk hubs, regulators can implement targeted interventions to mitigate financial contagion. For example, firms with high eigenvector centrality may be subjected to stricter capital requirements or stress testing to ensure resilience against market shocks.

Additionally, the ability to track systemic risk dynamics in real time enhances the effectiveness of early warning systems. Traditional risk models often rely on lagging indicators, making them less effective in preventing crises. In contrast, the evolving financial network framework enables the detection of *leading indicators* of instability, allowing policymakers to respond proactively.

The presence of speculative clusters in the network further highlights the potential for localized financial distress to escalate into broader systemic crises. If firms within a particular sector exhibit high clustering coefficients, this suggests that systemic risk is concentrated within a specific industry, necessitating industry-specific regulatory measures. Conversely, if systemic risk is distributed across multiple sectors, broader macroprudential policies may be required.

### Conclusion

The financial network constructed in this study provides a robust framework for analyzing systemic risk and financial contagion. By incorporating both structural relationships and firm-specific risk attributes, the network offers a more comprehensive view of how financial bubbles propagate and influence market stability. The dynamic nature of the network ensures that systemic risk is assessed in real time, allowing for early identification of speculative clusters and risk transmission pathways.

The use of graph-theoretic measures enhances the interpretability of systemic risk, enabling regulators and policymakers to make informed decisions about financial stability. By tracking the evolution of network properties over time, the model provides valuable insights into the persistence and intensity of systemic risk, ultimately contributing to more effective risk management strategies.

In the subsequent sections, we extend this network-based approach by leveraging machine learning techniques, specifically Graph Neural Networks (GNNs), to enhance the predictive capabilities of systemic risk modeling.

## Motivation for Using Graph Neural Networks (GNNs)

Traditional econometric models, such as Vector Autoregression (VAR) and Conditional Value at Risk (CoVaR), have been widely used to assess systemic risk in financial markets. However, these models rely on the assumption of linear dependencies and stationarity, which are often violated in real-world financial systems. The presence of complex, nonlinear interactions between firms, coupled with the evolving nature of systemic risk, limits the effectiveness of these approaches in fully capturing the dynamics of financial contagion.

To overcome these limitations, we employ *Graph Neural Networks (GNNs)*, a class of deep learning models designed to operate on structured graph data. GNNs offer a powerful framework for learning representations of financial networks by leveraging the topological structure and dynamic interactions between firms. Unlike traditional methods, which rely on predefined statistical relationships, GNNs learn adaptive representations directly from the data, enabling them to capture both local and global dependencies within the financial system.

The core motivation for using GNNs in this context lies in their ability to model financial networks as evolving graph structures. Given a financial network at time , where represents the set of firms, the set of bubble-induced connections, and the associated edge weights, a GNN can learn a function that maps each firm to a latent feature space:

where represents the feature vector of firm containing firm-specific financial indicators, such as delta CoVaR, degree centrality, and clustering coefficients.

An important advantage of GNNs is their ability to propagate information across the network through iterative message passing. By aggregating information from neighboring firms, GNNs can encode systemic dependencies that are not directly observable through traditional econometric techniques. The general form of message passing in a GNN is given by:

where represents the feature embedding of node at layer , is the set of neighboring nodes, and are learnable weight matrices, and is a nonlinear activation function.

Another crucial aspect of financial markets is their temporal evolution, where the relationships between firms change over time. Traditional models struggle to incorporate this dynamic aspect effectively. GNNs, however, can be extended to temporal settings by introducing mechanisms such as recurrent memory updates and attention mechanisms, allowing them to capture time-varying dependencies in systemic risk propagation.

In summary, GNNs provide a flexible and expressive approach to modeling financial contagion by integrating network structure, firm-level risk indicators, and temporal dependencies. Their ability to learn adaptive representations from evolving financial networks makes them a powerful tool for understanding systemic risk and predicting the propagation of financial instability. The next section formalizes the mathematical framework used in this study to implement GNN-based systemic risk modeling.

## Graph Neural Network Formulation

Graph Neural Networks (GNNs) provide a powerful framework for learning representations of financial networks by propagating information across the network structure. In this study, we apply GNNs to the time-evolving financial network , where each firm is represented as a node, and edges encode the co-occurrence of financial bubbles between firms. The objective of the GNN is to learn a meaningful low-dimensional representation for each firm , which captures both its local financial attributes and the structural dependencies in the network.

At each time step , the financial network is represented as a graph, where each firm is associated with a feature vector containing firm-specific attributes, including delta CoVaR, degree centrality, and clustering coefficients. The goal is to transform these features into informative node embeddings that reflect the firm’s systemic importance. This is achieved through an iterative message-passing mechanism, where each node updates its representation based on the information received from its neighbors. The general update rule for a node at layer is given by:

where:

* represents the embedding of node at layer ,
* is the set of neighboring firms in the financial network,
* and are learnable weight matrices that determine how information is aggregated,
* is a nonlinear activation function, such as ReLU.

The iterative nature of this update rule allows each firm to incorporate higher-order dependencies in the network, beyond its immediate neighbors. After layers, each node’s embedding captures information from up to -hop neighbors, making it possible to model indirect systemic risk spillovers.

To ensure that the learned representations incorporate both financial attributes and network structure, we initialize the node embeddings using firm-specific features:

These initial embeddings are then iteratively refined through the message-passing mechanism, leading to a hierarchical representation of systemic risk propagation in the financial system.

In addition to standard message passing, we introduce an attention mechanism that allows each firm to weigh the influence of different neighbors dynamically. The attention coefficient between firms and is computed as:

where is a learnable parameter vector, and denotes vector concatenation. This mechanism allows the GNN to assign different importance weights to different firms, improving its ability to capture asymmetric risk propagation.

The final node embeddings serve as inputs to a predictive model that estimates each firm’s systemic risk contribution. By incorporating both firm-specific attributes and network topology, this approach provides a robust framework for systemic risk modeling in financial markets.

In the next section, we extend this formulation to a temporal setting by introducing a mechanism for capturing time-dependent dependencies in systemic risk evolution.

## Temporal Graph Attention Mechanism

Financial markets are inherently dynamic, with systemic risk and firm-level dependencies evolving over time. A static graph representation fails to capture these time-varying interactions, making it necessary to extend the graph neural network framework to a temporal setting. To achieve this, we introduce a *temporal graph attention mechanism* that allows the model to dynamically weigh historical observations when predicting future systemic risk.

### Time-Varying Node Representations

In a static GNN, the representation of each firm at time is based solely on the network structure and node features at that specific time step. However, systemic risk is influenced by past market conditions, requiring a formulation that incorporates historical information. We define the time-evolving representation of a firm as:

where:

* is the financial network at time ,
* represents the firm’s systemic risk attributes at time ,
* is the firm’s previous representation, capturing historical market conditions,
* is the learnable function mapping node features and past states to the new representation.

The inclusion of ensures that past systemic risk levels influence the current prediction, improving the model’s ability to track risk accumulation and dissipation over time.

### Temporal Attention Mechanism

Not all historical time steps contribute equally to systemic risk forecasting. Market shocks, regulatory changes, and external macroeconomic events create fluctuations where certain time periods are more relevant than others. To model this, we employ a *temporal attention mechanism* that assigns an adaptive weight to each past time step, allowing the model to focus on the most informative historical observations.

For each firm , the attention score assigned to past time step when predicting at time is given by:

Here:

* is the query vector at the current time step,
* is the key vector associated with past time step ,
* The softmax function ensures that attention scores sum to one.

Using these attention weights, the time-aware node embedding is computed as a weighted sum of past embeddings:

This formulation allows the model to dynamically adapt to changing market conditions by focusing on the most relevant historical risk patterns.

### Integration with Graph Convolutions

To integrate temporal attention with the GNN framework, we replace the static node embedding update with a time-aware update rule:

This ensures that each node embedding incorporates both:

1. Structural information from neighboring firms at the current time step,
2. Temporal dependencies from past observations, weighted by relevance.

### Impact on Systemic Risk Modeling

By incorporating temporal attention, the model is able to:

* Adaptively focus on critical historical periods that drive systemic risk,
* Capture long-term dependencies in financial contagion,
* Improve predictive performance by dynamically adjusting to market shifts.

In the next section, we describe how the learned representations are used to predict future systemic risk levels, integrating GNN-based embeddings into a predictive framework for delta CoVaR estimation.

## Prediction of Systemic Risk Using GNNs

After learning firm-level representations that incorporate both network structure and temporal dependencies, the next step is to use these embeddings to predict systemic risk. In this study, we employ a supervised learning framework where the goal is to estimate the future systemic risk contribution of each firm, measured by its delta Conditional Value at Risk (CoVaR). Given a firm’s historical risk profile and its position in the financial network, the GNN is trained to output a risk score that reflects its expected contribution to market instability.

### Predicting Future Systemic Risk

Let be the learned embedding of firm at time , obtained from the temporal graph attention model. The systemic risk prediction function is defined as:

where:

* is the predicted systemic risk score for firm at time ,
* is a fully connected neural network with learnable parameters ,
* is the final time-aware node embedding from the GNN.

The function consists of multiple layers of nonlinear transformations that map the latent representation of the firm to its risk level. Specifically, we employ a feedforward neural network with activation functions such that:

where:

* and are weight matrices,
* and are bias terms,
* is a nonlinear activation function, such as ReLU.

### Loss Function and Optimization

To train the model, we minimize the difference between the predicted systemic risk scores and the actual delta CoVaR values. The loss function is defined as the Mean Squared Error (MSE) between predictions and true values:

where:

* is the actual delta CoVaR value for firm at time ,
* is the predicted systemic risk score,
* is the number of firms in the dataset.

To optimize the parameters , we use the Adam optimizer, which adjusts learning rates adaptively:

where is the learning rate.

### Training Process and Data Splitting

The model is trained using historical data, where the dataset is divided into:

1. A **training set**, consisting of past systemic risk values for parameter optimization,
2. A **validation set**, used to fine-tune hyperparameters and prevent overfitting,
3. A **test set**, containing the most recent data for evaluating predictive accuracy.

During training, the model iteratively updates the parameters to minimize the loss function. Regularization techniques, such as dropout and weight decay, are applied to prevent overfitting.

### Evaluation Metrics

To assess the predictive performance of the GNN-based systemic risk model, we employ the following evaluation metrics:

* **Mean Absolute Error (MAE)**: Measures the absolute deviation between predicted and actual risk values:
* **Root Mean Squared Error (RMSE)**: Evaluates prediction accuracy while penalizing large errors:
* **R-Squared () Score**: Assesses the proportion of variance in systemic risk explained by the model:
* **Pearson Correlation Coefficient**: Measures the linear correlation between predicted and actual delta CoVaR values:

### Impact of GNN-Based Risk Prediction

By leveraging GNN-based representations, this approach improves systemic risk forecasting by:

* Incorporating both firm-specific attributes and network effects,
* Learning dynamic dependencies between firms over time,
* Enhancing predictive accuracy compared to traditional econometric models.

In the next section, we discuss the advantages of the proposed methodology and compare it with traditional approaches to systemic risk assessment.

## Benefits of GNN-Based Systemic Risk Modeling

The application of Graph Neural Networks (GNNs) in systemic risk modeling introduces several advantages over traditional econometric and statistical approaches. Unlike classical methods that rely on predefined assumptions about financial relationships, GNNs provide a data-driven framework that dynamically learns complex, nonlinear dependencies among firms. This section highlights the key benefits of the proposed methodology in systemic risk assessment.

### Capturing Nonlinear Dependencies

Traditional models, such as Vector Autoregression (VAR) and CoVaR, assume linear dependencies between financial entities, limiting their ability to model intricate contagion effects. In contrast, GNNs utilize multiple layers of transformation functions to capture highly nonlinear relationships between firms, allowing for a more realistic representation of systemic risk propagation. The ability to encode these nonlinear dependencies enhances predictive accuracy, particularly during periods of market stress when financial relationships become more volatile.

### Learning Dynamic Market Patterns

Financial networks are inherently dynamic, with firm-level interactions evolving over time due to macroeconomic conditions, regulatory changes, and investor behavior. Traditional risk models often struggle to account for these temporal shifts, requiring frequent recalibration. By incorporating a temporal graph attention mechanism, the proposed GNN framework automatically adjusts to evolving market conditions, ensuring that risk predictions remain accurate even as financial dependencies change. This adaptability is particularly important for detecting emerging systemic risks before they materialize into full-scale financial crises.

### Integration of Structural and Financial Information

One of the fundamental limitations of classical risk models is their reliance on either purely network-based metrics (such as degree centrality) or purely financial indicators (such as delta CoVaR) without fully integrating both. The GNN approach overcomes this limitation by learning representations that jointly encode:

where captures structural dependencies, represents firm-specific financial indicators, and introduces temporal dependencies. This joint modeling ensures that risk predictions are based on a holistic understanding of both firm-level vulnerabilities and market-wide contagion mechanisms.

### Enhanced Predictive Performance

By leveraging network topology, firm-specific financial attributes, and historical risk patterns, GNN-based models achieve superior predictive performance compared to traditional approaches. The effectiveness of the model is evaluated using various performance metrics, including Mean Squared Error (MSE), Root Mean Squared Error (RMSE), and Pearson correlation. Empirical results demonstrate that the GNN approach consistently outperforms benchmark models in forecasting future systemic risk.

### Improved Interpretability of Systemic Risk

A key concern in financial modeling is the interpretability of predictions. Unlike black-box deep learning models, GNNs provide interpretable insights into systemic risk propagation by:

* Identifying firms that serve as major *contagion hubs* based on their learned centrality in the graph.
* Highlighting key financial and network-based risk factors that contribute most to systemic instability.
* Enabling visualization of risk transmission pathways over time, offering regulators an intuitive understanding of how crises unfold.

This level of interpretability facilitates effective policy interventions, allowing financial authorities to implement targeted risk-mitigation strategies.

### Robustness to Market Shocks

A major limitation of traditional models is their instability during financial crises, where abrupt structural changes can invalidate prior statistical assumptions. The GNN framework, by continuously updating its representations based on new market conditions, ensures robustness to unexpected shocks. This adaptability makes the model particularly useful for real-time systemic risk monitoring and early-warning systems.

### Comparative Analysis with Traditional Methods

Traditional systemic risk models, such as Vector Autoregression (VAR) and Conditional Value at Risk (CoVaR), have long been used to assess financial stability. These models rely on predefined statistical relationships and econometric techniques to estimate risk propagation. However, they often struggle with capturing nonlinear dependencies, adapting to evolving market conditions, and incorporating the complex interconnections inherent in financial systems .

In contrast, Graph Neural Networks (GNNs) offer a more flexible and data-driven approach by leveraging graph structures to model systemic risk dynamically. Unlike econometric models, which require explicit assumptions about dependencies, GNNs learn the relationships between firms directly from market data, improving predictive accuracy and adaptability .

To highlight the advantages of the GNN-based approach, we compare it with traditional systemic risk models across several dimensions, as summarized in Table [1](#tab:comparison).

Comparison of Traditional Systemic Risk Models vs. GNN-Based Approach

| **Aspect** | **Traditional Systemic Risk Models** | **GNN-Based Approach** |
| --- | --- | --- |
| **Methodology** | Rely on statistical measures and econometric models to assess systemic risk . | Utilize deep learning techniques to capture complex patterns in financial networks . |
| **Data Requirements** | Depend on aggregated financial data and predefined risk metrics . | Can process raw, high-dimensional data, including transaction networks and market microstructure data . |
| **Scalability** | Struggle with large-scale financial networks due to computational complexity . | Efficiently handle large graphs, enabling real-time systemic risk monitoring . |
| **Adaptability** | Typically static, requiring manual updates to incorporate new data . | Dynamic, capable of learning evolving financial relationships automatically . |
| **Predictive Performance** | Limited in capturing nonlinear dependencies and complex interdependencies . | Excels at modeling nonlinearities, improving systemic risk forecasting . |

The table illustrates that while traditional models provide valuable insights, they face limitations in handling the complexities of modern financial networks. Studies have shown that financial contagion often follows nonlinear paths, with abrupt transitions that econometric models fail to capture . In contrast, GNN-based approaches offer a more expressive representation of financial markets by dynamically adjusting to new information and uncovering hidden patterns of systemic instability .

Furthermore, GNNs facilitate the integration of network topology with firm-specific risk measures, allowing for a holistic analysis of systemic risk propagation. This is particularly relevant in stress-testing scenarios, where traditional models rely on predefined stress scenarios, whereas GNNs can simulate risk transmission dynamically .

By leveraging machine learning techniques, the GNN-based approach not only enhances predictive accuracy but also improves interpretability. Unlike black-box deep learning models, the incorporation of attention mechanisms and explainable AI methods in GNNs enables regulators to identify key risk drivers in financial networks . This interpretability is crucial for regulatory decision-making and the development of targeted policy interventions.

Overall, the findings from recent research suggest that GNNs represent a significant advancement in systemic risk modeling. By addressing the shortcomings of traditional methods and providing a more adaptive, data-driven framework, they offer a promising avenue for improving financial stability assessments and early-warning systems. .

### Implications for Financial Regulation and Policy

The insights obtained from GNN-based systemic risk modeling have several implications for financial stability monitoring:

* **Early Warning Systems:** The ability to track systemic risk propagation in real-time enables regulators to identify potential crises before they escalate.
* **Targeted Risk Mitigation:** Identifying firms that act as systemic risk hubs allows policymakers to implement proactive interventions, such as capital buffer requirements.
* **Stress Testing Enhancement:** Traditional stress tests rely on predefined scenarios, whereas GNN-based models dynamically adjust to changing market conditions, improving stress test reliability.

### Conclusion

The integration of GNNs into systemic risk modeling represents a significant advancement over traditional econometric approaches. By combining firm-specific financial data, network effects, and temporal dependencies, the proposed methodology provides a more comprehensive, accurate, and interpretable framework for assessing financial contagion. This approach not only improves systemic risk forecasting but also enhances the ability of policymakers to detect and mitigate potential financial crises in a timely manner.