MA2110 - Probabilty
Assignment 2 - Random Variables I
Hints and Answers for some problems

2. $P_N(k) = \begin{cases} \frac{1}{3}(\frac{2}{3})^{k-1} & k = 1,2,3... \\ 0 & otherwise \end{cases}$

3. (a) $\frac{2}{n(n+1)}$
(b) 1

5. $P_X(k) = \begin{cases} \binom{10}{k-10}(\frac{1}{4})^{k-10}(\frac{3}{4})^{20-k} & k = 10, 11, 12.....20 \\ 0 & otherwise \end{cases}$

7. (a) $c = \frac{1}{9}$
(b) $P(1 < X < 2) = \frac{7}{27}$
(c) $F(x) = \begin{cases} 0 & x < 0 \\ cx^2 & 0 \leqslant x < 3 \\ 1 & x \geqslant 3 \end{cases}$
(d) $\frac{7}{27}$

8. Use $\sum f(x) = 1$ to solve for c.If the summation is diveregent, then there exists no such c
(a) not a PMF
(b) $\sum_{i=1}^{\infty} \frac{1}{x}$ is divering. Hence, not a PMF

11. (1 + s) / (7 - 5s)

12. 4

13. 0.122

14. https://www.math.ucdavis.edu/g̃ravner/MAT135A/resources/chpr.pdf

15. If the board has radius a,
$f_Y(y) = \frac{2}{\pi a^2}(a^2 - y^2)^{\frac{1}{2}}$,
$F_Y(y) = \frac{1}{2} + \frac{1}{\pi} \arcsin \frac{y}{a} + \frac{y}{\pi a^2}(a^2 - y^2)^{\frac{1}{2}}$, $y \leqslant a$;
$f_R(r) = \frac{2r}{a^2}$, $F_R(r) = \frac{r^2}{a^2}$, $0 \leqslant r \leqslant a$, $E(R) = \frac{2}{3}a$

16. $p_X(x) = \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \cdots \times \frac{x-1}{x} \frac{1}{x+1}$, $1 \leqslant x \leqslant 9$,
$p_X(10) = \frac{1}{10}$; $X = 1 + \frac{1}{2} \cdots + \frac{1}{9} + \frac{1}{10}$,
$p_Y(y) = y(y+1)^{-1}$, $y \geqslant 1$; $E(Y) = \infty$

17. After r readings a character is erroneous with probability $p(1-\delta)^r$; there are fn characters. So the number of errors X is binomial $B(fn, p(1-\delta)^r)$
$P(X = 0) = 1 - p(1-\delta)^{rfn}$, which exceeds$\frac{1}{2}$ for the given values if $(1 - 2^{-8-r})^{2^{17}} > \frac{1}{2}$

18. $p^k q^{n-k} \binom{n-1}{k-1}$

19. $P(X = x) = p\left\{(q+r)^{x-1} - r^{x-1}\right\} + q\left\{(p+r)^{x-1} - r^{x-1}\right\}$, $x \leqslant 2$
$P(Y = y) = \binom{y-1}{j-1} p^j \sum_{i=k}^{y-j} \binom{y-j}{i} q^i r^{y-j-i} + \binom{y-1}{k-1} q^k \sum_{i=j}^{y-k} \binom{y-k}{i} p^i r^{y-k-i}$, $y \geqslant j = k$