Photonics: Laser -II

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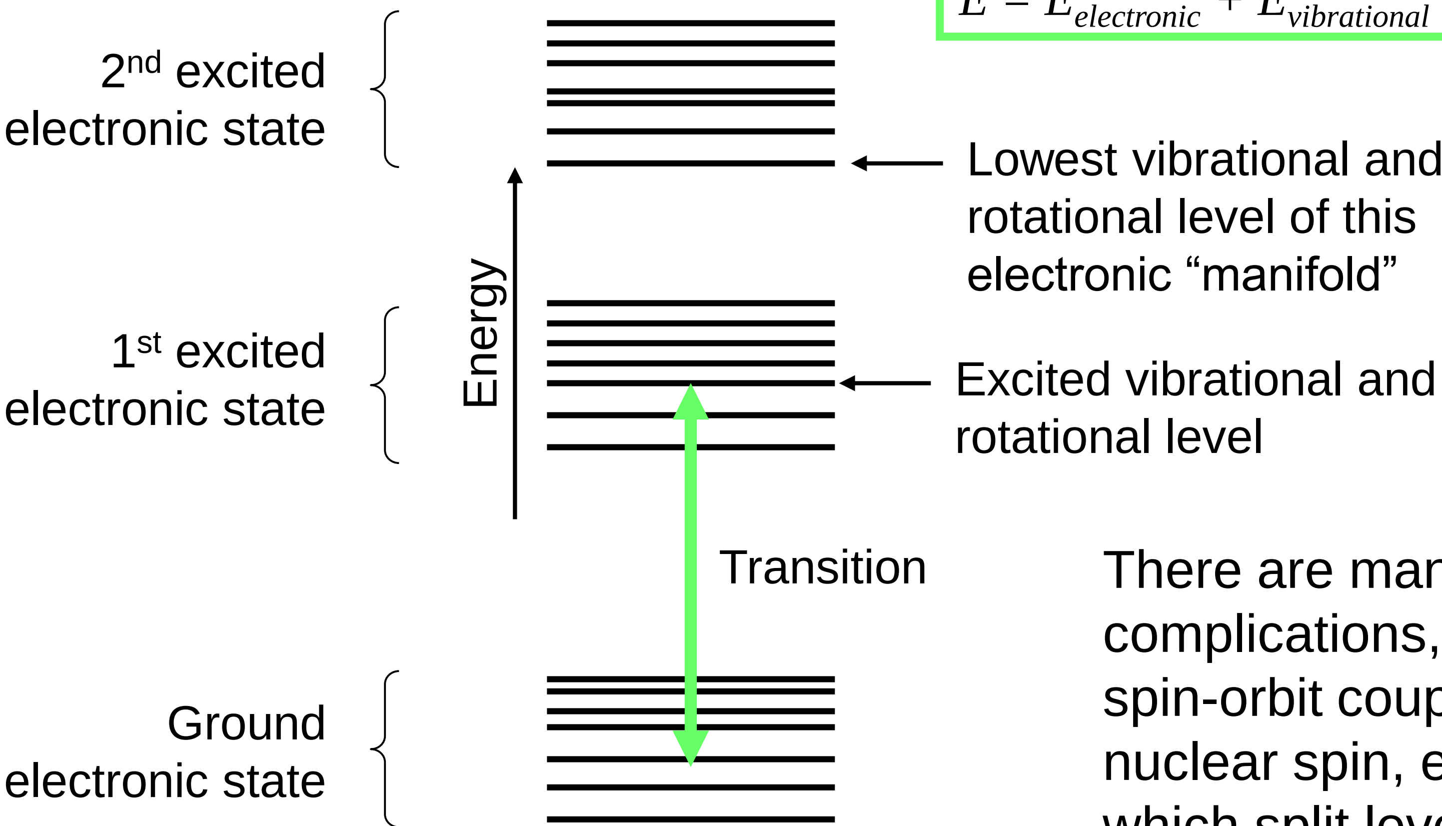
A very little mathematics of what we had learned so far!
$\frac{A}{B} = \frac{8\pi h\nu^3}{c^2}$

Molecules have many energy levels.

- A typical molecule’s energy levels:

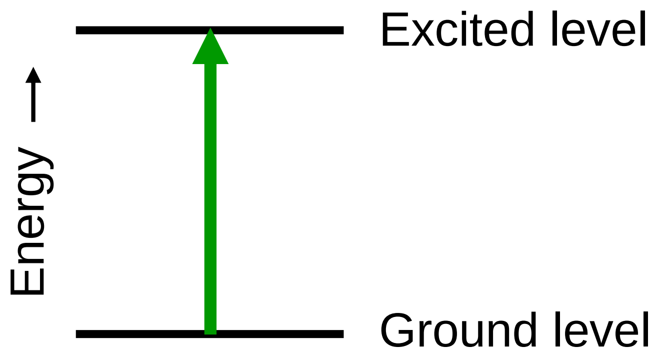
$$\boxed{E = E_{electronic} + E_{vibrational} + E_{rotational}}$$

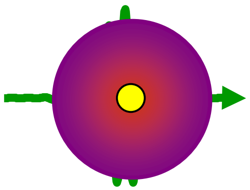


There are many other complications, such as spin-orbit coupling, nuclear spin, etc., which split levels.

As a result, molecules generally have very complex spectra.

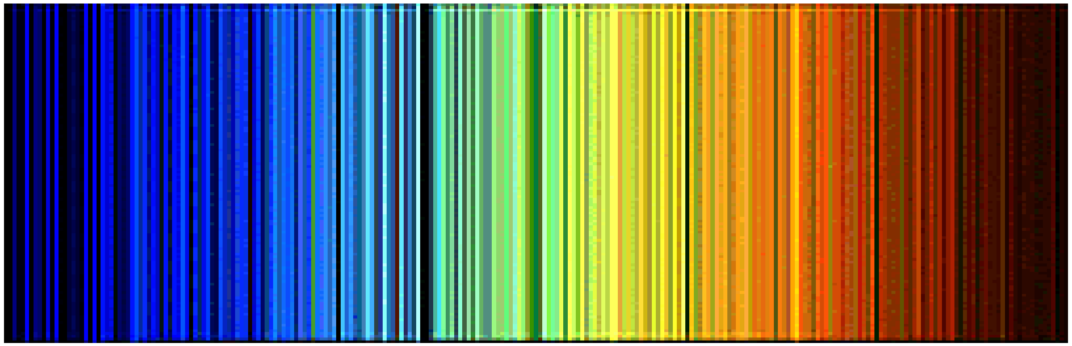
Atoms and molecules can also absorb photons, making a transition from a lower level to a more excited one.



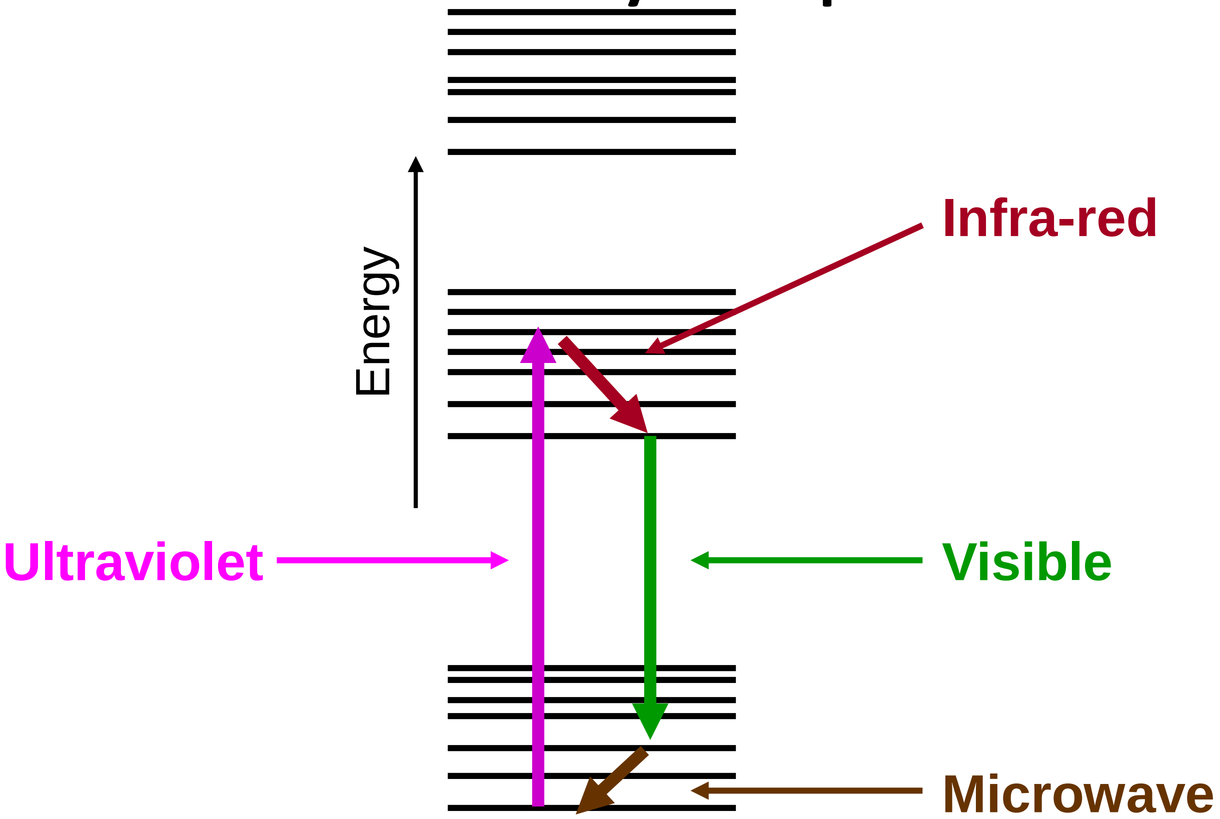


This is, of course, absorption.

Absorption lines in an otherwise continuous light spectrum due to a cold atomic gas in front of a hot source.

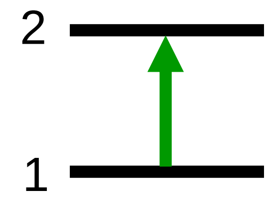


Decay from an excited state can occur in many steps.



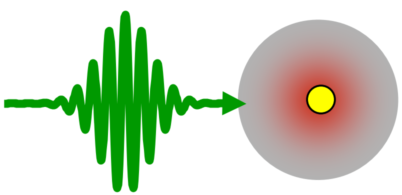
The light that's eventually re-emitted after absorption often occurs at other colors.

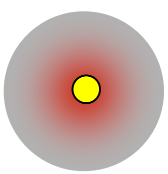
Calculating the gain: Einstein A and B coefficients



Einstein considered the various transition rates between molecular states (say, 1 and 2) involving light of irradiance, I:

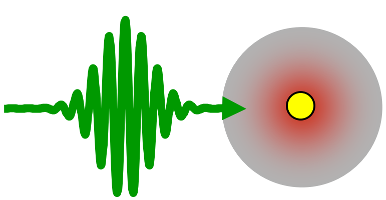
Stimulated Absorption rate $= B\,N_1\,I$ $\longrightarrow$





$\longleftarrow$ Spontaneous emission rate $= A\,N_2$

Stimulated emission rate $= B\,N_2\,I$ $\longrightarrow$

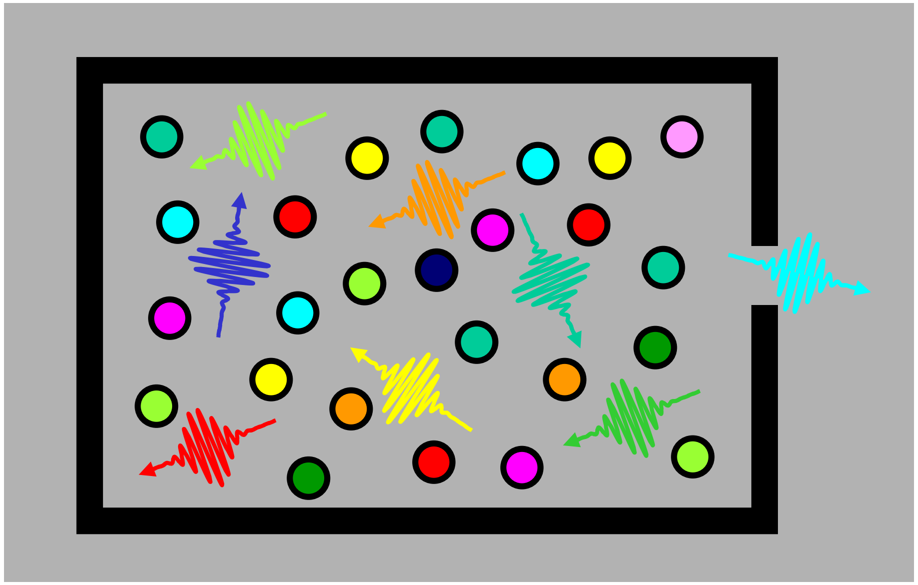


where N_i is the number density of molecules in the i^{th} state, and I is the irradiance.

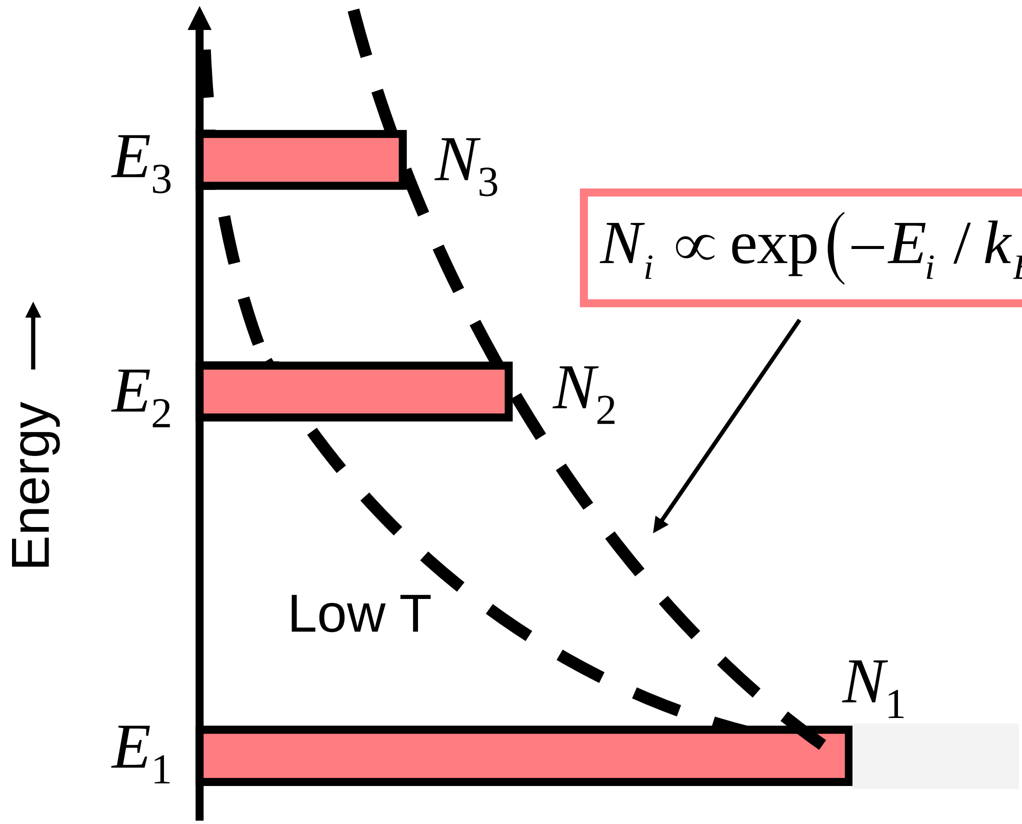
Blackbody Radiation

Blackbody radiation results from a combination of **spontaneous emission, absorption, and stimulated emission** occurring in a medium at a given temperature.

It assumes that the box is filled with many different molecules that together have transitions (absorptions) at every wavelength.



In what energy levels do molecules reside? Boltzmann Population Factors



Population density ⟶
(Number of molecules per unit volume)

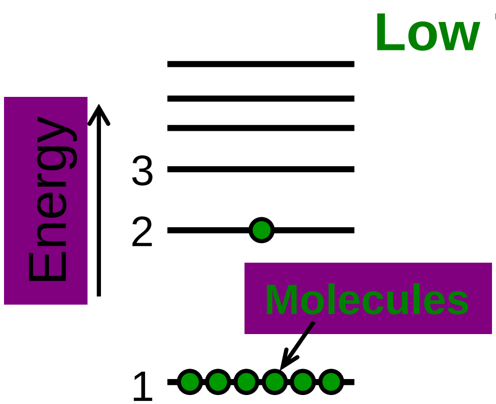
N_i is the number density (also known as the **population density**) of molecules in state i (i.e., the number of molecules per cm^3).

T is the temperature, and k_B is Boltzmann's constant = 1.3806503 × 10^{-23} J/^{o}K

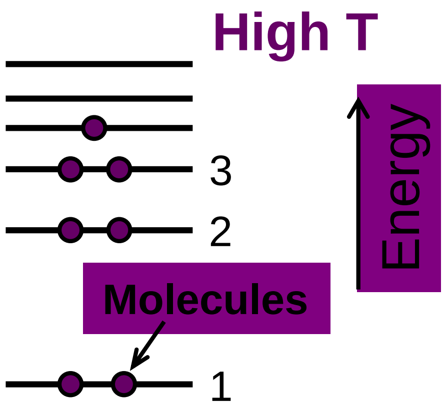
The Maxwell-Boltzmann Distribution

In the absence of collisions, (low T) molecules tend to remain in the lowest energy state available.

Collisions can knock a molecule into a higher-energy state. The higher the temperature, the more this happens.



$$\frac{N_2}{N_1} = \frac{\exp\left[-E_2 / k_B T\right]}{\exp\left[-E_1 / k_B T\right]} = \exp\left[-(E_2 - E_1) / k_B T\right]$$



The ratio of the population densities of two states is:

$$\boxed{N_2 / N_1 = \exp(-\Delta E / k_B T), \text{ where } \Delta E = E_2 - E_1 = h\nu}$$

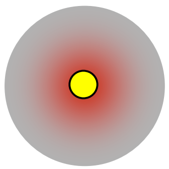
frequency of a photon for an $E_2 - E_1$ transition

As a result, higher-energy states are always less populated than the ground state, and absorption is stronger than stimulated emission.

Einstein A and B Coefficients

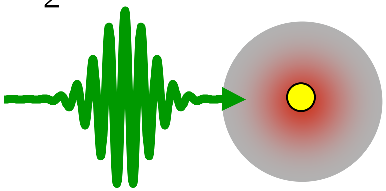
In 1916, Einstein considered the various transition rates between molecular states (say, 1 and 2) involving light of intensity, I:

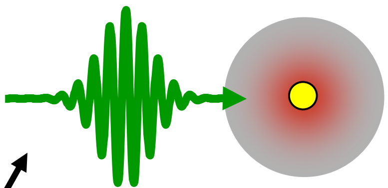
at frequency ν



Spontaneous emission rate $= A\,N_2$

Absorption rate $= B_{12}\,N_1\,I$





Stimulated emission rate $= B_{21}\,N_2\,I$

In equilibrium, the rate of upward transitions equals the rate of downward transitions:

$$A\,N_2 + B_{21}\,N_2\,I = \text{Down} = \text{Up} = B_{12}\,N_1\,I$$

Dividing by $N_1\,(A + B_{21}I\,)$ yields N_2/N_1:

Recalling the Maxwell-Boltzmann Distribution

$$(B_{12}\,I\,)\,/\,(A + B_{21}\,I\,) = N_2\,/\,N_1 = \exp(-h\nu/k_BT)$$

Now solve for the intensity in: $(B_{12}\, I) / (A + B_{21}\, I) = \exp(-h\nu / k_B T)$

Multiply by $(A + B_{21}\, I) \exp(h\nu / k_B T)$: $B_{12}\, I \exp(h\nu / k_B T) = A + B_{21}\, I$

Solve for I: $I = A / [B_{12} \exp(h\nu / k_B T) - B_{21}]$

Dividing numerator and denominator by B_{21}:

$$I = (A/B_{21}) / [(B_{12} / B_{21}) \exp(h\nu / k_B T) - 1]$$

Now, when $T \to \infty$, I should also. As $T \to \infty$, $\exp(h\nu / k_B T) \to 1$.

So: $(B_{12} / B_{21}) - 1 = 0$

And: $B_{12} = B_{21} \equiv B$ $\leftarrow$ **Coeff up = coeff down!**

And: $I = (A/B) / [\exp(h\nu / k_B T) - 1]$

$$I = (A/B) / [\exp(h\nu/k_B T) - 1]$$

Compare the above expression with Planck's expression for I_ν:

$$\boxed{I_\nu = \frac{8\pi h\nu^3 / c^2}{\exp(h\nu / k_B T) - 1}}$$

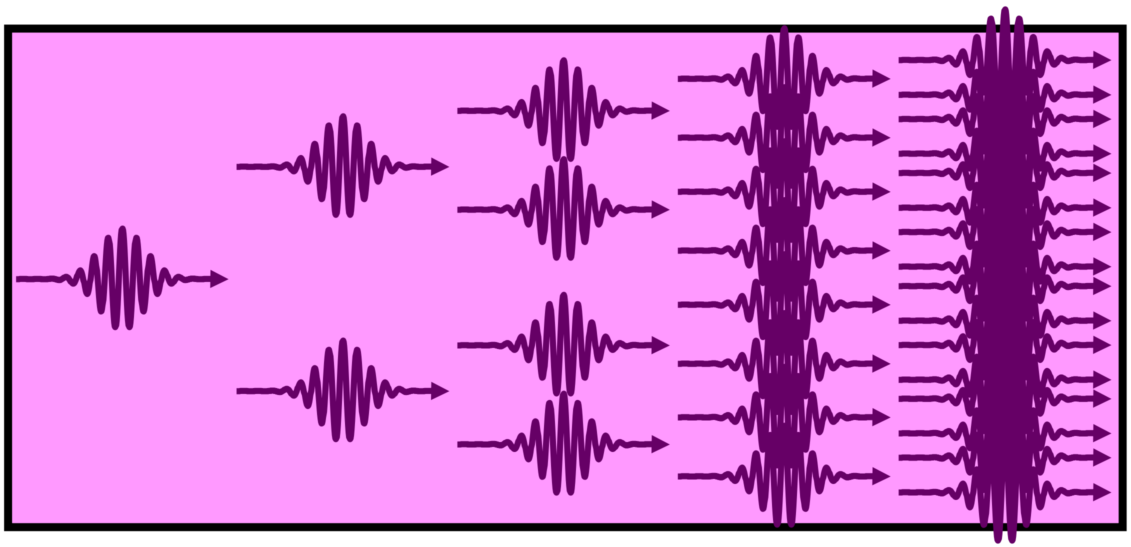
$$\boxed{\frac{A}{B} = \frac{8\pi h\nu^3}{c^2}}$$

The X-ray lasers are difficult to make.

Stimulated emission leads to a chain reaction and laser emission.

If many molecules in a medium are excited, one photon can become many.

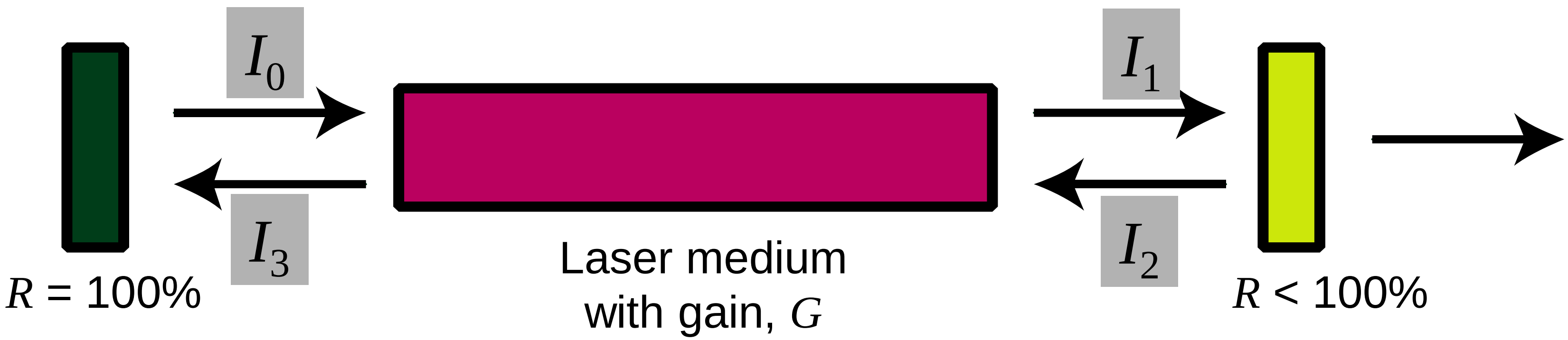
Excited medium



This is the essence of the laser. The factor by which an input beam is amplified by a medium is called the **gain** and is represented by G.

Gain

A laser is a medium that stores energy, surrounded by two mirrors. A partially reflecting output mirror lets some light out.



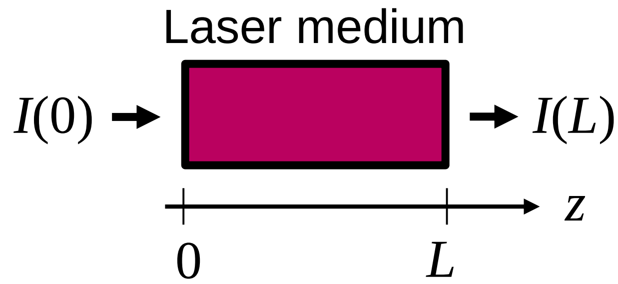
A laser will lase if the beam increases in intensity during a round trip: that is, if $I_3 \geq I_0$

Usually, additional **losses** in intensity occur, such as absorption, scattering, and reflections. In general, the laser will lase if, in a round trip:

Gain > Loss

This is called achieving **Threshold**.

Laser gain



Neglecting spontaneous emission:

$$\frac{dI}{dt} = c\frac{dI}{dz} \propto BN_2I - BN_1I$$

[Stimulated emission minus absorption]

$$\propto B\left[N_2 - N_1\right]I$$

The solution is:

Proportionality constant is the **absorption/gain cross-section, σ**

$$I(z) = I(0)\exp\left\{\sigma\left[N_2 - N_1\right]z\right\}$$

There can be exponential gain or loss in irradiance.
Normally, $N_2 < N_1$, and there is loss (absorption).
But if $N_2 > N_1$, there's gain, and we define the gain, G:

$$\boxed{G \equiv \exp\left\{\sigma\left[N_2 - N_1\right]L\right\}}$$

If $N_2 > N_1$: $\quad g \equiv \left[N_2 - N_1\right]\sigma$

If $N_2 < N_1$: $\quad \alpha \equiv \left[N_1 - N_2\right]\sigma$

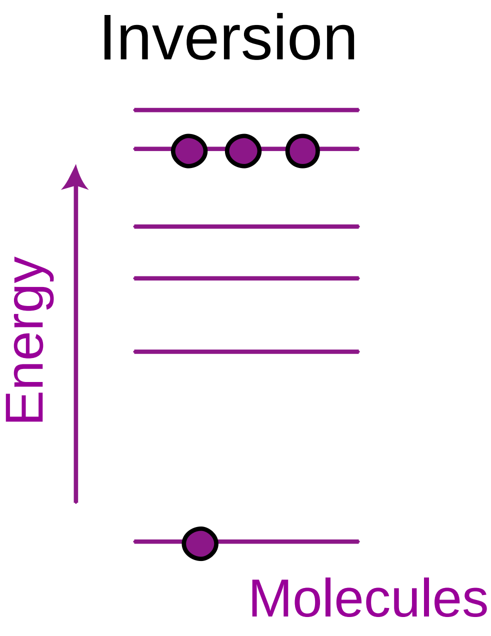
Inversion

In order to achieve $G > 1$, stimulated emission must exceed absorption:

$$B\,N_2\,I \;>\; B\,N_1\,I$$

Or, equivalently,

$$\boxed{N_2 \;>\; N_1}$$

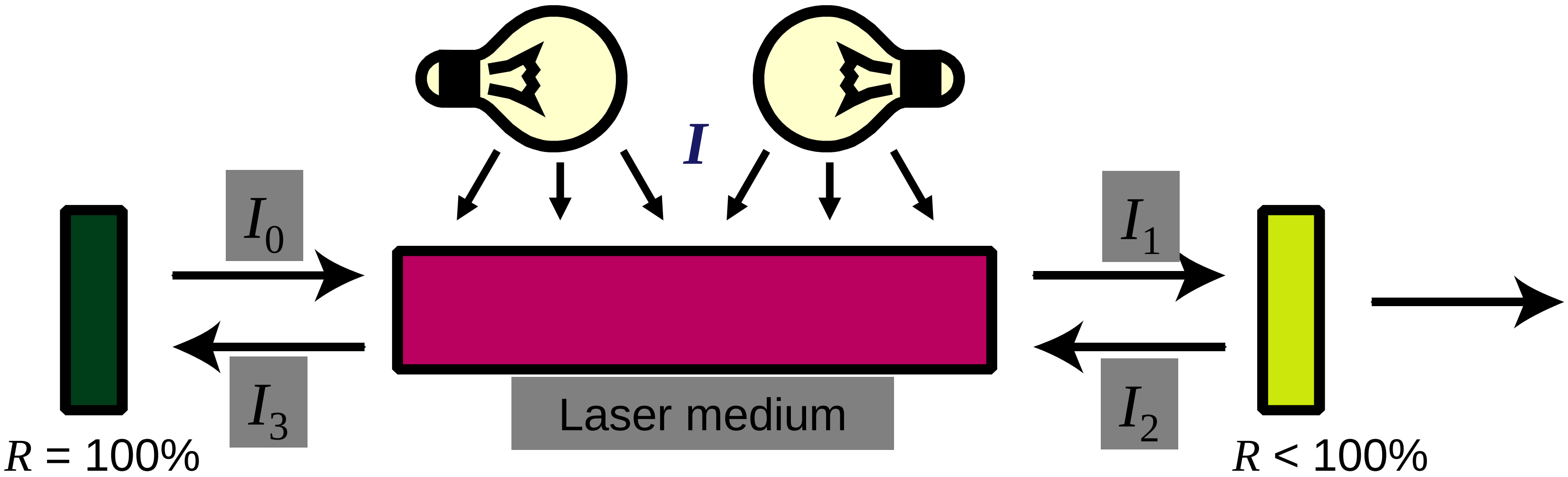


This condition is called **inversion**.
It does not occur naturally. It is inherently a non-equilibrium state.

In order to achieve inversion, we must hit the laser medium very hard in some way and choose our medium correctly.

Achieving inversion: Pumping the laser medium

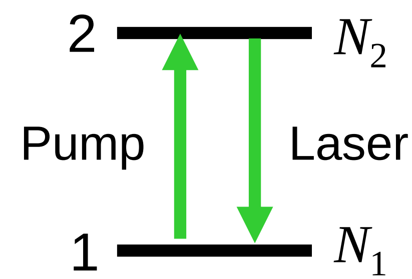
Now let I be the intensity of (flash lamp) light used to pump energy into the laser medium:



Will this intensity be sufficient to achieve inversion, $N_2 > N_1$?

It'll depend on the laser medium's energy level system.

Rate equations for a two-level system



Rate equations for the densities of the two states:

Absorption Stimulated emission Spontaneous emission

$$\frac{dN_2}{dt} = BI(N_1 - N_2) - AN_2$$

Pump intensity

$$\frac{dN_1}{dt} = BI(N_2 - N_1) + AN_2$$

If the total number of molecules is N:

$$N \equiv N_1 + N_2$$

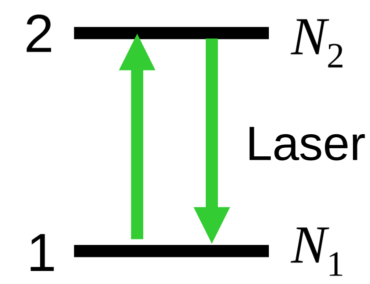
$$\Delta N \equiv N_1 - N_2$$

$$\Rightarrow \quad \frac{d\Delta N}{dt} = -2BI\Delta N + 2AN_2$$

$$2N_2 = (N_1 + N_2) - (N_1 - N_2) = N - \Delta N$$

$$\Rightarrow \quad \frac{d\Delta N}{dt} = -2BI\Delta N + AN - A\Delta N$$

Why inversion is impossible in a two-level system



$$\frac{d\Delta N}{dt} = -2BI\Delta N + AN - A\Delta N$$

In steady-state: $0 = -2BI\Delta N + AN - A\Delta N$

$$\Rightarrow (A + 2BI)\Delta N = AN$$

$$\Rightarrow \Delta N = AN/(A + 2BI)$$

$$\Rightarrow \Delta N = N/(1 + 2BI/A)$$

$$\Rightarrow \boxed{\Delta N = \frac{N}{1 + 2I/I_{sat}}}$$

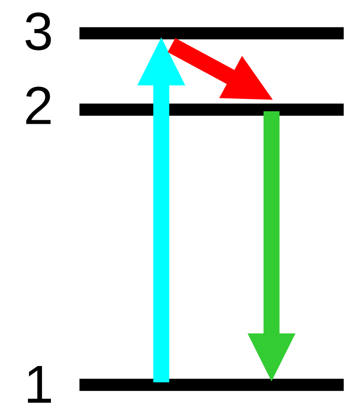
where: $I_{sat} = A/B$

I_{sat} is the **saturation intensity**.

ΔN is always positive, no matter how high I is!

It's impossible to achieve an inversion in a two-level system!

Rate equations for a three-level system



Assume we pump to a state 3 that rapidly decays to level 2. No pump stimulated emission!

Spontaneous emission

$$\frac{dN_2}{dt} = BIN_1 - AN_2$$

Absorption

$$\frac{dN_1}{dt} = -BIN_1 + AN_2$$

The total number of molecules is N:

$$N \equiv N_1 + N_2$$

Level 3 decays fast and so is zero.

$$\Delta N \equiv N_1 - N_2$$

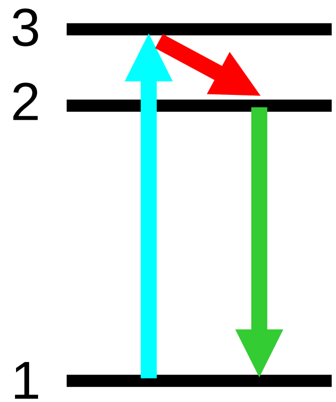
$$\frac{d\Delta N}{dt} = -2BIN_1 + 2AN_2$$

$$2N_2 = N - \Delta N$$

$$2N_1 = N + \Delta N$$

$$\Rightarrow \frac{d\Delta N}{dt} = -BIN - BI\Delta N + AN - A\Delta N$$

Why inversion is possible in a three-level system



$$\frac{d\Delta N}{dt} = -BIN - BI\Delta N + AN - A\Delta N$$

In steady-state: $0 = -BIN - BI\Delta N + AN - A\Delta N$

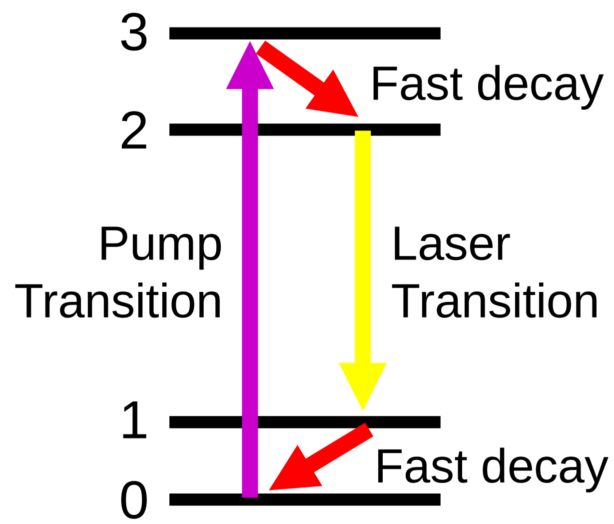
$$\Rightarrow (A + BI)\Delta N = (A - BI)N$$

$$\Rightarrow \Delta N = N(A - BI)/(A + BI)$$

$$\Rightarrow \boxed{\Delta N = N\,\frac{1 - I/I_{sat}}{1 + I/I_{sat}}}$$

Now if $I > I_{sat}$, ΔN is negative!

Rate equations for a four-level system



Now assume the lower laser level 1 also rapidly decays to a ground level 0. So $N_1 \approx 0$! And $\Delta N \approx -N_2$

As before: $$\frac{dN_2}{dt} = BIN_0 - AN_2$$

$$\frac{dN_2}{dt} = BI(N - N_2) - AN_2$$

The total number of molecules is N:

$$N \equiv N_0 + N_2$$

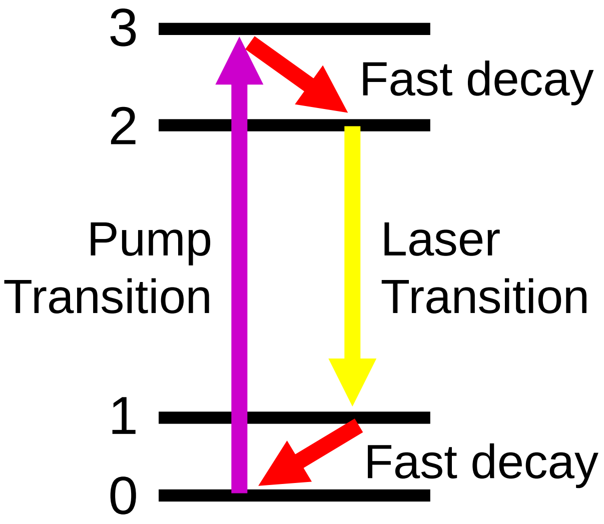
$$N_0 = N - N_2$$

Because $\Delta N \approx -N_2$

$$-\frac{d\Delta N}{dt} = BIN + BI\Delta N + A\Delta N$$

At steady state: $$0 = BIN + BI\Delta N + A\Delta N$$

Why inversion is easy in a four-level system (cont'd)



$$0 = BIN + BI\,\Delta N + A\Delta N$$

$$\Rightarrow \; (A + BI)\Delta N = -BIN$$

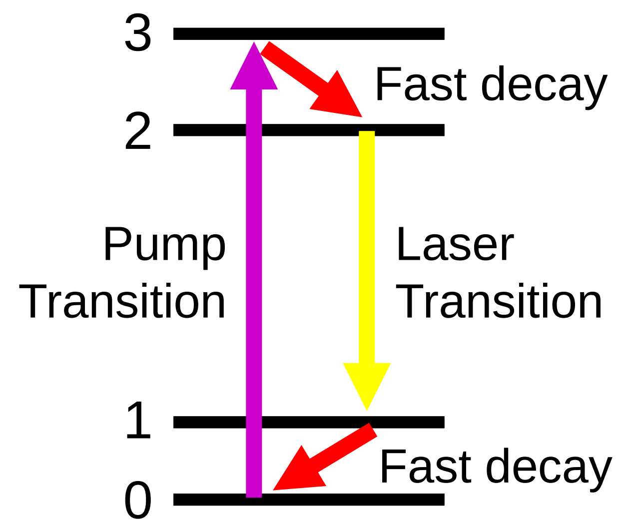
$$\Rightarrow \; \Delta N = -BIN/(A + BI)$$

$$\Rightarrow \; \Delta N = -(BIN/A)/(1 + BI/A)$$

$$\Rightarrow \; \boxed{\Delta N = -N \frac{I/I_{sat}}{1 + I/I_{sat}}}$$

Now, ΔN is negative—always!

What about the saturation intensity?



$$I_{sat} = A / B$$

A is the excited-state relaxation rate: $1/\tau$

B is the absorption cross-section, σ, divided by the energy per photon, $\hbar\omega$: $\sigma / \hbar\omega$

Both σ and τ depend on the molecule, the frequency, and the various states involved.

$$\boxed{I_{sat} = \frac{\hbar\omega}{\sigma\tau}}$$

$\hbar\omega$ ~10^{-19} J for visible/near IR light

τ ~10^{-12} to 10^{-8} s for most molecules
10^{-9} to 10^{-3} s for laser molecules

σ ~10^{-20} to 10^{-16} cm^2 for molecules (on resonance)

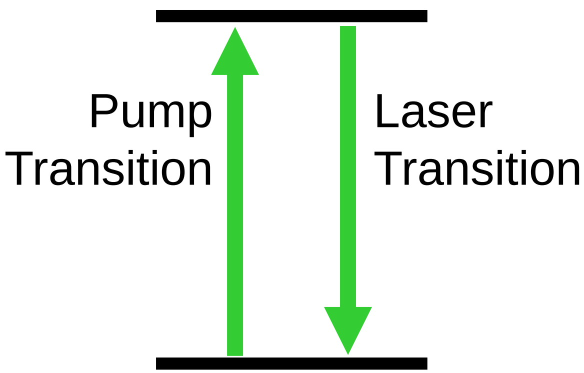
1 to 10^{13} W/cm^2

The saturation intensity plays a key role in laser theory.

Two-, three-, and four-level systems

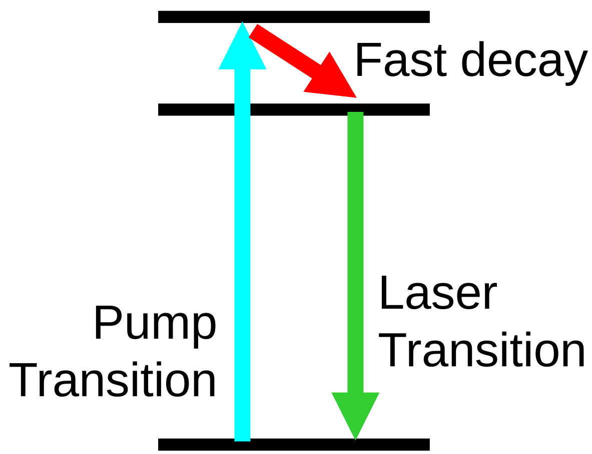
It took laser physicists a while to realize that four-level systems are best.

Two-level system



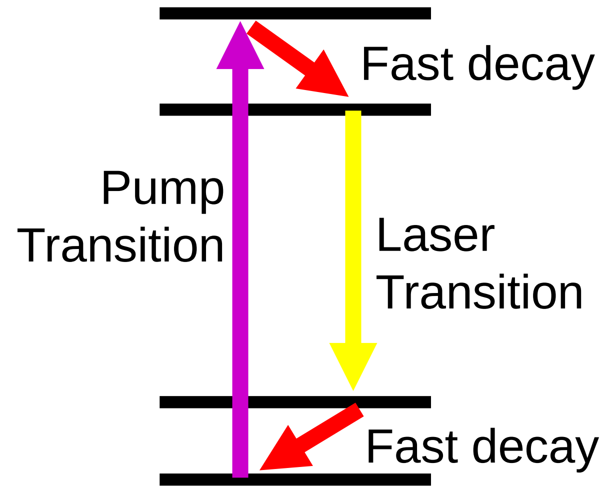
At best, you get equal populations. No lasing.

Three-level system



If you hit it hard, you get lasing.

Four-level system



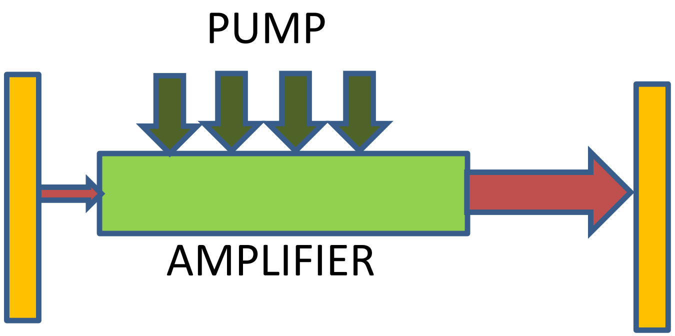
Lasing is easy!

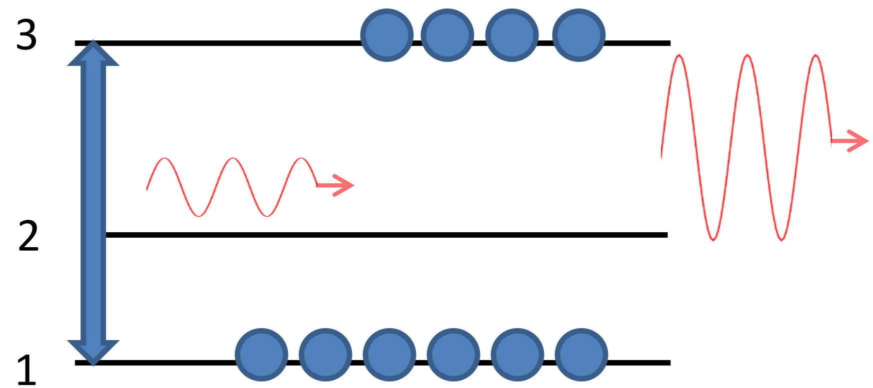
Types of lasers

- Gas
- Liquid
- Solid
- Semiconductor
- Excimer
- Gas dynamic
- E-beam
- Free electron
- Fiber
- Waveguide

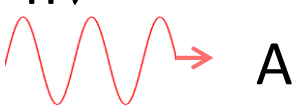
Laser Classification is according to pumping mechanism

Optical Pump



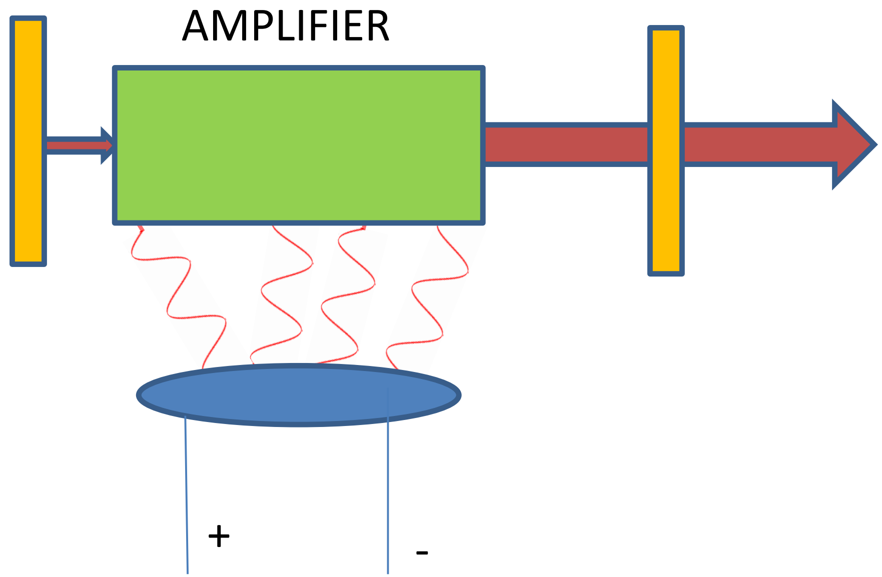


$h\nu$



$$A + h\nu \rightarrow A^*$$

Eg. Ruby, YAG, Glass, Fiber, Dye


AMPLIFIER
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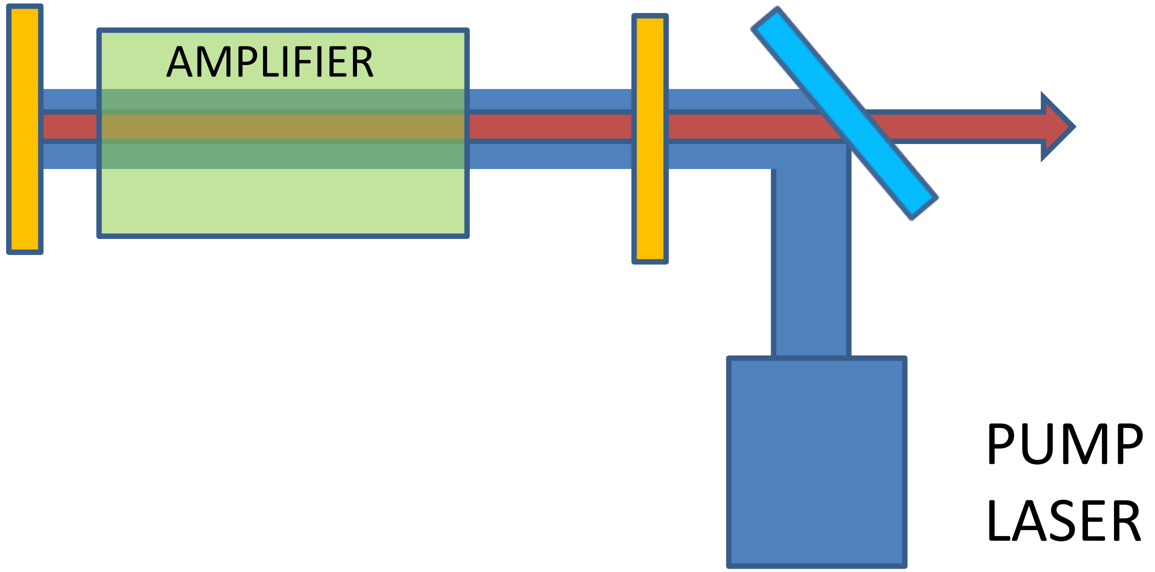
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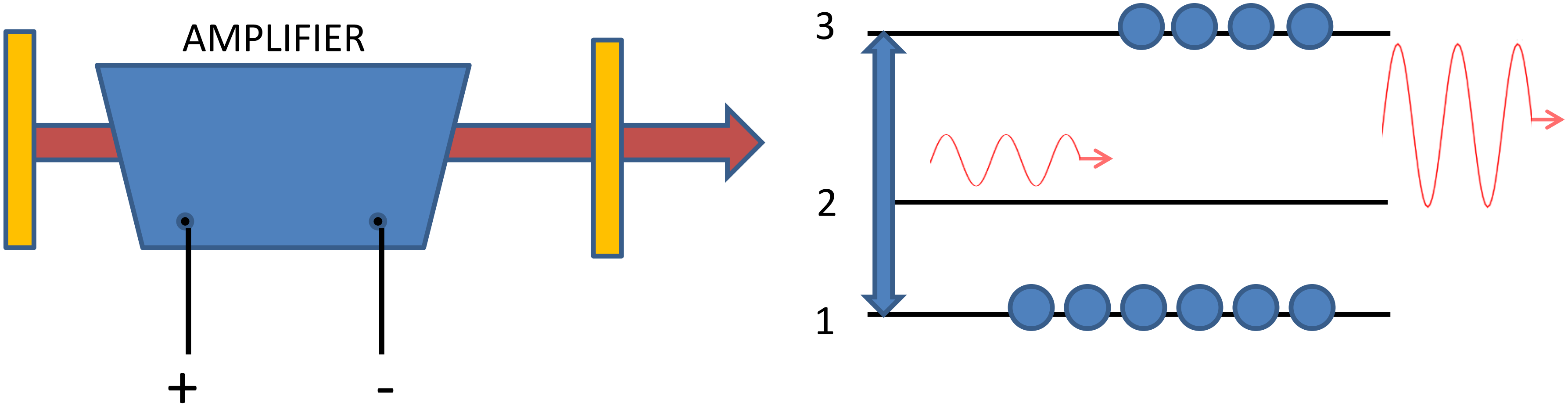
PUMP
LASER

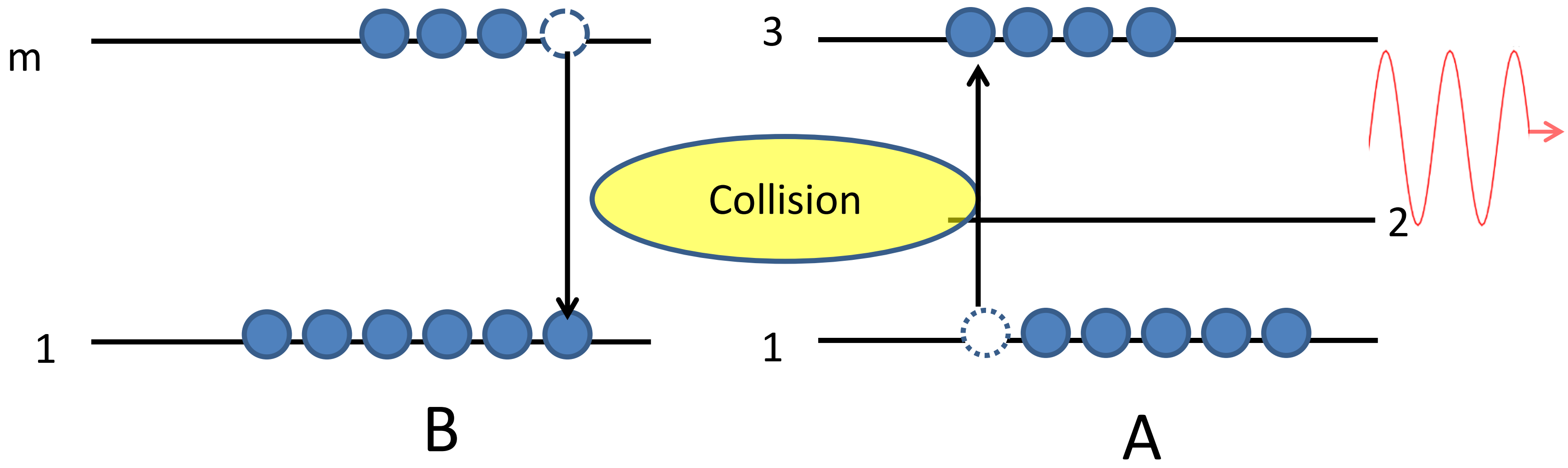
Electron-collision Pump



$$A + e\,(\varepsilon_2) \rightarrow A^* + e(\varepsilon_1)$$

Eg. Argon, Krypton, Xenon, Nitrogen, Copper

Atom-Collision Pump

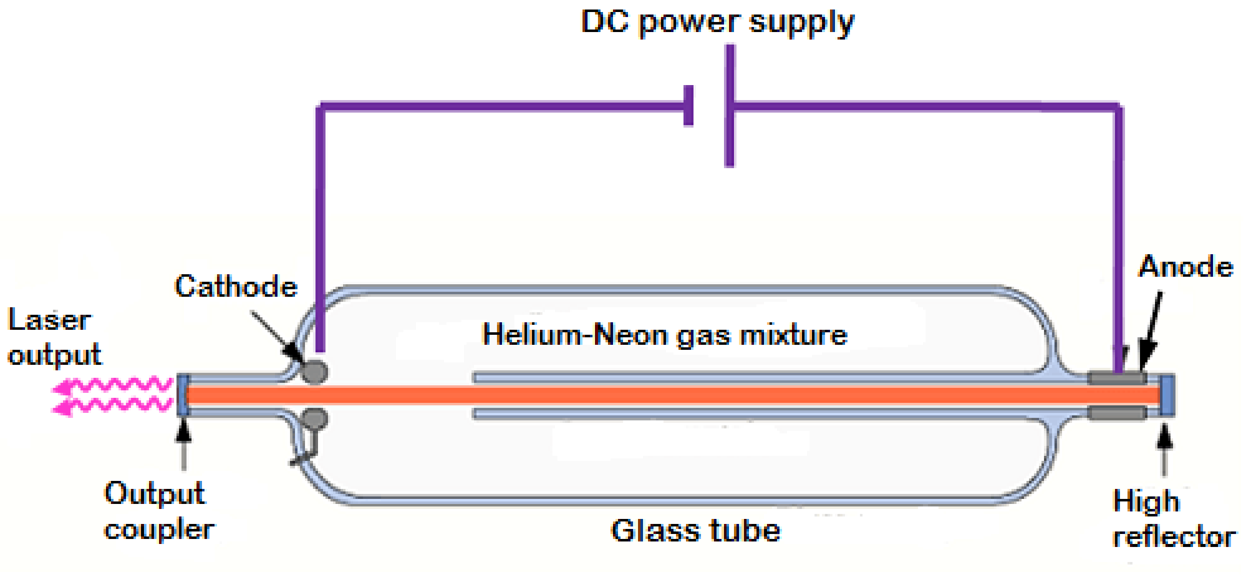


$$B^m \rightarrow A$$

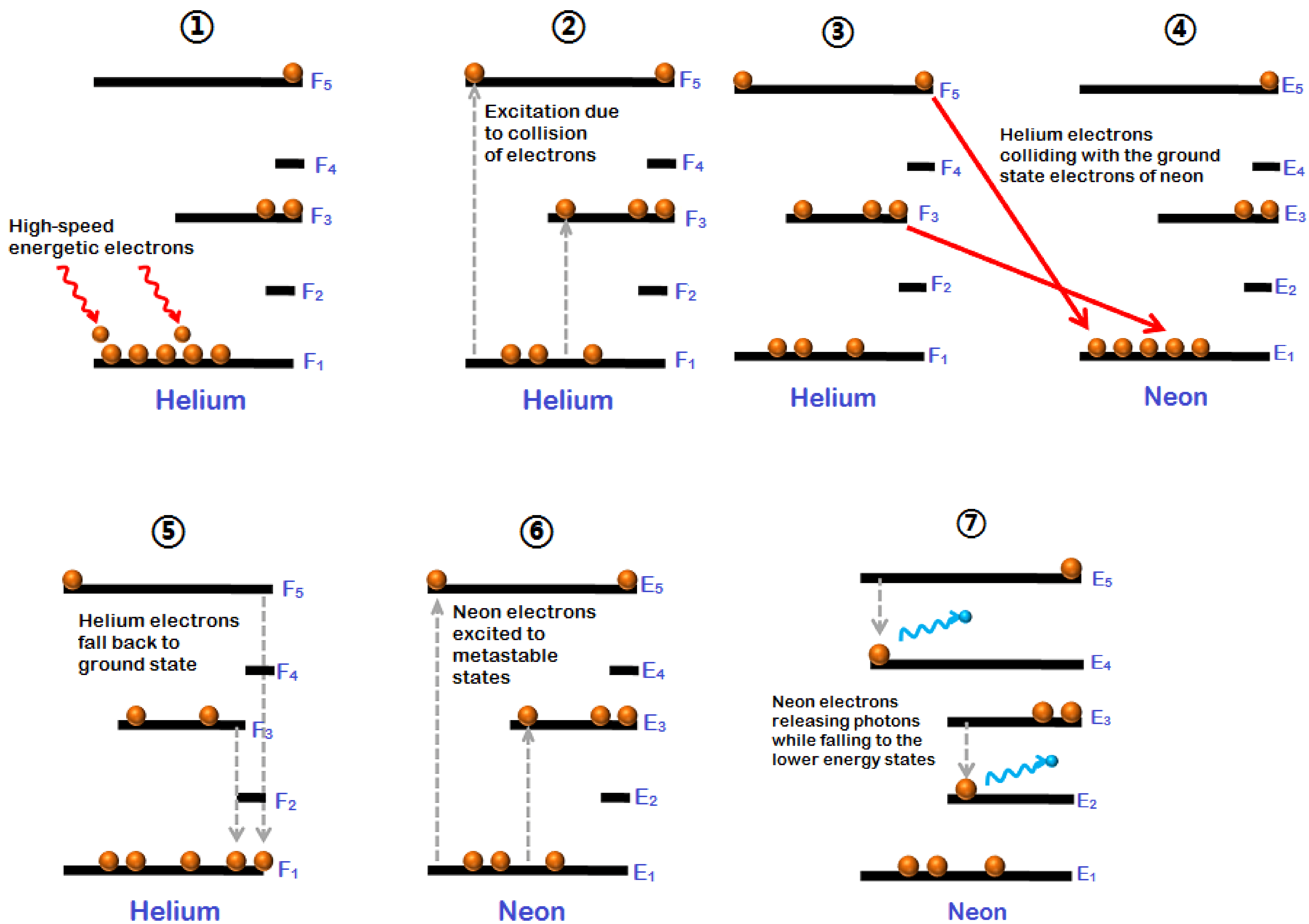
$$B^m + A \rightarrow B + A^*$$

Eg. He-Ne, CO_2-N_2, He-Cd

He-Ne Laser



The partial pressure of helium is 1 mbar whereas that of neon is 0.1 mbar.



①
F5
F4
F3
High-speed energetic electrons
F2
F1
Helium
②
F5
Excitation due to collision of electrons
F4
F3
F2
F1
Helium
③
F5
F4
F3
F2
F1
Helium
④
E5
Helium electrons colliding with the ground state electrons of neon
E4
E3
E2
E1
Neon
⑤
F5
Helium electrons fall back to ground state
F4
F3
F2
F1
Helium
⑥
E5
Neon electrons excited to metastable states
E4
E3
E2
E1
Neon
⑦
E5
E4
Neon electrons releasing photons while falling to the lower energy states
E3
E2
E1
Neon