**MA 2110 - Probability**

**Random Variables II - 28, August 2016**

(1) Let X be a continuous random variable with the following probability density function: $f(x) = 3(1-x)^2$ for $0 < x < 1$. What is the probability density function of $Y = (1-X)^3$ ?

(2) Let$X \sim \mathcal{U}(-2, 2, )$.
  (a) Find the CDF of $X$.
  (b) Find the PDF of the random variable $Y = X + 1$.
  (c) Find the CDF and PDF of $Z = 2X$.

(3) Suppose $Y$ is a normal random variable with mean $\mu$ and standard deviation $\sigma$. Let $X = e^Y$. $X$ is called a lognormal random variable since its log is a normal random variable.
  (a) Find the PDF and the expectation of X.
  (b) The median of a continuous random variable Z is defined as such number d that $P(Z \leq d) = P(Z \geq d) = \frac{1}{2}$. Find the median of X.

(4) Find the mean and variance of the **standard gamma distribution**.

(5) Let X be a continuous random variable with PDF

$$f_X(x) = \begin{cases} 3(\frac{1}{x})^4, & x >= 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the mean and variance of X.

(6) If the weather is good (with probability 0.6), Alice walks the 2 miles to class at a speed of V=5 miles per hour, and otherwise rides her motorcycle at a speech of V=30 miles per hour. What is the expected time E[T] to get to the class ?

(7) The probability distribution of X, the number of imperfections per 10 metres of synthetic fabric in continuous rolls of uniform width is given as

$$f(x) = \begin{cases} 0.41 & if \quad x = 0 \\ 0.37 & if \quad x = 1 \\ 0.16 & if \quad x = 2 \\ 0.05 & if \quad x = 3 \\ 0.01 & if \quad x = 4 \end{cases}$$

Find the avg number of imperfections per 10 m of this fabric.

(8) Y is an exponential random variable with parameter $\lambda = 0.2$. Given the event $A = Y < 2$, find the conditional expected value, $E[Y|A]$.

(9) A biased coin, which lands heads with probability $\frac{1}{10}$ each time it is flipped, is flipped 200 times consecutively. Give an upper bound on the probability that it lands heads at least 120 times.

(10) Suppose that we roll a standard fair die 100 times. Let $X$ be the sum of the numbers that appear over the 100 rolls. Use Chebyshevs inequality to bound $P[| X - 350 | \geq 50]$.

(11) Let $X \sim \mathcal{B}(n,p)$. Using the Chebyshev's inequality, find an upper bound on $P(X \geq \alpha n)$, where $p < \alpha < 1$. Evaluate the bound for $p = \frac{1}{2}$ and $\alpha = \frac{3}{4}$.

(12) Use Markov's or Chebyshev's inequality to solve the following:
  (a) A post office handles, on average, 10,000 letters a day. What can be said about the probability that it will handle at least 15,000 letters tomorrow?
  (b) A post-office handles 10,000 letters per day with a variance of 2,000 letters. What can be said about the probability that this post office handles between 8,000 and 12,000 letters tomorrow? What about the probability that more than 15,000 letters come in?

(13) In 1998, the average conventional first mortgage for new single family homes was for 195,000 dollars. Assume that mortgage amounts are normally distributed with a standard deviation of sigma = 30,000 dollars. Find the probability that a randomly selected mortgage amount is
  (a) between 140,000 dollars and 160,000 dollars
  (b) over 160,000 dollars
  (c) under 225,000 dollars
  (d) Find the 85th percentile mortgage amount.

(14) A taxi dispatcher has found that the time between successive calls for taxis is exponentially distributed with a mean time between calls of 5.30 minutes. The dispatcher much disconnect the telephone for 3 minutes in order to have the push-button mechanism repaired. What is the probability that a call will be received while the system is out of service? (You could assume that a call came in just before the system went out of service, although it actually doesnt matter.)

(15) Media researchers report the average daily TV viewing time for U.S. adult males to be 4.28 hours. Assuming a normal distribution with a standard deviation of 1.30 hours:
  (a) What is the probability that a randomly selected U.S. adult male watches TV less than 2.00 hours per day?
  (b) How much TV would a U.S. adult male have to watch per day in order to be at the 99th percentile?

(16) Consider the event that every bin receives exactly $k$ balls when $kn$ balls are thrown randomly into n bins.
  (a) Determine the exact probability of this event.
  (b) Compute the probability under the Poisson approximation.
  (c) The value upon dividing the expression in (b) by that in (a) equals the probability that a Poisson random variable with parameter lambda takes on some value $r$.
    - Explain briefly but precisely why this quotient matches a Poisson distribution.
    - Also, what are the values of lambda and $r$?