Light Matter Interaction

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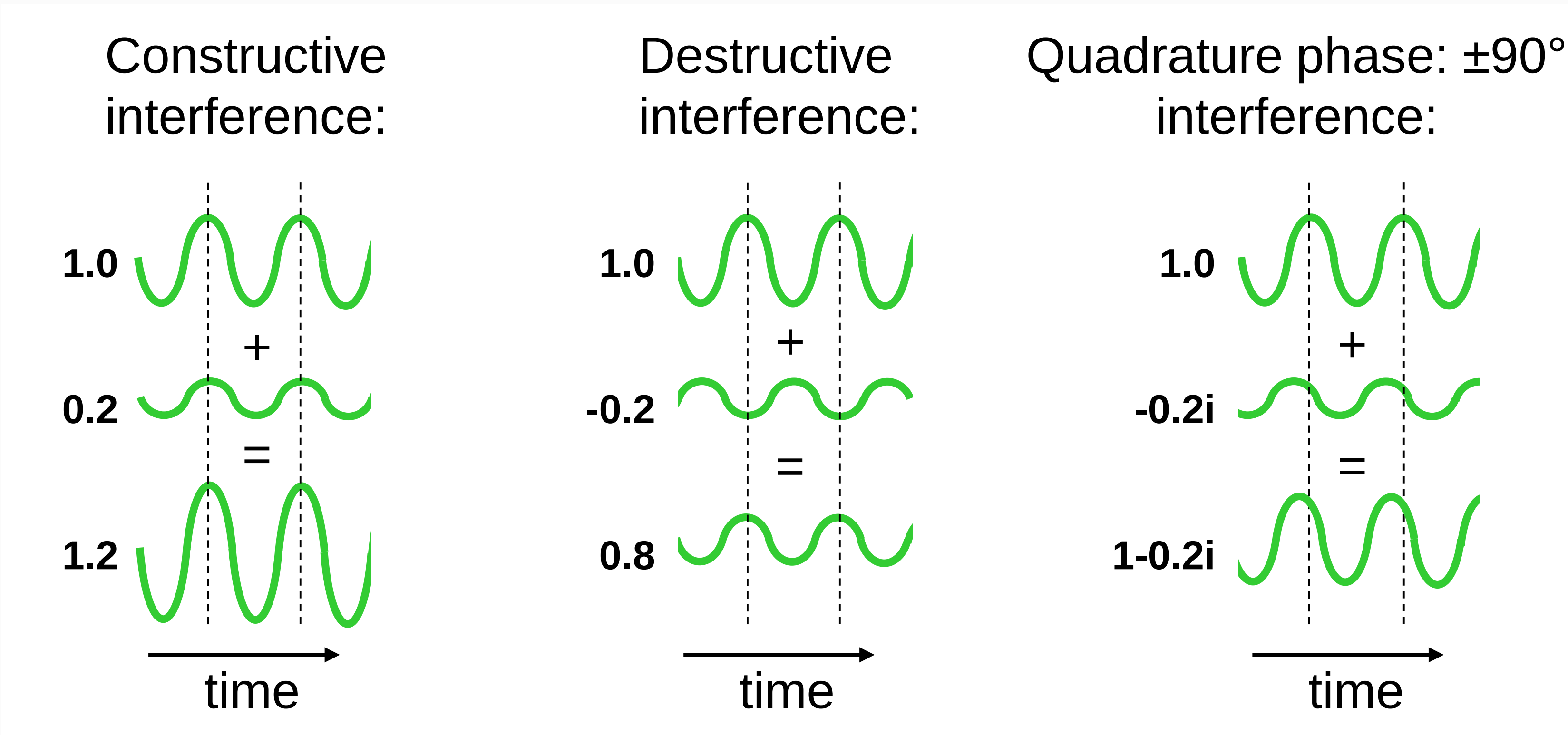
Adding Complex Amplitudes

When two waves add together with the same complex exponentials, we add the complex amplitudes, $\underset{\sim}{E}_0 + \underset{\sim}{E}_0{}'$.



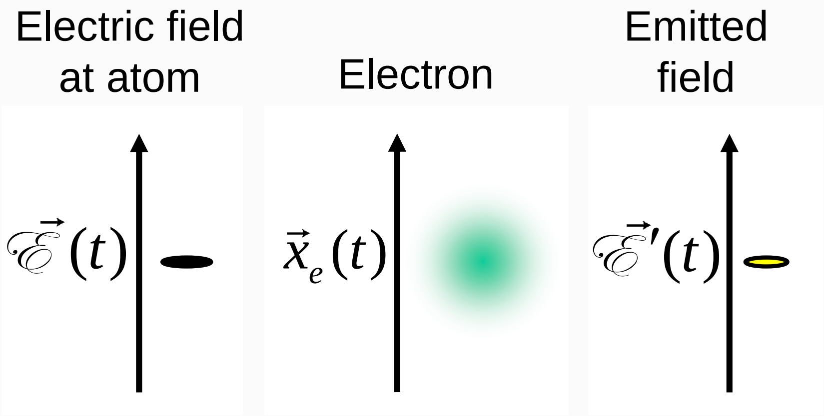
Laser

Absorption

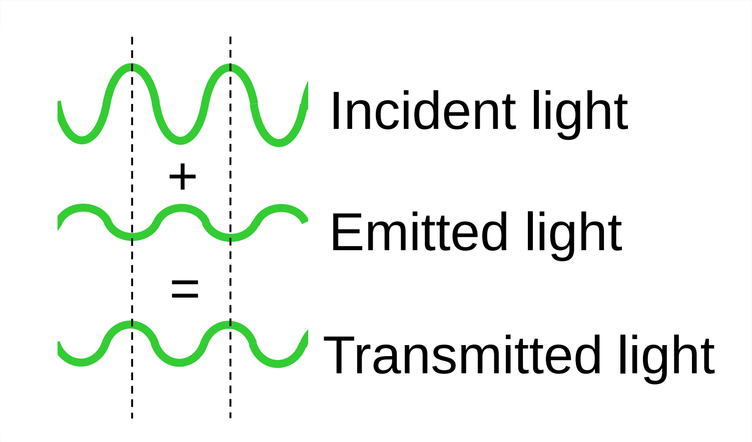
Slower phase velocity

Light excites atoms, which then emit light that interferes with the input light.

When light of frequency ω excites an atom with resonant frequency ω_0:



On resonance ($\omega = \omega_0$)



An excited atom vibrates at the frequency of the light that excited it and emits energy as light of that frequency.

The crucial issue is the **relative phase** of the incident light and this emitted light. For example, if these two waves are ~180° out of phase, the beam will be attenuated. We call this absorption.

The Forced Oscillator

When we apply a periodic force to a natural oscillator (such as a pendulum, spring, swing, or atom), the result is a **forced oscillator**.

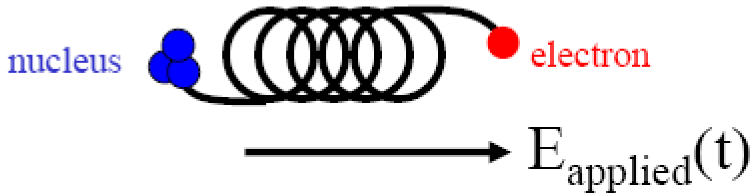
Examples:

- Child on a swing being pushed
- Periodically pushed pendulum
- Bridge in wind or an earthquake
- Electron in a light wave
- Nucleus in a light wave

The forced oscillator is one of the most important problems in physics. It is the concept of **resonance**.

The Forced Oscillator: Math

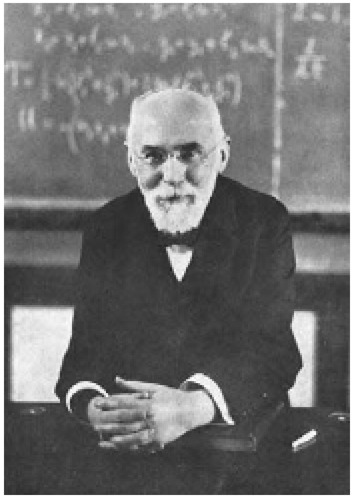
$$\mathrm{E}(t) = \underset{\sim}{E}_0 \exp(-i\omega t)$$



The forces on the electron are:

1. The restoring force of the spring: $-kx_e$
2. The force exerted by the electric field: eE

This model was first proposed by Hendrik Lorentz in the 1890's as a way to explain the emission of light by atoms.



Hendrik Lorentz
1853-1928

The Forced Oscillator: Math

Consider an electron on a spring with position $\tilde{x}_e(t)$, and driven by a light wave, $\tilde{E}_0 \exp(-i\omega t)$. Using Newton's Second Law (F = ma):

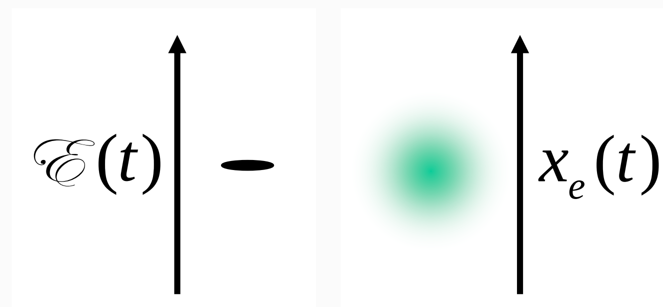
m a	Restoring force	Lorentz force from the light wave

$$m_e \, d^2 \tilde{x}_e / dt^2 \;+\; m_e \omega_0^2 \tilde{x}_e \;=\; e\tilde{E}_0 \exp(-i\omega t)$$

m_e is the electron mass, and e is the electron charge.

The solution is:

$$\tilde{x}_e(t) = \left[\frac{(e/m_e)}{\left(\omega_0^2 - \omega^2\right)} \right] \tilde{E}_0 \exp(-i\omega t)$$



So the electron oscillates at the incident light-wave frequency (ω), but with an amplitude that depends on the difference between the frequencies (and it can be either in phase or 180º out of phase).

Checking Our Solution

$$m_e d^2 \underset{\sim}{x}_e / dt^2 \; + \; m_e \omega_0^2 \underset{\sim}{x}_e \; = \; e\underset{\sim}{E}_0 \exp(-i\omega t)$$

$$\underset{\sim}{x}_e(t) \; = \; \left[\frac{(e/m_e)}{(\omega_0^2 - \omega^2)} \right] \underset{\sim}{E}_0 \exp(-i\omega t)$$

Substitute the solution for $\underset{\sim}{x}_e(t)$ to see if it works:

$$-m_e\omega^2 \left\{ \left[\frac{(e/m_e)}{(\omega_0^2 - \omega^2)} \right] \cancel{\underset{\sim}{E}_0 \exp(-i\omega t)} \right\} + m_e\omega_0^2 \left\{ \left[\frac{(e/m_e)}{(\omega_0^2 - \omega^2)} \right] \cancel{\underset{\sim}{E}_0 \exp(-i\omega t)} \right\} = e\cancel{\underset{\sim}{E}_0 \exp(-i\omega t)}$$

$$-\cancel{m_e}\omega^2 \left[\frac{(\cancel{e/m_e})}{(\omega_0^2 - \omega^2)} \right] + \cancel{m_e}\omega_0^2 \left[\frac{(\cancel{e/m_e})}{(\omega_0^2 - \omega^2)} \right] = \cancel{e}$$

$$-\omega^2 \left[\frac{1}{(\omega_0^2 - \omega^2)} \right] + \omega_0^2 \left[\frac{1}{(\omega_0^2 - \omega^2)} \right] = 1$$

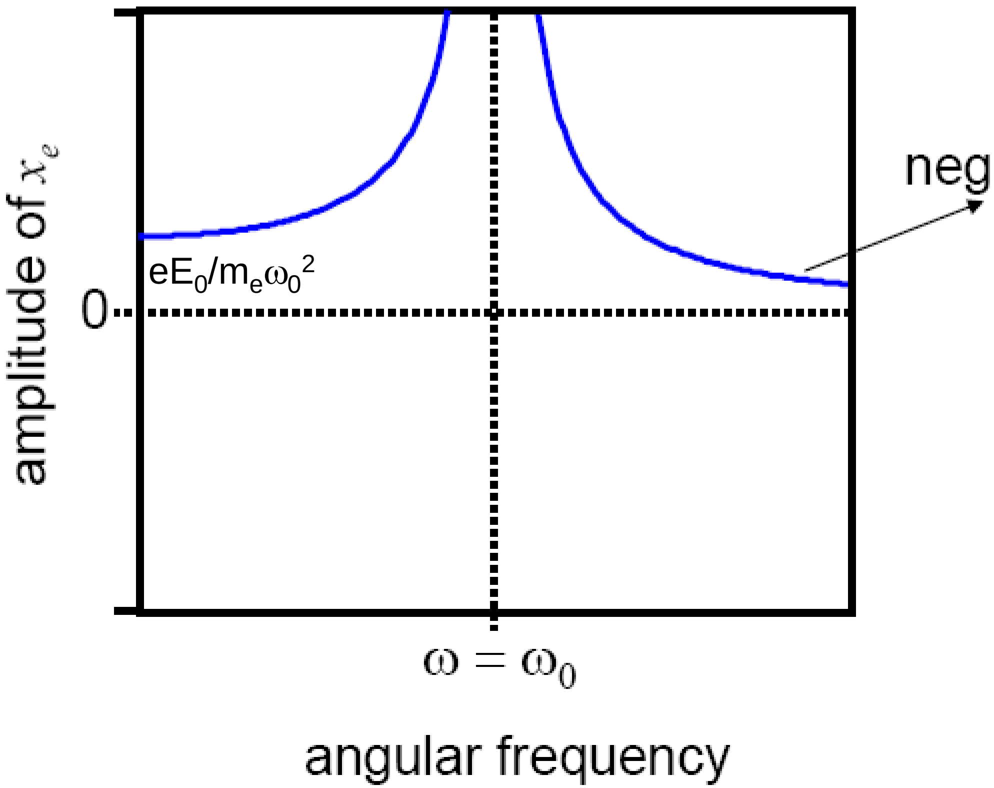
$$\left[\frac{(\omega_0^2 - \omega^2)}{(\omega_0^2 - \omega^2)} \right] = 1$$

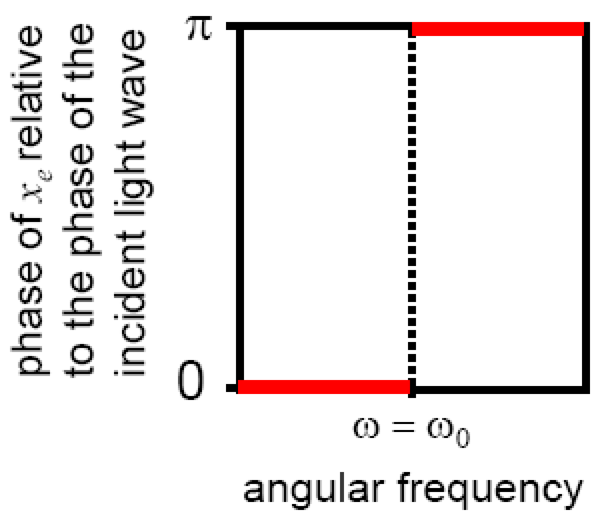
QED

Amplitude and phase response

How does the amplitude (and phase) of the motion of the charge depend on the frequency of the electric field?

$$\left|x_e(t)\right| = \frac{eE_0}{m_e} \cdot \frac{1}{\left(\omega_0^{\,2} - \omega^2\right)}$$





Question: what if the light wave oscillates at frequency $\omega = \omega_0$?

The Forced Oscillator: The amplitude and relative phase of the oscillator motion with respect to the input force depend on the frequencies.

Let the oscillator's reso-nant frequency be ω_0, and the forcing frequency be ω.

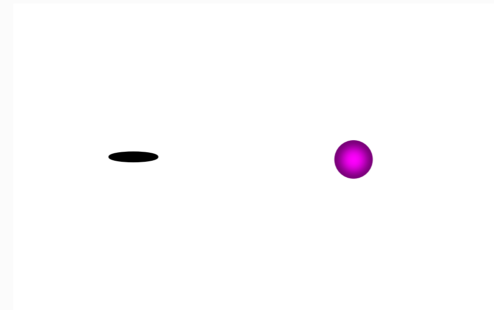
Let the forcing function be a light electric field and the oscillator a (positively charged) nucleus in a molecule.

Electric field at nucleus

Nucleus

Below resonance

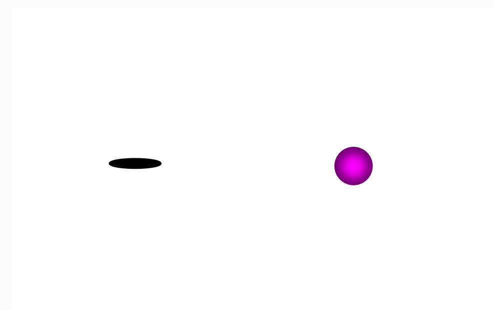
$\omega << \omega_0$



Weak vibration. **In phase**.

On resonance

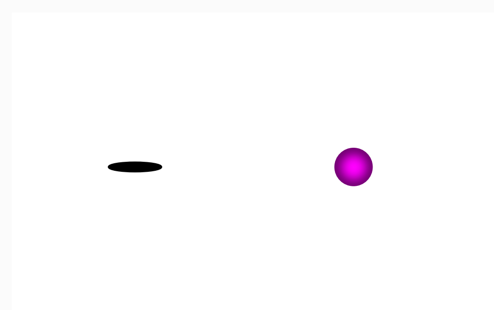
$\omega = \omega_0$



Strong vibration. **90° out of phase.**

Above resonance

$\omega >> \omega_0$



Weak vibration. **180° out of phase.**

The amplitude and relative phase of an electron's motion with respect to incident light also depend on the frequencies.

The electron charge is **negative**, so there's a **180° phase shift** in all cases (compared to the previous slide's plots).

Electric field at electron

Electron

Below resonance

$\omega << \omega_0$

On resonance

$\omega = \omega_0$

Above resonance

$\omega >> \omega_0$

Weak vibration. **180° out of phase.**

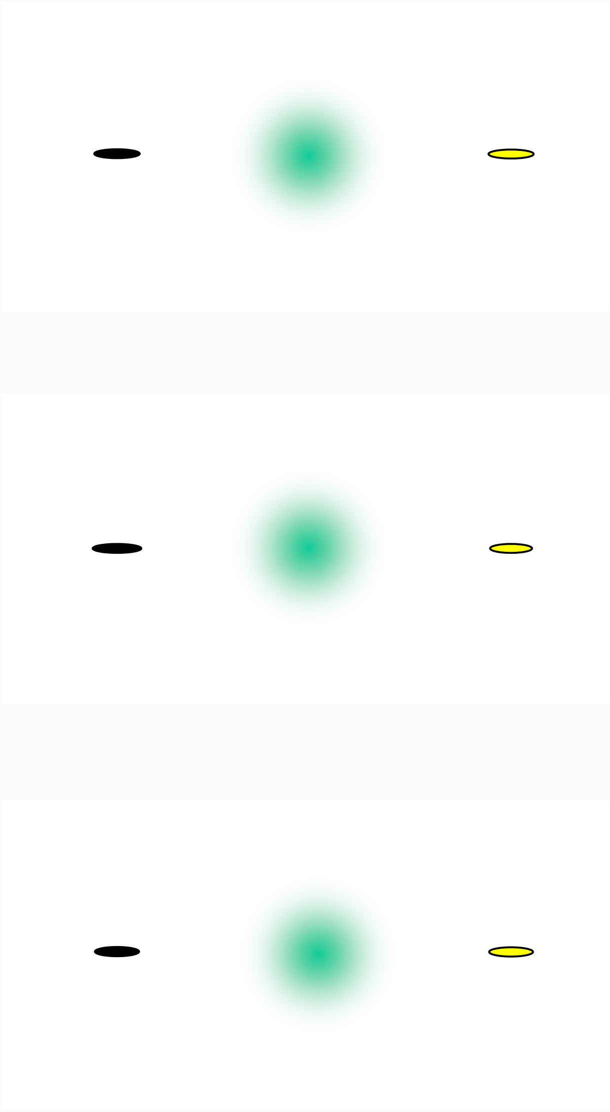
Strong vibration. **-90° out of phase.**

Weak vibration. **In phase.**

The amplitude and relative phase of emitted light with respect to the incident light depend on the frequencies.

Maxwell's Equations will require that the emitted light is **90° phase-shifted** with respect to the atom's motion.

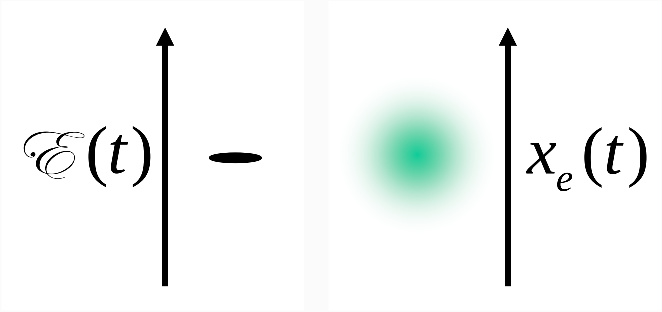
	Electric field at atom	Electron	Emitted field	
Below resonance $\omega << \omega_0$				Weak emission. **90° out of phase.**
On resonance $\omega = \omega_0$				Strong emission. **180° out of phase.**
Above resonance $\omega >> \omega_0$				Weak emission. **-90° out of phase.**



The Problem with this Model

$$\underset{\sim}{x}_e(t) = \left[\frac{(e/m_e)}{(\omega_0^2 - \omega^2)} \right] \underset{\sim}{E}_0 \exp(-i\omega t)$$

Exactly on resonance, when $\omega = \omega_0$, x_e goes to infinity.



This is unrealistic.

We'll need to fix this.

The Damped Forced Oscillator

Our solution has infinite amplitude on resonance, which is unrealistic. We fix this by using a **damped** forced oscillator: a harmonic oscillator experiencing a sinusoidal force **and viscous drag**.

We must add a viscous drag term: $2m_e\Gamma \dfrac{d\underset{\sim}{x}_e}{dt}$

$$\boxed{m_e \frac{d^2\underset{\sim}{x}_e}{dt^2} + 2m_e\Gamma \frac{d\underset{\sim}{x}_e}{dt} + m_e\omega_0^2 \underset{\sim}{x}_e = e\underset{\sim}{E}_0 \exp(-i\omega t)}$$

The solution is now:

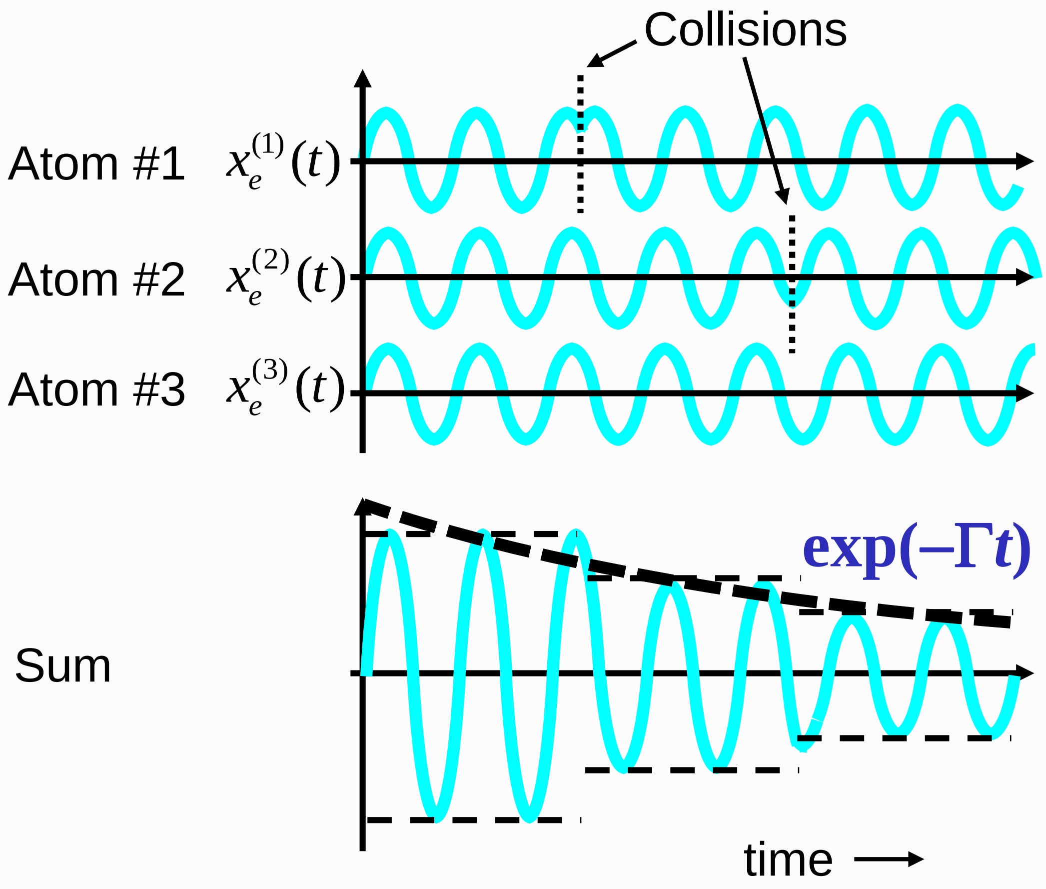
$$\boxed{\underset{\sim}{x}_e(t) = \left[\frac{(e/m_e)}{(\omega_0^2 - \omega^2 - 2i\omega\Gamma)}\right]\underset{\sim}{E}(t)}$$

The electron still oscillates at the light frequency and with a potential phase shift, but now with a finite amplitude for all ω.

Why We Included the Damping Factor, Γ

Atoms spontaneously decay to the ground state after a random time.

Also, the vibration of a medium is the sum of the vibrations of all the atoms in the medium, and collisions cause the sum to cancel.



Collisions **dephase** the vibrations, causing cancellation of the total medium vibration, typically exponentially.

We can use the same argument for the emitted light, too.

Damped-Forced-Oscillator Solution for Light-Driven Atoms

The forced-oscillator response is sinusoidal, with a relative phase that depends on the frequencies involved:

Here, $e < 0$. Also, ω, ω_0 >> Γ.

$$\boxed{\underset{\sim}{x}_e(t) = \left[\frac{(e/m_e)}{(\omega_0^2 - \omega^2 - 2i\omega\Gamma)}\right]\underset{\sim}{E}(t) \propto -\left[\frac{1}{(\omega_0^2 - \omega^2 - 2i\omega\Gamma)}\right]\underset{\sim}{E}(t)}$$

When $\omega << \omega_0$: $\underset{\sim}{x}_e(t) \propto -\left[\frac{1}{(\omega_0^2)}\right]\underset{\sim}{E}(t) \propto -\underset{\sim}{E}(t)$

The electron vibrates **180° out of phase** with the light wave.

When $\omega = \omega_0$: $\underset{\sim}{x}_e(t) \propto -\left[\frac{1}{(-i\Gamma)}\right]\underset{\sim}{E}(t) \propto -i\underset{\sim}{E}(t)$

The electron vibrates **-90° out of phase** with the light wave.

When $\omega >> \omega_0$: $\underset{\sim}{x}_e(t) \propto -\left[\frac{1}{(-\omega^2)}\right]\underset{\sim}{E}(t) \propto \underset{\sim}{E}(t)$

The electron vibrates **in phase** with the light wave.

The amplitude and relative phase of an electron's motion with respect to incident light depend on the frequencies.

Recall that the atom's resonant frequency is ω_0, and the light frequency is ω.

Electric field at atom | Electron

Below resonance $\omega \ll \omega_0$ — Weak vibration. **180° out of phase.**

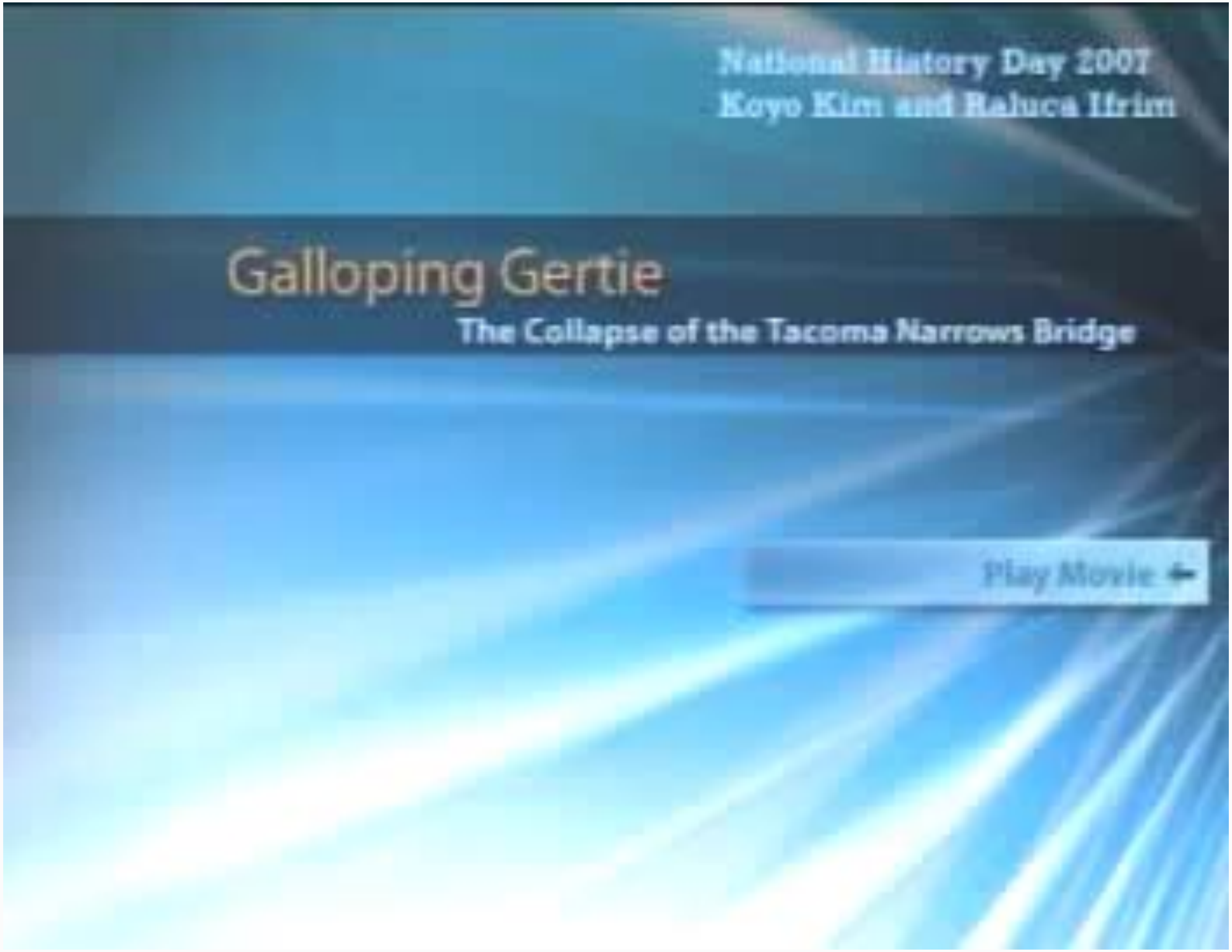
On resonance $\omega = \omega_0$ — Strong vibration. **-90° out of phase.**

Above resonance $\omega \gg \omega_0$ — Weak vibration. **In phase.**

Resonances can be disastrous..

Show a movie of Tacoma Narrow Bridge and Breaking of Glass.


LiveLeak
Breaking Glass
with Sound
MIT Department of Physics
Technical Services Group



Tacoma Narrows Bridge oscillating and collapsing because oscillatory winds blew at its resonance frequency.