MA 2110 - Introduction to Probability

Assignment 1

August 8, 2016

Question 1. A box contains three coins: two regular coins and one fake two-headed coin ($P(H) = 1$),
(a)You pick a coin at random and toss it. What is the probability that it lands heads up?
(b)You pick a coin at random and toss it, and get heads. What is the probability that it is the two-headed coin?

Question 2. I toss a coin repeatedly. The coin is unfair and $P(H) = p$. The game ends the first time that two consecutive heads (HH) or two consecutive tails (TT) are observed. I win if HH is observed and lose if TT is observed. For example if the outcome is $HTHTT$, I lose. On the other hand, if the outcome is $THTHTHH$, I win. Find the probability that I win.

Question 3. In a certain day care class, 30% of the children have grey eyes, 50% of them have blue and the other 20% 's eyes are in other colors. One day they play a game together. In the first run, 65% of the grey eyed ones, 82% of the blue eyed ones and 50% of the children with other eye color were selected. Now, if a child is selected randomly from the class, and we know that he/she was not in the first game, what is the probability that the child has blue eyes?

Question 4. Suppose you're on a game show, and you're given the choice of three doors labelled 1,2 and 3. Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2 or stick with door No. 1?" Is it to your advantage to switch your choice? Explain using Probabilities.

Question 5. Elvis Presley had a twin brother who died at birth. What is the probability that Elvis was an identical twin? To answer this one, you need some background information: According to the Wikipedia article on twins: "$\frac{1}{125}$ of all births were fraternal twins and $\frac{1}{300}$ were identical twins."

Hint: Identical twins have the same gender. Fraternal twins may or may not. Do NOT Ignore Genders.

Question 6. For three events A, B, and C, we know that

- A and C are independent,
- B and C are independent,
- A and B are disjoint
- $P(A \cup C) = \frac{2}{3}$
- $P(B \cup C) = \frac{3}{4}$
- $P(A \cup B \cup C) = \frac{11}{12}$

Find $P(A), P(B)$ and $P(C)$.

Question 7. In my town, it's rainy one third of the days. Given that it is rainy, there will be heavy traffic with probability 1/2, and given that it is not rainy, there will be heavy traffic with probability 1/4. If it's rainy and there is heavy traffic, I arrive late for work with probability 1/2. On the other hand, the probability of being late is reduced to 1/8 if it is not rainy and there is no heavy traffic. In other situations (rainy and no traffic, not rainy and traffic) the probability of being late is 0.25. You pick a random day.
a. What is the probability that it's not raining and there is heavy traffic and I am not late?
b. What is the probability that I am late?
c. Given that I arrived late at work, what is the probability that it rained that day?

Question 8. N guests arrive at a party. Each person is wearing a hat. We collect all hats and then randomly redistribute the hats, giving each person one of the N hats randomly. What is the probability that at least one person receives his/her own hat? Hint: Use the inclusion-exclusion principle.

Question 9. How many distinct solutions does the following equation have?

$$x_1 + x_2 + x_3 + x_4 = 100$$

such that $x_1 \in \{1, 2, 3..\}, \quad x_2 \in \{2, 3, 4, ..\}, \quad x_3, x_4 \in \{0, 1, 2, 3, ...\}$.

Question 10. An urn contains 30 red balls and 70 green balls. What is the probability of getting exactly k red balls in a sample of size 20 if the sampling is done
(a) With replacement (repetition allowed)
(b) Without replacement
(Assume k is between 0 and 20, both inclusive)

Question 11. A certain mathematician, his wife, and their teenage son all play a fair game of chess. One day when the son asked his father for 10 dollars for a Saturday night date, his father puffed his pipe for a moment and replied, "Let's do it this way. Today is Wednesday. You will play a game of chess tonight, tomorrow, and a third on Friday. If you win two games in a row, you get the money."

"Whom do I play first, you or mom?"

"You may have your choice," said the mathematician, his eyes twinkling.

The son knew that his father played a stronger game than his mother. To maximize his chance of winning two games in succession, should he play father-mother-father or mother-father-mother?

Question 12. Mike and James are arguing over who gets the last cookie in the jar, so their dad decides to create a game to settle their dispute. First, Mike flips a coin twice, and each time James calls heads or tails in the air. If James gets both calls right, he gets the last cookie. If not, Mike picks a number between one and six and then rolls a die. If he gets the number right, he gets the last cookie. If not, James picks two numbers between one and five, then spins a spinner with numbers one through five on it. If the spinner lands on one of James' two numbers, he gets the last cookie. If not, Mike does.

Who is more likely to win the last cookie, Mike or James? And what is the probability that person wins it?

Question 13. During an interview

- 50% of all people who receive a first interview receive a second interview.
- 95% of your friends that got a second interview felt they had a good first interview.
- 75% of your friends that DID NOT get a second interview felt they had a good first interview.

If you feel that you had a good first interview, what is the probability you will receive a second interview?

Question 14. The blue MM was introduced in 1995. Before then, the color mix in a bag of plain MM's was (30% Brown, 20% Yellow, 20% Red, 10% Green, 10% Orange, 10% Tan). Afterward it was (24% Blue , 20% Green, 16 % Orange, 14% Yellow, 13% Red, 13% Brown). A friend of mine has two bags of MMs, and he tells me that one is from 1994 and one from 1996. He won't tell me which is which, but he gives me one MM from each bag. One is yellow and one is green. What is the probability that the yellow MM came from the 1994 bag?

Question 15. The names of 100 people are placed in 100 boxes, one name in each box, and the boxes are lined up on a table in a room. One by one, those 100 people are led into the room. Each person may look in at most 50 boxes, but must then leave the room exactly as he or she found it and is permitted no further communication with the others. The 100 people only win the game if each and every one of them finds their own name. Before the game starts, they have a chance to plot their strategy.

Question 16. You are a prisoner sentenced to death. The Emperor offers you a chance to live by playing a simple game. He gives you 50 black marbles, 50 white marbles and 2 empty bowls. He then says, "Divide these 100 marbles into these 2 bowls. You can divide them any way you like as long as you use all the marbles. Then I will blindfold you and mix

the bowls around. You then can choose one bowl and remove ONE marble. If the marble is WHITE you will live, but if the marble is BLACK... you will die."
How do you divide the marbles up so that you have the greatest probability of choosing a WHITE marble?

Question 17. A hiker climbs all day up a steep mountain path and arrives at the mountain top where he camps overnight. The next day he begins the descent down the same trail to the bottom of the mountain when suddenly he looks at his watch and exclaims, "That is amazing! I was at this very same spot at exactly the same time of day yesterday on my way up."
What is the probability that a hiker will be at exactly the same spot on the mountain at the same time of day on his return trip, as he was on the previous day's hike up the mountain? Is the probability closest to (A) 99% or (B) 50% or (C) 0.1% ?

Question 18. A line of 100 airline passengers is waiting to board a plane. They each hold a ticket to one of the 100 seats on that flight. (For convenience, let's say that the nth passenger in line has a ticket for the seat number n.)
Unfortunately, the first person in line is crazy, and will ignore the seat number on their ticket, picking a random seat to occupy. All of the other passengers are quite normal, and will go to their proper seat unless it is already occupied. If it is occupied, they will then find a free seat to sit in, at random.
What is the probability that the last (100th) person to board the plane will sit in their proper seat (#100)?
Hint : The whole thing stops when someone else sits in Crazy Guy's seat.