Light and its important properties

V Sharma

The dual particle/wave nature of light

A beam of light can be thought of as ...

... a flux of particles

(Newton/Planck/Einstein)

photons

Zero mass, speed: $c = 3 \times 10^8 \mathrm{m/sec}$

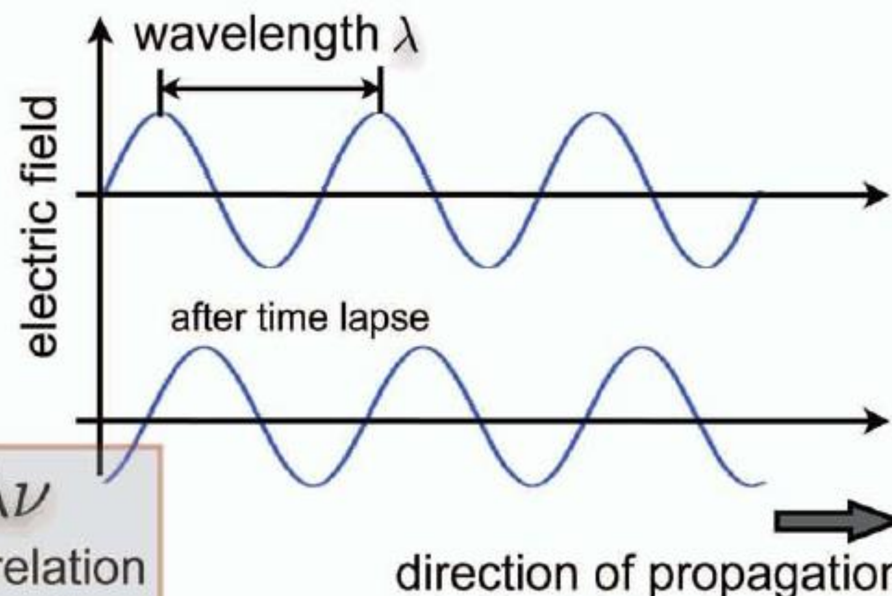
Energy carried by each particle: $E = h\nu$

$h = 6.6262 \times 10^{-34} \mathrm{J} \cdot \mathrm{sec}$ (Planck's const.)

$\nu \text{ (frequency)} = \frac{1}{T \text{ (period)}}$

... an electromagnetic wave

(Huygens/Maxwell/Hertz)



$c = \lambda\nu$

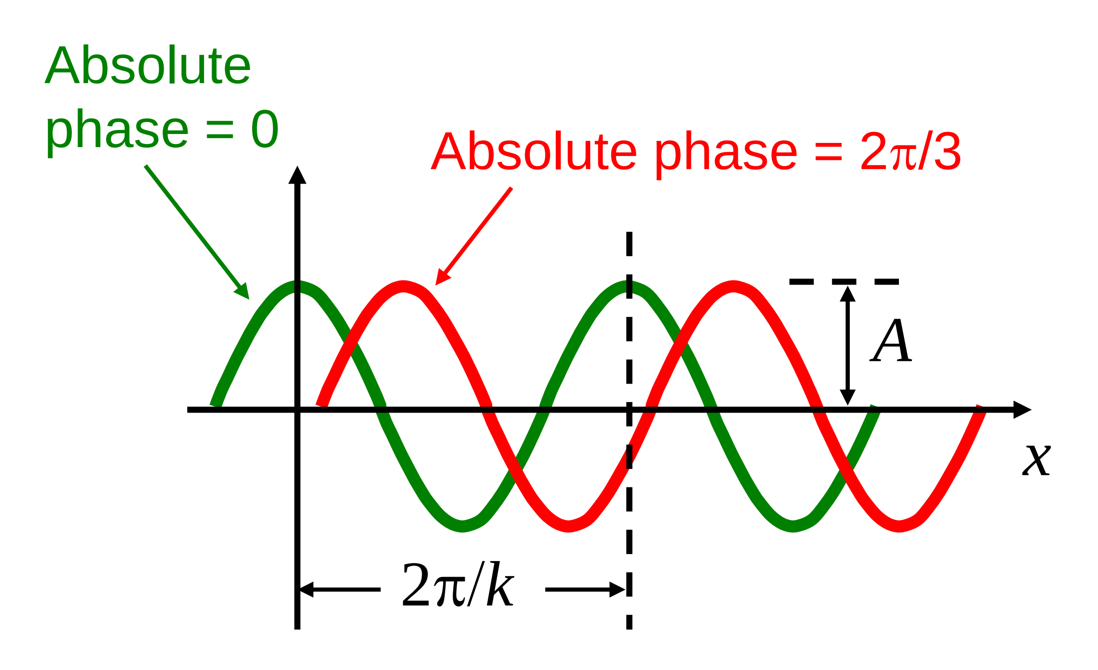
Dispersion relation
(free space)

Amplitude and Absolute Phase

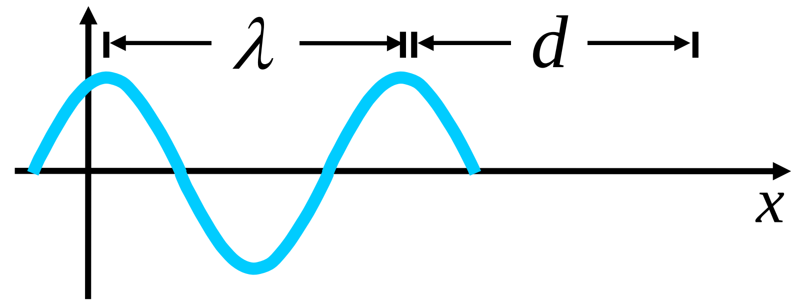
$$\mathscr{E}(x,t) = A\cos[(k\,x - \omega\,t\,) - \theta\,]$$

A = Amplitude

θ = Absolute phase (or initial phase)



Phase Delays



The wave moves a distance, d, in the time, T.

It's also helpful to define a **delay**, T, that a wave experiences after propagating a distance, d:

$$\boxed{T = d / \mathrm{v}}$$

This time can also be expressed as a **phase delay**, ϕ, which is the difference in phase before and after moving the distance, d:

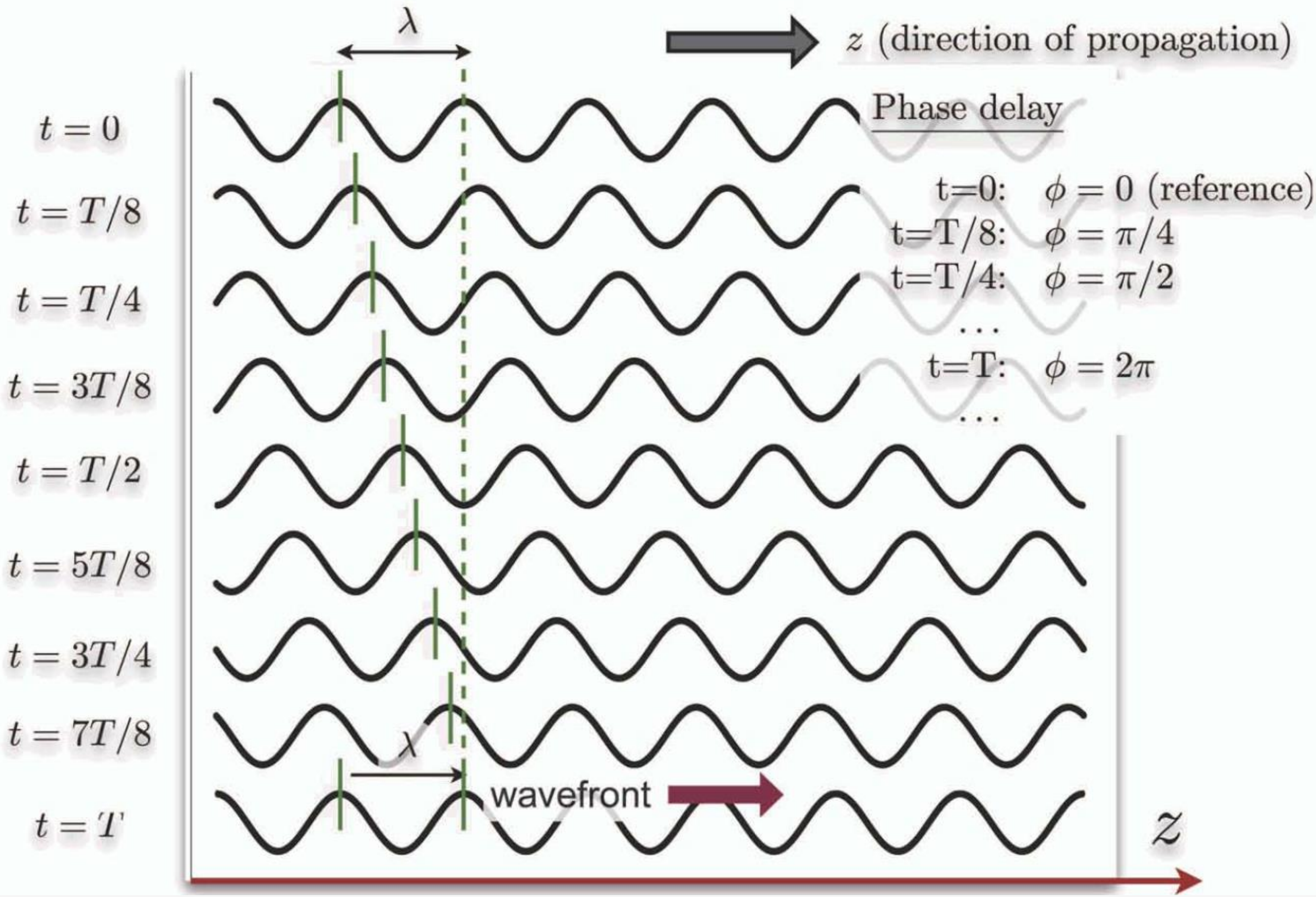
$$\varphi(x+d,t) - \varphi(x,t) = [k(x+d) - \omega t - \theta] - [k x - \omega t - \theta]$$

$$\boxed{\phi = k d = \omega T}$$

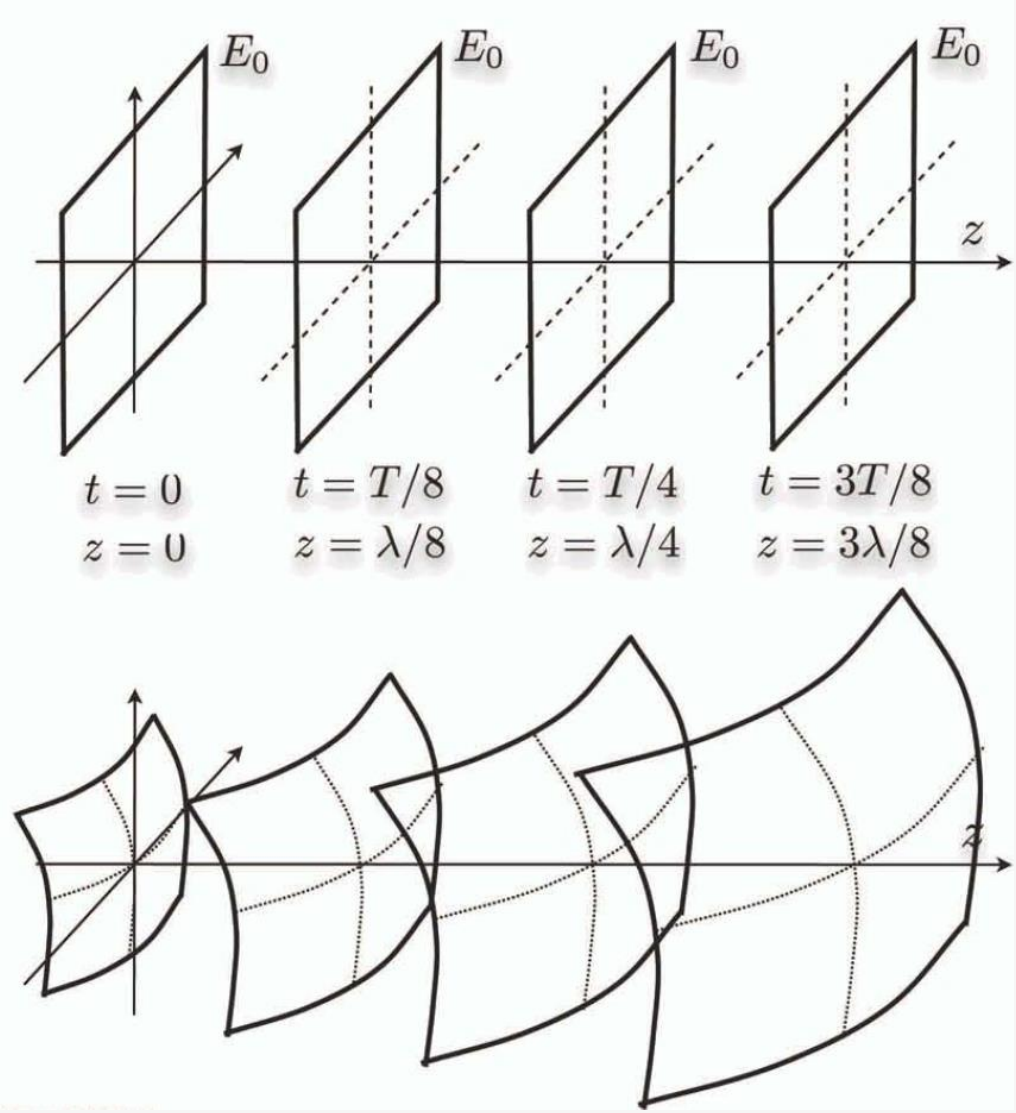
using $d = \mathrm{v}T$ and $k\mathrm{v} = \omega$

The idea of phase delay is used when we move a mirror a distance, d, in any interferometer.

1D wave propagation



3D wavefronts



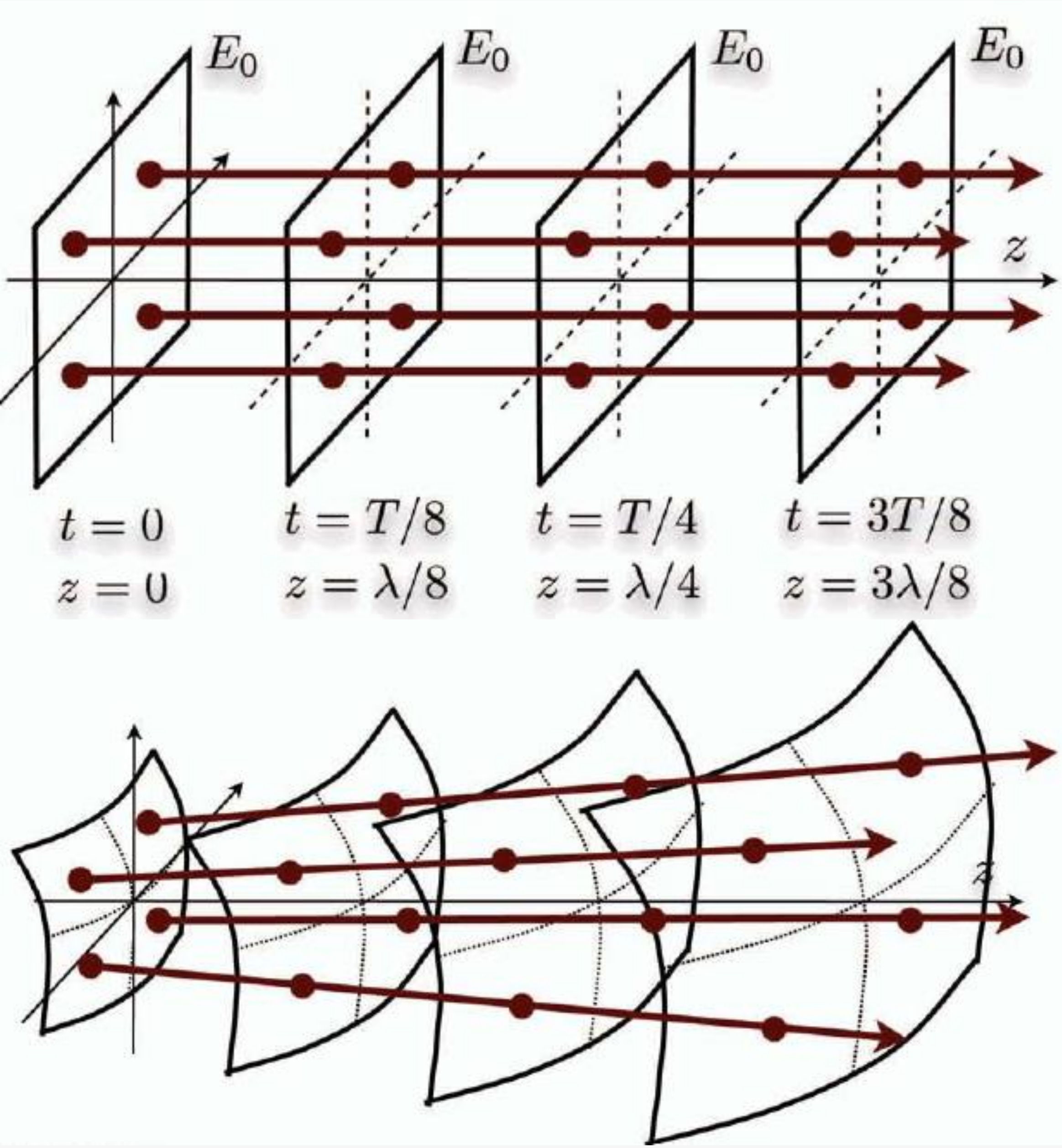
Plane wave:
The wave phase is constant along a planar surface (the wavefront).

As time evolves, the wavefronts propagate at the wave speed without changing; We say that the wavefronts are invariant to propagation in this case.

Spherical wave:
The wave phase is constant along a spherical surface (the wavefront).

As time evolves, the wavefronts propagate at the wave speed and expand outwards while preserving the wave's energy.

Rays



Rays are:

1) **Normals to the wavefront surfaces**
2) **Trajectories of "particles of light"**

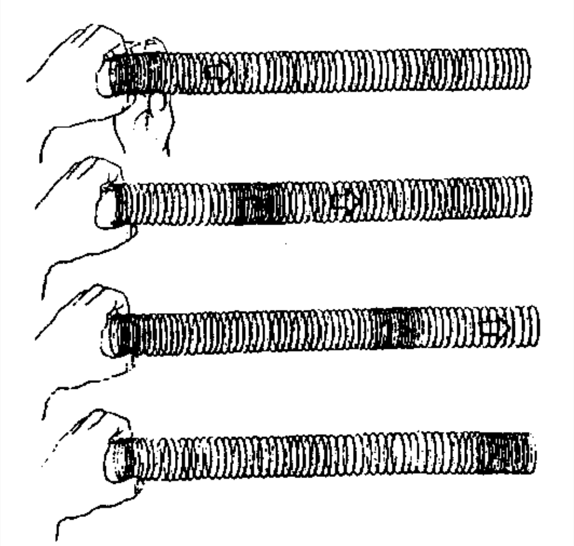
Properties of rays:

1. **Continuous and non differentiable**
2. **Ray trajectories are such as to minimize the optical path**

=> In free space, ray trajectories are straight lines

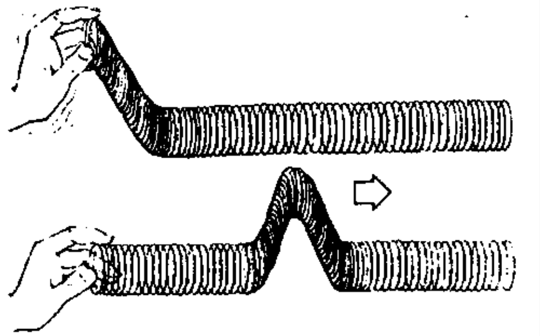
Longitudinal vs. Transverse Waves

Longitudinal:



Motion is along the direction of propagation—**longitudinal polarization**

Transverse:



Motion is transverse to the direction of propagation—**transverse polarization**

Photons Flux

The energy of a single photon is: $h\nu$ or $\hbar\omega = (h/2\pi)\omega$

where h is Planck's constant, 6.626 x 10^{-34} Joule-sec.

One photon of visible light contains about 10^{-19} Joules—not much!

Φ is the **photon flux**, or the number of photons/sec in a beam.

$$\Phi = P / h\nu$$

where P is the beam power.

Question: Calculate the photon flux in beam of a 1mW He-Ne laser with a mean wavelength of 632.8nm.

TABLE 3.1 The Mean Photon Flux Density for a Sampling of Common Sources

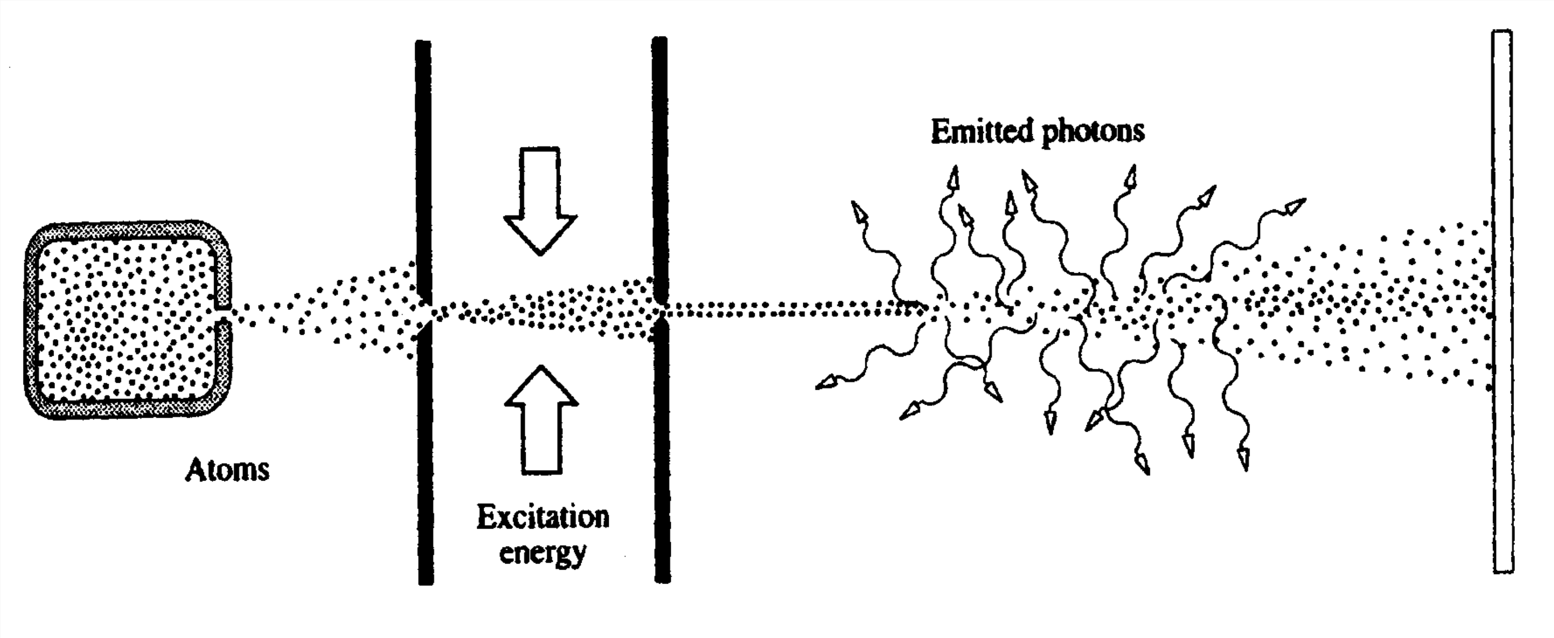
Light Source	Mean Photon Flux Density Φ/A in units of (photons/s·m^2)
Laserbeam (10 mW, He-Ne, focused to 20 μm)	10^{26}
Laserbeam (1 mW, He-Ne)	10^{21}
Bright sunlight	10^{18}
Indoor light level	10^{16}
Twilight	10^{14}
Moonlight	10^{12}
Starlight	10^{10}

Photons—Radiation Pressure

- Photons have no mass and always travel at the speed of light.
- The momentum of a single photon is: h/λ, or $\hbar k$
- Radiation pressure = Energy Density (Force/Area = Energy/Volume)
- When radiation pressure cannot be neglected:
- Comet tails (other forces are small)
- Viking space craft (would've missed Mars by 15,000 km)
- Laser cooling and trapping

Photons have momentum

If an atom emits a photon, it recoils in the opposite direction.



If the atoms are excited and then emit light, the atomic beam spreads much more than if the atoms are not excited and do not emit.

Light interaction with matter

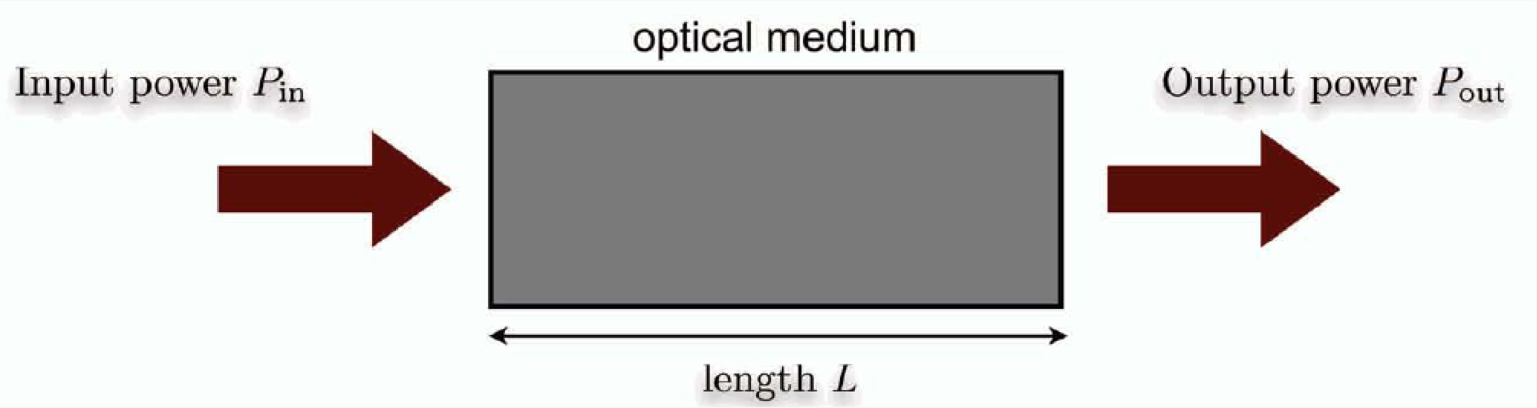
• We will consider three basic types of light-matter interaction:
– absorption
– dispersion
--scattering
– refraction
-- reflection

Fermat's Principle

• The type of interaction that occurs depends on
– the wavelength of the light
– other wave attributes of the light, e.g. polarization
– the atomic/molecular structure of matter
– the amount of incident light energy

• At high energies, "nonlinear" interactions occur, e.g. florescence and multi-photon scattering or even ionization.

Absorption



Physical origin of absorption: conversion of light energy to heat

Beer's Law:

$$P_{\text{out}} = P_{\text{in}} e^{-2\alpha L}$$

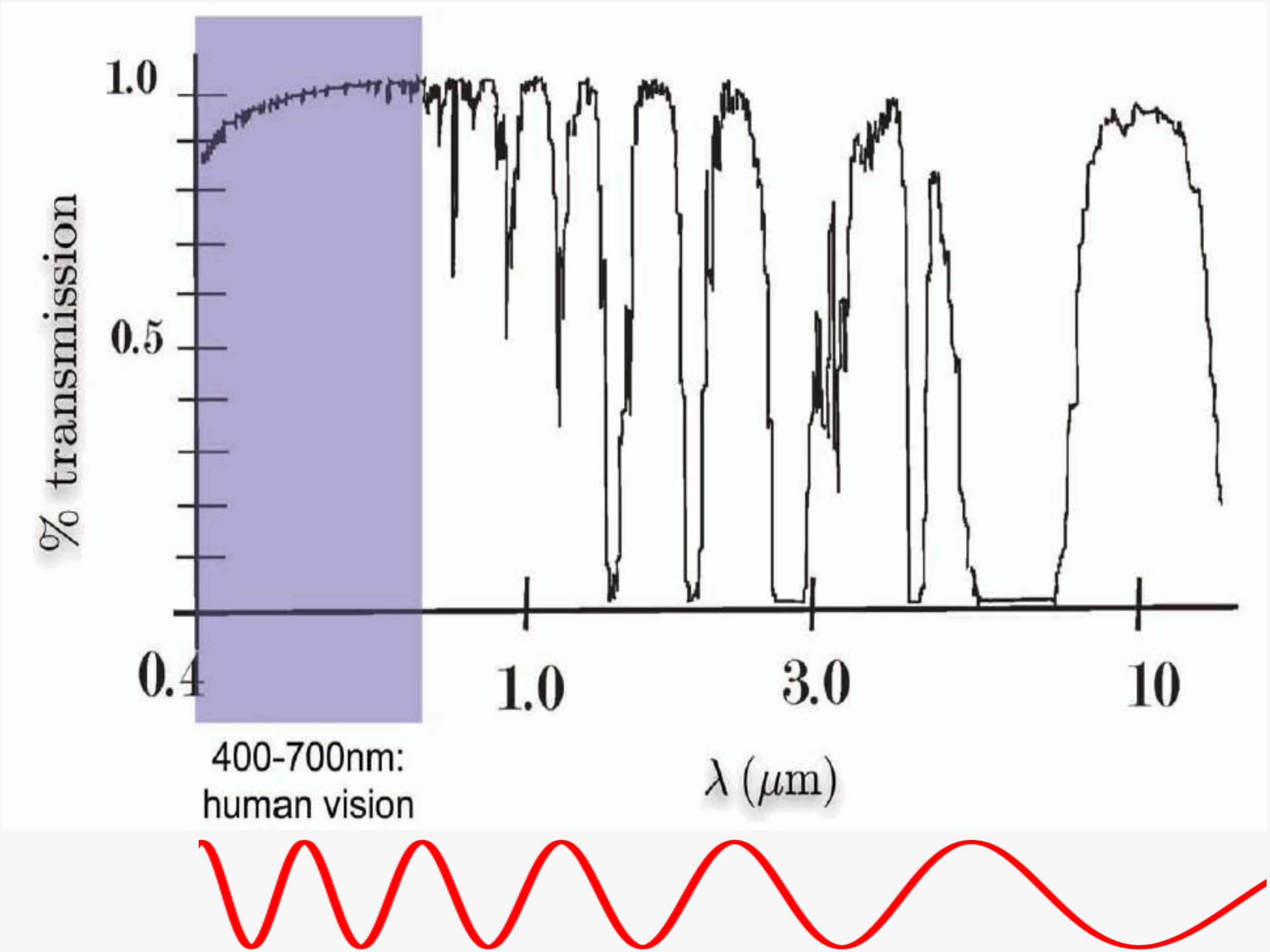
Conductors: very high absorption,

$\alpha \sim 10\text{dB}/\mu\text{m}$ or higher in metals

Dielectrics: very low absorption,

$\alpha \sim 0.1\text{dB}/\text{km}$ or lower in optical fibers

Light absorption through the Earth's atmosphere



Refraction: light speed in vacuum and matter

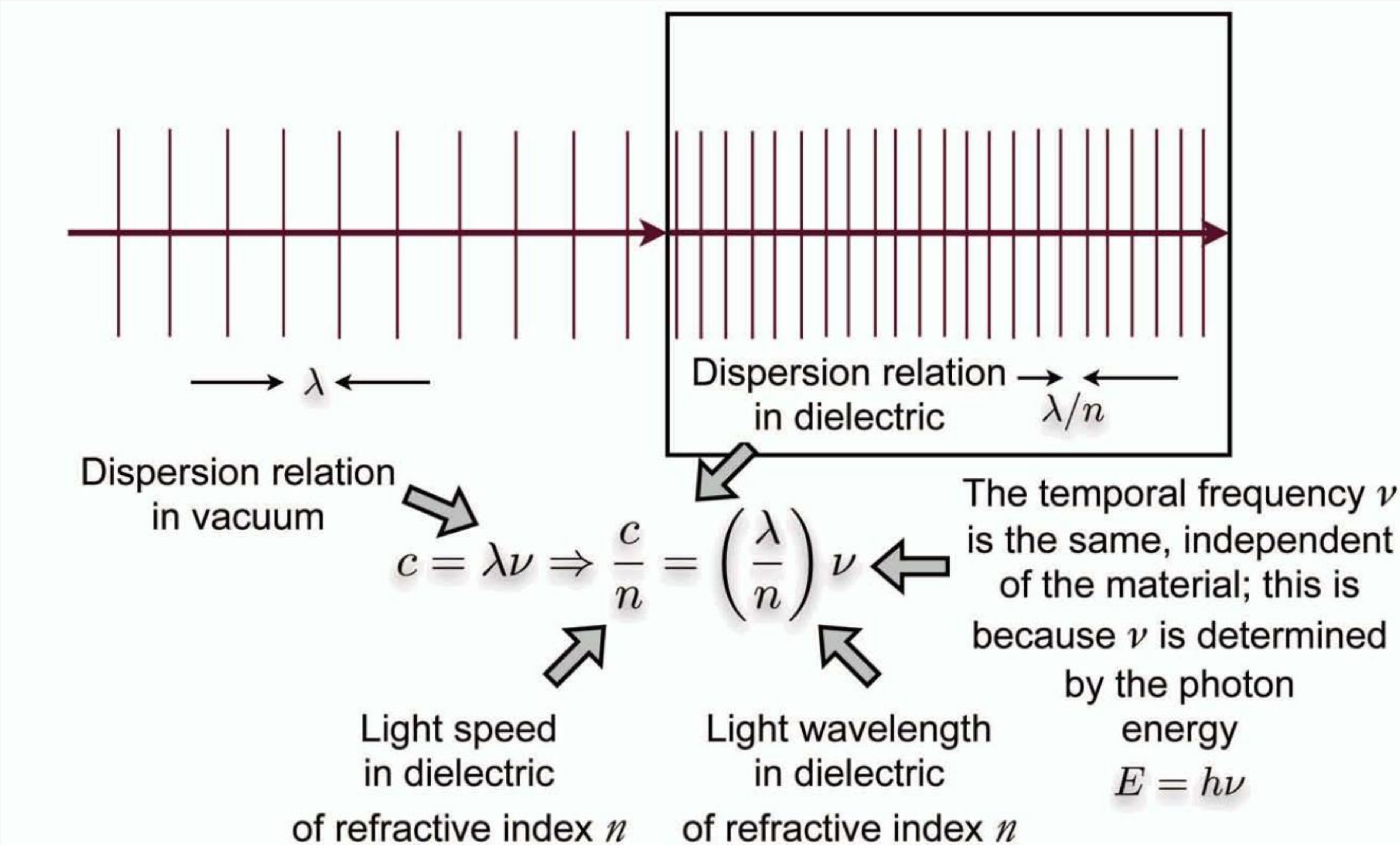
In free space:
$c = 3 \times 10^8$ m/sec

In dielectric materials (e.g. water, glass) : c/n where the quantity n is referred to as the refractive index

The refractive index expresses the optical "density" of a dielectric medium
Refractive index values of commonly used dielectrics:

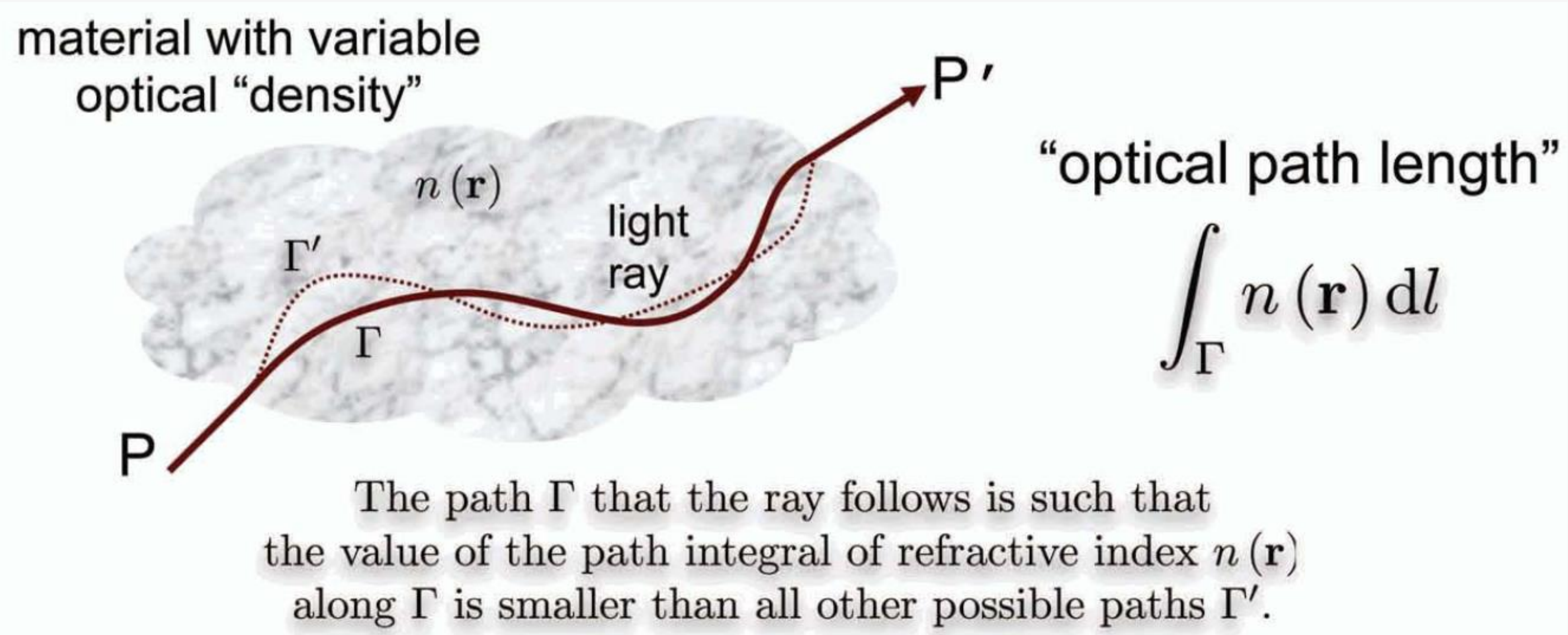
Air = 1
Water = 1.3
Glass = 1.5

Wavelength in vacuum and matter



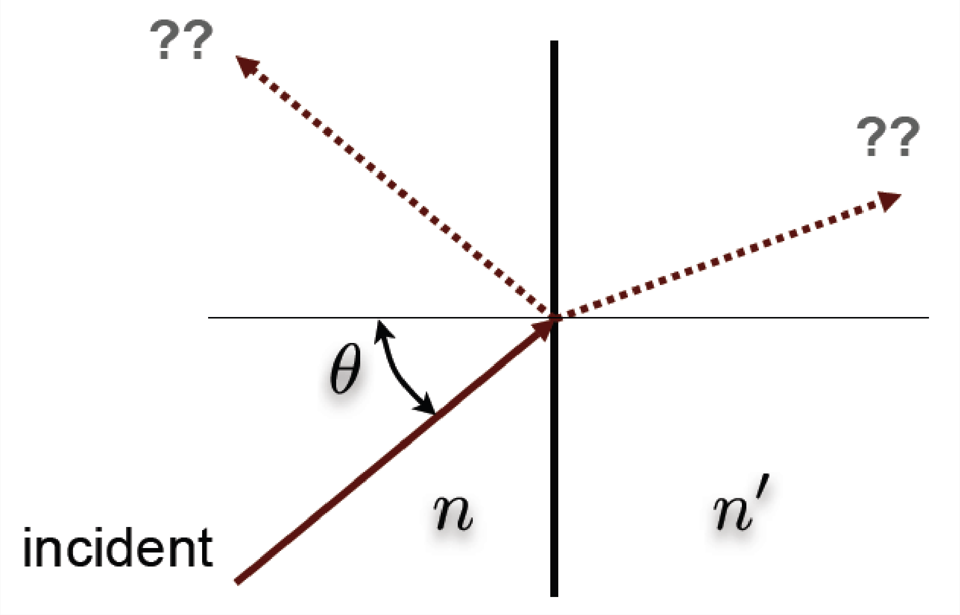
Fermat Principle of minimum path

Fermat Principle of minimum path



In free space or uniform space of constant refractive index n, light propagates in a straight line.
Consequences: the laws of reflection and refraction.

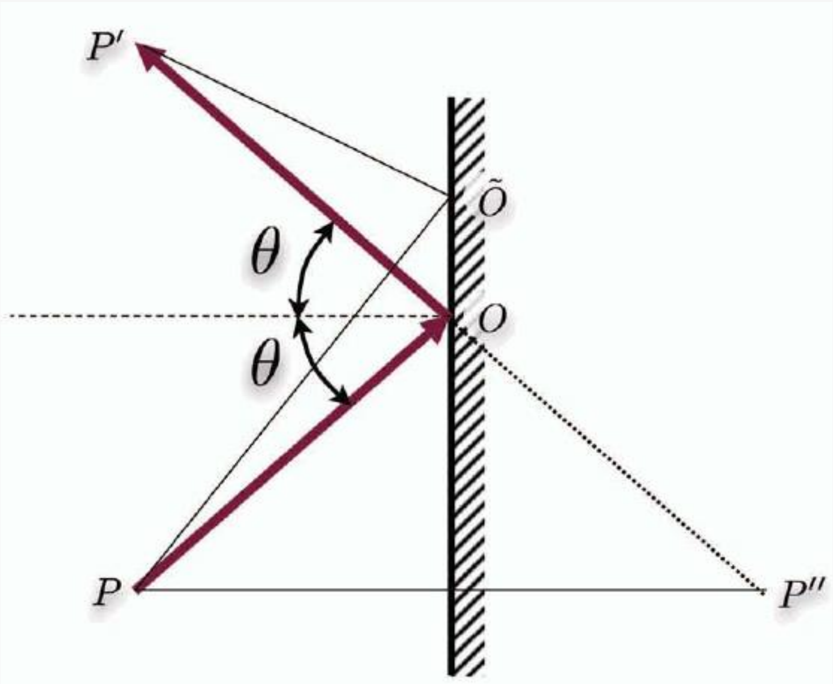
Incidence at dielectric interface



When a light ray is incident at a dielectric interface,
part of the light energy is *reflected* back into the material on the left-hand side

part of the light energy is *refracted* towards the material on the right-hand side

The law of reflection



mirror
or
dielectric interface

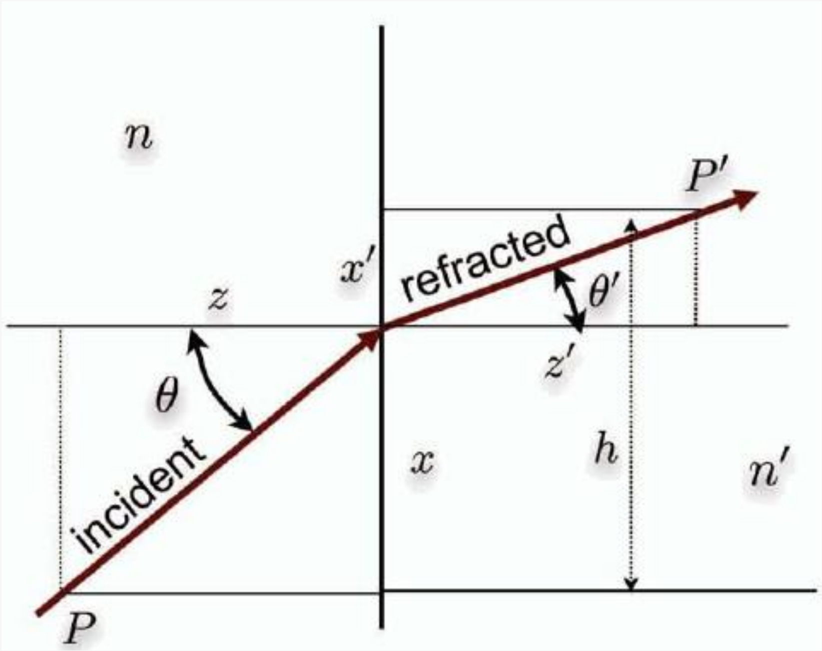
A ray departing from P in the direction Ɵ, with respect to the mirror normal (is reflected symmetrically at the same angle Ɵ*.*

This is because the symmetric path *POP'* has *minimum length.*

Compare, for example, the alternative *POP'.* Clearly, lPOl + l*OP'*l < lPÕl + lÕP'l.

Consider the continuation of *OP' backwards* through the mirror. To an observer in the direction of *P', th*e ray will appear to have originated at *P''.*

Snell's Law: The law of refraction



Let P, P' denote two points along the ray trajectory. According to the Fermat principle, the angle θ, must be such as to minimize the optical path length between PP'

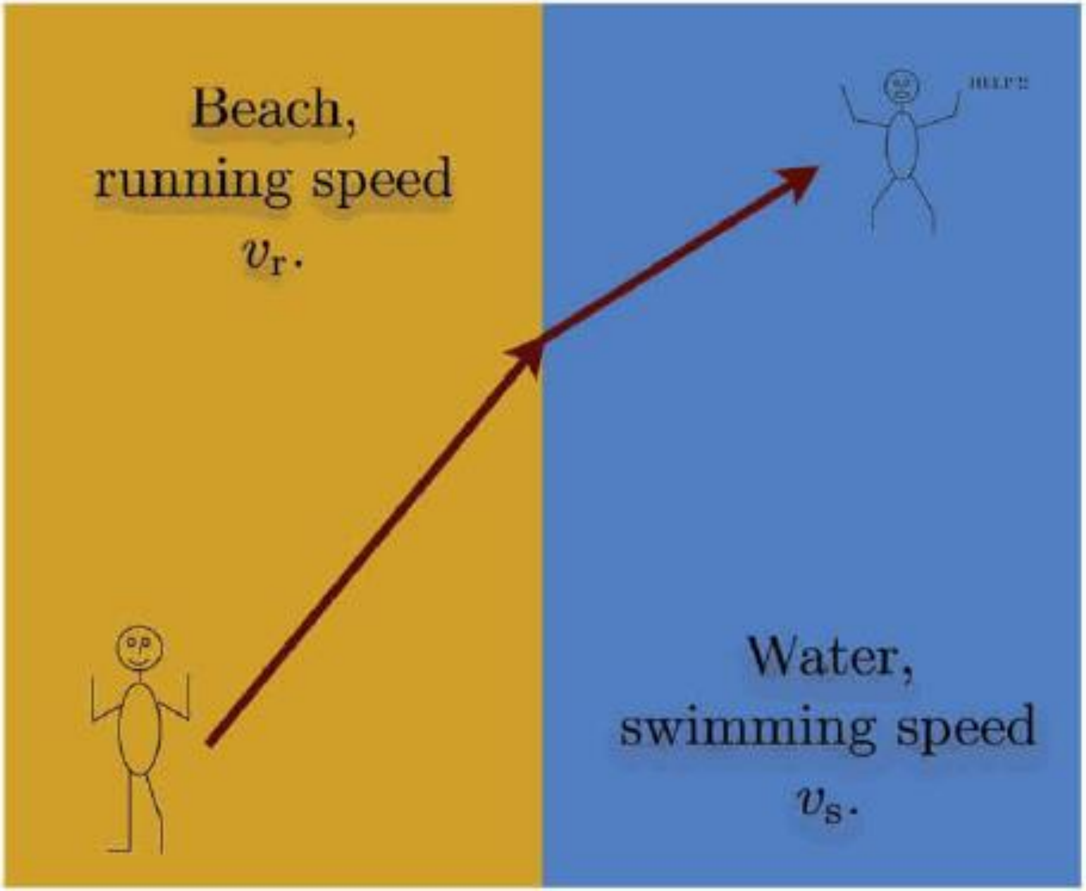
$$OPL = n\sqrt{x^2 + z^2} + n'\sqrt{(h-x)^2 + z'^2}$$

Taking derivatives wrt x,

$$\frac{\partial(OPL)}{\partial x} = n\frac{x}{\sqrt{x^2 + z^2}} - n'\frac{h-x}{\sqrt{(h-x)^2 + z'^2}} = n\sin\theta - n'\sin\theta' = 0$$

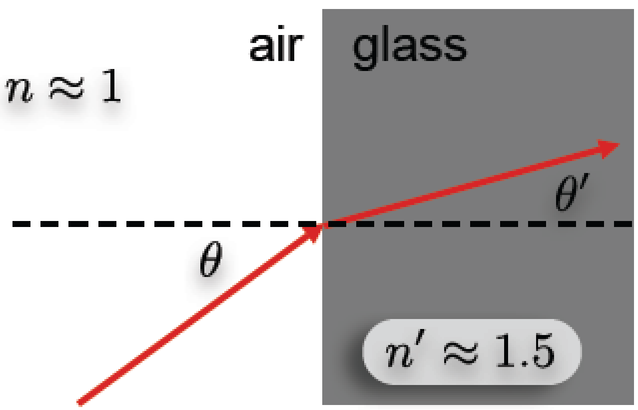
$\Rightarrow n\sin\theta = n'\sin\theta'$ **This is k/a the famous Snell's Law.**

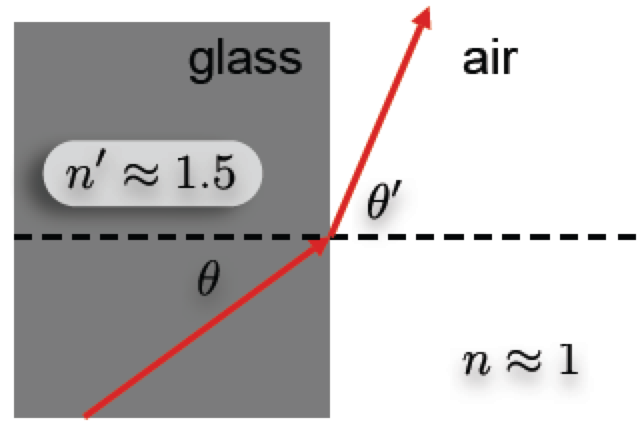
Question for you ☺



Which path should the lifeguard follow
to reach the drowning person in minimum time?

Two types of refraction





$$\boxed{\mathrm{n} \sin \theta = n' \sin \theta'}$$

from lower to higher index
(towards *optically denser* material)

angle wrt normal decreases

the maximum angle θ'
that can enter the optically dense medium
is such that

$$n' \sin \theta' = n$$

from higher to lower index
(towards *optically less dense* material)

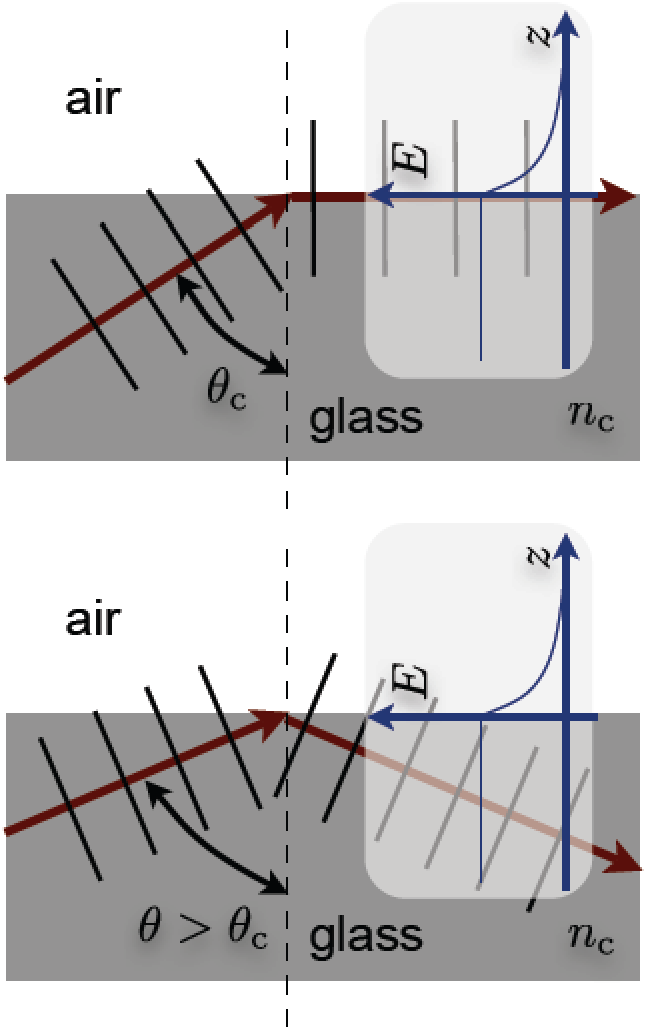
angle wrt normal increases

if the combination of n and θ
is such that

$$n \sin \theta > n'$$

then Snell's law in the less dense medium cannot be satisfied. This situation is known as **Total Internal Reflection (TIR)**

Total Internal Reflection (TIR)



$$n_c \sin\theta_c = 1$$

at critical angle, the refracted light propagates parallel to the interface

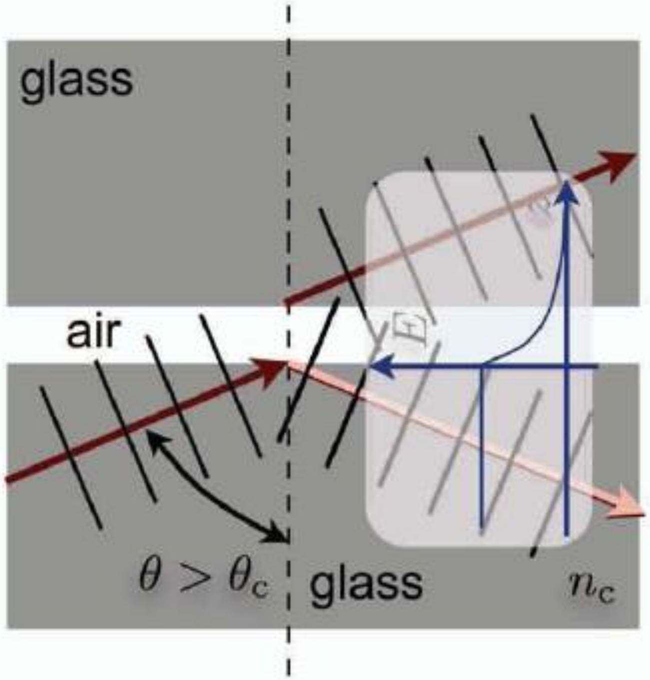
the result is called a "surface wave"

at angles of incidence higher than critical,
the light is totally internally reflected
almost as if
the glass-air interface were a mirror

in both cases of surface wave and TIR,
there is an exponential tail of electric field
leaking into the medium of lower optical density;
this is called the **evanescent wave.**

Typically, the evanescent wave is a few wavelengths long
but it can be much longer near the critical angle

Frustrated Total Internal Reflection (FTIR)

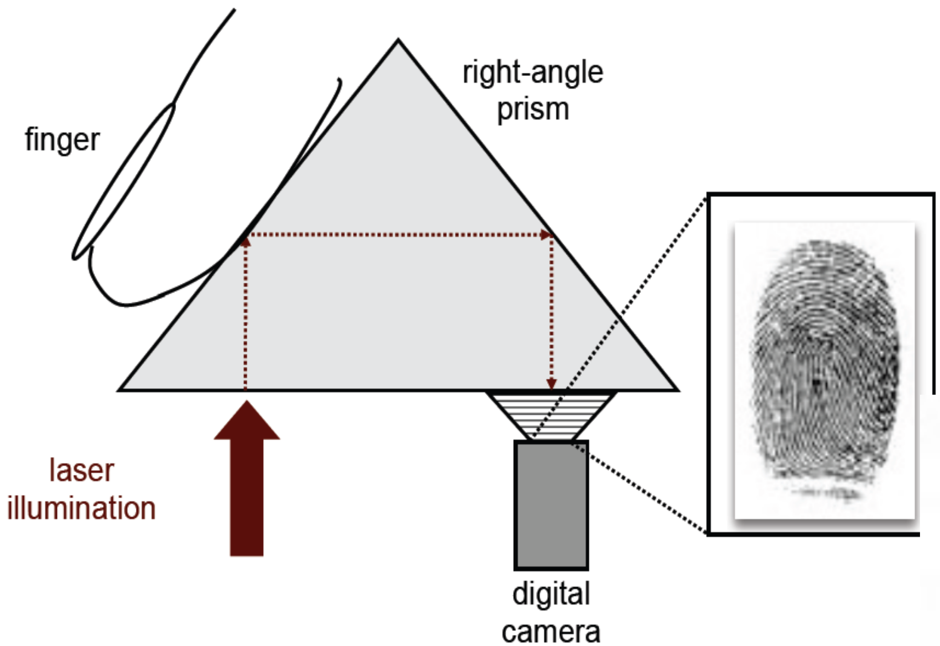


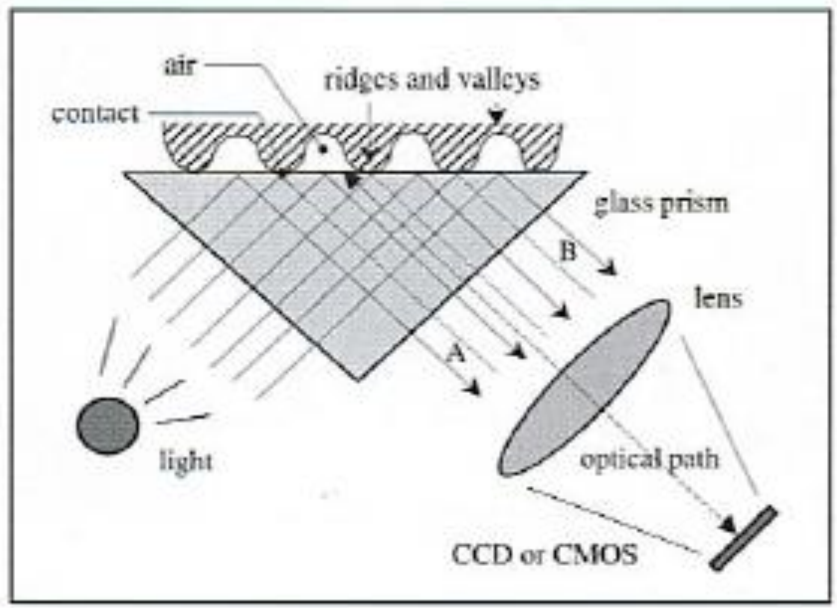
If another dielectric approaches within a few distant constants from the TIR interface, the tail of the exponentially evanescent wave becomes propagating; *i.e.* the light couples out of the medium.

This situation is known as *Frustrated Total Internal Reflection (FTIR)*.

In quantum mechanics, there is an analogous effect known as *tunneling*.

Fingerprint sensors



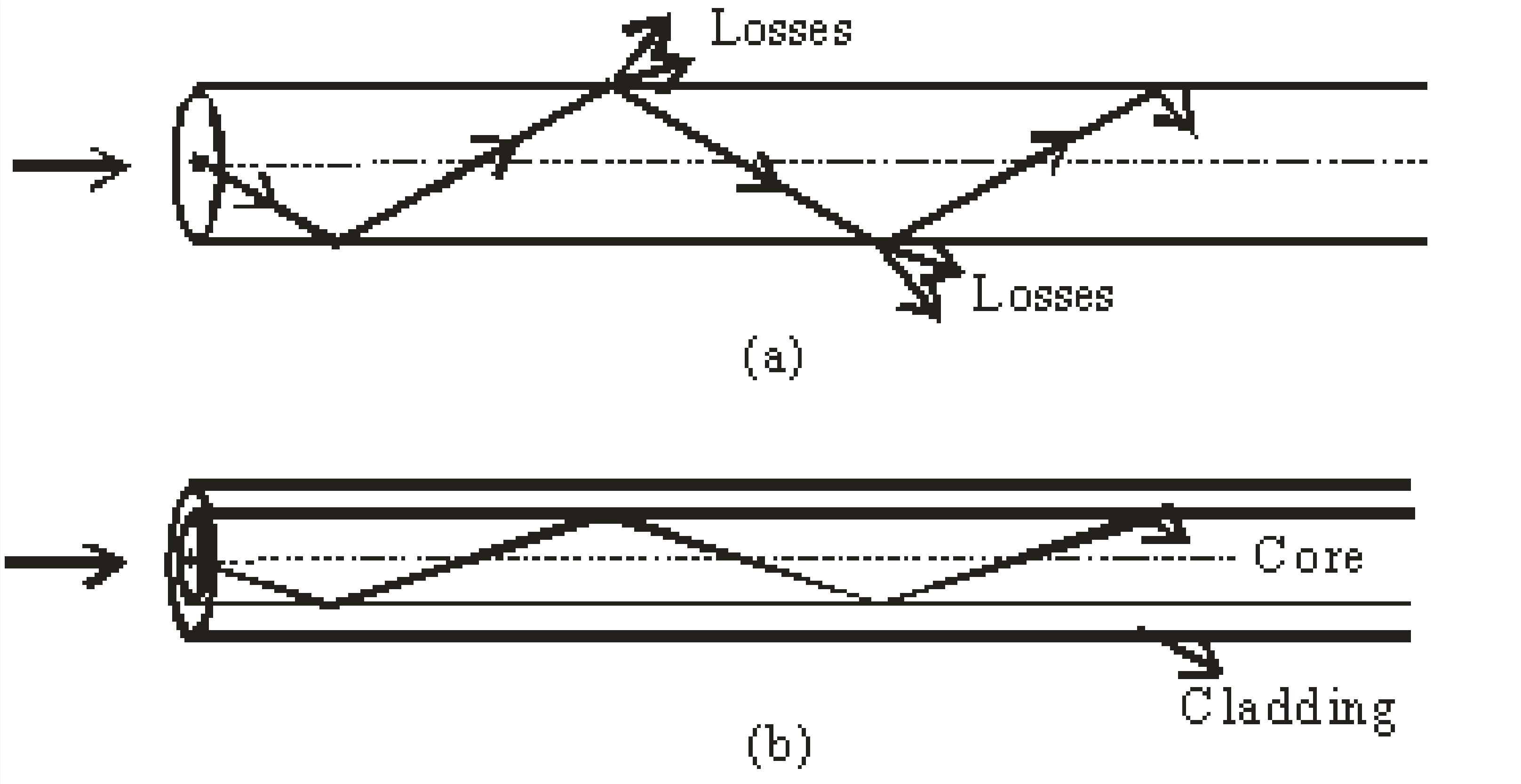


FTIR-based fingerprint sensing.

TOTAL INTERNAL REFLECTION
(where is it useful)

Optical waveguides

"planar" waveguide: high-index dielectric material sandwiched between lower-index dielectrics

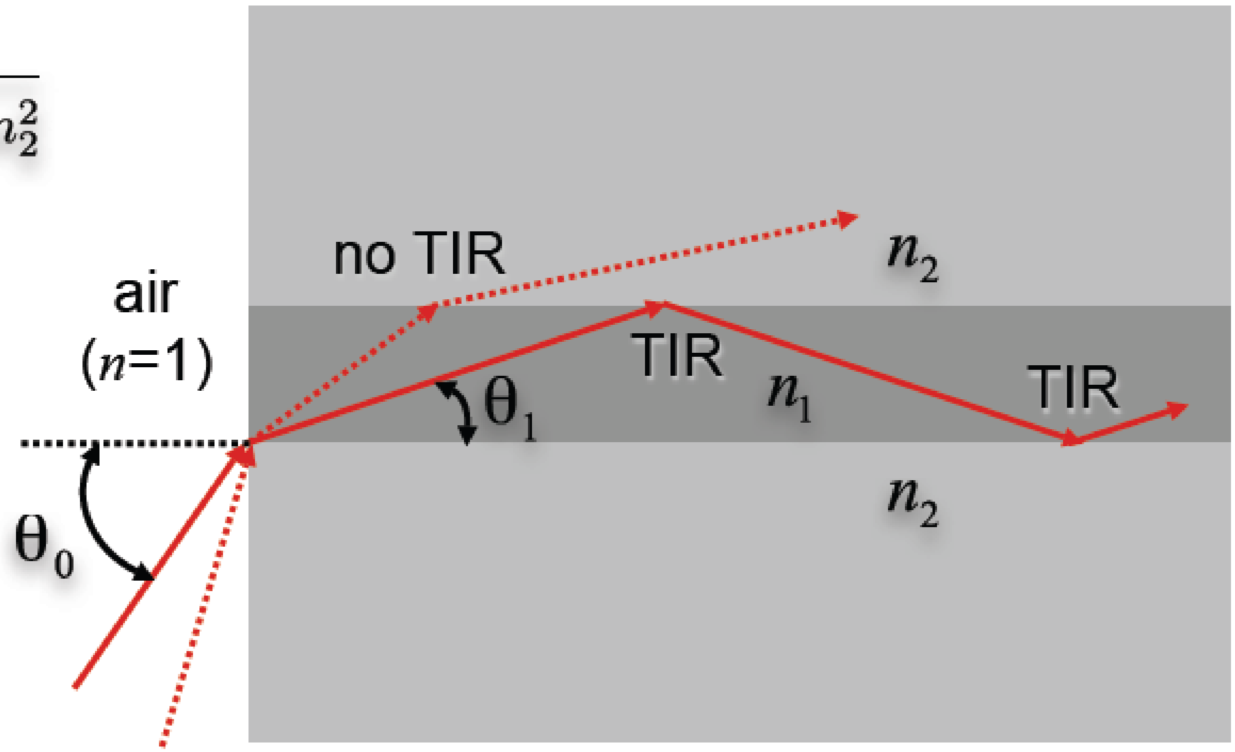


Light guides (a) Simple glass rod (b) Glass rod and cladding with different refraction qualities

Numerical Aperture (NA) of a waveguide

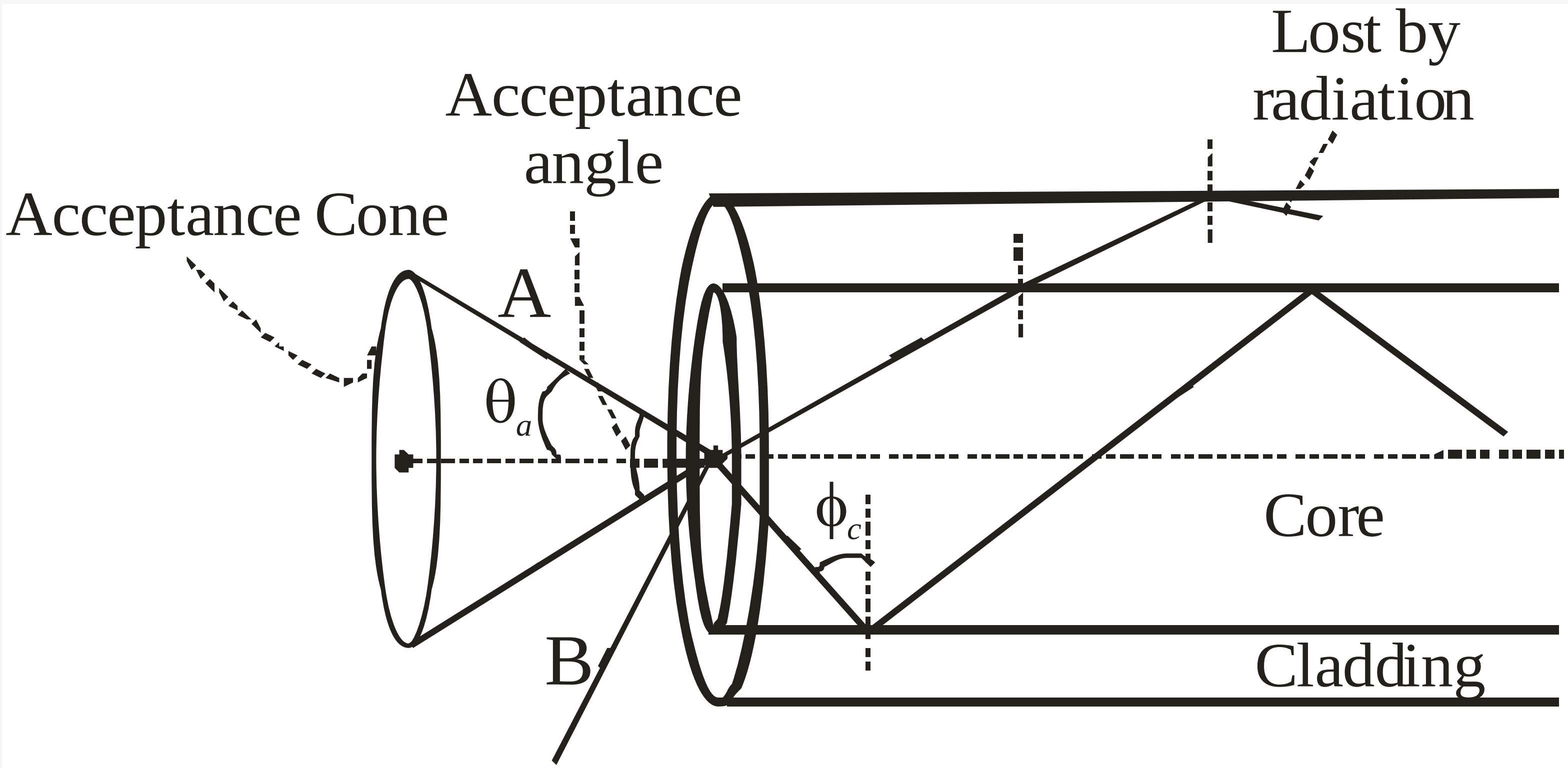
NA is the sine of the largest angle that is waveguided

$$(\mathrm{NA}) \equiv \sin\theta_0 \leq \sqrt{n_1^2 - n_2^2}$$



i.e., NA is the incident *angle of acceptance* of the waveguide

high index contrast (n_1/n_2) ⇔ high NA



Acceptance angle

$$N.A = (n_1^{\,2} - n_2^{\,2})^{1/2}$$

The half acceptance angle θ_0 is given by

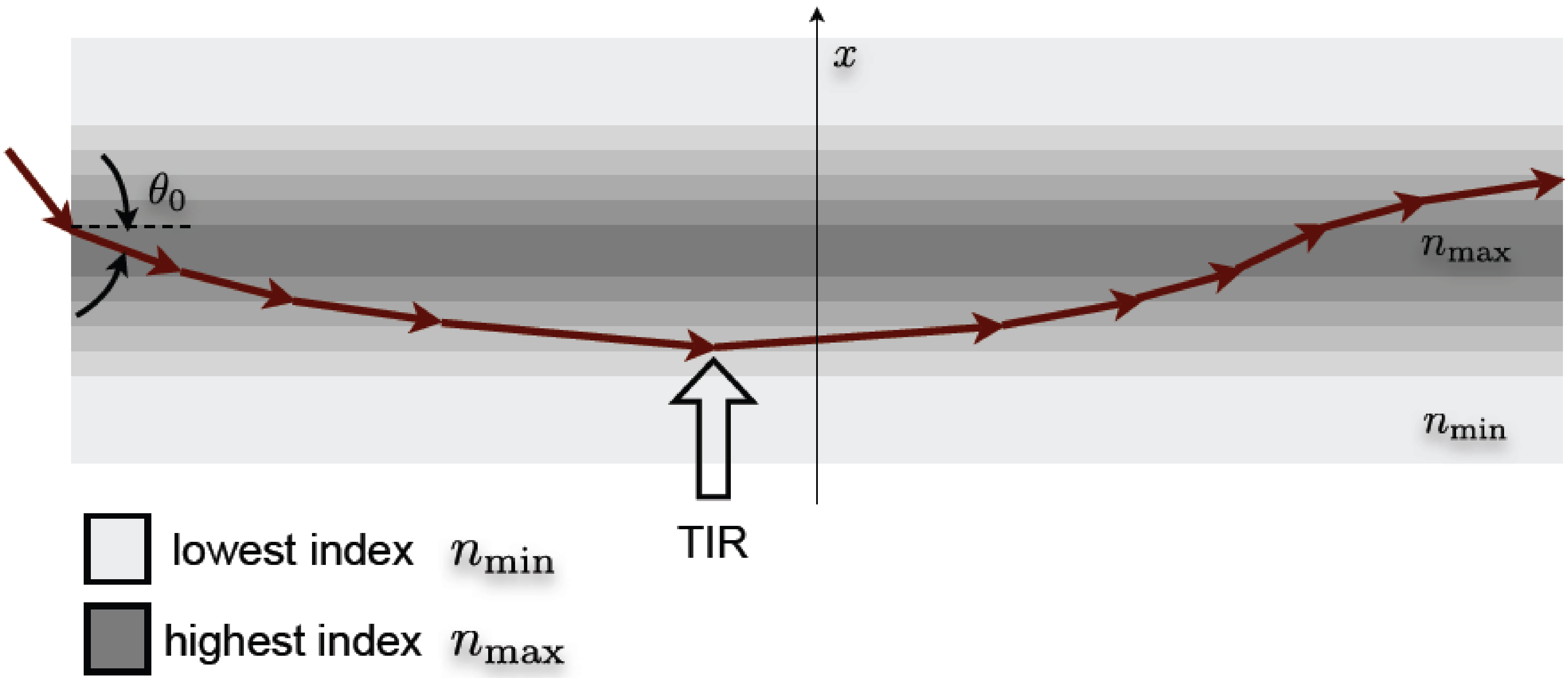
$$\theta_0 = \sin^{-1}(N.A)$$

$$\Delta = \frac{n_1^{\,2} - n_2^{\,2}}{2n_1^{\,2}}$$

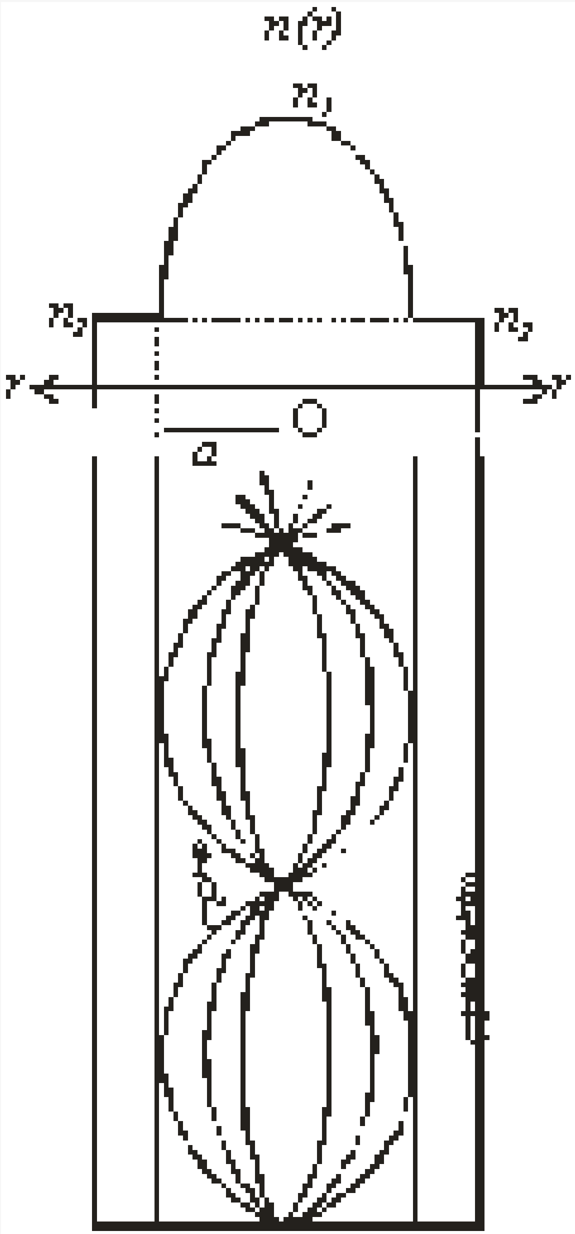
$$\mathrm{N.A} = n_1 \times (2\Delta)^{1/2}$$

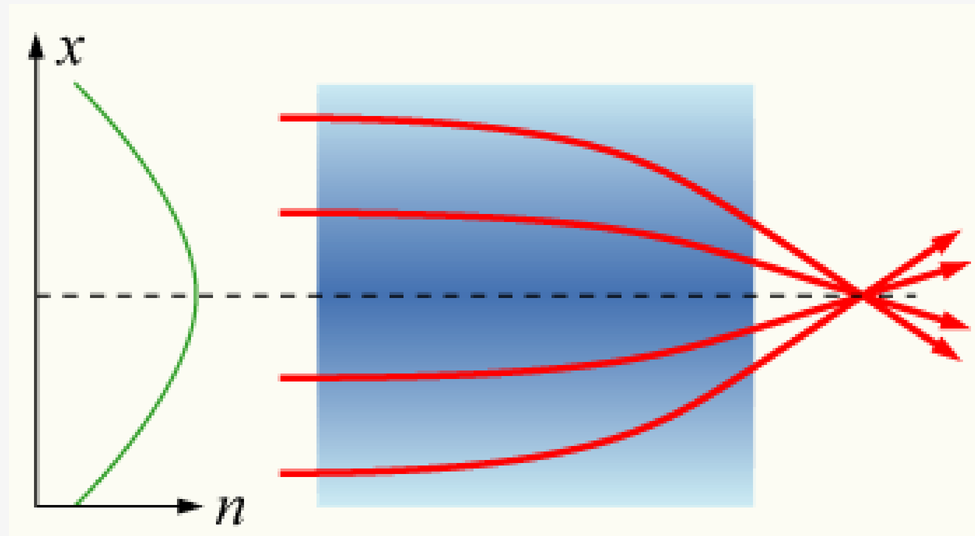
Δ is the relative refractive index

GRadient INdex (GRIN) waveguide



If the TIR condition $n_{\max} \sin\theta_0$ is satisfied,
TIR will *always* occur at one of the outer cladding interfaces;
therefore, the ray bends backwards and is guided by the GRIN structure.

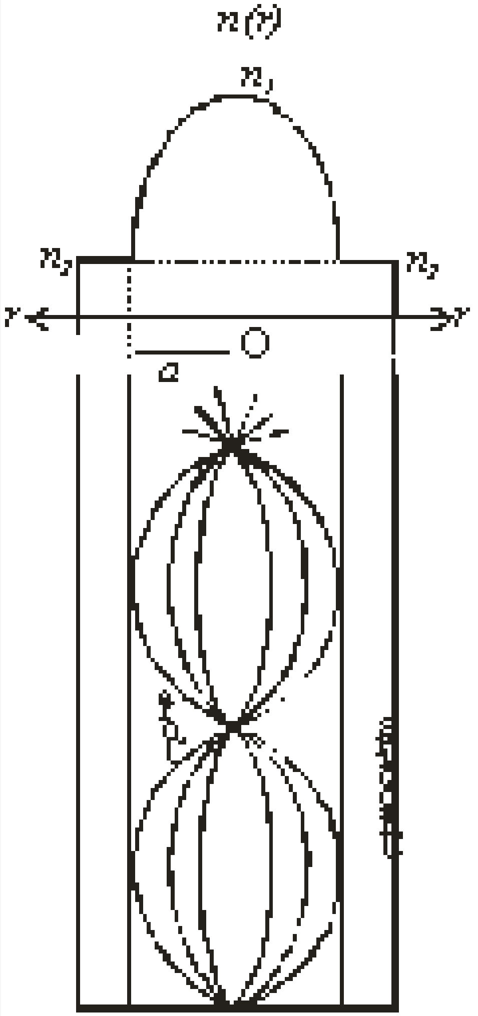




A gradient-index lens with a parabolic variation of refractive index (*n*) with radial distance (*x*). The lens focuses light in the same way as a conventional lens.

Ref: https://en.wikipedia.org/wiki/Gradient-index_optics

The refractive index (n) profile with reference to the radial distance (r) from the fiber axis is given as:



when $r = 0$, $n(r) = n_1$

$$r < a, \quad n(r) = n_1\left[1-\left(2\Delta\left[\frac{r}{a}\right]^2\right)\right]^{\frac{1}{2}}$$

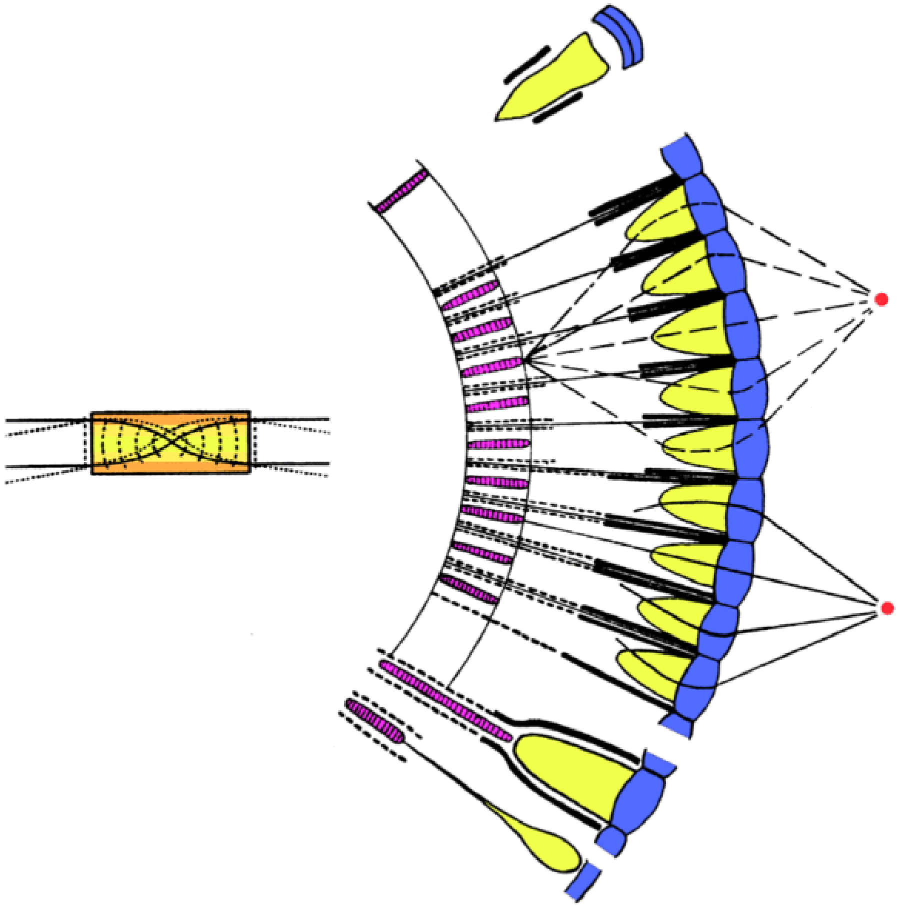
$$r \geq a, \quad n(r) = n_2 = n_1(1-2\Delta)^{\frac{1}{2}}$$

At the fiber center we have n_1; at the cladding we have n_2; and in between we have $n(r)$, where n is the function of the particular radius as shown in Fig. simulates the change in n in a stepwise manner. Δ = relative refractive indices difference.

The refractive index of the core is made to vary in the form of **parabolic manner** such that the maximum refractive index is present at the center of the core.

GRIN waveguides in nature: insect eyes

Image by Thomas Bresson at Wikimedia Commons.



Human eye is one of the examples of GRIN lenses.

1. Next Class is on How is light Generated.
2.Two classes after next class is on Geometrical Optics: Derivations based on Fermat's Principle and Ray transfer Matrix.
3. Next Class after that will be on Aberration.
4. And last two classes on Physics of Lasers.

Question for you ☺

1. Calculate the numerical aperture and acceptance angle of fiber with a core index of 1.50 and a cladding index of 1.48.

2. Calculate the numerical aperture and acceptance angle of a optical fiber having core refractive index of 1.48 and relative index of 0.02.