AERO-423, Spring 2024, Homework #1
(**Due date: 23:59 hours, Wednesday, February 7, 2024**)

Show all the work and justify your answer!
Make sure to upload your submission to the Canvas, in time.

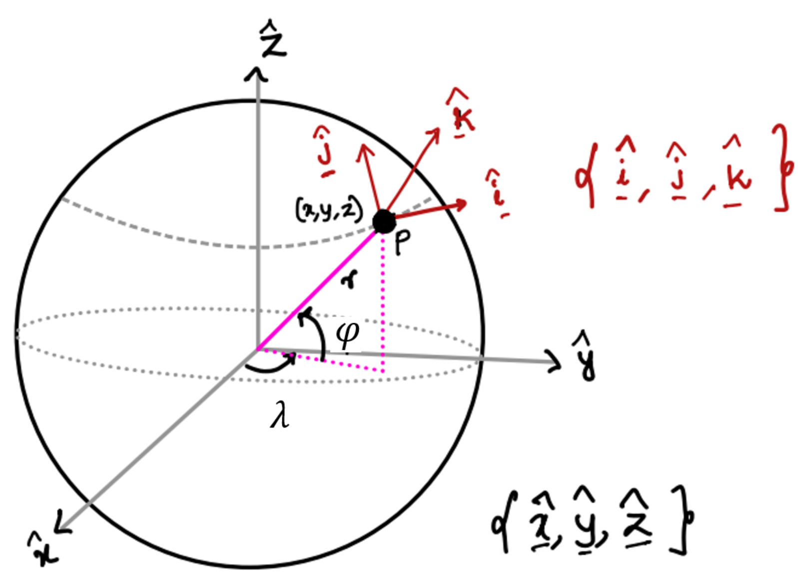
1. A direction cosine matrix (DCM) $C_{\mathcal{BA}}$ represents transformation from coordinate frame $\mathcal{A}$ to frame $\mathcal{B}$. The vectors $\boldsymbol{v}_{\mathcal{A}}$ and $\boldsymbol{v}_{\mathcal{B}}$ are the vector $\boldsymbol{v}$ in frames $\mathcal{A}$ and $\mathcal{B}$, respectively. Answer the following.

Part a. (2 points) Express the relationship between $\boldsymbol{v}_{\mathcal{B}}$ and $\boldsymbol{v}_{\mathcal{A}}$. Using $C_{\mathcal{BA}}$ write the expressions for both $\boldsymbol{v}_{\mathcal{B}}$ and $\boldsymbol{v}_{\mathcal{A}}$.

Part b. (2 points) If the direction cosine matrix C is parameterized using the 3-2-1 Euler angle sequence. The angles are ϑ_1, ϑ_2, and ϑ_3 . Show the steps in the computation of C.

Part c. (10 points) Determine the forward and inverse kinematic equations associated with the "3-2-1" Euler angle rotation sequence. Specify frame transformations and explain your approach. Is there any singularity?

2. The position of a point P can be specified using Cartesian coordinates (x, y, z) or spherical coordinates (r, φ, λ), where r is the radial distance of P from origin of the $\{\hat{\boldsymbol{x}}, \hat{\boldsymbol{y}}, \hat{\boldsymbol{z}}\}$ coordinate system. Angles λ and φ denote azimuth and elevation angles respectively as shown in the below figure. The unit vectors $\{\hat{\boldsymbol{i}}, \hat{\boldsymbol{j}}, \hat{\boldsymbol{k}}\}$ form the spherical coordinate system to represent any arbitrary point P on the sphere.



Part a. (2 points) Express the Cartesian coordinates (x, y, z) in terms of spherical coordinates (r, φ, λ) and vice-versa. For simplification, you may consider (x, y, z) are positive values.

Part b. (6 points) The equations transforming the unit-vectors $\{\hat{\boldsymbol{x}}, \hat{\boldsymbol{y}}, \hat{\boldsymbol{z}}\}$ to the unit-vectors $\{\hat{\boldsymbol{\imath}}, \hat{\boldsymbol{\jmath}}, \hat{\boldsymbol{k}}\}$ are,

$$\begin{aligned}
\hat{\boldsymbol{\imath}} &= -\sin\lambda\,\hat{\boldsymbol{x}} + \cos\lambda\,\hat{\boldsymbol{y}} \\
\hat{\boldsymbol{\jmath}} &= -\sin\varphi\cos\lambda\,\hat{\boldsymbol{x}} - \sin\varphi\sin\lambda\,\hat{\boldsymbol{y}} + \cos\varphi\,\hat{\boldsymbol{z}} \\
\hat{\boldsymbol{k}} &= \cos\varphi\cos\lambda\,\hat{\boldsymbol{x}} + \cos\varphi\sin\lambda\,\hat{\boldsymbol{y}} + \sin\varphi\,\hat{\boldsymbol{z}}
\end{aligned}$$

Prove that the angular velocity, $\boldsymbol{\omega}$, of the $\{\hat{\boldsymbol{\imath}}, \hat{\boldsymbol{\jmath}}, \hat{\boldsymbol{k}}\}$ frame relative to the inertial $\{\hat{\boldsymbol{x}}, \hat{\boldsymbol{y}}, \hat{\boldsymbol{z}}\}$ frame can be written as,

$$\boldsymbol{\omega} = a\,\hat{\boldsymbol{\imath}} + b\,\hat{\boldsymbol{\jmath}} + c\,\hat{\boldsymbol{k}}$$

by finding the expressions of a, b, and c, in terms of φ, λ, and their derivative, $\dot{\varphi}$ and $\dot{\lambda}$.

HINT: Use transport theorem, e.g., $\dfrac{\mathrm{d}\hat{\boldsymbol{k}}}{\mathrm{d}t} = \boldsymbol{\omega} \times \hat{\boldsymbol{k}}$.

Part c. (3 points) Validate that

$$\frac{\mathrm{d}\hat{\boldsymbol{\imath}}}{\mathrm{d}t} = \boldsymbol{\omega} \times \hat{\boldsymbol{\imath}} \qquad \frac{\mathrm{d}\hat{\boldsymbol{\jmath}}}{\mathrm{d}t} = \boldsymbol{\omega} \times \hat{\boldsymbol{\jmath}} \qquad \frac{\mathrm{d}\hat{\boldsymbol{k}}}{\mathrm{d}t} = \boldsymbol{\omega} \times \hat{\boldsymbol{k}}$$