AERO-423, Spring 2024, Homework #3
(Due date: 23:59 hours, Thursday, March 21, 2024)

NAME:______________________________

Show all the work and justify your answer.
Upload your submission to the CANVAS, in time.
This is a long homework. Start right now!

1-a. (8 points) An astronomer observes the following position $\boldsymbol{r}$ and velocity $\boldsymbol{v}$ vectors of the ISS in the inertial frame.

$$\boldsymbol{r} = \begin{Bmatrix} -10,063.829 \\ -473.07 \\ -12,487.599 \end{Bmatrix} \text{ (km)} \qquad \text{and} \qquad \boldsymbol{v} = \begin{Bmatrix} -0.359 \\ -4.950 \\ 0.475 \end{Bmatrix} \text{ (km/s)}$$

Derive the Keplerian ISS orbital elements: a, e, i, Ω, ω and the true anomaly. φ. Use $\mu = 398,600$ km/s^2.

1-b. (8 points) Convert the following orbital elements,

$$\begin{Bmatrix} h \\ i \\ \Omega \\ e \\ \omega \\ \theta \end{Bmatrix} = \begin{Bmatrix} 80,000\,\text{km}^2/\text{s} \\ 51.2^\circ \\ 250^\circ \\ 0.00083 \\ 240^\circ \\ 15^\circ \end{Bmatrix}$$

into Cartesian coordinates, position $\boldsymbol{r}$ (km) and velocity $\boldsymbol{v}$ (km/s).

2. Coding problem (14 pts). Two spacecrafts A and B are in a same circular orbit of radius $R = 13,600$ km as shown in the figure 1. Spacecraft B is initially at β angle ahead of A. Perform a lower rendezvous maneuver to rendezvous with B by $k = 5$ revolutions.

Write a code that plot the total Δv as a function of the angle $\beta \in [10, 350]$ deg and the altitude (in km) of the perigee of the transfer orbit

3. (15 pts) For the following two cases determine the required launch azimuth for a satellite it is launched from Cape Canaveral (latitude = 28.5°N). Also answer the following for each of the cases (a) is direct launch to specified parking orbit feasible and why?, (b) if applicable, specify when to apply orbit plane corrections, (c) if applicable, specify the Δv impulse for the correction. Consider $\mu = 398600$ km^3/s^2. Angular momentum direction $\hat{\boldsymbol{h}} = \left\{-\sin i\,\sin\Omega,\ \ \sin i\,\cos\Omega,\ \ \cos i\right\}^{\mathrm{T}}$.

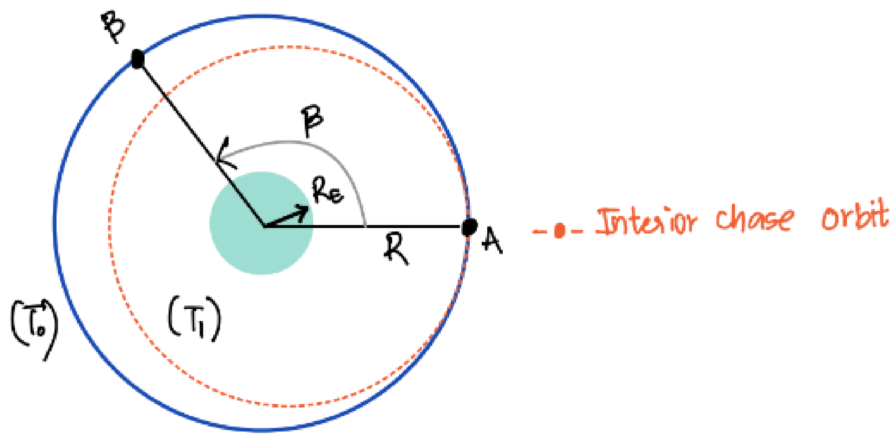

Figure 1: Rendezvous geometry for problem #3

- **3-a. (5 points)** A satellite is to be launched into a circular orbit with a period of 100 minutes, inclination of 98.43°, $\Omega = 105°$.
- **3-b. (10 points)** A satellite is to be launched into a circular orbit with a period of 100 minutes, inclination of 28° and $\Omega = 105°$.

4. Coding problem (15 pts) A spacecraft is in a 300 km circular Earth orbit. Use $\mu = 398,600.4415$ km^3/s^2 and $R_E = 6,378.135$ km. Calculate the total Δv required for the bi-elliptic transfer to a 3,000 km altitude co-planar circular orbits. Use bisection approach to find the root (unknown radius "r_b").

5. (15 pts) Using the data provided in fig. 2, consider the mission Saturn to Mars and back to Saturn. Consider all planets in circular orbits (radius = semi-major axis) and at inclination $i = 0$. Compute the following,

1. The departure angle;
2. The total Δv
3. the waiting time in Mars for starting an Homann transfer back to Saturn.

Object	Radius (km)	Mass (kg)	Sidereal Rotation Period	Inclination of Equator to Orbit Plane	Semimajor Axis of Orbit (km)	Orbit Eccentricity	Inclination of Orbit to the Ecliptic Plane	Orbit Sidereal Period
Sun	696,000	1.989×10^{30}	25.38d	7.25°	—	—	—	—
Mercury	2440	330.2×10^{21}	58.65d	0.01°	57.91×10^{6}	0.2056	7.00°	87.97d
Venus	6052	4.869×10^{24}	243d*	177.4°	108.2×10^{6}	0.0067	3.39°	224.7d
Earth	6378	5.974×10^{24}	23.9345h	23.45°	149.6×10^{6}	0.0167	0.00°	365.256d
(Moon)	1737	73.48×10^{21}	27.32d	6.68°	384.4×10^{3}	0.0549	5.145°	27.322d
Mars	3396	641.9×10^{21}	24.62h	25.19°	227.9×10^{6}	0.0935	1.850°	1.881y
Jupiter	71,490	1.899×10^{27}	9.925h	3.13°	778.6×10^{6}	0.0489	1.304°	11.86y
Saturn	60,270	568.5×10^{24}	10.66h	26.73°	1.433×10^{9}	0.0565	2.485°	29.46y
Uranus	25,560	86.83×10^{24}	17.24h*	97.77°	2.872×10^{9}	0.0457	0.772°	84.01y
Neptune	24,760	102.4×10^{24}	16.11h	28.32°	4.495×10^{9}	0.0113	1.769°	164.8y
(Pluto)	1195	12.5×10^{21}	6.387d*	122.5°	5.870×10^{9}	0.2444	17.16°	247.7y

Figure 2: Solar system data (from Curtis)