AERO-423, Spring 2024, Homework 5
(Due date: Thursday April 25, 2024)

NAME:_____

Show all work and justify your answer!

For all problems use: $J_2 = 1.082645 \cdot 10^{-3}$, $\mu = 398,600.4415$ km^3/s^2, and (Earth radius) $R_\oplus = 6,378.135$ km.

1. (8 pts) A satellite is in a "Molniya" orbit with an apogee altitude of $39,906$ km over the Russian territory, with an inclination of $63.435°$, and with argument of perigee $270°$. The period of this orbit is half a sidereal day. Considering only the J_2 effect, answer the following:

a. (1 points) Is Russia toward Northern point of the orbit or is it towards the South?

b. (3 points) Find the perturbation (rate change) in the argument of perigee (degrees/day). How does the perigee position change over time?

c. (2 points) Find the perturbation (rate change) of node line (degrees/day).

c. (2 points) Change in orbital period.

2. (5 pts) Compute the yearly perigee maintenance cost of a satellite that is almost in a Tundra orbit. The orbital parameters are:

$a = 42,164$ km, $e = 0.24$, $i = 70$ deg, $\omega = 270$ deg. Compute the annual perigee maintenance cost using a radial impulse in terms of percentage $\dfrac{\Delta m}{m_0}$ using $I_{\text{sp}} = 300$ s, $g = 9.81$ m/s^2, 1 year = 365.25 days and use $\dfrac{\Delta m}{m_0} = 1 - e^{-\Delta v/(g\, I_{\text{sp}})}$. Why do you think the perigee maintenance cost is low or high?

Consider only the J_2 effect.

3. (6 points) The following are the estimated attitudes of a spacecraft using three different coordinates (at different instances).

i. quaternion

$$\boldsymbol{q} = \begin{Bmatrix} 0.0921, & 0.2306, & 0.4191, & 0.8733 \end{Bmatrix}^T$$

ii. Principal axis-angle ($\boldsymbol{e}$ and ϕ)

$$\boldsymbol{e} = \begin{Bmatrix} 0.1382, & 0.4075, & 0.9027 \end{Bmatrix}^T \qquad \phi = 61.1403°$$

iii. Gibbs vector

$$\boldsymbol{\rho} = \begin{Bmatrix} 0.0890, & 0.2976, & 0.4994 \end{Bmatrix}^T$$

If the true attitude quaternion is

$$\boldsymbol{q}_{\mathrm{t}} = \begin{Bmatrix} 0.0742, & 0.2363, & 0.4418, & 0.8623 \end{Bmatrix}^{T}$$

a. (3 points) Compute the attitude errors in the estimated attitudes.

b. (3 points) If $\boldsymbol{r} = \begin{Bmatrix} 0.1250, & -0.5735, & -0.8096 \end{Bmatrix}^{\mathrm{T}}$ is a measured inertial direction, compute the direction errors of $\boldsymbol{r}$ that is measured using the estimated attitudes (i, ii, iii).

Report all angles in degrees.

4. (3+3 points) Determine the attitude (quaternion and DCM) using the Davenport's q-method and the TRIAD method. The inertial ($\boldsymbol{r}$) and the observed ($\boldsymbol{b}$) unit-vectors are as follows:

$$\begin{aligned}
\boldsymbol{r}_1^{\mathrm{T}} &= \begin{Bmatrix} 0.1732, & 0.3293, & -0.9282 \end{Bmatrix} \\
\boldsymbol{r}_2^{\mathrm{T}} &= \begin{Bmatrix} -0.4056, & -0.5613, & 0.7214 \end{Bmatrix} \\
\boldsymbol{b}_1^{\mathrm{T}} &= \begin{Bmatrix} -0.3306, & -0.3173, & 0.8888 \end{Bmatrix} \\
\boldsymbol{b}_2^{\mathrm{T}} &= \begin{Bmatrix} 0.5563, & 0.5208, & -0.6475 \end{Bmatrix}
\end{aligned}$$

Use the weights: $[\alpha_1, \alpha_2] = [1/3, 1/4]$.