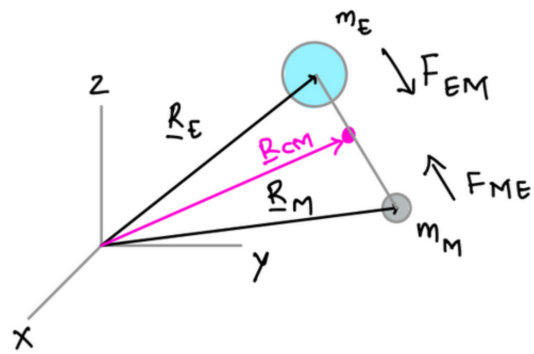
AERO-423, Spring 2024, Homework #2
(Due date: 23:59 hours, Sunday, February 25, 2024)

Show all the work and justify your answer!
Make sure to upload your submission to the Canvas, in time.

1. Masses of the Earth and the Moon are given as m_{E} and m_{M} respectively. Their respective position vectors are given as $\boldsymbol{R}_{\mathrm{E}}$ and $\boldsymbol{R}_{\mathrm{M}}$.



For the Earth-Moon system described in the figure, answer the following questions:

Part a. (1 points) Position vector of the center of mass, $\boldsymbol{R}_{\mathrm{CM}}$

Part b. (1 points) If the relative distance between the Earth and the Moon is $\boldsymbol{r} = \boldsymbol{R}_{\mathrm{E}} - \boldsymbol{R}_{\mathrm{M}}$, write the force of Moon on the Earth $\boldsymbol{F}_{\mathrm{EM}}$ and the force of Earth on the Moon $\boldsymbol{F}_{\mathrm{ME}}$. The gravitational constant is G.

Part c. (2 points) Derive relative equations of motion

Part d. (4 points) Prove that the equation of motion of the Earth with respect to the center of mass of the Earth-Moon system is

$$\ddot{\boldsymbol{r}} = -\frac{G m_{\mathrm{M}}^3}{(m_{\mathrm{E}} + m_{\mathrm{M}})^2 r^3} \boldsymbol{r}$$

where $\boldsymbol{r} = \boldsymbol{R}_{\mathrm{E}} - \boldsymbol{R}_{\mathrm{CM}}$.
Also, derive the equation of motion for m_{M} with respect to $\boldsymbol{R}_{\mathrm{CM}}$ with $\boldsymbol{r}' = \boldsymbol{R}_{\mathrm{M}} - \boldsymbol{R}_{\mathrm{CM}}$.

2. Consider a satellite orbiting a planet of radius R_p with gravitational parameter μ. At perigee, if the position (vector) $\boldsymbol{r}$ and velocity $\boldsymbol{v}$ of the satellite are given as

$$\boldsymbol{r} = 3R_p\,\hat{\boldsymbol{e}} \qquad \text{and} \qquad |\boldsymbol{v}| = \sqrt{\frac{5\mu}{12\,R_p}}$$

where $\hat{\boldsymbol{e}}$ is the unit-vector pointing to perigee. Using only the given information answer the following:

Part a. (3 points) Semi-major axis, a

Part b. (1 point) Eccentricity, e

Part c. (1 point) Radius at apogee, r_a

Part d. (1 point) Semi-latus rectum, p

Part e. (1 point) Magnitude of angular momentum, h

Part f. (1 point) Magnitude of velocity at apogee, v_a

Part g. (1 point) True anomaly, φ, (in degrees) at $r = 3.5\,R_p$

Part h. (1 point) Tangential and radial velocities, $v_\perp$ and v_r, at φ

Part i. (1 point) Eccentric anomaly, E (in degrees)

Part j. (1 point) Mean anomaly, M (in degrees)

3. (5 points) Using the mean anomaly M and eccentricity e obtained in problem 2, write a MATLAB/Python *Kepler solver function* to solve for eccentric anomaly using Newton's method. Write the equations, and comment on the choice of initial value of E, convergence, tolerance, and number of iterations. Plot the values of E (y-axis) vs iterations (x-axis). Attach your code and the plot to this submission.