AERO-423, Spring 2024, Homework #4
(Due date: 23:59 hours, Thursday, April 11, 2024)

Show all the work and justify your answer!
Make sure to upload your submission to the Canvas, in time.

1. Consider the following interplanetary trajectory problems using the interplanetary data given in the Curtis book (Figure 4).

Part a. (4 points) Consider a Venus orbiter launched via a Hohmann transfer from the Earth. The orbiter is to be placed in a circular capture orbit around Venus. Calculate: (i) altitude of the capture orbit that minimizes the Δv needed for the capture, (ii) the minimum/optimal Δv, and (iii) optimum approach distance d.

Solution:

- Hyperbolic excess speed $v_\infty = \sqrt{\frac{\eta_{\text{sun}}}{R_V}}\left(1 - \sqrt{\frac{2R_E}{R_E+R_V}}\right) = 2.7074$ km/s.
- Capture radius: $r_p = \dfrac{2\mu_V}{v_\infty^2} = 88,648$ km.
- Altitude of the parking orbit: $88,648.15 - 6,052 = 82,596$ km.
- $\Delta v_{\min} = \dfrac{v_\infty}{\sqrt{2}} = 1.9144$ km/s
- Approach distance $d_{\text{opt}} = 2\sqrt{2}\dfrac{\mu_V}{v_\infty^2} = 125,376$ km.

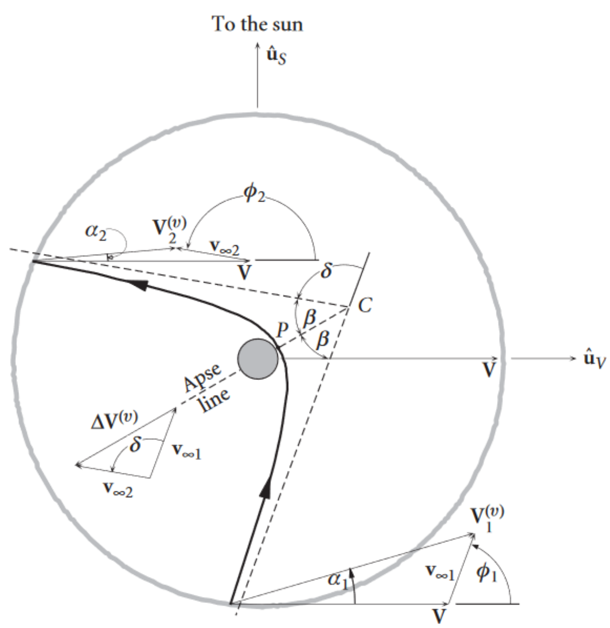


Figure 1: Planetary flyby example.

Part b. (6 points) A spacecraft on a Hohmann transfer from the Earth, performs a flyby maneuver at Venus at an altitude 500 km. The following information is given on the inbound and outbound velocity vectors $\boldsymbol{V_1}^{(v)}$ and $\boldsymbol{V_2}^{(v)}$ in the geocentric coordinate system $\{\hat{\boldsymbol{u}}_V, \hat{\boldsymbol{u}}_S\}$ as shown in figure 1:

$$\boldsymbol{V_1}^{(v)} = 36.5\hat{\boldsymbol{u}}_V + 2.5\hat{\boldsymbol{u}}_S$$
$$\boldsymbol{V_2}^{(v)} = 32.117\hat{\boldsymbol{u}}_V + 0.119\hat{\boldsymbol{u}}_S$$

Compute the following:

1. Angular momentum and eccentricity of the flyby hyperbola.
2. Hyperbolic excess velocities at the inbound and the outbound crossings.
3. Compute the turning angle and state whether it is a leading or a trailing flyby.
4. Calculate the ΔV imparted to the spacecraft during the flyby maneuver (change in the velocity of the spacecraft between when it enters at inbound and crosses at outbound).

Solution: Velocity of Venus relative to the Sun is

$$\boldsymbol{V}_{\text{Venus}} = \sqrt{\frac{\eta_s}{R_v}}\hat{\boldsymbol{u}}_V + 0\hat{\boldsymbol{u}}_S = 35.02\hat{\boldsymbol{u}}_V$$

Hyperbolic excess velocities at the inbound (1) and the outbound crossings are

$$\boldsymbol{V}_{\infty 1} = \boldsymbol{V_1}^{(v)} - \boldsymbol{V}_{\text{Venus}} = 1.48\hat{\boldsymbol{u}}_V + 2.5\hat{\boldsymbol{u}}_S$$
$$v_{\infty 1} = 2.905\,\mathbf{km/s}$$
$$\boldsymbol{V}_{\infty 2} = \boldsymbol{V_2}^{(v)} - \boldsymbol{V}_{\text{Venus}} = -2.9\hat{\boldsymbol{u}}_V + 0.1196\hat{\boldsymbol{u}}_S$$
$$v_{\infty 2} = 2.905\,\mathbf{km/s}$$

Periapse radius

$$r_p = R + 500 = 6552\text{km}$$

1. Angular momentum and eccentricity of the flyby hyperbola.

$$h = r_p\sqrt{v_{\infty 1}^2 + \frac{2\eta_v}{r_p}} = 67969.29\,\text{km}^2/\text{s}$$
$$e = 1 + \frac{r_p v_{\infty 1}^2}{\eta_v} = 1.1702$$

2. Hyperbolic excess velocities at the inbound and the outbound crossings. $v_{\infty 1} = v_{\infty 2} = 2.9054$ km/s.
3. Compute the turning angle and state whether it is a leading or a trailing flyby.

$$\delta = \sin^{-1}(1/e) = 117.4194°$$

$\delta => 0 \implies$ leading-side flyby.
Alternately, compute angles between $\boldsymbol{V}_{\infty 1}$ and the planet's heliocentric velocity, as ϕ_1. Similarly, at the outbound, ϕ_2.

$$\phi_i = \tan^{-1}\frac{(v_{\infty_i})_S}{(v_{\infty_i})_V}; \qquad \phi_1 = 59.37^\circ, \quad \phi_2 = 176.79^\circ$$
$$\delta = \phi_2 - \phi_1 \approx 117.419^\circ$$

4. Calculate the ΔV imparted to the spacecraft during the flyby maneuver. Outbound crossing speed is 4.468 km/s less than the inbound crossing speed.

$$\Delta V = |\boldsymbol{V_2}^{(v)}| - |\boldsymbol{V_1}^{(v)}| = -4.4681\,\text{km/s}$$

Note: $\Delta\mathcal{E} < 0$ for this problem. Not practical.

2. Assume that ISS is in a 400 km circular orbit. An approaching Dragon spacecraft aims to rendezvous with the ISS by executing a Δv burn. At $t = 0$, the position vector $\delta\boldsymbol{r}_0$ and the before burn velocity $\delta\boldsymbol{v}_0^-$ relative to the ISS (for part a.) are as follows.

$$\delta\boldsymbol{r}_0 = \{0,\ -2,\ 0.5\}^{\mathrm{T}}\ \text{km}$$
$$\delta\boldsymbol{v}_0^- = \{0,\ 0,\ 5\}^{\mathrm{T}}\ \text{m/s}$$

Part a. (4 points) Calculate the total Δv required for the Dragon spacecraft to rendezvous with ISS in one-third of the ISS's orbital period.

Solution:

For the circular orbit

$$v = \sqrt{\frac{\eta}{r}} = \sqrt{\frac{398,600}{6778}} = 7.6686\,\text{km/s}$$
$$T = 2\pi\sqrt{\frac{R^3}{\eta}} = 5,553.5\,\text{s}$$

Time to rendezvous $t = T/3 = 1,388.4$ s.

From the computed Clohessey-Wiltshire matrices, and that $\delta\boldsymbol{r}_f = \{0,\ 0,\ 0\}^{\mathrm{T}}$, and $\delta\boldsymbol{v}_f^+ = \{0,\ 0,\ 0\}^{\mathrm{T}}$

$$\delta\boldsymbol{v}_0^+ = -[\Phi_{rv}]^{-1}[\Phi_{rr}]\delta\boldsymbol{r}_0$$
$$\delta\boldsymbol{v}_f^- = [\Phi_{vr}]\delta\boldsymbol{r}_0 + [\Phi_{vv}]\delta\boldsymbol{v}_0^+$$
$$\delta\boldsymbol{v}_0 = \boldsymbol{v}_0^+ - \delta\boldsymbol{v}_0^- = \{-1.03,\ 0.298,\ -0.467\}^{\mathrm{T}}\,\text{m/s}$$
$$\delta\boldsymbol{v}_f = \boldsymbol{v}_f^+ - \delta\boldsymbol{v}_f^- = \{-1.035,\ -0.298,\ 0.6532\}^{\mathrm{T}}\,\text{m/s}$$
$$\Delta v = ||\delta v_f|| + ||\delta v_0|| = 6.1\,\text{m/s}$$

Part b. (4 points) Suppose if Dragon spacecraft is in the same 400 km circular orbit as the ISS but behind it by 5 km. Find the total Δv requirement for Dragon to rendezvous with the ISS in 5 hours. Plot the motion of the Dragon relative to the ISS (for 5 orbits).

Solution:

Spacecraft in the circular orbit as the ISS. Initial and final positions in the Clohessey-Wiltshire frame are

$$\begin{aligned}\delta \boldsymbol{r}_0 &= [0 \quad -5 \quad 0]^{\mathrm{T}}\,\mathrm{km}\\ \delta \boldsymbol{r}_f &= [0 \quad 0 \quad 0]^{\mathrm{T}}\end{aligned}$$

Also, since Dragon is in the same circular orbit as the ISS, $\delta \boldsymbol{v}_0^- = \{\boldsymbol{0}\}$. From the CW matrices, compute $\delta \boldsymbol{v}_0^+$, $\delta \boldsymbol{v}_f^-$, $\delta \boldsymbol{v}_0$, and $\delta \boldsymbol{v}_f$ as

$$\begin{aligned}\delta \boldsymbol{v}_0^+ &= [0.2 \quad -0.1057 \quad 0]\\ \delta \boldsymbol{v}_f^- &= [-0.2 \quad -0.1057 \quad 0]\\ \delta \boldsymbol{v}_0 &= [0.2 \quad -0.1057 \quad 0]\\ \delta \boldsymbol{v}_f &= [0.2 \quad 0.1057 \quad 0]\end{aligned}$$

Units: m/s Evaluate total $\Delta v = 0.45247$ m/s.

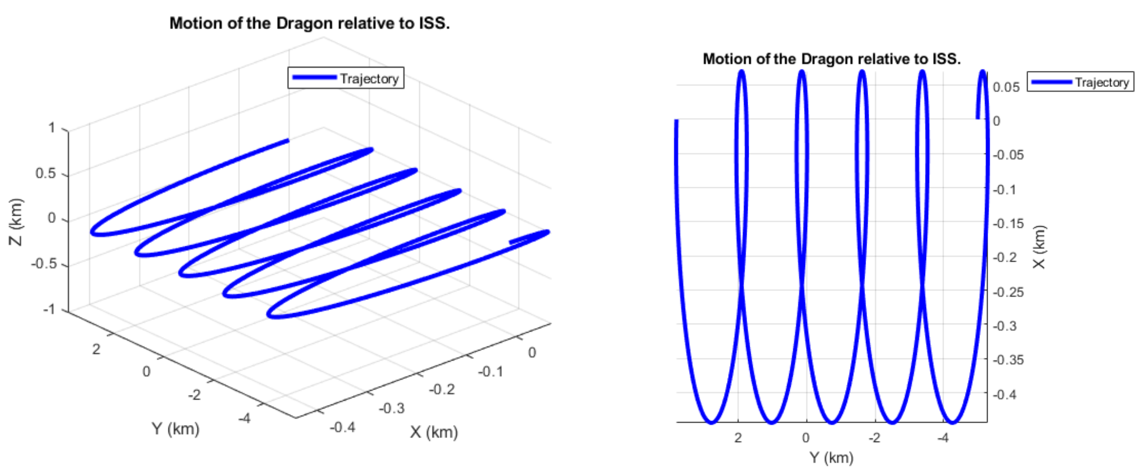


Figure 2: Co-planar rendezvous trajectory (no out of plane motion).

3. (7 pts) Integrate using Runge-Kutta the circular-restricted 3-body problem equations,

$$\begin{aligned}\ddot{x} - 2\omega\dot{y} - \omega^2 x &= -\left(\frac{\mu_1}{r_1^3} + \frac{\mu_2}{r_2^3}\right) x - \left(\frac{\mu_1 m_2}{r_1^3} - \frac{\mu_2 m_1}{r_2^3}\right) \frac{r_{12}}{m_1 + m_2}\\ \ddot{y} + 2\omega\dot{x} - \omega^2 y &= -\left(\frac{\mu_1}{r_1^3} + \frac{\mu_2}{r_2^3}\right) y\\ \ddot{z} &= -\left(\frac{\mu_1}{r_1^3} + \frac{\mu_2}{r_2^3}\right) z\end{aligned}$$

where $\omega = 2.662 \cdot 10^{-6}$, $\mu_1 = 398600.44$, $\mu_2 = 4904.87$, and $r_{12} = 384,748$ (r_{12} is the combined distance $r_1 + r_2$ from the center of each of the two massive bodies to the origin). These values and all other values in the problem refer to those of the Earth-Moon system.

Part a. (4 points) Use the following initial conditions (run several trials varying the $\dot{y}$ velocity from 0 to -3 in increments of 0.5):

Axis	Position	Velocity
X	$1.05 \cdot r_2$	0
Y	0	vary in $[-3, 0]$ with 0.5 step
Z	0	0.05

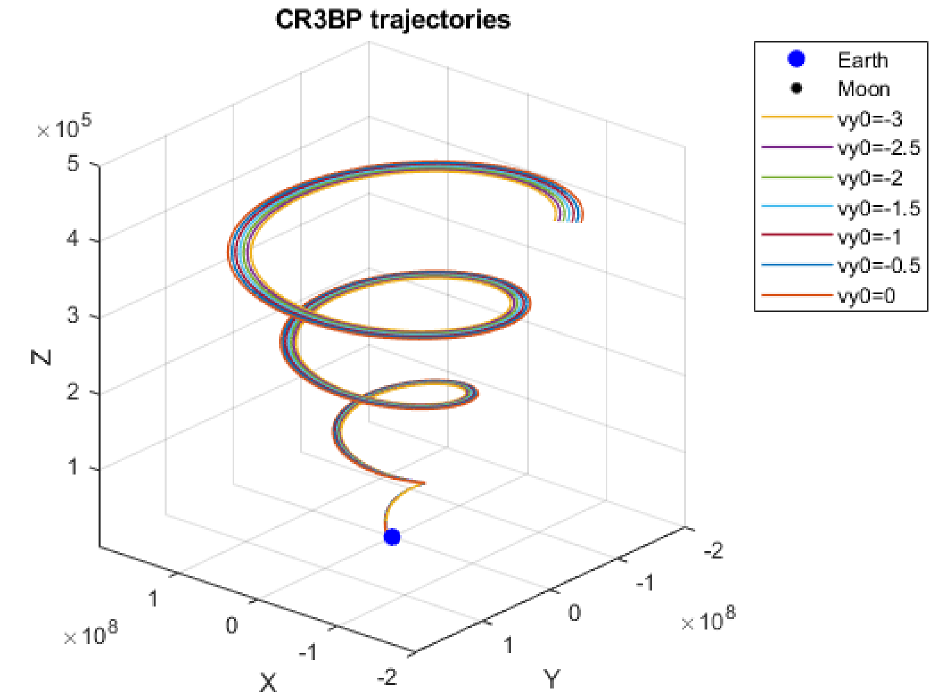
Plot the seven resulting trajectories after integrating over 100 days of simulation time.

Part b. (3 points) Plot the trajectories for the following initial conditions.

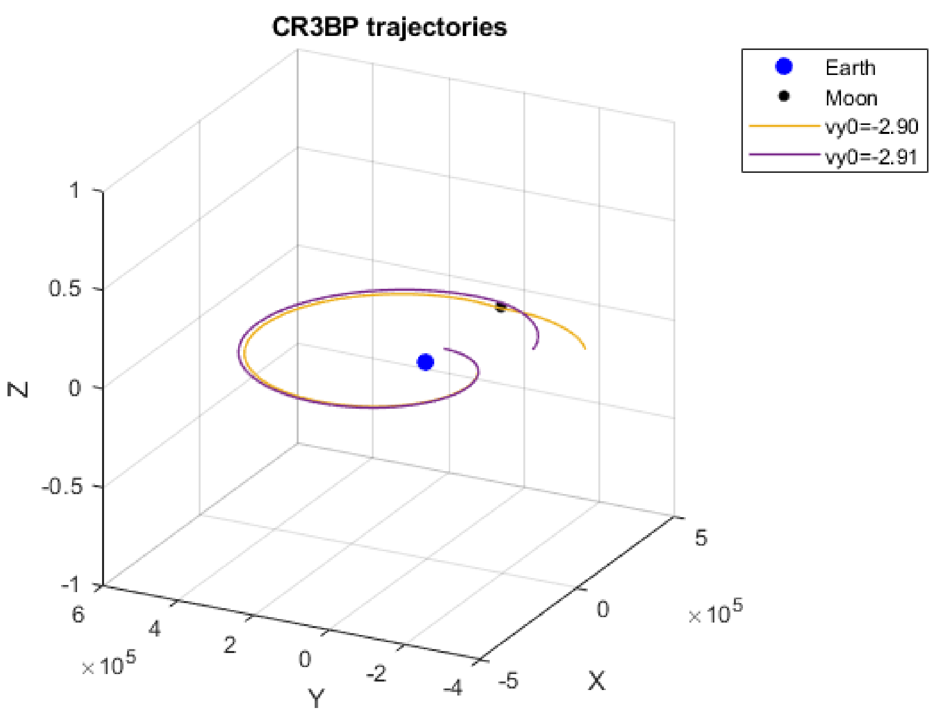
Axis	Position	Velocity
X	$0.95 \cdot r_1$	0
Y	0	-2.90 & -2.91
Z	0	0

Propagate trajectories for 14 days and plot the resulting trajectories with two different initial velocities in y direction, $v_y = [-2.90, -2.91]$ km/s.

Solution:



(a) Part a



(b) Part b

Object	Radius (km)	Mass (kg)	Sidereal Rotation Period	Inclination of Equator to Orbit Plane	Semimajor Axis of Orbit (km)	Orbit Eccentricity	Inclination of Orbit to the Ecliptic Plane	Orbit Sidereal Period
Sun	696,000	1.989×10^{30}	25.38d	7.25°	—	—	—	—
Mercury	2440	330.2×10^{21}	58.65d	0.01°	57.91×10^{6}	0.2056	7.00°	87.97d
Venus	6052	4.869×10^{24}	243d*	177.4°	108.2×10^{6}	0.0067	3.39°	224.7d
Earth	6378	5.974×10^{24}	23.9345h	23.45°	149.6×10^{6}	0.0167	0.00°	365.256d
(Moon)	1737	73.48×10^{21}	27.32d	6.68°	384.4×10^{3}	0.0549	5.145°	27.322d
Mars	3396	641.9×10^{21}	24.62h	25.19°	227.9×10^{6}	0.0935	1.850°	1.881y
Jupiter	71,490	1.899×10^{27}	9.925h	3.13°	778.6×10^{6}	0.0489	1.304°	11.86y
Saturn	60,270	568.5×10^{24}	10.66h	26.73°	1.433×10^{9}	0.0565	2.485°	29.46y
Uranus	25,560	86.83×10^{24}	17.24h*	97.77°	2.872×10^{9}	0.0457	0.772°	84.01y
Neptune	24,760	102.4×10^{24}	16.11h	28.32°	4.495×10^{9}	0.0113	1.769°	164.8y
(Pluto)	1195	12.5×10^{21}	6.387d*	122.5°	5.870×10^{9}	0.2444	17.16°	247.7y

Figure 4: Solar system data (from Curtis)

Homework 4

- AERO423, Spring 2024

Problem 1a. Planetary Rendezvous

```
% Constants
G = 6.6743 * 1e-11; % N m2 / kg2
M_sun = 1.989*10^(30);

mu_sun = 132.71 * 1e9;
mu_earth = 398600;
mu_venus = 324900;

r_earth = 6378;
r_venus = 6052;

R1 = 149.6*1e6; % semi-major axis: Earth
R2 = 108.2*1e6; % semi-major axis: Venus

% Hyperbolic excess speed
v_infty = abs(sqrt(mu_sun / R2) * (1 - sqrt(2*R1 /(R1+R2))) );

% Optimum radius the capture orbit
r_p = 2*mu_venus/v_infty^2;

fprintf('Capture radius: %f\n', r_p);
```

```
Capture radius: 88648.151182
```

```
% altitude
altitude = r_p - r_venus;
fprintf('Altitude of the parking orbit: %f\n', altitude);
```

```
Altitude of the parking orbit: 82596.151182
```

```
% Delta - v
delta_v = v_infty / sqrt(2);
fprintf("Optimal delta-v: %f\n", delta_v);
```

```
Optimal delta-v: 1.914432
```

```
% optimal approach distance
d_opt = 2*sqrt(2)*mu_venus / v_infty^2;

fprintf("Approach distance: %f\n", d_opt);
```

```
Approach distance: 125367.417681
```

1b) Flyby

```
% Backward calculations (not the homework problem but the setup)
% Homework problem is solved in tje next section

mu_sun = 132.71 * 1e9;
mu_earth = 398600;
mu_venus = 324900;

r_earth = 6378;
r_venus = 6052;

R1 = 149.6*1e6; % semi-major axis: Earth
R2 = 108.2*1e6; % semi-major axis: Venus

h1 = 4.05*1e9;
e = 0.17;
theta = -30; % degrees

Vp1 = mu_sun / (h1 * (1+e*cosd(theta)));
Vr1 = mu_sun/h1 * e*sind(theta);

V1 = [Vp1; Vr1];

Vp = sqrt(mu_sun / R2);
V = [Vp; 0];

V_infty1 = V1 - V;
V_inty1_norm = norm(V_infty1);

rp = 300 + r_venus;

h = rp*sqrt(V_inty1_norm^2 + 2*mu_venus/rp);
e = 1 + rp*V_inty1_norm^2/mu_venus;
```

```
% 1b) Actual question
V1 = [36.5; 2.5];
V = [35.02; 0];
V2 = [32.117; 0.119];

vi1 = V1 - V;
vi2 = V2 - V;

vi1_norm = norm(vi1);
vi2_norm = norm(vi2); % Also, vi2 = V2 - V_venus

rp = r_venus + 500;

h = rp*(sqrt( vi1_norm^2 + 2*mu_venus / rp ) );
fprintf("Angular momentum h=%f\n", h);
```

```
Angular momentum h=67969.294783
```

```
e = 1 + rp*vi1_norm^2 / mu_venus;
fprintf("Eccentricity e=%f\n", e);
```

```
Eccentricity e=1.170211
```

```
turn_angle = 2*asind(1/e);
fprintf("Turn angle delta=%f\n", turn_angle);
```

```
Turn angle delta=117.419420
```

```
phi1 = atan2d(vi1(2),vi1(1));

% vi2 = vi1_norm * [cosd(phi1 + turn_angle); sind(phi1 + turn_angle)]
% vi2_norm = norm(vi2); % Also, vi2 = V2 - V_venus

fprintf("V_infty1=%f, V_infty2=%f\n", vi1_norm, vi2_norm);
```

```
V_infty1=2.905237, V_infty2=2.905438
```

```
% V2 = V + vi2;

% vi2 = V2 - Vp

deltaV = vi2 - vi1;
deltaV_norm = norm(V2) - norm(V1);
fprintf("delta v=%f\n", deltaV_norm);
```

```
delta v=-4.468296
```

```
phi1 = atan2d(vi1(2),vi1(1));

phi2 = atan2d(vi2(2),vi2(1));

turn_angle = phi2 - phi1;

if turn_angle > 0
    fprintf("Leading-side flyby with turn angle: %f", turn_angle);
else
    fprintf("Trailing-side flyby with turn angle: %f", turn_angle);
end
```

```
Leading-side flyby with turn angle: 118.278174
```

Relative Motion

```
mu_earth = 398600;
Re = 6378;
orbit_radius = 400;

v = sqrt(mu_earth/(Re + orbit_radius)) ;
```

```
n = v/(Re + orbit_radius);

T = 2*pi*sqrt( (Re+orbit_radius)^3 / mu_earth );
t = T/3;

[phi_rr, phi_rv, phi_vr, phi_vv] = CW_Matrices(n, t);

dr0 = [0 -2 0.5]';
drf = [0 0 0]';

dv0_plus = -phi_rv \ phi_rr * dr0;
dvf_minus = phi_vr*dr0 + phi_vv*dv0_plus;

dv0_minus = 1e-3*[0 0 5]';

delta_v0 = dv0_plus - dv0_minus;

dvf_plus = [0 0 0]';

delta_vf = dvf_plus - dvf_minus ;

% delta_vf = norm(delta_vf);

delta_v_total = norm(delta_vf) + norm(delta_v0);

fprintf("2a) Delta-V total = %f", delta_v_total);
```

```
2a) Delta-V total = 0.006056
```

```
% Part b
mu_earth = 398600;
Re = 6378;
orbit_radius = 400;

v = sqrt(mu_earth/(Re + orbit_radius))
```

```
v = 7.6686
```

```
n = v/(Re + orbit_radius);

t = 5*60*60;
% T = 2*pi*sqrt( (Re+orbit_radius)^3 / mu_earth );
% t = T/3;

[phi_rr, phi_rv, phi_vr, phi_vv] = CW_Matrices(n, t);

dr0 = [0 -5 0]';
drf = [0 0 0]';

dv0_plus = -phi_rv \ phi_rr * dr0
```

dv0_plus = 3×1

10^{-3} ×

```
    0.2000
```

```
   -0.1057
         0
```

```
dvf_minus = phi_vr*dr0 + phi_vv*dv0_plus
```

```
dvf_minus = 3×1
10^-3 ×
   -0.2000
   -0.1057
         0
```

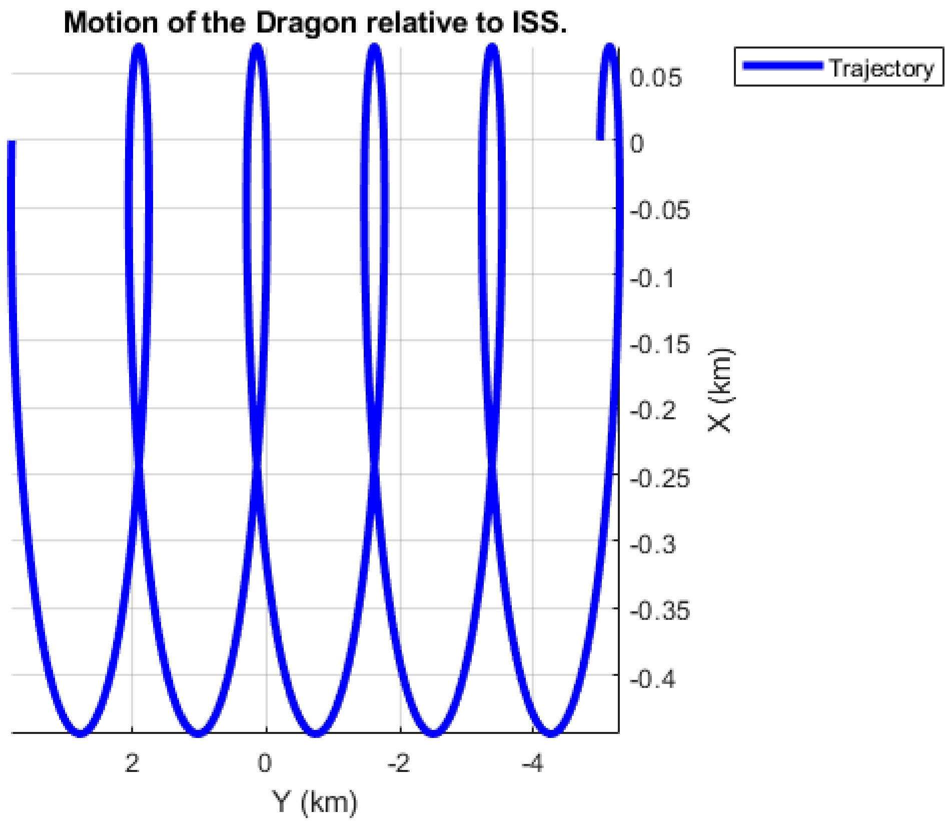
```
dv0_minus = [0 0 0]';

delta_v0 = dv0_plus - dv0_minus
```

```
delta_v0 = 3×1
10^-3 ×
    0.2000
   -0.1057
         0
```

```
dvf_plus = [0 0 0]';

delta_vf = dvf_plus - dvf_minus ;

% delta_vf = norm(delta_vf);

delta_v_total = norm(delta_vf) + norm(delta_v0);

fprintf("2b) Delta-V total = %f", delta_v_total);
```

```
2b) Delta-V total = 0.000452
```

```
% Relative motion
n_orbits = 5;
t = linspace(0, n_orbits*2*pi/n, 1000);
delta_r = zeros(3, length(t));

for ii=1:length(t)
    [phi_rr, phi_rv, phi_vr, phi_vv] = CW_Matrices(n, t(ii));
    delta_r(:,ii) = phi_rr*dr0 + phi_rv*dv0_plus;
end

figure
plot3(delta_r(1,:), delta_r(2,:), delta_r(3,:), '-b', 'LineWidth',3);
xlabel('X (km)'); ylabel('Y (km)'); zlabel("Z (km)");
title('Motion of the Dragon relative to ISS.')
legend('Trajectory');
grid on; axis tight
```

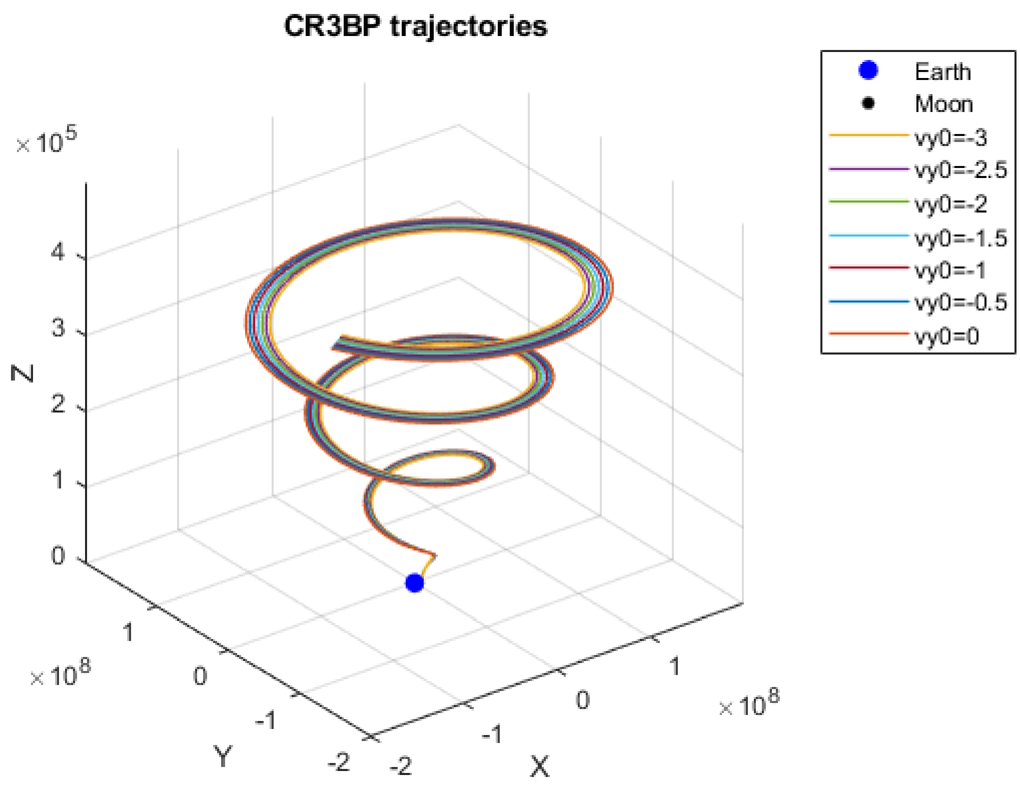


CR3BP

```
m1= 5.9742e24;                 % Mass of earth (kgs)
m2= 7.347e22;                 % Mass of moon (kgs)
pi1 = m1/(m1+m2);
pi2 = m2/(m1+m2);


r12 = 385748;

x1 = -pi2*r12;
x2 = pi1*r12;

r2 = x2 / 0.05;

r1 = -x1 / 0.05;
```

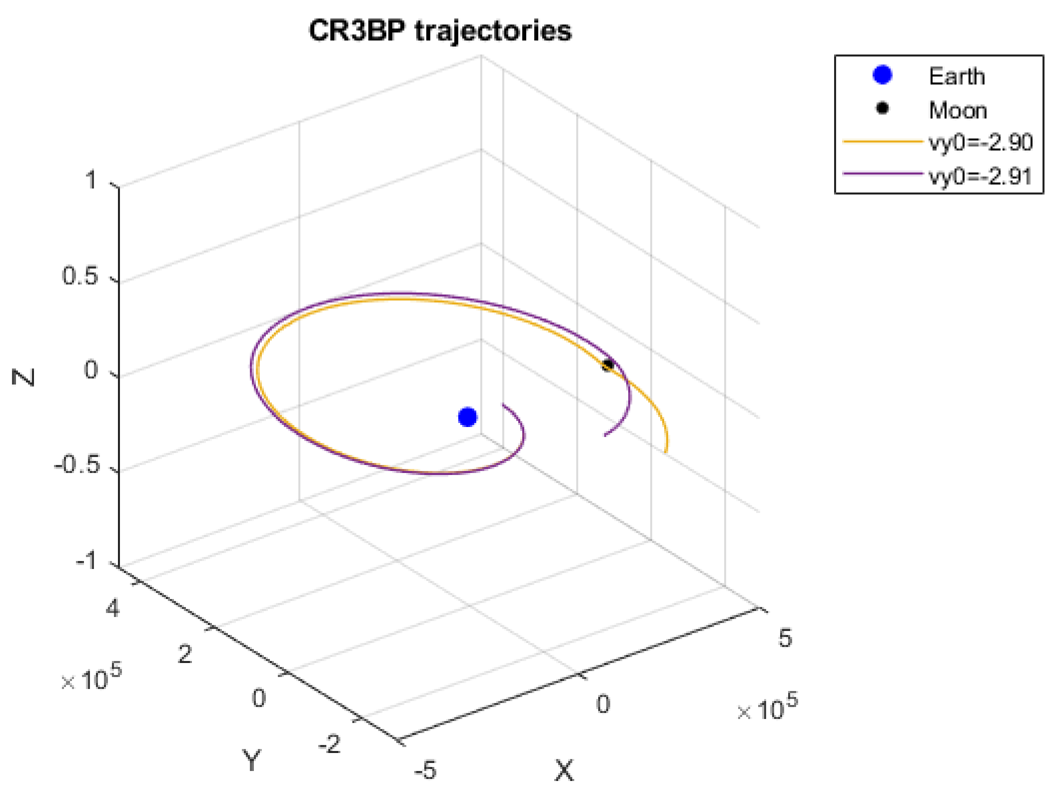
```
% A: initial conditions 1
x0 = 1.05*r2;
y0 = 0;
z0 = 0;

vx0 = 0;
vz0 = 0.05;
```

```
vy_init = -3:0.5:0;
tspan = [0 100*24*60*60];

options = odeset('RelTol', 1.0e-5, 'AbsTol', 1.0e-5);

figure
plot3(x1,0,0,'.b','Markersize',25);
hold on;
plot3(x2,0,0,'.k','Markersize',15);

for ii=1:length(vy_init)

    vy0 = vy_init(ii);
    INIT_y = [x0 y0 z0 vx0 vy0 vz0]';
    [t, y] = ode45(@CRTBP, tspan, INIT_y, options);
    plot3(y(:,1), y(:,2), y(:,3),'Linewidth', 1);
    hold on;

end

grid on; title('CR3BP trajectories');  hold off;
xlabel('X'); ylabel('Y'); zlabel('Z');
legend('Earth','Moon','vy0=-3','vy0=-2.5', 'vy0=-2','vy0=-1.5','vy0=-1','vy0=-0.5','vy0=0')
```




```
% B: initial conditions 2
x0 = 0.95*r1;
y0 = 0;
```

```
z0 = 0;

vx0 = 0;
vz0 = 0.0;
vy_init = [-2.90, -2.91];
tspan = [0 14*24*60*60];

options = odeset('RelTol', 1.0e-5, 'AbsTol', 1.0e-5);

figure
plot3(x1,0,0,'.b','Markersize',25);
hold on;
plot3(x2,0,0,'.k','Markersize',15);

for ii=1:length(vy_init)

    vy0 = vy_init(ii);
    INIT_y = [x0 y0 z0 vx0 vy0 vz0]';
    [t, y] = ode45(@CRTBP, tspan, INIT_y, options);
    plot3(y(:,1), y(:,2), y(:,3),'Linewidth', 1);
    hold on;

end

grid on; title('CR3BP trajectories');  hold off;
xlabel('X'); ylabel('Y'); zlabel('Z');
legend('Earth','Moon','vy0=-2.90','vy0=-2.91')
```



Functions

```
function ydot = CRTBP(~,y_)

% input

%  y(1) = x-component of position
%  y(2) = y-component of position
%  y(3) = z-component of position

%  y(4) = x-component of velocity
%  y(5) = y-component of velocity
%  y(6) = z-component of velocity

% output

%  ydot(1) = x component of velocity
%  ydot(2) = y component of velocity
%  ydot(3) = z component of velocity

%  ydot(4) = x component of acceleration
%  ydot(5) = y component of acceleration
%  ydot(6) = z component of acceleration


mu1 = 398600.44; mu2 = 4904.87;
omega = 2.662*1e-6; r12 = 385748;

m1= 5.9742E24;               % Mass of earth (kgs)
m2= 7.3459E22;               % Mass of moon (kgs)
pi1 = m1/(m1+m2);
pi2 = m2/(m1+m2);

 x = y_(1);
 y = y_(2);
 z = y_(3);
 vx = y_(4);
 vy = y_(5);
 vz = y_(6);

r1 = norm([ x+pi2*r12, y, z ]);
r2 = norm([ x-pi1*r12, y, z ]);


ax = 2*omega*vy + omega^2*x - ...
           ( mu1/r1^3 + mu2/r2^3 )*x - ...
           ( mu1*pi2/r1^3 - mu2*pi1/r2^3 ) * r12;

ay = -2*omega*vx + omega^2*y - ...
            ( mu1/r1^3 + mu2/r2^3 )*y;
```

```
az = -( mu1/r1^3 + mu2/r2^3 )*z;

ydot = [vx; vy; vz; ax; ay; az];

end
```

```
function [phi_rr, phi_rv, phi_vr, phi_vv] = CW_Matrices(n, t)

% Clohessy Wiltshire
phi_rr = [4-3*cos(n*t) 0 0;
            6*(sin(n*t) - n*t) 1 0;
            0 0 cos(n*t)];

phi_rv = (1/n) * [sin(n*t)  2*(1-cos(n*t)) 0;
        2*(cos(n*t)-1)   4*sin(n*t)-3*n*t 0;
        0 0 sin(n*t)];

phi_vr = n* [3*sin(n*t) 0 0;
    6*(cos(n*t) - 1) 0 0;
    0 0 -sin(n*t)];

phi_vv = [cos(n*t) 2*sin(n*t) 0;
    -2*sin(n*t) 4*cos(n*t)-3 0;
    0 0 cos(n*t)];

end
```