

**AERO-423, Spring 2024, Homework #4**  
 (Due date: 23:59 hours, Thursday, April 11, 2024)

**Show all the work and justify your answer!**  
**Make sure to upload your submission to the Canvas, in time.**

- Consider the following interplanetary trajectory problems using the interplanetary data given in the Curtis book (Figure 2).

**Part a. (4 points)** Consider a Venus orbiter launched via a Hohmann transfer from the Earth. The orbiter is to be placed in a circular capture orbit around Venus. Calculate: (i) altitude of the capture orbit that minimizes the  $\Delta v$  needed for the capture, (ii) the minimum/optimal  $\Delta v$ , and (iii) optimum approach distance  $d$ .

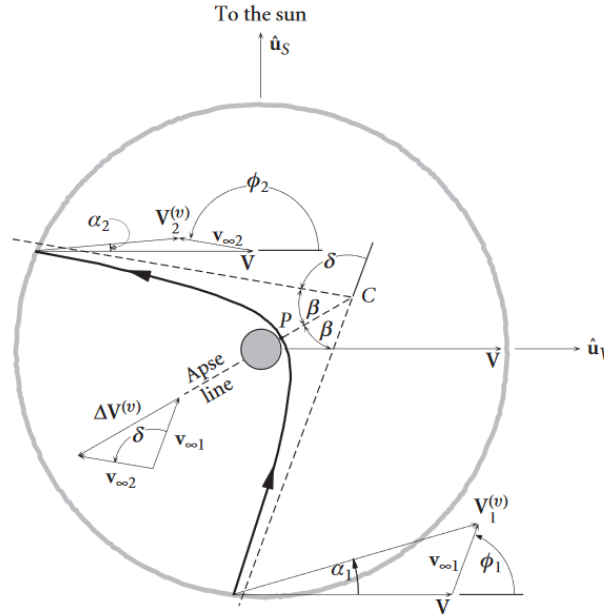


Figure 1: Planetary flyby example.

**Part b. (6 points)** A spacecraft on a Hohmann transfer from the Earth, performs a flyby maneuver at Venus at an altitude 500 km. The following information is given on the inbound and outbound velocity vectors  $\mathbf{V}_1^{(v)}$  and  $\mathbf{V}_2^{(v)}$  in the geocentric coordinate system  $\{\hat{\mathbf{u}}_V, \hat{\mathbf{u}}_S\}$  as shown in figure 1:

$$\mathbf{V}_1^{(v)} = 36.5\hat{\mathbf{u}}_V + 2.5\hat{\mathbf{u}}_S$$

$$\mathbf{V}_2^{(v)} = 32.117\hat{\mathbf{u}}_V + 0.119\hat{\mathbf{u}}_S$$

Compute the following:

- Angular momentum and eccentricity of the flyby hyperbola.

2. Hyperbolic excess velocities at the inbound and the outbound crossings.
  3. Compute the turning angle and state whether it is a leading or a trailing flyby.
  4. Calculate the  $\Delta V$  imparted to the spacecraft during the flyby maneuver (change in the velocity of the spacecraft between when it enters at inbound and crosses at outbound).
2. Assume that ISS is in a 400 km circular orbit. An approaching Dragon spacecraft aims to rendezvous with the ISS by executing a  $\Delta v$  burn. At  $t = 0$ , the position vector  $\delta \mathbf{r}_0$  and the before burn velocity  $\delta \mathbf{v}_0^-$  relative to the ISS (for part a.) are as follows.

$$\delta \mathbf{r}_0 = \{0, -2, 0.5\}^T \text{ km}$$

$$\delta \mathbf{v}_0^- = \{0, 0, 5\}^T \text{ m/s}$$

**Part a. (4 points)** Calculate the total  $\Delta v$  required for the Dragon spacecraft to rendezvous with ISS in one-third of the ISS's orbital period.

**Part b. (4 points)** Suppose if Dragon spacecraft is in the same 400 km circular orbit as the ISS but behind it by 5 km. Find the total  $\Delta v$  requirement for Dragon to rendezvous with the ISS in 5 hours. Plot the motion of the Dragon relative to the ISS (for 5 orbits).

3. (7 pts) Integrate using Runge-Kutta the circular-restricted 3-body problem equations,

$$\ddot{x} - 2\omega\dot{y} - \omega^2 x = - \left( \frac{\mu_1}{r_1^3} + \frac{\mu_2}{r_2^3} \right) x - \left( \frac{\mu_1 m_2}{r_1^3} - \frac{\mu_2 m_1}{r_2^3} \right) \frac{r_{12}}{m_1 + m_2}$$

$$\ddot{y} + 2\omega\dot{x} - \omega^2 y = - \left( \frac{\mu_1}{r_1^3} + \frac{\mu_2}{r_2^3} \right) y$$

$$\ddot{z} = - \left( \frac{\mu_1}{r_1^3} + \frac{\mu_2}{r_2^3} \right) z$$

where  $\omega = 2.662 \cdot 10^{-6}$ ,  $\mu_1 = 398600.44$ ,  $\mu_2 = 4904.87$ , and  $r_{12} = 384,748$  ( $r_{12}$  is the combined distance  $r_1 + r_2$  from the center of each of the two massive bodies to the origin). These values and all other values in the problem refer to those of the Earth-Moon system.

**Part a. (4 points)** Use the following initial conditions (run several trials varying the  $\dot{y}$  velocity from 0 to  $-3$  in increments of 0.5):

Axis	Position	Velocity
X	$1.05 \cdot r_2$	0
Y	0	vary in $[-3, 0]$ with 0.5 step
Z	0	0.05

Plot the seven resulting trajectories after integrating over 100 days of simulation time.

**Part b. (3 points)** Plot the trajectories for the following initial conditions.

Axis	Position	Velocity
$X$	$0.95 \cdot r_1$	0
$Y$	0	$-2.90$ & $-2.91$
$Z$	0	0

Propagate trajectories for 14 days and plot the resulting trajectories with two different initial velocities in  $y$  direction,  $v_y = [-2.90, -2.91]$  km/s.

Object	Radius (km)	Mass (kg)	Sidereal Rotation Period	Inclination of Equator to Orbit Plane	Semimajor Axis of Orbit (km)	Orbit Eccentricity	Inclination of Orbit to the Ecliptic Plane	Orbit Sidereal Period
Sun	696,000	$1.989 \times 10^{30}$	25.38d	$7.25^\circ$	—	—	—	—
Mercury	2440	$330.2 \times 10^{21}$	58.65d	$0.01^\circ$	$57.91 \times 10^6$	0.2056	$7.00^\circ$	87.97d
Venus	6052	$4.869 \times 10^{24}$	243d*	$177.4^\circ$	$108.2 \times 10^6$	0.0067	$3.39^\circ$	224.7d
Earth	6378	$5.974 \times 10^{24}$	23.9345h	$23.45^\circ$	$149.6 \times 10^6$	0.0167	$0.00^\circ$	365.256d
(Moon)	1737	$73.48 \times 10^{21}$	27.32d	$6.68^\circ$	$384.4 \times 10^3$	0.0549	$5.145^\circ$	27.322d
Mars	3396	$641.9 \times 10^{21}$	24.62h	$25.19^\circ$	$227.9 \times 10^6$	0.0935	$1.850^\circ$	1.881y
Jupiter	71,490	$1.899 \times 10^{27}$	9.925h	$3.13^\circ$	$778.6 \times 10^6$	0.0489	$1.304^\circ$	11.86y
Saturn	60,270	$568.5 \times 10^{24}$	10.66h	$26.73^\circ$	$1.433 \times 10^9$	0.0565	$2.485^\circ$	29.46y
Uranus	25,560	$86.83 \times 10^{24}$	17.24h*	$97.77^\circ$	$2.872 \times 10^9$	0.0457	$0.772^\circ$	84.01y
Neptune	24,760	$102.4 \times 10^{24}$	16.11h	$28.32^\circ$	$4.495 \times 10^9$	0.0113	$1.769^\circ$	164.8y
(Pluto)	1195	$12.5 \times 10^{21}$	6.387d*	$122.5^\circ$	$5.870 \times 10^9$	0.2444	$17.16^\circ$	247.7y

Figure 2: Solar system data (from Curtis)