Neutrino P_z Calculation:

This is done in two scenario

1) Where we did not negelected lepton mass as compared to other terms.

2) Where we negelected lepton mass as compared to the other terms.

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1) Lepton Mass Not Negelected:

$$P_\omega = P_\nu + P_\ell$$

$$\Rightarrow P_\omega^2 = (P_\nu + P_\ell)^2$$

$$\Rightarrow M_\omega^2 = \cancel{M_\nu^2}^{\,\to 0} + M_\ell^2 + 2 P_\nu P_\ell \qquad \text{--- (1)}$$

$$P_\ell = (E_\ell, p_{x\ell}, p_{y\ell}, p_{z\ell})$$

$$P_\nu = (E_\nu, p_{x\nu}, p_{y\nu}, p_{z\nu})$$

$$\Rightarrow M_\omega^2 = M_\ell^2 + 2\left(E_\ell E_\nu - p_{\ell x} p_{\nu x} - p_{\ell y} p_{\nu y} - p_{\ell z} p_{\nu z}\right)$$

$$\Rightarrow \frac{M_\omega^2 - M_\ell^2}{2} = E_\ell E_\nu - p_{\ell x} p_{\nu x} - p_{\ell y} p_{\nu y} - p_{\ell z} p_{\nu z}$$

Also, $E_\ell E_\nu = E_\ell \sqrt{E_{\nu T}^2 + E_{\nu z}^2} = E_\ell \sqrt{p_{\nu T}^2 + p_{\nu z}^2}$ --- (2)

$$\Rightarrow \frac{M_\omega^2 - M_\ell^2}{2} + p_{\ell x} p_{\nu x} + p_{\ell y} p_{\nu y} + p_{\ell z} p_{\nu z} = E_\ell \sqrt{p_{\nu T}^2 + p_{\nu z}^2}$$

$$\Rightarrow \frac{1}{E_\ell}\left[\frac{M_W^2 - M_\ell^2}{2} + p_{\ell x}p_{\nu x} + p_{\ell y}p_{\nu y} + p_{\ell z}p_{\nu z}\right] = \sqrt{p_{\nu T}^2 + p_{\nu z}^2}$$

$$\frac{1}{2E_\ell}\left[\frac{M_W^2 - M_\ell^2}{\cancel{2}} + 2\left(p_{\ell x}p_{\nu x} + p_{\ell y}p_{\nu y}\right)\right] + \frac{p_{\ell z}p_{\nu z}}{E_\ell} = \sqrt{p_{\nu T}^2 + p_{\nu z}^2}$$

Put $\boxed{a = M_W^2 - M_\ell^2 + 2\left(p_{\ell x}p_{\nu x} + p_{\ell y}p_{\nu y}\right)}$ — (3)

$$\Rightarrow \frac{a}{2E_\ell} + \frac{p_{\ell z}p_{\nu z}}{E_\ell} = \sqrt{p_{\nu T}^2 + p_{\nu z}^2}$$

$$\Rightarrow \left(a + 2p_{\ell z}p_{\nu z}\right)^2 = 4E_\ell^2\left(p_{\nu T}^2 + p_{\nu z}^2\right)$$

$$\Rightarrow a^2 + 4p_{\ell z}^2 p_{\nu z}^2 + 4a\,p_{\ell z}p_{\nu z} = 4E_\ell^2 p_{\nu T}^2 + 4E_\ell^2 p_{\nu z}^2$$

$$\Rightarrow 4p_{\nu z}^2\left(E_\ell^2 - p_{\ell z}^2\right) - 4a\,p_{\ell z}p_{\nu z} + 4E_\ell^2 p_{\nu T}^2 - a^2 = 0$$

— (4)

Compare it with

$$Ax^2 + bx + c = 0 \Rightarrow x = \frac{-b}{2A} \pm \frac{\sqrt{b^2 - 4Ac}}{2A}$$

$$\Rightarrow \boxed{\begin{aligned} A &\equiv 4\left(E_\ell^2 - p_{\ell z}^2\right) \\ b &= -4a\,p_{\ell z} \\ c &= 4E_\ell^2 p_{\nu T}^2 - a^2 \end{aligned}}$$

— (5)

$\Rightarrow$ Roots for $p_{\nu z}$ ~~are~~

$$p_{\nu z} = \frac{1}{2\cdot 4(E_\ell^2 - p_{\ell z}^2)}\left[4ap_{\ell z} \pm \sqrt{16a^2p_{\ell z}^2 - 16(E_\ell^2 - p_{\ell z}^2)\times(4E_\ell^2 p_{\nu T}^2 - a^2)}\right]$$

$$\Rightarrow p_{\nu z} = \frac{1}{2A}\left[-b \pm \sqrt{b^2 - 4AC}\right] \quad \text{— (6)}$$

Case-I if Roots are real

i.e. $b^2 - 4AC > 0$

two solution exist

$$sol1 = \frac{1}{2A}\left(-b + \sqrt{b^2 - 4AC}\right)$$

$$sol2 = \frac{1}{2A}\left(-b - \sqrt{b^2 - 4AC}\right)$$

type 0: pick one solution which is closest to the p_z of lepton. If $p_{\nu z} > 300$ then pick most central roots.

if $[\,|sol2 - p_{z\ell}| < |sol1 - p_{z\ell}|\,] \Rightarrow p_{z\nu} = sol2$

else $p_{z\nu} = sol1$

if $(p_{z\nu} > 300) \Rightarrow$ if $(|sol1| < |sol2|) \Rightarrow p_{z\nu} = sol1$

else $p_{z\nu} = sol2$

type 1 ~~two roo~~ pick the root which is closest to p_z of lepton.

$$\text{if}\left(|sol2 - p_{z\ell}| < |sol1 - p_{z\ell}|\right)$$

$$p_{z\nu} = sol2$$

$$\text{else} \quad p_{z\nu} = sol1$$

type 2: pick the most central root

$$\text{if}\left(|sol1| < |sol2|\right) \Rightarrow p_{z\nu} = sol1$$

$$\text{else} \quad p_{z\nu} = sol2$$

type: 3: pick the largest value of the cosine.

Case - II if Roots are imaginary

$\Rightarrow$ Take real part of complex root

$$\boxed{p_{z\nu} = -\frac{b}{2A}} \quad \text{--- (7)}$$

$\Rightarrow$ Recalculate neutrino p_T. Solve quadratic eq. discriminator $= 0$ for p_T of neutrino.

$$D = b^2 - 4AC = 0$$

$$\Rightarrow 16a^2 p_{lz}^2 - 16\left(E_l^2 - p_{lz}^2\right)\left(4E_l^2 p_{\nu T}^2 - a^2\right) = 0$$

$$\Rightarrow \cancel{a^2 p_{lz}^2} - \left[4E_l^4 p_{\nu T}^2 - E_l^2 a^2 - 4p_{lz}^2 E_l^2 p_{\nu T}^2 + \cancel{p_{lz}^2 a^2}\right] = 0$$

$$\Rightarrow 4E_l^2\left(p_{lz}^2 - E_l^2\right) p_{\nu T}^2 + E_l^2 a^2 = 0$$

$$\Rightarrow 4\left(p_{lz}^2 - E_l^2\right) p_{\nu T}^2 + a^2 = 0 \quad \text{--- (8)}$$

from eq. (3)

$$a = M_W^2 - M_l^2 + 2\left(p_{lx} p_{\nu x} + p_{ly} p_{\nu y}\right)$$

$$p_{\nu x} = p_{T\nu} \cos\phi_\nu \qquad p_{lx} = p_{Tl} \cos\phi_l$$

$$p_{\nu y} = p_{T\nu} \sin\phi_\nu \qquad p_{ly} = p_{Tl} \sin\phi_l$$

$$a = M + 2p_{T\ell}\,p_{T\nu}\left(\cos\phi_\nu \cos\phi_\ell + \sin\phi_\nu \sin\phi_\ell\right)$$

where $M = M_W^2 - M_\ell^2$ — (9)

$$\Rightarrow a = M + 2p_{T\ell}\,p_{T\nu}\cos(\phi_\nu - \phi_\ell)$$

$$\Rightarrow a^2 = M^2 + 4p_{T\ell}^2\,p_{T\nu}^2\cos^2(\phi_\nu - \phi_\ell) + 4M\,p_{T\ell}\,p_{T\nu}\cos(\phi_\nu - \phi_\ell) \quad \text{— (10)}$$

Using (10) in eqn (8), we get

$$\cancel{4\left(p_{\ell z}^2 - E_\ell^2\right)p_{\nu T}^2 +}$$

$$4\left(p_{\ell z}^2 - E_\ell^2 + p_{T\ell}^2\cos^2(\phi_\nu - \phi_\ell)\right)p_{T\nu}^2 + 4p_{\ell T}\cos(\phi_\nu - \phi_\ell)\,p_{T\nu} + M^2 = 0 \quad \text{— (11)}$$

Compare it with $ax^2 + bx + c = 0$

$$\Rightarrow a = 4\left(p_{\ell z}^2 - E_\ell^2 + p_{T\ell}^2\cos^2(\phi_\nu - \phi_\ell)\right)$$

$$b = 4p_{\ell T}\cos(\phi_\ell - \phi_\nu)$$

$$c = M^2 = M_W^2 - M_\ell^2$$

Also $\Rightarrow$ $p_{x\ell} p_{x\nu} + p_{y\ell} p_{y\nu} = p_{\ell T} p_{\nu T} \cos(\phi_\nu - \phi_\ell)$

$$\Rightarrow p_{x\ell} p_{x\nu} + p_{y\ell} p_{y\nu} = p_{\ell T} \not{E} \cos(\phi_\nu - \phi_\ell)$$

$$\Rightarrow \alpha = p_{T\ell} \cos(\phi_\nu - \phi_\ell) = \frac{p_{x\ell} p_{x\nu} + p_{y\ell} p_{y\nu}}{\not{E}} \quad \text{— (12)}$$

Thus,

$$\left.\begin{aligned} a &= 4\left(p_{\ell z}^2 - E_\ell^2 + \alpha^2\right) \\ b &= 4\alpha \\ c &= M_W^2 - M_\ell^2 \end{aligned}\right\} \quad \text{— (13)}$$

where $\alpha = \dfrac{p_{\ell x} p_{\nu x} + p_{\ell y} p_{\nu y}}{\not{E}}$ — (14)

Solution:

$$p_{T\nu} = \frac{1}{2a}\left[-b \pm \sqrt{b^2 - 4ac}\right] \quad \text{— (15)}$$

Take root which is closest to $p_{T\nu}$ obtained from detector.

2) Lepton Mass Neglected (run2)

$$P_\omega = P_\nu + P_\ell$$

$$\Rightarrow P_\omega^2 = M_\omega^2 = (P_\nu + P_\ell)^2 = \cancel{P_\nu^2}^{\,0} + \cancel{P_\ell^2}^{\,0} + 2P_\nu P_\ell$$

$$\Rightarrow M_\omega^2 = 2\left[E_\ell E_\nu - p_{\ell x} p_{\nu x} - p_{\ell y} p_{\nu y} - p_{\ell z} p_{\nu z}\right] \quad \text{— (1)}$$

$$E_\ell E_\nu = E_\ell \sqrt{E_{\nu T}^2 + E_{\nu z}^2} = E_\ell \sqrt{p_{\nu T}^2 + p_{\nu z}^2} \quad \text{— (2)}$$

$$\Rightarrow \frac{M_\omega^2}{2} = E_\ell \sqrt{p_{\nu T}^2 + p_{\nu z}^2} - p_{\ell x} p_{\nu x} - p_{\ell y} p_{\nu y} - p_{\ell z} p_{\nu z}$$

$$\Rightarrow \sqrt{p_{\nu T}^2 + p_{\nu z}^2} = \frac{1}{E_\ell}\left[\frac{M_\omega^2}{2} + p_{\ell x} p_{\nu x} + p_{\ell y} p_{\nu y} + p_{\ell z} p_{\nu z}\right]$$

Put $$d = \frac{1}{E_\ell}\left[\frac{M_\omega^2}{2} + p_{\ell x} p_{\nu x} + p_{\ell y} p_{\nu y}\right] \quad \text{— (3)}$$

$$\Rightarrow \sqrt{p_{\nu T}^2 + p_{\nu z}^2} = d + \frac{p_{\ell z} p_{\nu z}}{E_\ell}$$

$$\Rightarrow p_{\nu T}^2 + p_{\nu z}^2 = d^2 + \frac{p_{\ell z}^2 p_{\nu z}^2}{E_\ell^2} + 2d\,\frac{p_{z\ell} p_{\nu z}}{E_\ell}$$

$$\Rightarrow p_{\nu z}^2\left(\frac{p_{\ell z}^2}{E_\ell^2}-1\right)+2d\left(\frac{p_{\ell z}}{E_\ell}\right)p_{\nu z}+d^2-p_{\nu T}^2=0$$

Put $\frac{p_{\ell z}}{E_\ell}=a_z$ — (4)

$$\Rightarrow p_{\nu z}^2\left(a_z^2-1\right)+2d\,a_z\,p_{\nu z}+d^2-p_{\nu T}^2=0$$

$$\Rightarrow p_{\nu z}^2+\frac{2d\,a_z}{(a_z^2-1)}p_{\nu z}+\frac{d^2-p_{\nu T}^2}{(a_z^2-1)}=0 \qquad \text{— (5)}$$

Compare it with
$ax^2+bx+c=0$

$$\Rightarrow \left.\begin{aligned} a &= 1\\ b &= \frac{2d\,a_z}{(a_z^2-1)}\\ c &= \frac{d^2-p_{\nu T}^2}{(a_z^2-1)}\end{aligned}\right\} \qquad \text{— (6)}$$

Sol:
$$p_{\nu z}=\frac{1}{2}\left[-b\pm\sqrt{b^2-4c}\right] \qquad \text{— (7)}$$

Case - I Real roots: