Inverse Kinematics of Redundant Robot

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***Abstract*—The goal of this project is to investigate the inverse kinematics of redundant robots in planar situations, where the robots have more degrees of freedom (DoF) than are necessary for the end-effector's orientation and location. The objective of this project is to use algorithmic development and mathematical modeling to create a robust numerical solution for motion planning optimization in redundant robots. Through investigating and evaluating several numerical approaches to the inverse kinematics problem, we aim to find the most suitable solution that maximizes performance and adaptability over a broad spectrum of uses.**

***Index Terms*—redundant robots, graphical, inverse kinematics, jacobian ,forward kinematics, Redundancy Resolution, pseudo-inverse**

INTRODUCTION

Redundant robots, characterized by possessing more degrees of freedom (DoF) than those required to perform the main task(s). These additional DoFs enable the integration of kinematics functions, encompassing various desirable characteristics such as posture control, joint limiting, and obstacle avoidance[3].To effectively utilizing the additional DoF requires sophisticated inverse kinematics solutions. This project focuses on developing robust numerical solutions for optimizing motion planning in redundant robots operating in planar environments. By using various mathematical modeling techniques like analytical method and graphical method, we aim to figure out the best one among them which enhance efficiency.

PLANNED WORK

1. Further delve into the theoretical understanding of redundant robots, Jacobean matrices, forward and inverse kinematics, and other relevant terms crucial for this project's implementation.
2. Investigate different strategies and methodologies for effectively leveraging redundancy in manipulators operating within planar environments, considering factors such as task complexity, workspace limitations, and end-effector constraints.
3. Develop mathematical models to represent the relationship between the manipulator's configuration and the desired kinematics functions.
4. Design algorithms to optimize the manipulator's motion planning while simultaneously fulfilling primary task requirements and user-defined kinematic functions.
5. Conduct a comprehensive comparison of the various mathematical models and algorithms developed throughout the project.

PROPOSED METHOD

We are analyzing the inverse Kinematics of 3 link(3 dof) planner redundant robot. We are mainly focusing on the 4 method to perform the action which are :

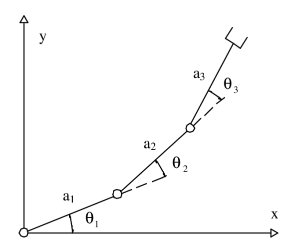

Fig. 1. Three Link Arm

1) **Analytical Method**
The analytical method offers a systematic approach to solving the inverse kinematics problem. Here we are using it for the redundant robots in planar environments. Here we desired to find the joint angles θ1, θ2 and θ3 of three link planar arm robot corresponding to a given end-effector's position but not orientation. We will assume some value of orientation of end-effector and then proceed The position of the end-effector is specified by $(p_x,\ p_y)$ and the orientation of the frame attached to the end-effector with respect to the X_1 axis is assumed to be angle φ.
From the forward position kinematics analysis,

$$\varphi = \theta_1 + \theta_2 + \theta_3 \tag{1}$$

$$p_x = a_1c_1 + a_2c_{12} + a_3c_{123} \tag{2}$$

$$p_y = a_1s_1 + a_2s_{12} + a_3s_{123} \tag{3}$$

where a_1, a_2 and a_3 is the length of the link1, link2 and link3 respectively.

To simplify the inverse kinematic problem, the task is subdivided to position the wrist before orienting the end-effector. This simplification equates to finding the inverse kinematic solution of a two-link planar arm with the wrist point (w_x, w_y) instead of (p_x, p_y).

$$w_x = p_x - a_3 c\varphi = a_1 c_1 + a_2 c_{12} \tag{4}$$

$$w_y = p_y - a_3 s\varphi = a_1 s_1 + a_2 s_{12} \tag{5}$$

Squaring and adding Eqs.(4-5), and then solving for c_2, we will get:

$$c_2 = \frac{w_x^2 + w_y^2 - a_1^2 - a_2^2}{2a_1 a_2} \tag{6}$$

To ensure a solution,c_2 is constrained from -1 to 1. Then,

$$s_2 = \pm\sqrt{1 - c_2^2} \tag{7}$$

where the positive sign is relative to the elbow-up posture and the negative sign is to the elbow-down posture. Hence, the angle θ2 is computed as

$$\theta_2 = atan2(s_2, c_2) \tag{8}$$

The atan2 function computes the arctangent value in the appropriate quadrant.
Now, θ1 is determined by expanding and rearranging Eqs.(4) to yield:

$$s_1 = \frac{(a_1 + a_2 c_2) w_y - a_2 s_2 w_x}{\Delta} \tag{9}$$

where Δ= $a_1^2 + a_2^2 + 2a_1 a_2 c_2 = w_x^2 + w_y^2$. Similarly, c1 is obtained as

$$c_1 = \frac{(a_1 + a_2 c_2) w_x + a_2 s_2 w_y}{\Delta} \tag{10}$$

Then,

$$\theta_1 = atan2(s_1, c_1) \tag{11}$$

Finally, θ3 is obtained from Eq.(1):

$$\theta_3 = \varphi - \theta_2 - \theta_1 \tag{12}$$

2) **Geometrical Method**
In the geometric solution method, we utilize the orientation angle equation(1) along with the coordinates of frame 3's origin derived from equation(2-3). By applying the cosine theorem to the angle formed by the arm's links a_1 and a_2, and the segment connecting points O_1 and W, gives

$$w_x^2 + w_y^2 = a_1^2 + a_2^2 - 2a_1 a_2 \cos(\pi - \theta_2) \tag{13}$$

The two admissible configuration of the triangle are shown in Fig1. Observing cos(π- θ2) = -cos(θ2) = -c_2, we obtain,

$$w_x^2 + w_y^2 = a_1^2 + a_2^2 + 2a_1 a_2 c_2 \tag{14}$$

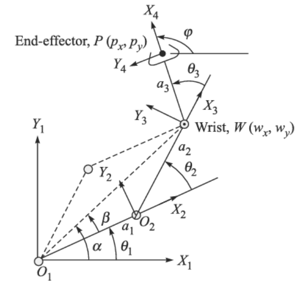

Fig. 2. Three Link Arm

whereas

$$\sqrt{w_x^2 + w_y^2} \le a_1 + a_2 \tag{15}$$

based on the assumption of admissible solutions, the angle θ2 is obtained as

$$\theta_2 = \arccos(c_2) \tag{16}$$

where the elbow-up posture is obtained when θ_2 is in between $-\pi$ and 0, and the elbow-down posture is obtained for θ_2 between 0 and π. To find q_1, consider the angles α and β in Fig. 1, which are computed from

$$\alpha = \text{atan2}(w_y, w_x) \quad \text{and} \quad \cos\beta = \frac{a_1 + a_2 c_2}{\sqrt{w_x^2 + w_y^2}} \tag{17}$$

Substituting c_2, Eq.2 into Eq. 17, yields the angle β, i.e.,

$$\beta = \arccos \frac{w_x^2 + w_y^2 + a_1^2 - a_2^2}{2a_1\sqrt{w_x^2 + w_y^2}} \tag{18}$$

In Eq.18 β should be within 0 and π so as to preserve the existence of the triangle. Then,

$$\theta_1 = \alpha \pm \beta \tag{19}$$

the positive sign applies to θ_2 within the range $(-\pi, 0)$, and the negative sign applies to θ_2 within the range $(0, \pi)$.
Finally, θ_3 is computed using Equation (12).

3) **Numerical Method**
   a) **Newton Raphson Method**
   Suppose we express the end-effector frame using a coordinate vector x governed by the forward kinematics $x = f(\theta)$, a nonlinear vector equation mapping the n joint coordinates to the m end-effector coordinates. Assume that $f : \mathbb{R}^n \to \mathbb{R}^m$ is differentiable, and let x_d be the desired end-effector coordinates. Then $g(\theta)$ for the Newton-Raphson method is defined as $g(\theta) = x_d - f(\theta)$, and the goal is to find joint coordinates θ_d such that

$$g(\theta_d) = x_d - f(\theta_d) = 0$$

Given an initial guess θ_0 which is "close to" a solution θ_d, the kinematics can be expressed as the Taylor expansion:

$$x_d = f(\theta_d) = f(\theta^0) + \left.\frac{\partial f}{\partial \theta}\right|_{\theta^0} (\theta_d - \theta^0) + \text{h.o.t.}$$

where

$$\left.\frac{\partial f}{\partial \theta}\right|_{\theta^0} = J(\theta^0)$$

$$(\theta_d - \theta^0) = \Delta\theta$$

where $J(\theta^0) \in \mathbb{R}^{m \times n}$ is the coordinate Jacobian evaluated at θ^0. Truncating the Taylor expansion at first order:

$$J(\theta^0)\Delta\theta = x_d - f(\theta^0)$$

Assuming that $J(\theta^0)$ is square ($m = n$) and invertible, we can solve for $\Delta\theta$ as:

$$\Delta\theta = J^{-1}(\theta^0)\left(x_d - f(\theta^0)\right)$$

If the Jacobian is not square we can find the pseudo-jacobian using the Moore-Penrose pseudoinverse formula:

$$J^{\dagger} = (J^T J)^{-1} J^T$$

If the forward kinematics is linear in θ, then the new guess $\theta^1 = \theta^0 + \Delta\theta$ exactly satisfies $x_d = f(\theta^1)$.
If the forward kinematics is not linear in θ, as is usually the case, the new guess θ^1 should still be closer to the root than θ^0, and the process is then repeated, producing a sequence $\{\ \theta^0, \theta^1, \theta^2, \dots\ \}$ converging to θ.

b) **Redundancy Resolution Method**
The mathematical formulation of the redundancy resolution problem in the position level is described as finding **q** such that

$$x^d = f(q)$$

where x^d represents the desired position of the end-effector. Since this problem is to be solved by integrating the joint velocities, an initial condition is needed. In the physical sense, this initial condition represents the initial posture of the manipulator from which the motion toward the desired position starts. Assume that the initial posture of the manipulator is described by q_1. At this initial posture, the end-effector is located at x_1, where

$$x_1 = f(q_1)$$

Assume an initial posture, q_1, for the manipulator and calculate the initial position of the end-effector, x_1. Plan a trajectory from x_1 to $x_{N+1} = x^d$ with N intervals, and assume the period of the motion, T.
Calculate a planned velocity at the interval k that moves the end-effector toward the desired position as

$$\dot{x}_k = \alpha \frac{(x_d - x_k)}{(N+1-k)\Delta t}, \Delta t = \frac{T}{N}$$

where α is the deceleration factor greater than 1, which results in faster velocities at the beginning of the motion and slower velocities closer to the desired position.
Find the joint rates that generate the planned end-effector velocity at step k. Any of the methods discussed so far can be used for joint rate calculation, for example, the exact redundancy resolution method using a pseudo-inverse Jacobian matrix.

$$\dot{q}_k = J_e^{\dagger}(q_k)\dot{x}_k$$

$$q_{k+1} = q_k + \dot{q}_k \Delta t$$

Find the new end-effector position

$$x_{k+1} = f(q_{k+1})$$

Repeat all these steps for k = 1....N
When the above algorithm runs to the end, when k = N, the last step represents x_{N+1} = x_d = f(q_{N+1}). This indicates that the joint posture corresponding to the last step, q_{N+1}, is the solution to the redundancy resolution problem at the position level.

RESULTS

- Newton-Raphson method:-
  - RRR Manipulator:- MATLAB Code
  - PRR Manipulator:- MATLAB Code
  - RRP Manipulator:- MATLAB Code
- Redundancy Resolution Method:-
  - RRR Manipulator:- MATLAB Code
  - PRR Manipulator:- MATLAB Code
  - RRP Manipulator:- MATLAB Code
- Algebraic Method:-MATLAB Code

Graphs showing the Joint angle Trajectory and end effector trajectory using redundancy resolution method:-

1. **3 link RRR redundant robot**

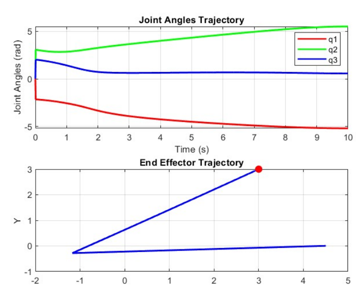

Fig. 3. Three Link Arm

2. **3 link PRR redundant robot**

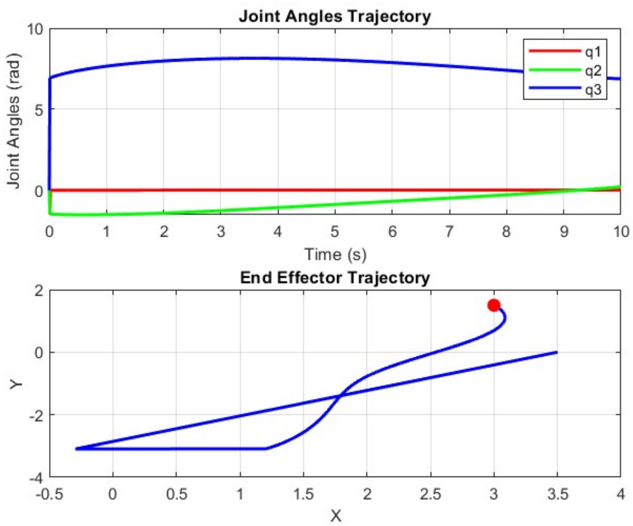

Fig. 4. Three Link Arm

3. **3 link RRP redundant robot**

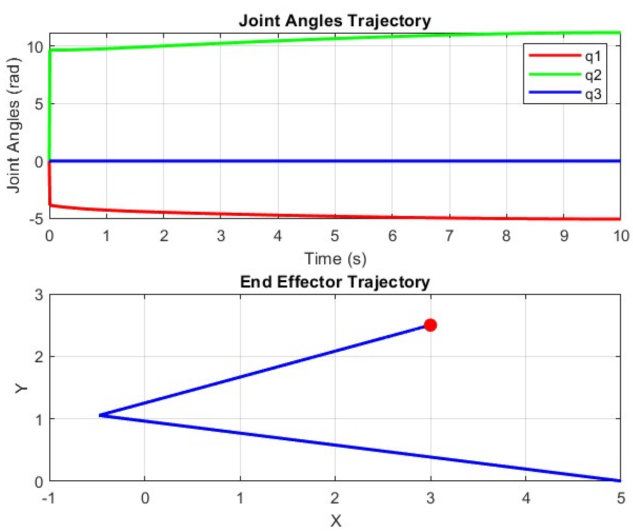

Fig. 5. Three Link Arm

Graphs showing the End effector trajectory using Newton Raphson method:-

1. **3 link RRR redundant robot**

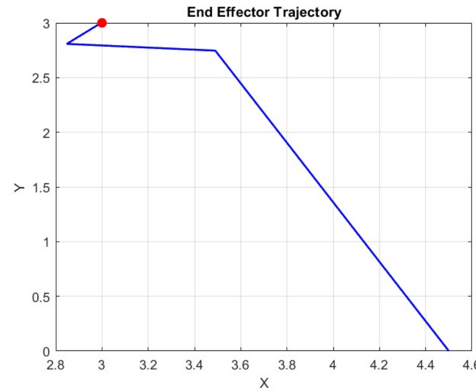

Fig. 6. Three Link Arm

2. **3 link PRR redundant robot**

Fig. 7. Three Link Arm

3. **3 link RRP redundant robot**

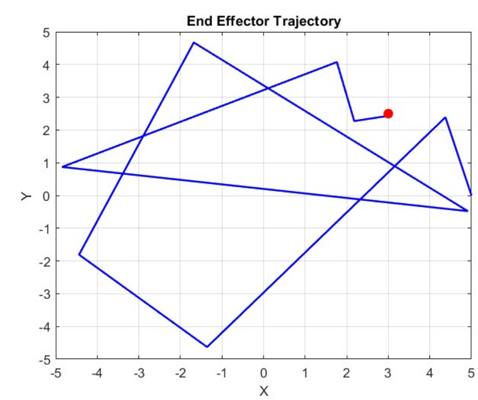

Fig. 8. Three Link Arm

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- Mota Ram:- Redundancy Resolution Method

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CONCLUSION

In conclusion, the investigation of inverse kinematics for a 3-link redundant robot in planar environments yielded valuable insights into the analytical, geometrical, and numerical methods. Each method offers distinct advantages and limitations, influencing their suitability for different scenarios. The analytical and geometrical methods provide efficient and intuitive solutions for simpler scenarios, while the numerical method, particularly through redundancy resolution techniques, offers flexibility and robustness in handling complex environments. We have plotted the behaviour of the joint angles with time and end effector trajectory in X-Y plain. Since we are working with the 3 dof manipulator only, so we got not much difference in the result from the all methods. So for low dof, any of the 4 method will give the same result with very minor difference. But if we go for higher dof, the result will not be the same.

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