Applied Algorithms

CSCI-B505 / INFO-I500

Lecture 3.

Review of Basic Data Structures - 1

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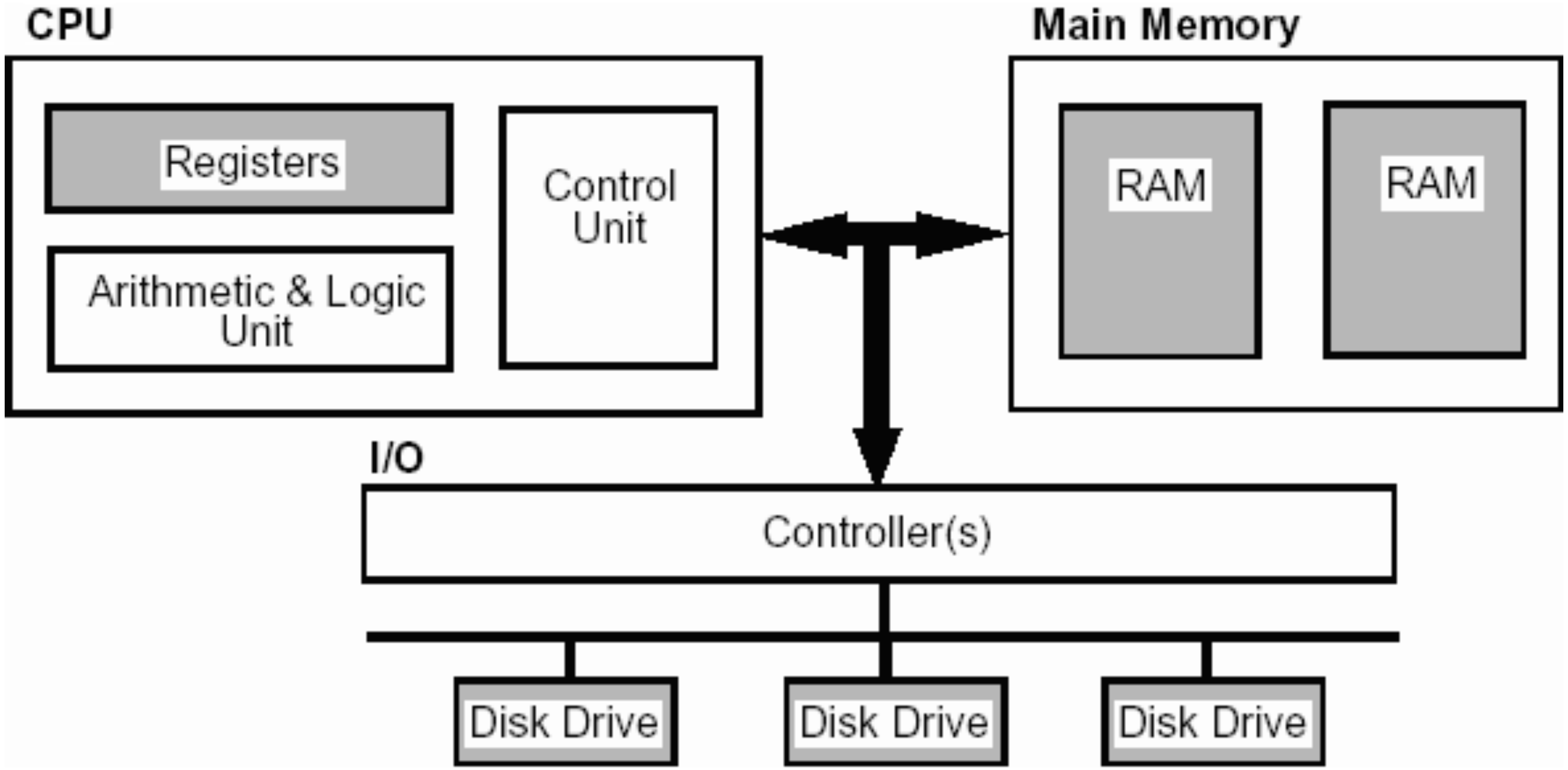
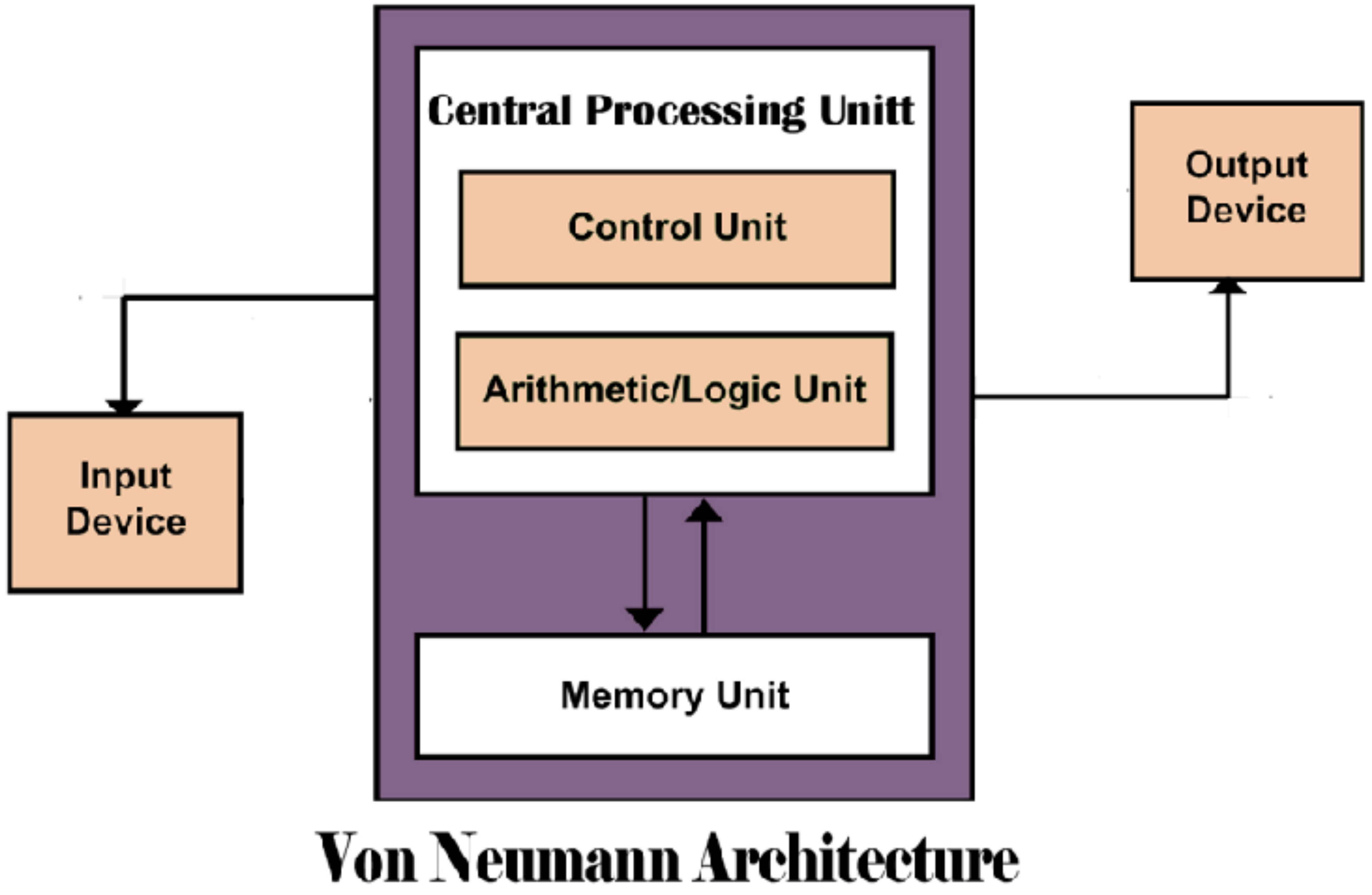
- An overview of the basic memory architecture
- Locality of reference principle
- Arrays
    - *Range-Minimum-Queries*
- Linked Lists

Computing Model

- The architecture of a machine that executes the software (= DS + Algorithms, you know:)).
- Specs are important while devising an algorithm !



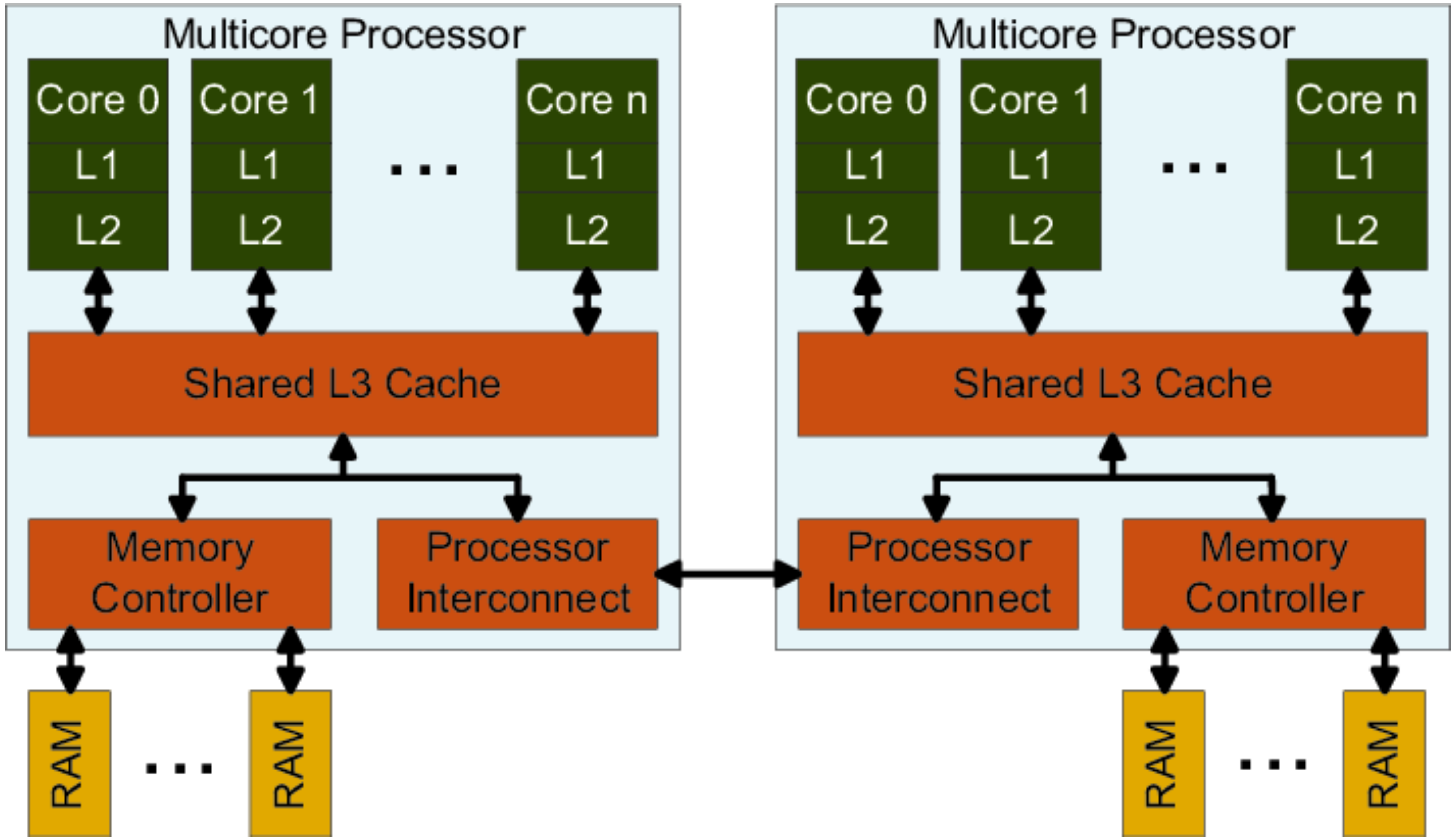
RAM Model



- In-order (!) **sequential** execution of instructions
- Unbounded memory of ***words***
- A word is of size $O(\log n)$, for any n
- Constant-time access to each word in memory
- Equal amount of execution time per instruction

Real computing environments are a bit (!!!) different than these assumptions ?

Modern Computer Systems…



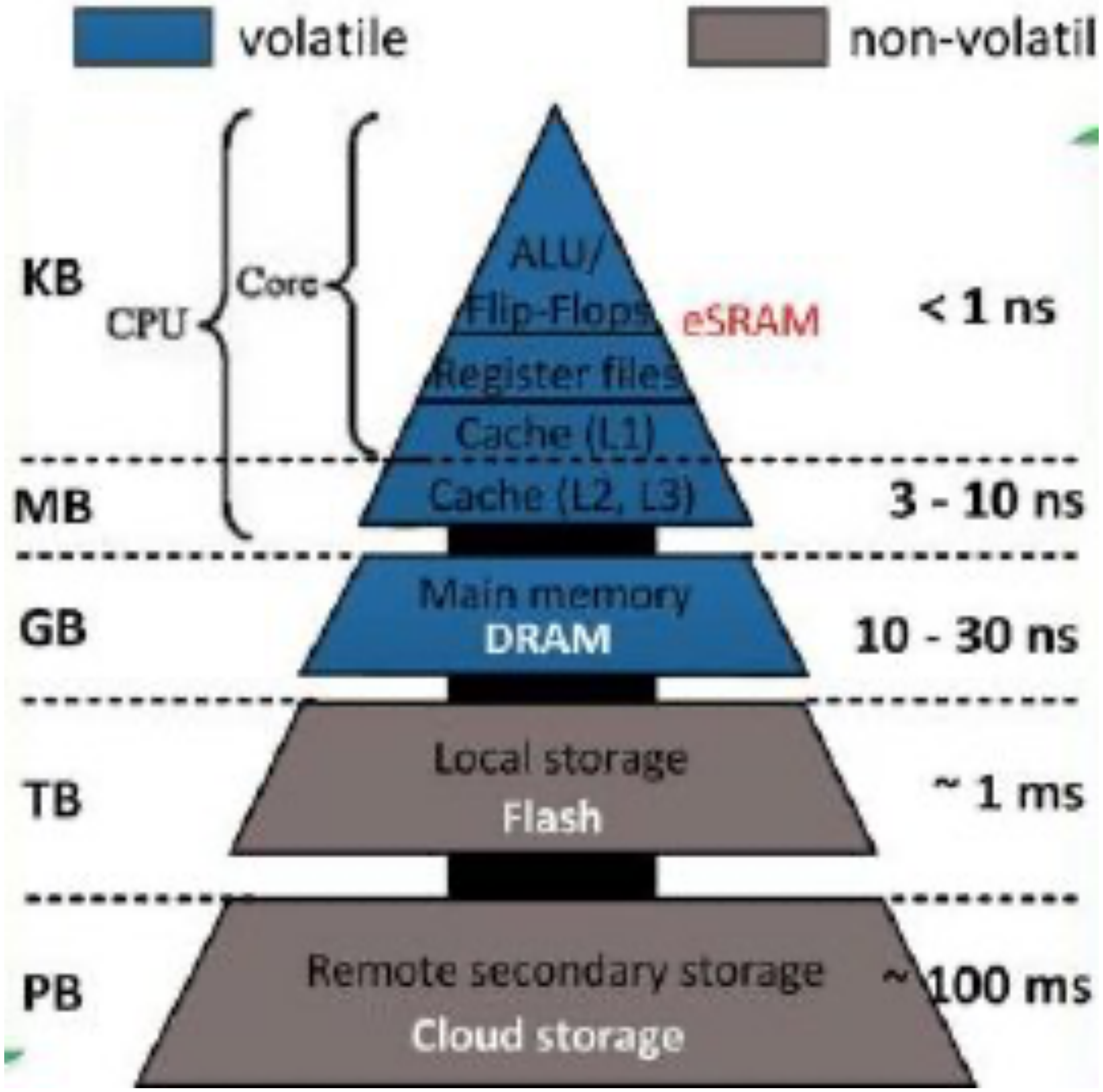
- Sequential execution
- Unbounded memory of ***words***
- A word is of size $O(\log n)$
- Constant-time access to each word in memory
- Equal amount of execution time per instruction

THEORY VS PRACTICE

- **Parallel** execution (out-of-order, multi-core, vectorization, pipelining, etc…)
- **Limited** memory of ***words***
- A **word** is of **constant** number of bits
- **Variable-time** access due to **memory hierarchy**
- **Different** amount of execution time per instruction

Memory Hierarchy

- CPUs operate on data that resides in their registers !
- In a computer, data flows respecting the memory hierarchy.
- It is a lot more efficient to have the data to be processed next as close as possible to CPU.
- Cache-efficient, cache-friendly, cache-aware, cache-oblivious data structures and related algorithms…



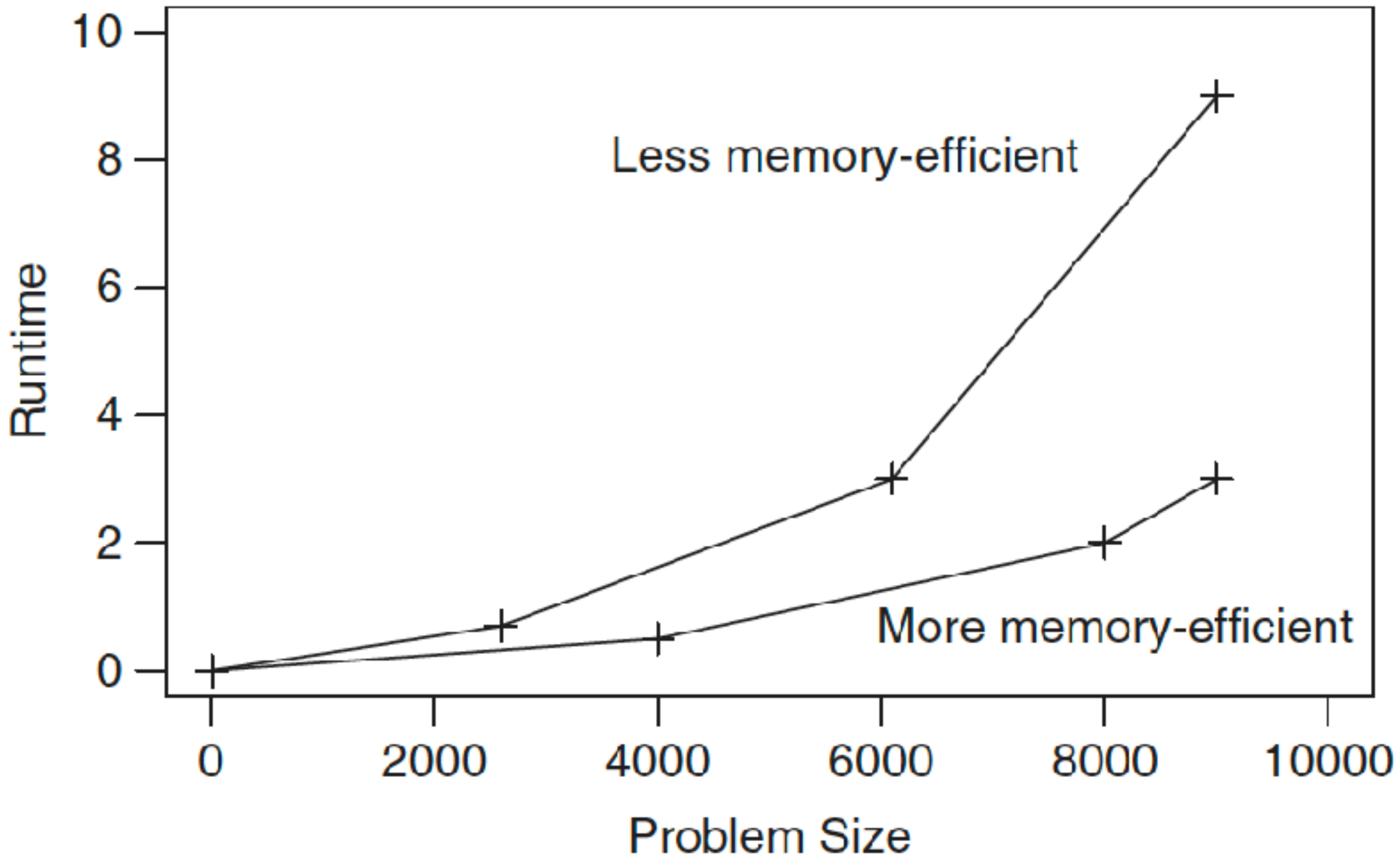
Locality of Reference

- **Temporal Locality *or Locality in Time:***

If a memory location is accessed, it is likely that it will be accessed again.

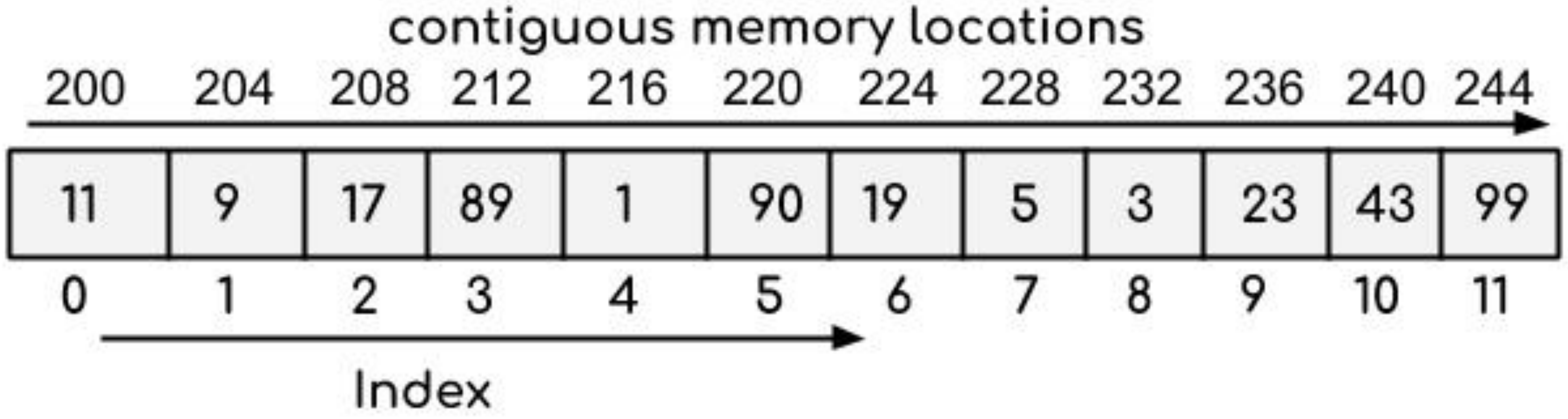
- **Spatial Locality *or Locality in Space:***

During a time interval, the items near the recently accessed items are more likely to be accessed.



Arrays

- A **contiguous** memory block, in which data resides in **fixed-size** chunks.
- Respects the locality of reference (particularly the *spatial* one)
- Most basic data structure that is used in many others…



- **Random access in O(1)-time.**
- **Insertion/deletion is O(n)-time.**

Range Minimum Queries on Arrays

- RMQ: Given an array of integers $A[1..n]$, find the minimum in the region $A[x\,..\,y]$, for the queried x and y positions, where $1 \leq x \leq y \leq n$.
- A primitive building block of many algorithms and related applications

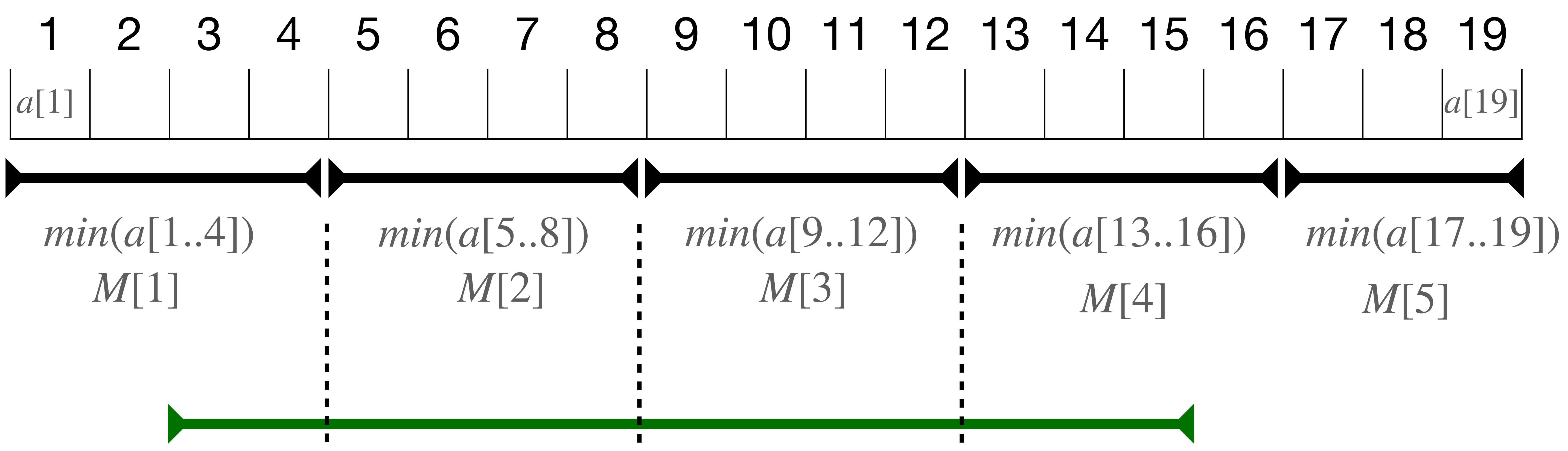
Naive solution: Pass over $A[x\,..\,y]$ and return the minimum.

Worst-case time complexity is $O(n)$. Why?

We can do better …

Range Minimum Queries

$O(\sqrt{n})$-time , $O(\sqrt{n})$-space



Keep the minimum of the $\sqrt{n}$ - length intervals in an additional array, which introduces $\left\lceil \frac{n}{\sqrt{(n)}} \right\rceil \in O(\sqrt{n})$ **space complexity**

$RMQ(A[3..15]) = min(A[3..4], M[2], M[3], A[13,14,15])$

$RMQ(A[3..15])$

What can be the maximum number of comparisons ?

Number of items in the first and last sections $< 2\sqrt{n}$

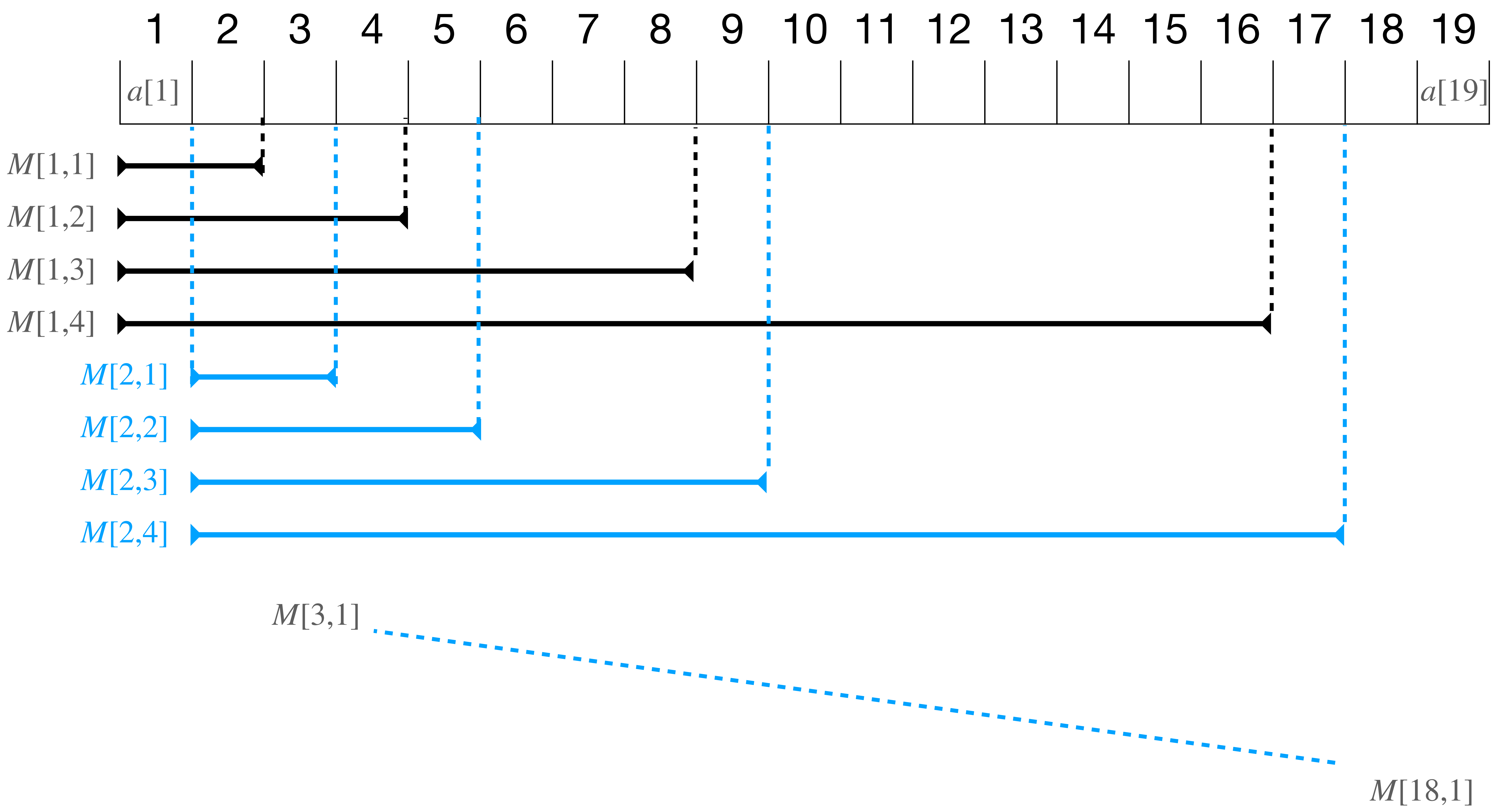
Number of items from the saved minimums $\leq 1 + \sqrt{n}$

Total number of comparisons $\leq 1 + 3\sqrt{n}$, which makes $O(\sqrt{n})$ - **time complexity.**

Range Minimum Queries

$O(1)$-time , $O(n \log n)$-space

For each $i = 1$ to $n-1$, and $j = 1$ to $\log n$, such that $i + 2^j - 1 \leq n$,
store $M[i,j] = min(a[i\,..\,i + 2^j - 1])$ in the look up table M.



$M[i,j]$	1	2	3	4
1				
2				
...				
18		X	X	X

$O(n \log n)$ space,
as n rows with at most $\log n$ entries

Range Minimum Queries

$O(1)$-time , $O(n \log n)$-space

For each $i = 1$ to $n-1$, and $j = 1$ to $\log n$, such that $i + 2^j - 1 \leq n$,
store $M[i,j] = min(a[i\,..\,i+2^j-1])$ in the look up table M.

$a[1]$ ⋯ $a[\ell]$ ⋯ $a[r']$ ⋯ $a[\ell']$ ⋯ $a[r]$ ⋯ $a[n]$

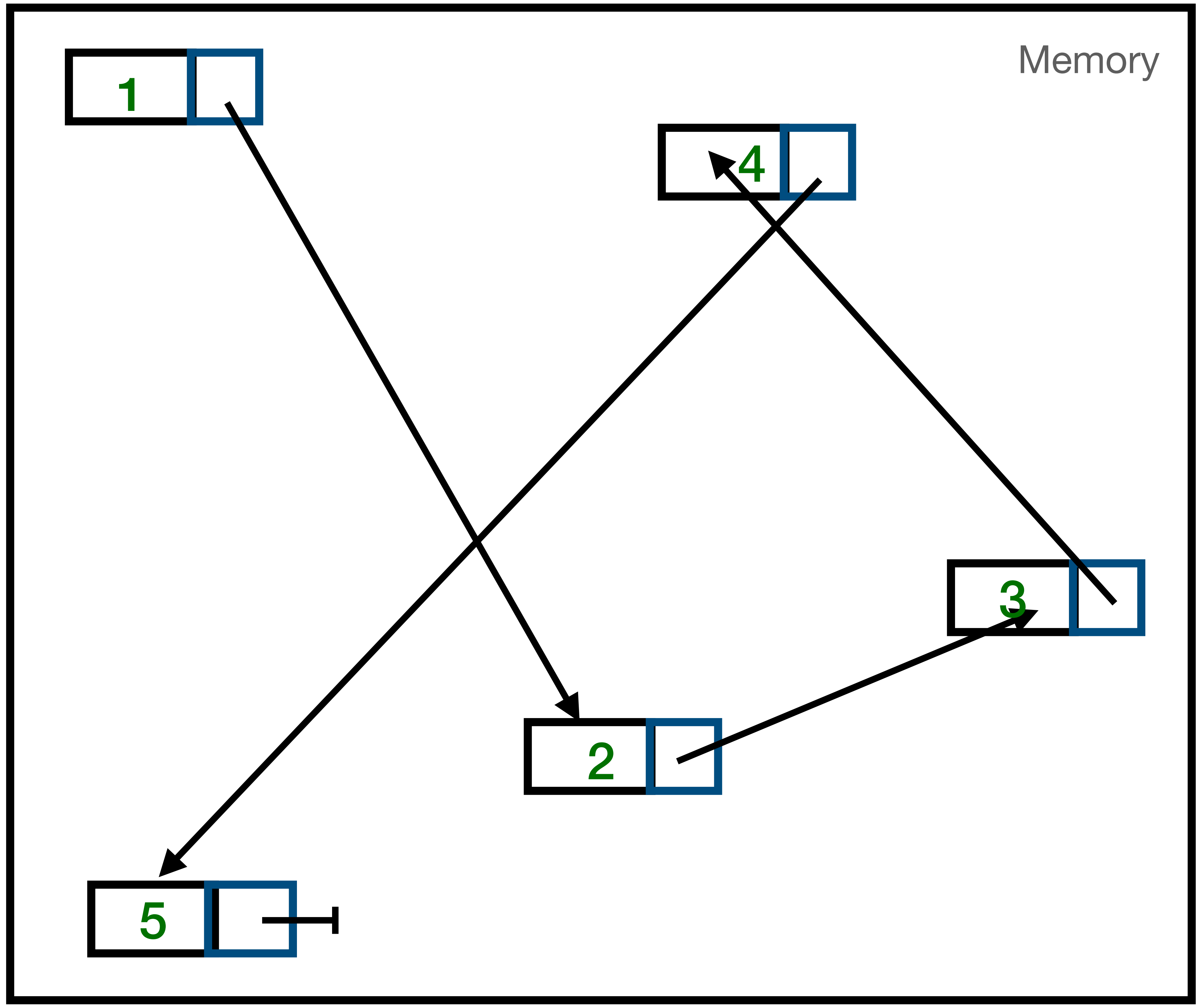
$$\left.\begin{array}{r} \ell' = \ell + 2^i - 1 \text{ for largest i such that } \ell + 2^{i+1} - 1 > r \\ r' = r - 2^j + 1 \text{ for largest j such that } r - 2^{j+1} + 1 < \ell \end{array}\right\} \boxed{(\ell' - r') \geq -1\ ?}$$

$$RMQ(\ell, r) = min(M[\ell, i], M[r', j])$$

Linked List

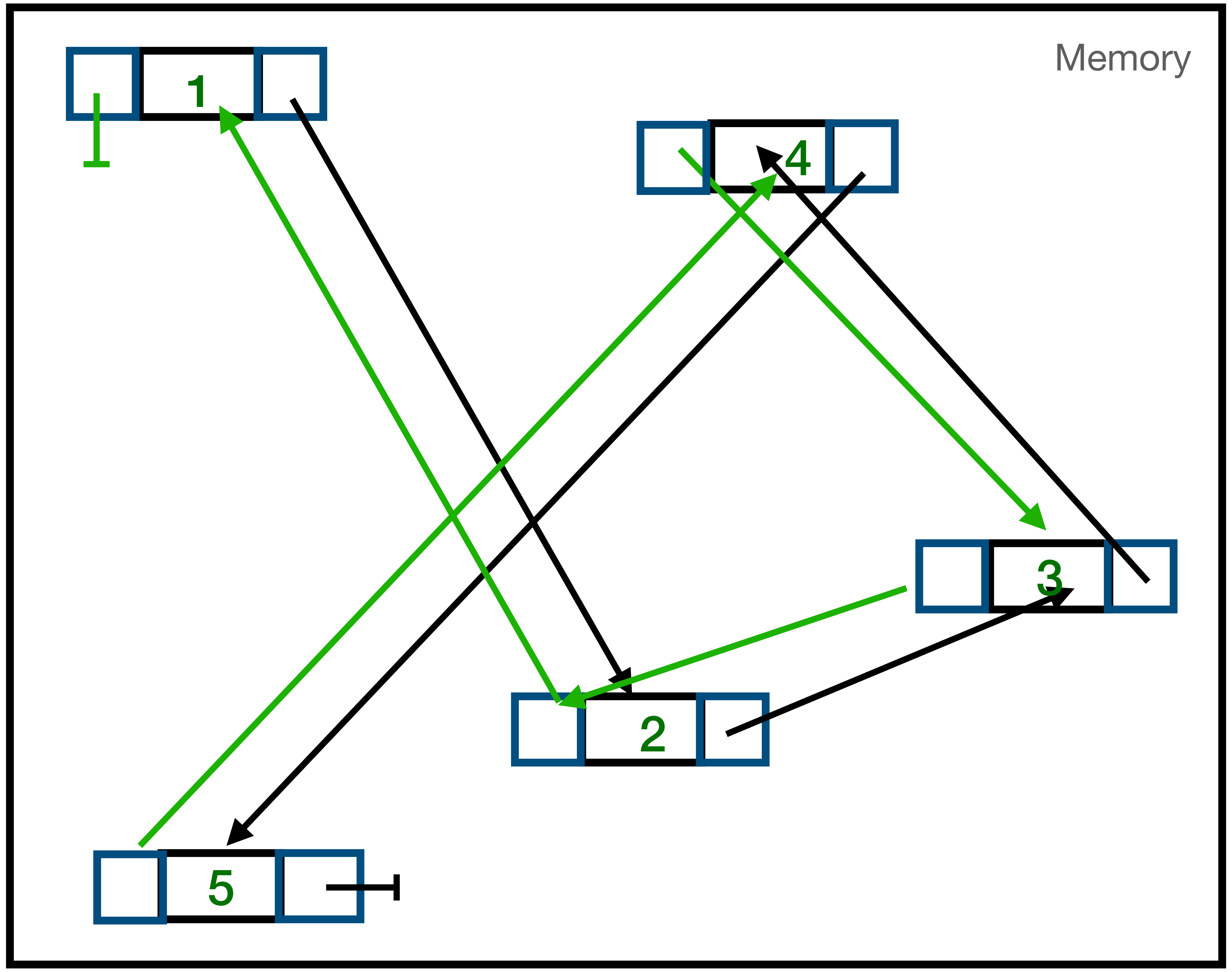
Singly linked list

next ptr, head ptr…



Doubly linked list

next ptr, prev ptr, head ptr, tail ptr…



- Access in O(n) - time
- Insert/delete in O(1)-time *(assuming we are at the position to insert/delete)*

Linked List

On a given doubly linked list of sorted integers, find the pairs that sum up to a queried value K.

On an input 1 <-> 2 <-> 4 <-> 5 <-> 6 <-> 8 <->9 and K=7, then (1,6), (2,5) are the pairs.

How can you find? Complexity?

Linked List vs Array

- Random access : O(n) vs O(1)
- Insert/delete: O(1) vs O(n)
- Space usage
- Sensitivity to the locality of reference

- **Search on sorted sequences: O(n) vs O(log n)**
- **Insert/delete on sorted sequences: O(1) vs O(n)**
- **We can have O(log n) time search with O(log n)-time insert/delete**

- Please read related array/linked list topics from the suggested text books.
- For the range minimum queries, you can find many resources on the internet.
- Next week we will start with the Skip List data structure. You can find a good coverage related to it in the Goodrich et. al.’s book.
- We will continue with the stacks, queues, trees …