# Applied Algorithms

# CSCI-B505 / INFO-I500

## Lecture 13.

## Huffman Codes

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- Introduction to data compression
- Huffman Coding
  - Pre-Huffman with Shannon-Fano Codes
  - Huffman Coding
  - Implementation with heap

# Efficient Transmission & Storage of Information

**CARVING**

**WRITING**

**DIGITIZING**

- Has been receiving interest for a long time, really :)
- Capabilities improve, but also the requirements, and demand
- How to do it most efficiently ?

# WHAT IS DATA COMPRESSION ?




- **Data size always increases proportional to available resource !**
- **Thus, find ways to represent it as concise as possible.**

**Remove the redundancy, and squeeze the data down to its information content (entropy) , which is analogous to a vacuum storage bag.**

# MAIN STEPS OF DATA COMPRESSION ?

## 1. MODELING

- Find a good way to describe your data, which helps to make implicit redundancies explicit.
- Very important since we can compress the data as much as we understand it!

## 2. ENTROPY CODING

- Encode your transformed data with a chosen entropy coder (Huffman, Arithmetic, ANS).
- The effect of entropy coders with respect to modeling is less significant !

**Today we will focus on a widely used entropy coding technique : The Huffman Codes**

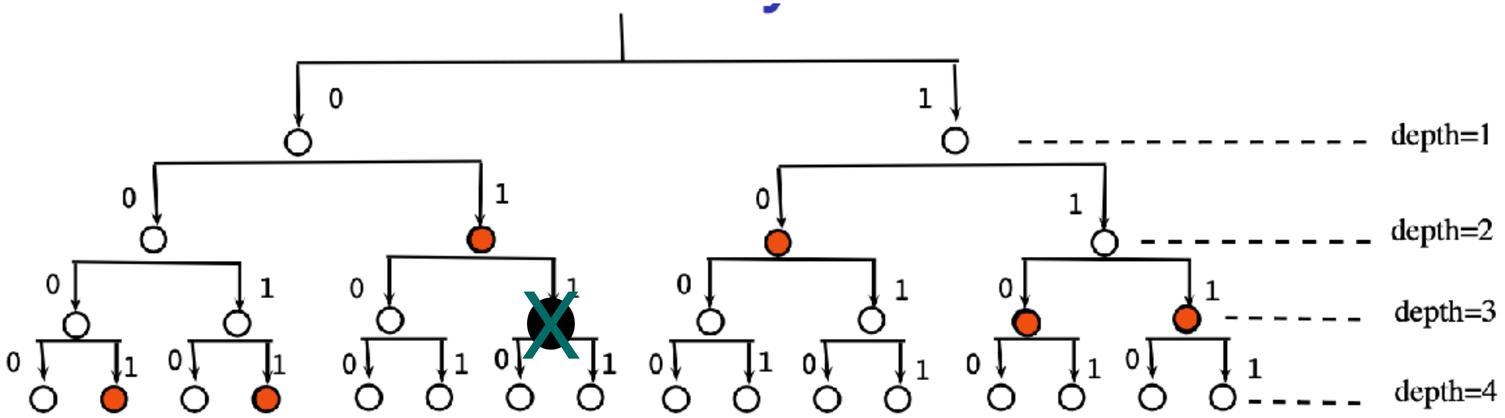
# Main Idea

- Assign a binary codeword to each symbol
- Represent frequent symbols with shorter codewords
- How to make such an  assignment ?

**`abracadabra`**

- We have five symbols so need 3-bits per each with fixed-length coding
- Frequencies: 5 a, 2 b,  2 r, 1 c, 1 d
- To make it compact, say $a \rightarrow 0,\ \ b \rightarrow 1,\ \ c \rightarrow 00,\ \ d \rightarrow 01,\ \ r \rightarrow 10$
- Then, abracadabra = 01100000101100, much better than 11x3=33 bit fixed length representation.
- But, how to decode it !!! Codeword boundaries are lost.
- So, assigning codewords is not only a matter of frequency !

# Prefix-Free Codes




- Let ’s mark the codewords on a binary tree.
- What happens if the assigned codewords are not leaves? **Ambiguity !!!**
- Thus, no codeword should appear as a prefix of another.
- Being **prefix free** guarantees unique decodability.

**Now, the question is how to assign prefix-free codewords respecting the frequency of the symbols ?**

# Initial Attempts: Shannon - Fano Coding

- Shannon-1948 and Fano-1949, independently
- SF does not guarantee optimal codes (unlike Huffman as we will see)
- SF codes guarantee that the codeword of each symbol is at most 1 bit larger than its entropy *(out of our scope today)*

# Shannon's Way

| Symbol | A | B | C | D | E |
|---|---|---|---|---|---|
| Probabilities | 0.385 | 0.179 | 0.154 | 0.154 | 0.128 |
| $-\log_2 p_i$ | 1.379 | 2.480 | 2.700 | 2.700 | 2.963 |
| Word lengths $\lceil -\log_2 p_i \rceil$ | 2 | 3 | 3 | 3 | 3 |
| Codewords | 00 | 010 | 011 | 100 | 101 |

Figure from https://en.wikipedia.org/wiki/Shannon–Fano_coding

# Fano's Way

| Symbol | A | B | C | D | E |
|---|---|---|---|---|---|
| Probabilities | 0.385 | 0.179 | 0.154 | 0.154 | 0.128 |
| First division | 0 | | 1 | | |
| Second division | 0 | 1 | 0 | 1 | |
| Third division | | | | 0 | 1 |
| Codewords | 00 | 01 | 10 | 110 | 111 |

Figure from https://en.wikipedia.org/wiki/Shannon–Fano_coding

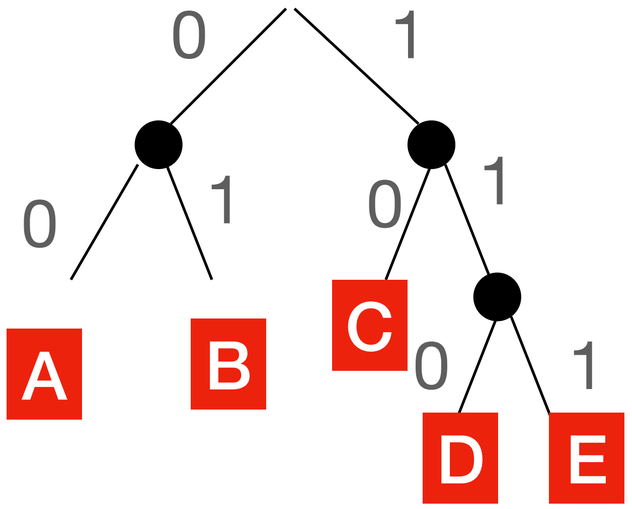
# Shannon - Fano Example

| Symbol: | A | B | C | D | E |
|---|---|---|---|---|---|
| Frequency: | **15** | **7** | **6** | **6** | **5** |

- Sort symbols in decreasing frequency,
- Split into two most balanced partitions
- Add 0 to the codes of the lower part, and 1 to upper part
- Recurse same on the partitions until they become single symbols

# Shannon - Fano Example

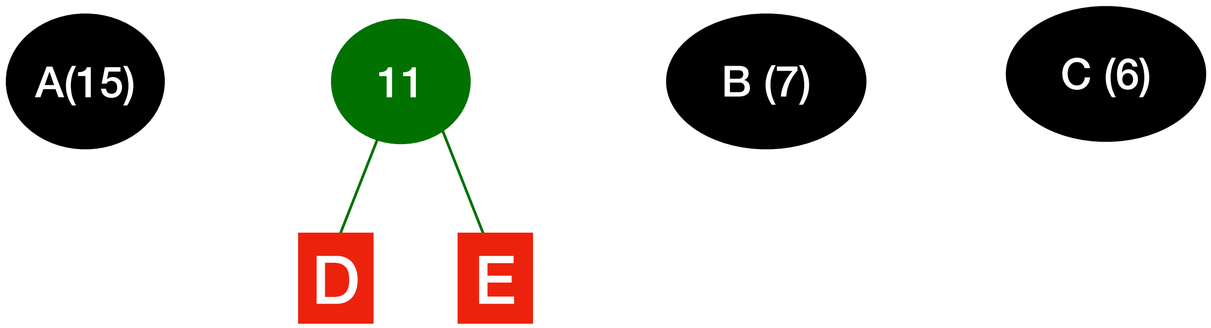
| | 15 - 24 | **22 - 17** | 28 - 11 | 34 - 5 |
|---|---|---|---|---|
| **Symbol:** | **A** | **B** | **C** | **D** | **E** |
| Frequency: | **15** | **7** | **6** | **6** | **5** |
| | **15 (0)** | **7 (0)** | **6 (1)** | **6 (1)** | **5 (1)** |
| | **15 (00)** | **7 (01)** | **6 (10)** | **6 (11)** | **5 (11)** |
| | | | | **6 (110)** | **5 (111)** |

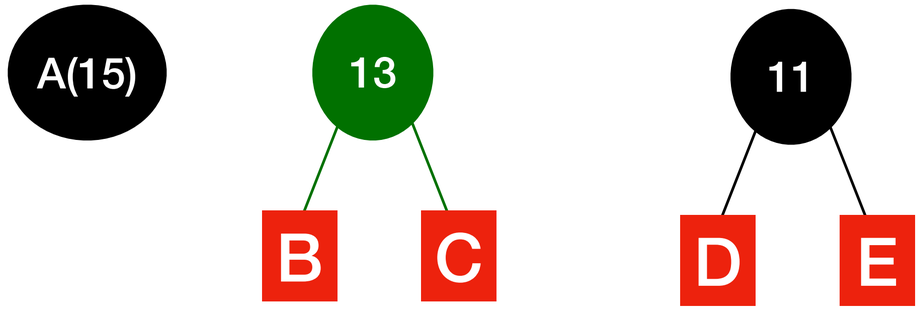



- Shannon-Fano is a top-down approach
- Optimality is not guaranteed !
- Huffman solved the optimality issue with a bottom-up approach

# Huffman

| Symbol: | A | B | C | D | E |
|---|---|---|---|---|---|
| Frequency: | 15 | 7 | 6 | 6 | 5 |



# Huffman

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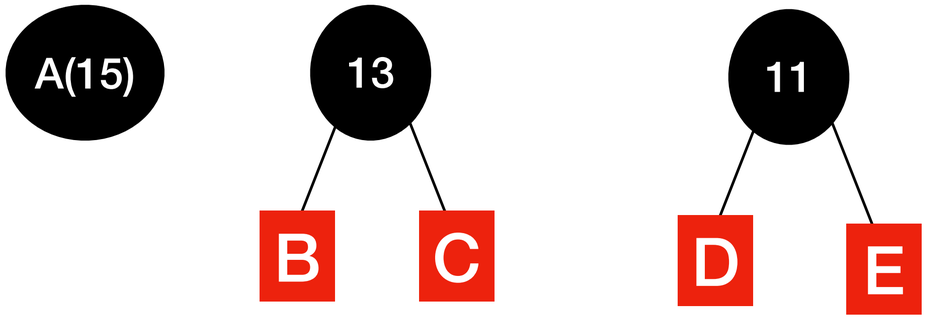
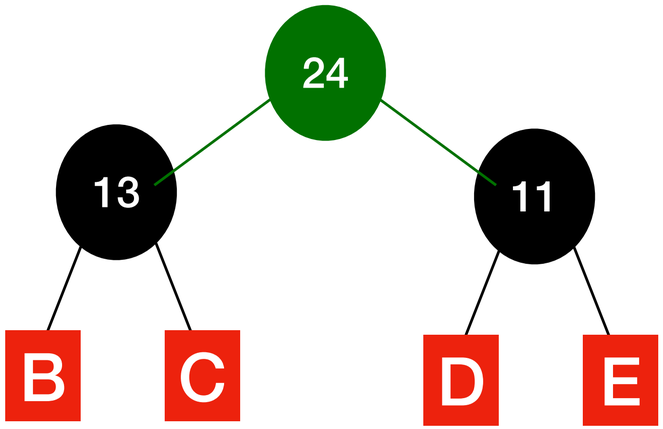
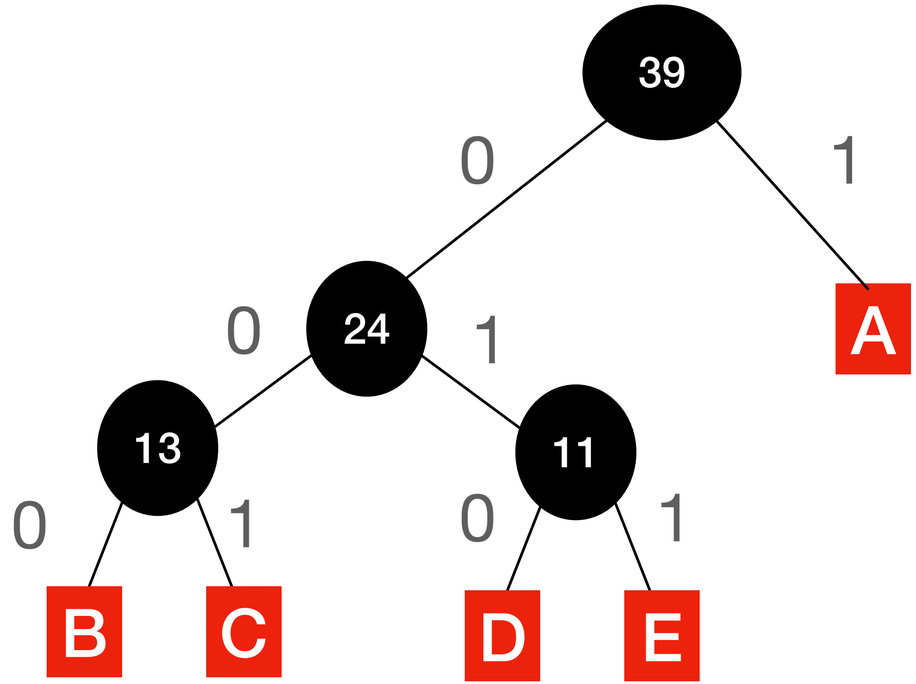




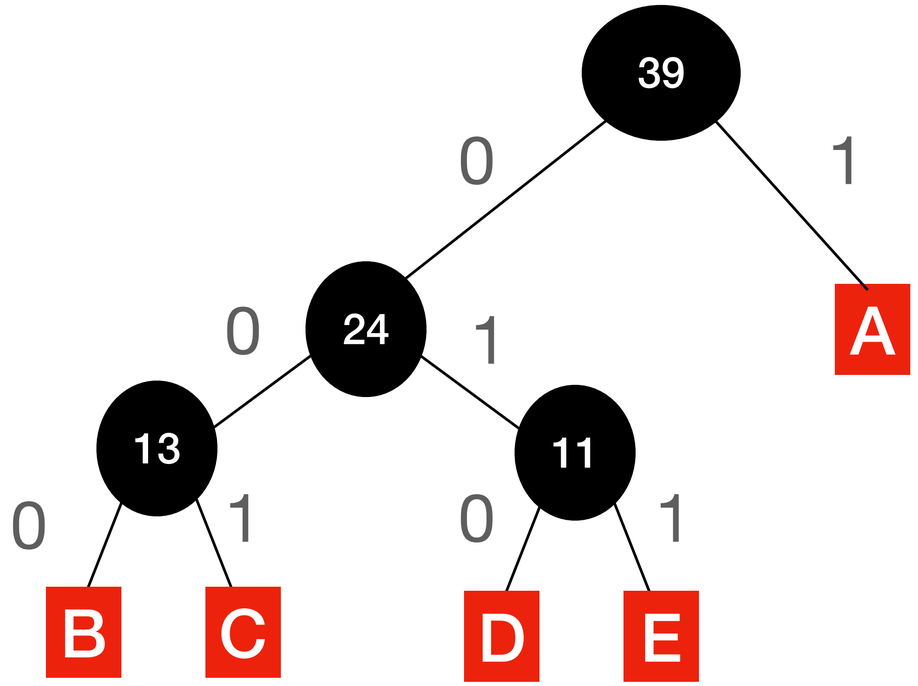


# Huffman

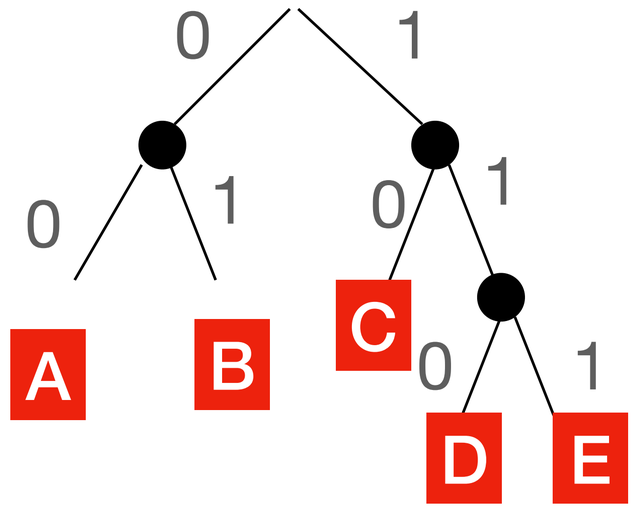
00101010000111001000

| Symbol: | A | B | C | D | E |
|---|---|---|---|---|---|
| Frequency: | **15** | **7** | **6** | **6** | **5** |
| Huffman | **1** | **000** | **001** | **010** | **011** |
| Shannon-Fano | **00** | **01** | **10** | **110** | **111** |

$$15 \cdot 1 + 24 \cdot 3 = 87$$




$$28 \cdot 2 + 11 \cdot 3 = 89$$



# Implementing Huffman

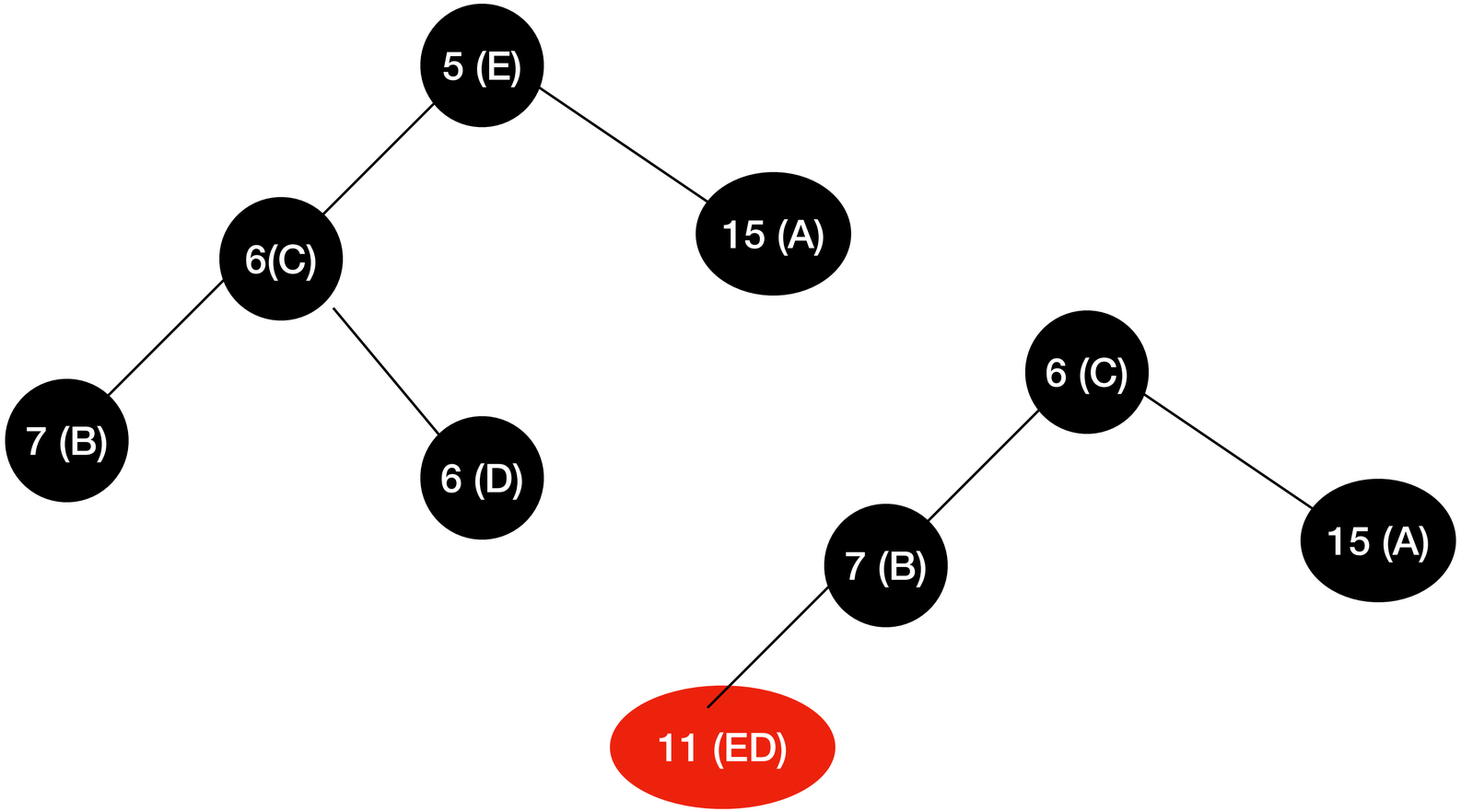
| Symbol: | A | B | C | D | E |
|---|---|---|---|---|---|
| Frequency: | **15** | **7** | **6** | **6** | **5** |

1. Create a min-heap from the frequency counts.

2. Extract the minimum from the heap, and append 0 to their codewords
3. Extract the minimum from the heap again, and append 1 to their codewords
4. Merge these two least frequent symbols into a new symbol, whose frequency is the sum of these symbols.
5. Insert the new symbol into the min-heap with its corresponding frequency count.
6. Go to step 2 until there remains only one node in the min-heap.
7. *Reverse all codewords generated! Why?*

# Implementing Huffman

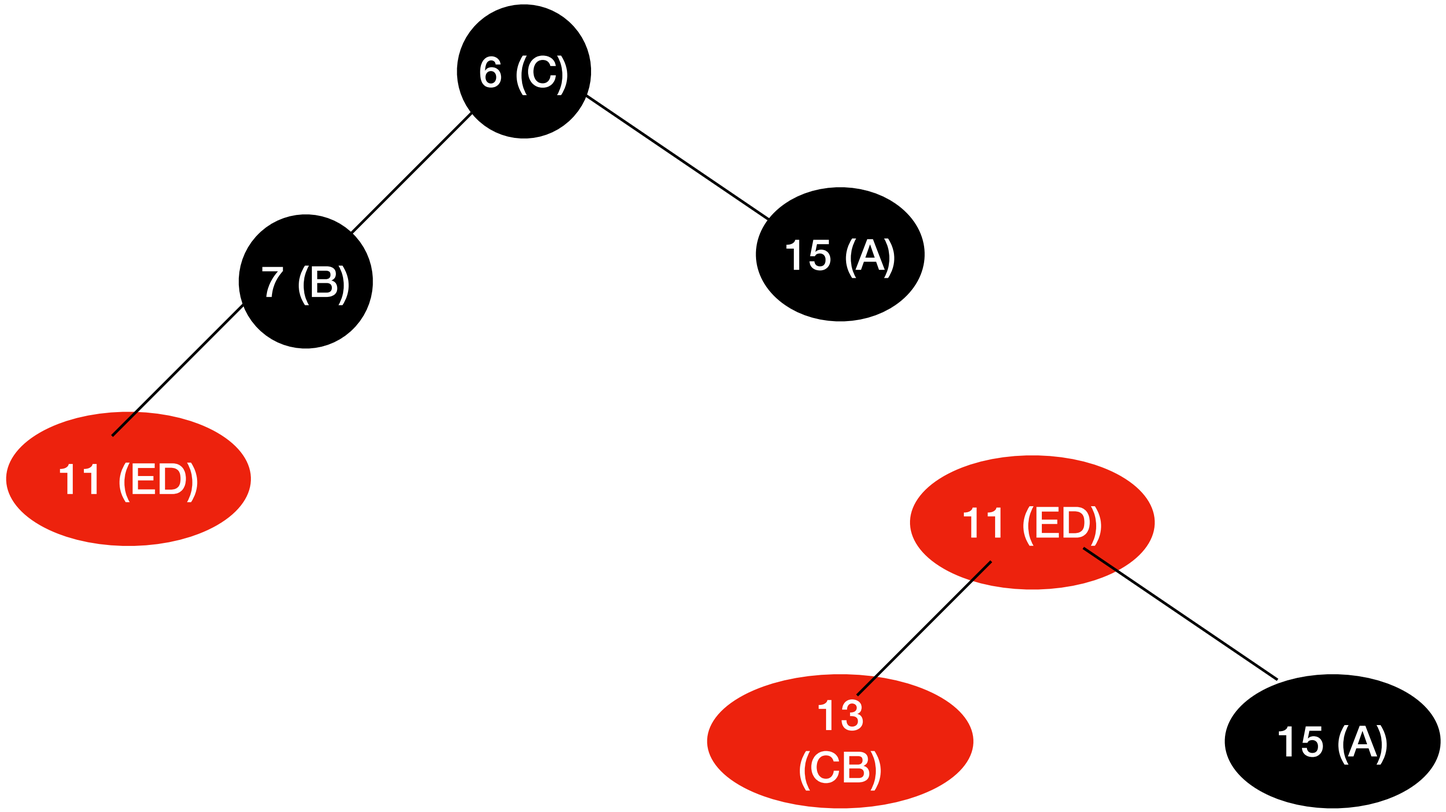
| Symbol: | A | B | C | D | E |
|---|---|---|---|---|---|
| Frequency: | **15** | **7** | **6** | **6** | **5** |




| Symbol: | Codeword |
|---|---|
| A | |
| B | |
| C | |
| D | **1** |
| E | **0** |

# Implementing Huffman

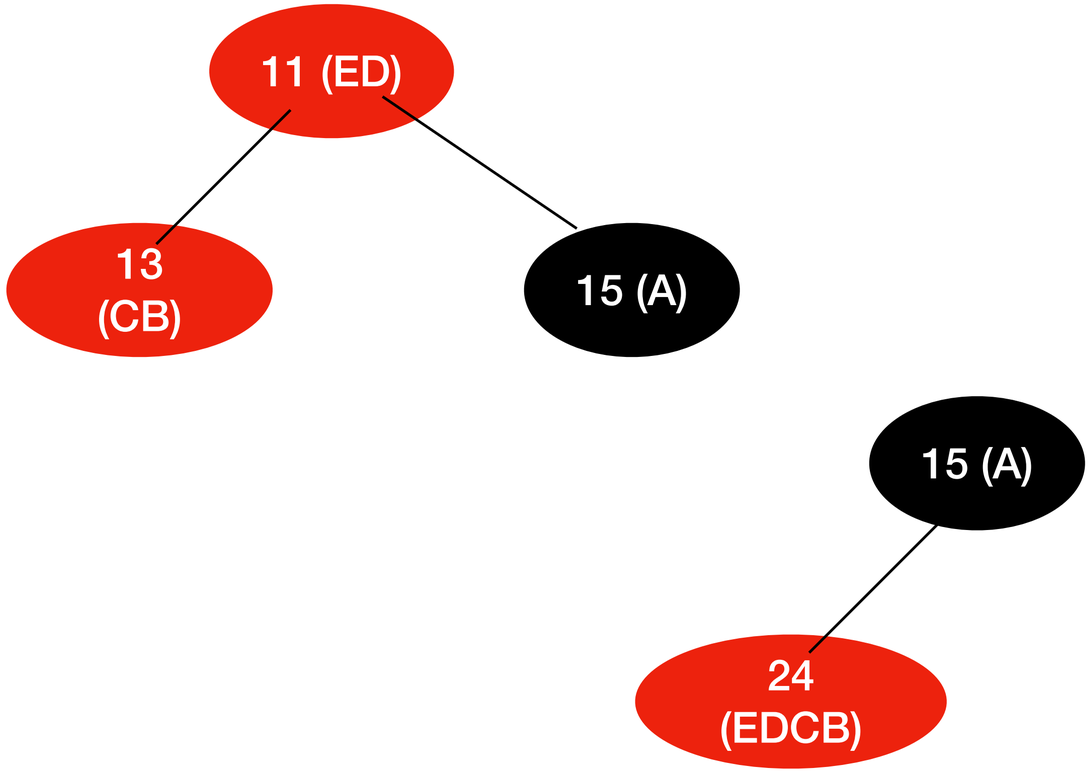
| Symbol: | A | B | C | D | E |
|---|---|---|---|---|---|
| Frequency: | **15** | **7** | **6** | **6** | **5** |




| Symbol: | Codeword |
|---|---|
| A | |
| B | **1** |
| C | **0** |
| D | **1** |
| E | **0** |

# Implementing Huffman

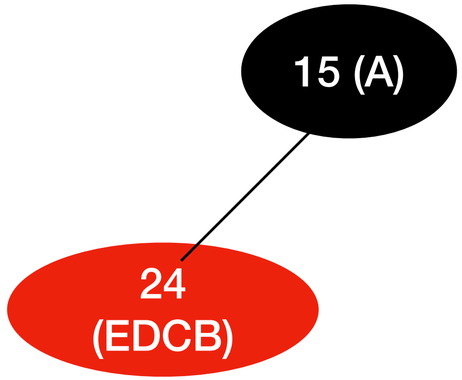
| Symbol: | A | B | C | D | E |
|---|---|---|---|---|---|
| Frequency: | **15** | **7** | **6** | **6** | **5** |




| Symbol: | Codeword |
|---|---|
| A | |
| B | **11** |
| C | **01** |
| D | **10** |
| E | **00** |

# Implementing Huffman

| Symbol: | A | B | C | D | E |
|---|---|---|---|---|---|
| Frequency: | **15** | **7** | **6** | **6** | **5** |




| Symbol: | Codeword |
|---|---|
| A | **0** |
| B | **111** |
| C | **011** |
| D | **101** |
| E | **001** |

# Decoding Huffman

- Receiver needs additional information to correctly decode the compressed data.
- Many different ways to achieve this (out of our scope today…)
- All need to maintain a header before the actual Huffman codes to let receiver generate the same codes of the sender.

# Reading assignment

- Read the chapter 4.8 from Kleinberg&Tardos, chapter 16.3 from Cormen.