

# **Applied Algorithms**

## **CSCI-B505 / INFO-I500**

**Lecture 15.**

**Selection and Beyond**

**with**

**Rank/Select and Wavelet Tree Data Structures**

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- Predecessor/Successor with R/S
- Range Quantile Queries

# Predecessor/Successor

Given a sequence of items,

- Predecessor: Find the largest item less than a queried one
- Successor :Find the smallest item larger than a queried one.

$$S = \{5, 7, 3, 10, 9, 1, 20, 8\}$$

$$Pred(3) = 1 \quad Succ(3) = 5$$

$$Pred(10) = 9 \quad Succ(10) = 20$$

# Predecessor/Successor

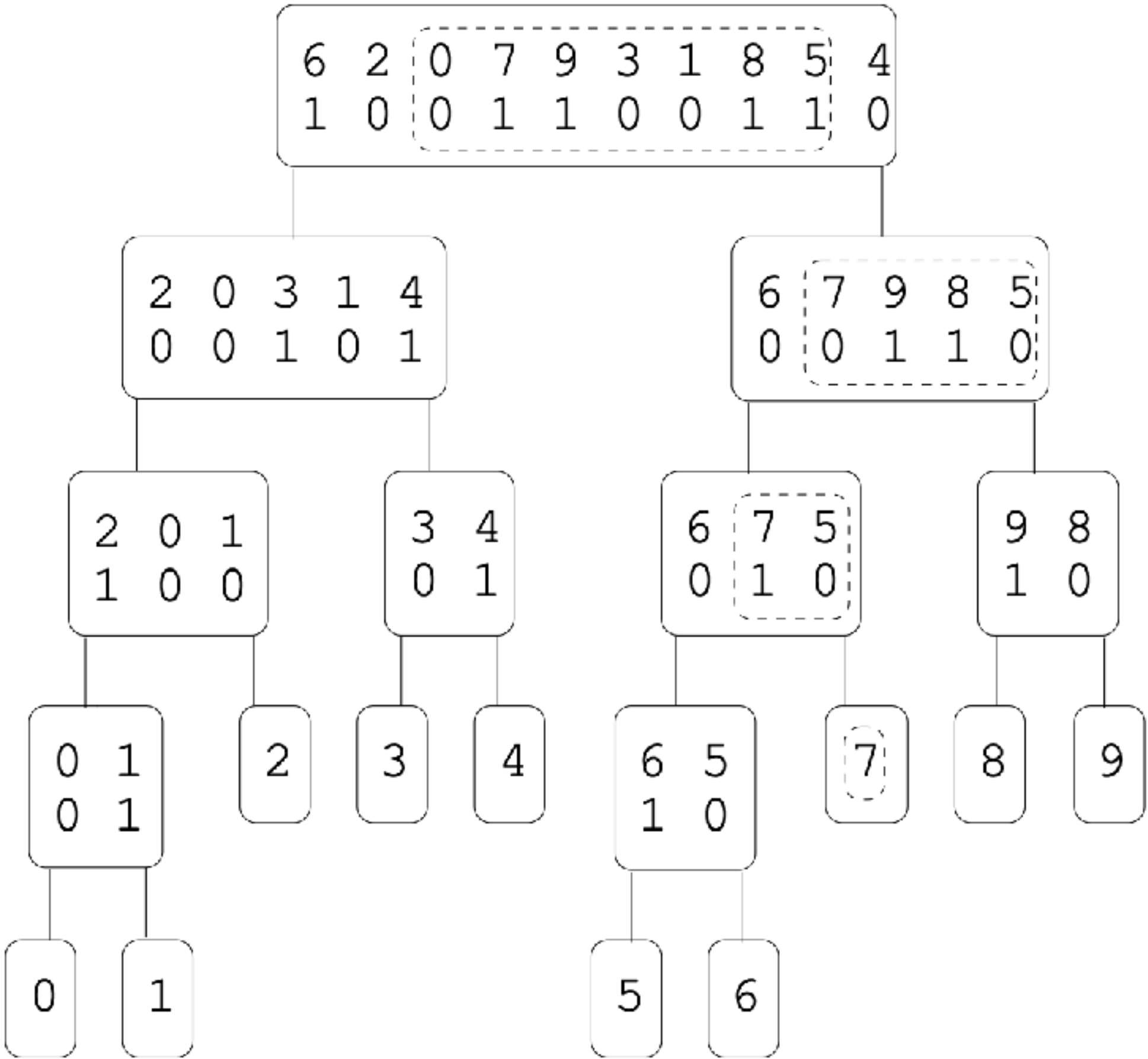
- Keeping a sorted copy of the array with some auxiliary data can solve it. Efficient ??
- There are many different data structures for such queries
- We will focus on a bit-vector solution, assuming we have R/S support

$$S = \{5, 7, 3, 10, 9, 1, 20, 8\}$$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	0	1	0	1	0	1	1	1	1	0	0	0	0	0	0	0	0	0	1

- If  $BV[k]$  is not 1, then  $\text{Pred}(k) = \text{select}(\text{rank}(k))$ ,  
    else  $\text{Pred}(k) = \text{select}(\text{rank}(k) - 1)$
- $\text{Succ}(k) = \text{select}(\text{rank}(k) + 1)$

# Range Quantile Queries with WT



$k = 5$   
 $\ell = 3$   
 $r = 9$

$k = 2$   
 $\ell = 2$   
 $r = 5$

$k = 2$   
 $\ell = 2$   
 $r = 3$

$k = 1$   
 $\ell = 1$   
 $r = 1$

**Time Complexity:**  $O(\log n)$   
**Space complexity:**  $O(n \log n)$

**Fig. 1.** A wavelet tree  $T$  (left) for  $s = 6, 2, 0, 7, 9, 3, 1, 8, 5, 4$ , and the values (right) the variables  $k$ ,  $\ell$  and  $r$  take on as we search for the 5th smallest element in  $s[3..9]$ . The dashed boxes in  $T$  show the ranges from which we recursively select.

# Compact Integer Codes with R/S Dictionary

$$X = \langle 3, 6, \boxed{11}, 5, 1, 3, 15, 9, 13 \rangle$$

1	<b>1</b>
3	<b>011</b>
5	<b>00101</b>
6	<b>00110</b>
9	<b>0001001</b>
11	<b>0001011</b>
13	<b>0001101</b>
15	<b>0001111</b>

Elias Integer Codes  
 $2 \lfloor \log x \rfloor + 1$  bits long

**011001100001011001011011000111100010010001101**

L = 01001**0001**01101000100010001

P = 110**011**001111001101

What is the third integer, given L and P ?

select(3,L) = 9  
select(2,L) = 5  
Bits 4,5,6 on P = 011

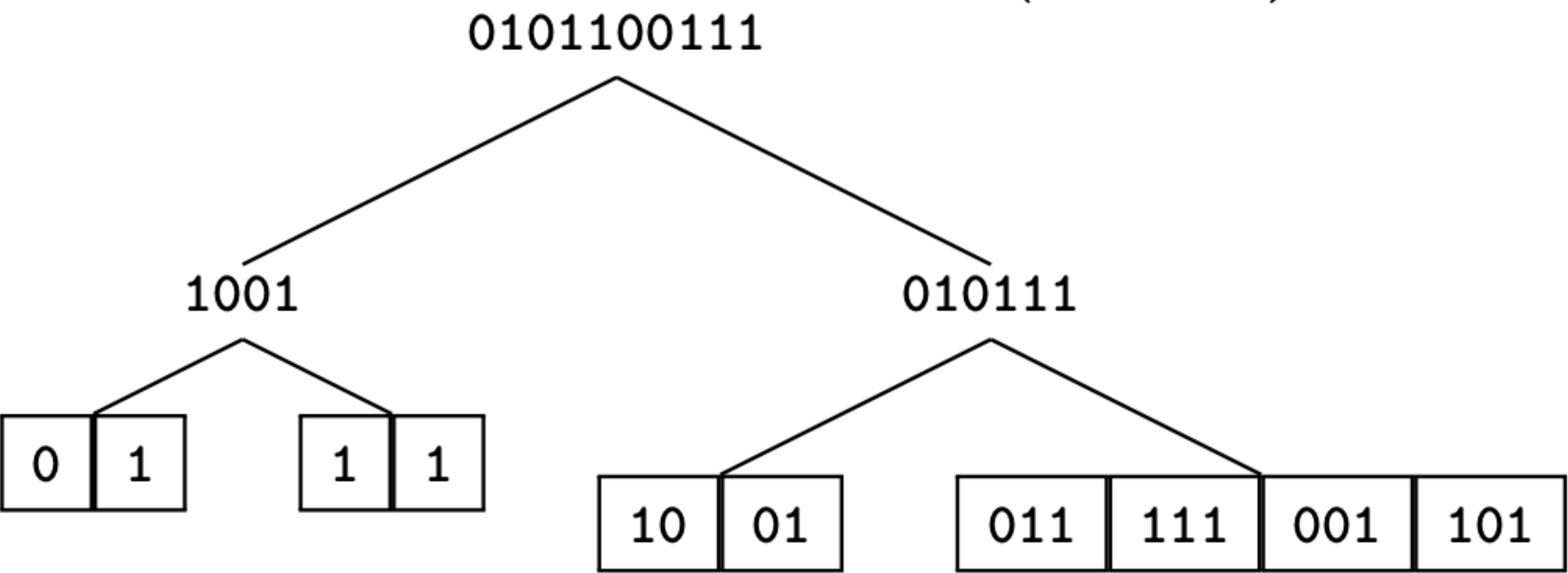
→

0001011 = 11

# Compact Integer Codes with WT

$$X=\{3,6,0,11,5,1,3,15,9,13\}$$

$$\text{Labels\_EliasW}=\{1,2,0,3,2,0,1,3,3,3\} \quad (\lfloor \log x_i \rfloor)$$



0	0
1	1
3	<b>11</b>
5	<b>101</b>
6	<b>110</b>
9	<b>1001</b>
11	<b>1011</b>
13	<b>1101</b>
15	<b>1111</b>

# Reading assignment

- Range quantile queries <https://arxiv.org/pdf/0903.4726.pdf>
- Integer representations with R/S and Wavelet Trees <https://ieeexplore.ieee.org/abstract/document/6824444>