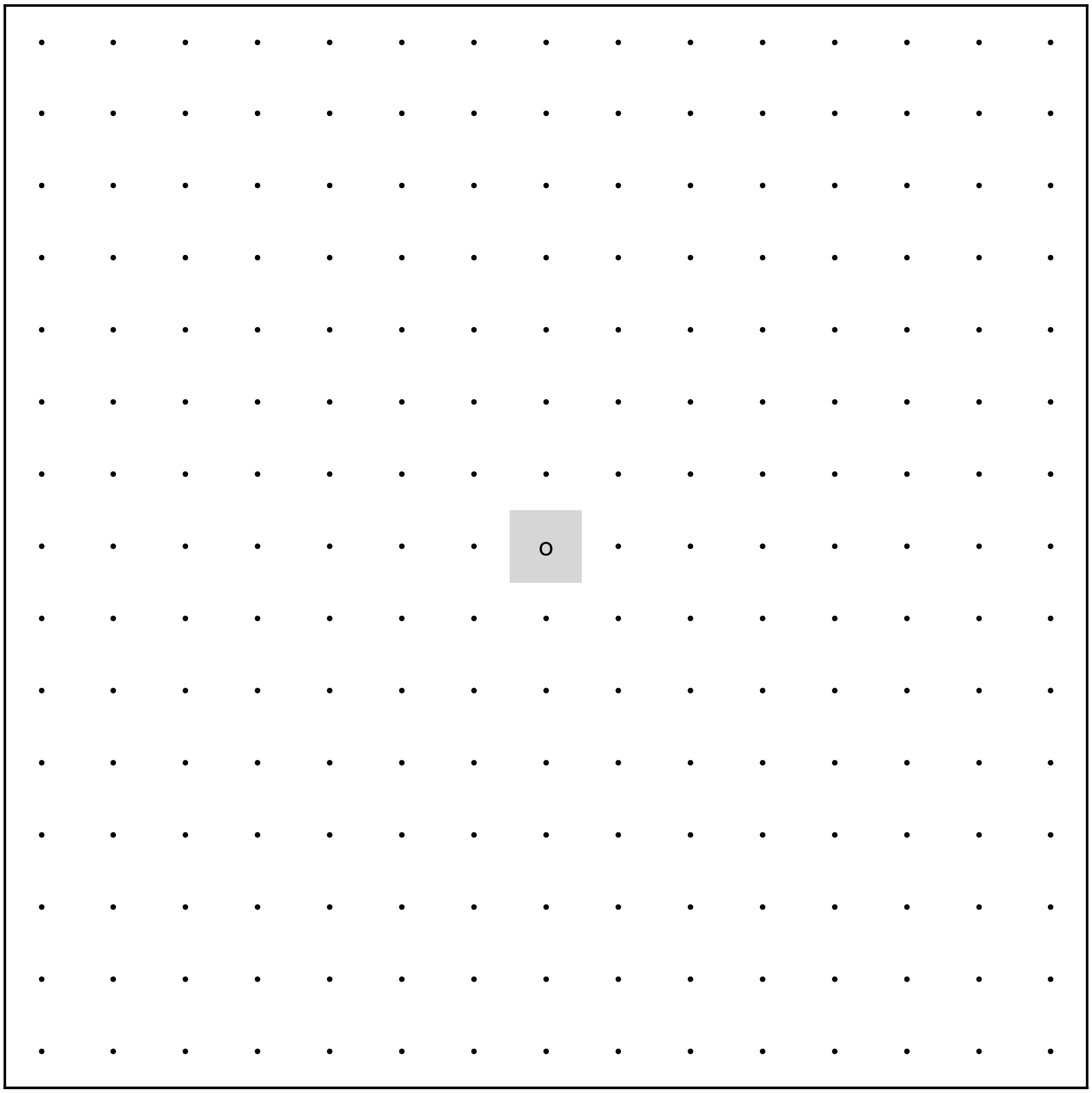
The Drunkard's Walk

Introduction

A drunkard is trying to make his way home after having wasted several hours of his mortal life in a tavern. He finds himself, in the dark, at the center of a field, which we can model as an n x n grid, where n is an odd number (so that the drunkard can be placed at the exact center). The drunkard then repeatedly stumbles one cell in a random direction (north, south, east or west) such that each direction is equally likely but under the constraint that he cannot stumble to a cell he has already visited. If he stumbles off of the field, we say that he has successfully escaped and made his way home. If he stumbles to a cell such he can no longer stumble to a cell he hasn't already visited, however, we say that he has blocked himself in. In this lab, we're interested in finding the probability that he will escape a field of given dimensions.

Task 1: Play

With a pencil, use the 15 x 15 grid below and the four-sided die provided to manually perform the simulation several times. After each simulation, record whether the drunkard escaped or was blocked in, then erase the path and start again.



Based on your results and the results of other groups, estimate the probability that a marker will escape in this scenario. What do think will happen to the probability as the size of the grid grows?

Task 2: Code

Let's now write a script that performs these simulations:

(a) Write a Boolean function with signature *simulate_walk(n)*, where *n* is an odd integer, that simulates the walk of a drunkard placed at the center of an *n* x *n* grid, and returns whether the drunkard escapes.

(b) Write a function with signature *estimate_probability(n, trials)*, where *n* is an odd integer and *trials* is an integer, that performs *trials* simulations and uses the simulation results to return an estimate of the probability that a drunkard placed at the center of an *n* x *n* grid will escape.

Check whether *estimate_probability* is working correctly by comparing its output to your results from part 1. (Of course, *estimate_probability* is a stochastic function, so we would expect different results from different calls, but we should have similar results if we did a large number of experiments.)

Task 3: Explore

Use the code you wrote in part 2 to help you answer the following questions.

(a) Estimate the probability that the drunkard will escape in an n x n grid for:

(i) n = 15
(ii) n = 17
(iii) n = 19

Choose a number of simulation trials that allows you to be fairly confident that your answers are correct to the nearest percent.

(b) At what point does the probability of escape drop below 0.5? Justify your answer with a probability plot that shows how the probability changes as the size of the grid increases.

Task 4: Present

Create a brief (~5 minutes) video that:

- Displays an appropriate title and the group member names for around 5 seconds
- Explains the self-avoiding random walk algorithm we encountered in this lab
- Explains the goal of the lab and how simulations can be used to estimate probabilities that might otherwise be difficult to calculate
- Uses animations, illustrations or other techniques to present an example walk that helps the viewer understand the algorithm better
- Presents and explains the code your group wrote
- Walks through the code you used to solve problem (a) in part 3
- Walks through the code you used to solve problem (b) in part 3 (including how it was used to generate your plot)

Assessment

Your group's video will be peer-assessed according to agreement with the following statements:

- The video title is clear.
- The self-avoiding random walk algorithm used is clearly explained.
- The goal of the lab is clearly explained.
- An example simulation is presented, which makes the algorithm easy to understand.
- The code presented is easy to read and presented without distractions.
- The code is well-written: variable names are well chosen, data types are explicitly given or obvious from the context.
- How the code was used to solve problem 3 (a) is clear.
- The answer to problem 3 (a) is presented and appears to be correct.
- How the code was used to solve problem 3 (b), including how the plot was created, is clear.
- The answer to problem 3 (b) is presented and appears to be correct.
- The narration of the video is clear and smooth.