Pig, a game of decisions

Activity Reflection

Mathematical Analysis

In *risk analysis* we often determine the *value* of a decision.

Where there are random effects involved, we need to use probability to determine the value of a decision.

Example

Consider a gambling game: it costs $3 to play, a fair die is thrown, if the die comes up a ⚀ or a ⚅ then the player wins $15, otherwise the player gets nothing.

What is the value, in dollars, of the decision to play this game?

Since we pay $3 to play, it is possible that we lose the game and lose $3. It is also possible that we win the game and gain $15 - $3 = $12. We can weight these outcomes by the respective probabilities to get the value of the decision to play this game:

$$\frac{2}{3}(-3) + \frac{1}{3}(12) = -2 + 4 = 2$$

The value of the decision to play this game, then, is $2. If we were to play the game many times, we would sometimes (usually) lose $3 and sometimes gain $12; on average, however, we would gain $2 per game.

Risk analysis in Pig

- Use similar reasoning to determine the value of the decisions to *take the loot* and to *throw again*, given that the amount of loot is x. For which values of x does the value of the decision to *throw again* exceed the value of the decision to *take the loot*?

Limitations

- How might we interpret the result of this analysis? Why is this analysis naive in a real game of *Pig*?

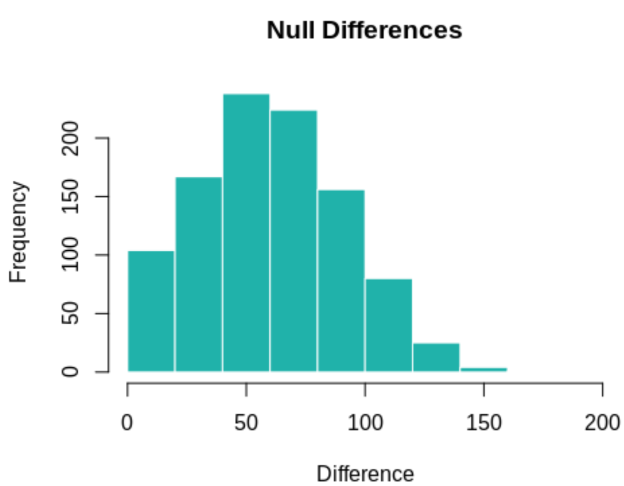
(If you're interested in reading more about the optimal rules for playing *Pig*, see the UMAP Journal's 2004 article by Neller, Presser: *Optimal Play of the Dice Game Pig*.)

Scientific Analysis to Compare Decision Processes

During our activity, each group created a set of rules to make decisions while playing *Pig*. We are now interested in determining which group created the *best* rules.

Before we can answer this question, we need to *define* what we mean by "good" and "best" when using these terms to describe a set of rules.

- What do you think we mean by these terms when describing a set of rules to play *Pig* by?
- Given that group A's rules beat group B's rules in a game of *Pig*, can we say that group A's rules are better than group B's rules? Explain.
- Given that group A's rules played 1000 games against group B's rules, and beat group B's rules for the majority of those games, can we say that group A's rules are better than group B's rules? Explain.
- Mr Ambler conducted the following experiment:
  - The *Random-Pig* rules played 1000 games against *itself* (to ensure that both players played *equally well*, or rather, equally poorly).
  - After the 1000 games, Mr Ambler recorded the *difference* between the number of wins of each player (for example, if one player won 525 times and the other player won 475 times then Mr Ambler would record "50").
  - This process was repeated 1000 times (in other words, 1 000 000 games were played in total).
  - Mr Ambler then plotted the following *histogram* of differences, to get an idea of the typical differences that might come up when two *equal* players play 1000 games against each other.



How might this information be useful if we are trying to decide whether one group's set of rules plays a better game of Pig than another group's rules after the groups have played 1000 games?