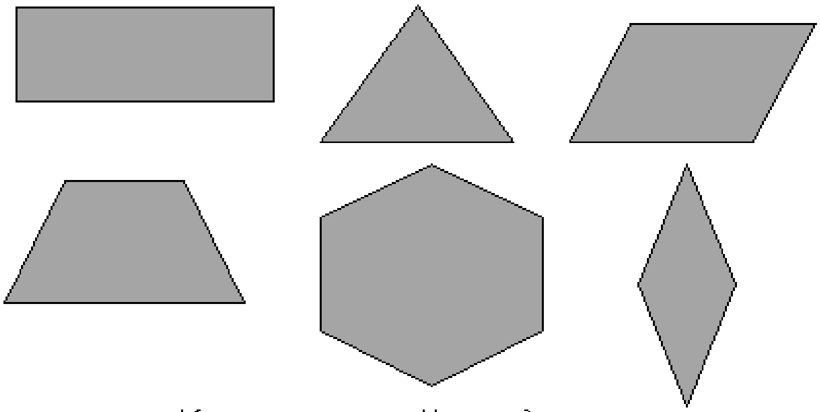
10. Mensuration

Mensuration [2-D]

Polygon

A **polygon** is a plan figure enclosed by line segments which are called the sides of the polygon. A polygon of five sides is called a pentagon that with six sides is called a hexagon and that with ten sides is called decagon and so on. Some examples of polygons are given in the following figures.



Important Points:

- The **perimeter is** length of any curve or polygon
- A **vertex** of a polygon is the point where its two adjacent sides meet.
- **Consecutive sides:** Consecutive sides are those which have a vertex in common.
- A **diagonal** of a polygon is a line segment connecting two nonadjacent vertices
- In any polygon, (n - 2) triangles are formed by drawing diagonals from one vertex.
- A polygon of 3 sides is called a **triangle.**
- A polygon of 4 sides is called a **quadrilateral.**
- A polygon of 5 sides is called a **pentagon.**
- A polygon of 6 sides is called a **hexagon.**
- A polygon of 7 sides is called a **heptagon.**
- A polygon of 8 sides is called a **octagon.**
- A polygon of 9 sides is called a **nonagon.**
- A polygon of 10 sides is called a **decagon.**
- A polygon of 11 sides is called a **undecagon**.

Regular polygons: If all the sides of a polygon are equal as well as all angles are equal then the polygon is called a regular polygon. A regular polygon call' always be inscribed in a circle. The centre of this circle circumscribing the polygon is also the centre of a regular polygon. Therefore, the centre of a regular polygon is equidistant from all its vertices. A regular polygon can also circumscribe a circle.

For a regular polygon

- Interior Angle + Exterior angle = 180^0

- Sum of exterior angles = 360^0
- Sum of Interior angles = (n - 2) 180°
- Perimeter P = n x a (n = numb of sides, a = length of side)
- Each Interior angle = $\frac{2n-4}{n} \times 90^0$
- Each exterior angle = $(\frac{360}{n})^0$
- Area = $\frac{1}{2} \times P \times r = \frac{1}{2} \times n \times a \times r$ where a is the length of sides, n is e number of sides and r is the radius of the inscribed circle.
- Area = $\frac{na^2}{4} \times \cot(\frac{180}{n})^0$

Example: Find the sum of the interior angles of a six, eight and ten sided polygon

Sol.

Sum of interior angles = (n - 2) 180°

So, sum of the interior angles of a six sided polygon = (6 - 2) x 80° = 720°;

Sum of the interior angles of a eight sided polygon = (8 - 2) x 80° = 1080° and

Sum of the interior angles of a ten sided polygon = (10 - 2) 1800 = 1440°

Example: Find the sum, of the external angles for a twelve sided polygon.

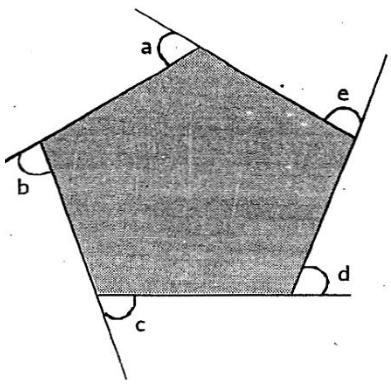
Sol. The sum of the exterior angles of any Polygon is always 360°

Example: If the sum of the internal angles of a polygon is 1260°, Find the number of sides.

Sol. Using the formula (n - 2) x 180° = Sum of the Interior Angles. Thus we can say

(n- 2) 180° = 1260° Solving n = 9 sides.

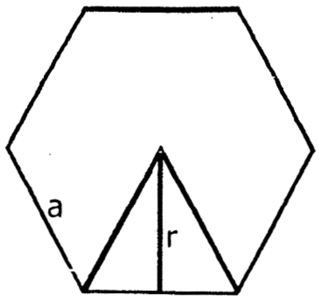
Pentagon



Important Points:

- Its external angle are named from a to e.
- The sum of internal angles = (5 - 2) x 180° = 540°
- So, each interior angle of the pentagon measures 108°
- The interior and exterior angles form linear pairs.
- Each exterior angle measures 180° - 108° = 72°.

Regular Hexagon

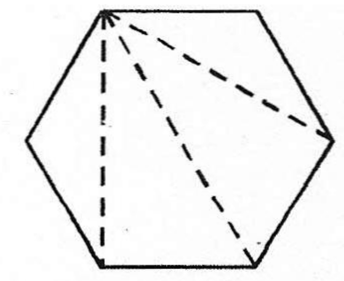


Important Points:

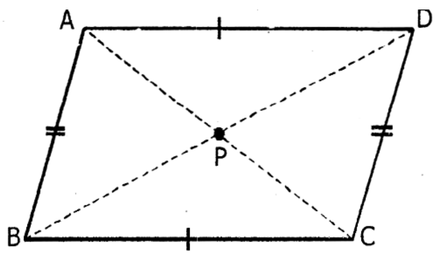
- Sum of Interior angles = 720°
- Each interior angle = 120°
- Each exterior angle = 60°
- Area = $\frac{3\sqrt{3}}{2}$ x a^2 (a = side).

Example: What is the measure of one interior angle of the following given regular hexagon?

Sol. Find the sum of the interior angles and divide by the number of interior angles or 6.



Parallelogram



If the opposite sides of a quadrilateral are parallel, the quadrilateral is a parallelogram. Opposite sides of a parallelogram are equal and so are opposite angles. Any two consecutive angles of a parallelogram are supplementary. A Diagonal of a parallelogram divides the parallelogram into congruent triangles. The diagonals of a parallelogram bisect each other.

AD || BC	∠A + ∠B = 180°
AD = BC	ΔABD ≈ ΔCDB
AB || DC	ΔABC ≈ Δ CDA

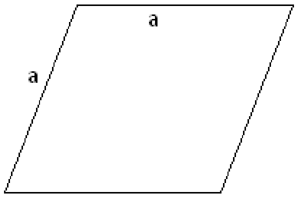
AB = DC AP = PC

∠D = ∠B BP = PD

∠A = ∠C.

Rhombus

If in a parallelogram II sides are equal, it is a Rhombus.



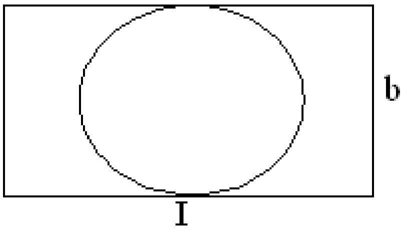
Important Points:

- Opposite angles are equal.
- Diagonals bisect each other at 90°.
- Diagonals bisect angles at vertices.
- Sum of any two adjacent angles = 180°.
- Figure formed b joining the mid points of the adjacent sides of a rhombus is rectangle.

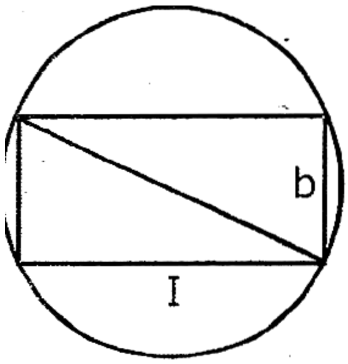
Rectangle

Important Points:

- Opposite sides equal, each angle = 90°.
- Diagonals bisect each other (not at 90°).
- Of all rectangles of given perimeter, a square has maximum area.
- When inscribed in a circle, the figure will have maximum area when it's square.
- Figure formed by joining the midpoints of a rectangle is a rhombus.
- If P is any point with in a rectangle ABCD then PA^2 + PC^2 = PB^2 + PD^2
- The biggest circle that can be inscribed in a rectangle will have the diameter equal to the breadth of the rectangle.

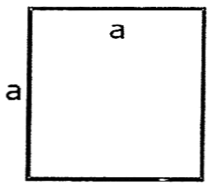


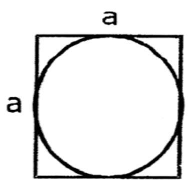
- When a rectangle is inscribed in a circle, the diameter of the circle is equal to the diagonal of the rectangle.

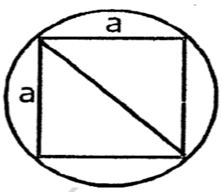


- Diagonals of a rectangle are equal.

Square



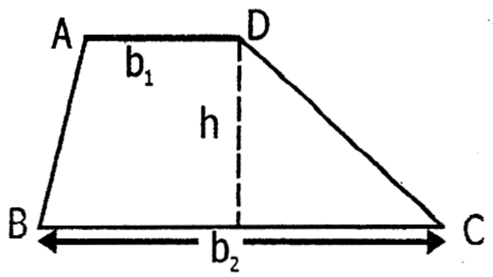




Important Points:

- All sides are equal.
- All angles are equal and of 90° each.
- Diagonals bisect each other at 90° and are equal.
- When inscribed in a circle, diagonal = diameter of circle.
- When circumscribed about a circle, Side of square = Diameter of circle.

Trapezium (Trapezoid)



AD || BC

AD and BC are bases.

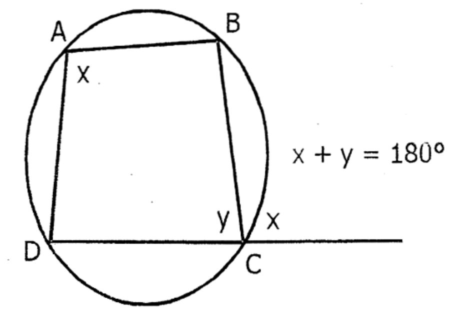
AB and DC are legs.

h = altitude

Important Points:

- If inscribed in a circle, it becomes an isosceles Trapezium. The oblique sides are equal, angles made by each parallel side with oblique sides are equal. Diagonals are equal.
- Diagonals intersect proportionally in the ratio = ratio of length of parallel sides.
- If the figure ABCD is a trapezium, then Ac^2 + BD^2 = BC^2 + AD^2 + 2 AB.CD
- In the above figure, if x & y are mid points of diagonals, then xy = $\frac{1}{2}$ (AB - CD).

Cyclic Quadrilateral



Important Points:

- The four vertices lie on a circle.
- Opposite C angles are supplementary.
- If anyone side is produced, Exterior angle = Remote interior angle.
- If one pair of opposite sides are equal, diagonals are also equal.
- The quadrilateral formed by angle bisectors of a cyclic quadrilateral is also cyclic.
- Sum of products of opposite sides = product of diagonals.

Area of plane figures

Any figure bounded by three or more than three straight lines or bounded by a closed circular line (e.g. a circle) is a **plane figure.** The space enclosed within that figure is called its area.

The measurement of the length of the lines enclosing the space is called its **perimeter.**

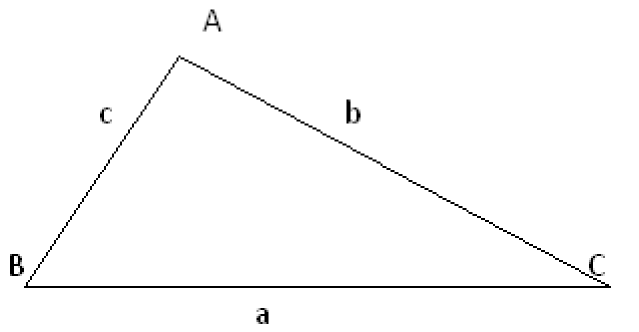
Types of plane figures: There are the following types of plane figures

- Triangle is plane figure bounded by three straight lines
- Quadrilateral is a plane figure bounded by four straight lines
- Polygon is closed figure bounded by 3 or more than 3 straight lines
- Circle is the path traced by a point which moves in such a way that it distance from a fixed point remains constant.

Throughout this chapter

- A = Area, P = Perimeter,
- C = Circumference V = Volume,
- R = circum-radius r = In-radius of a polygon or radius of circle
- H = Height Hyp = Hypotenuse

Triangles



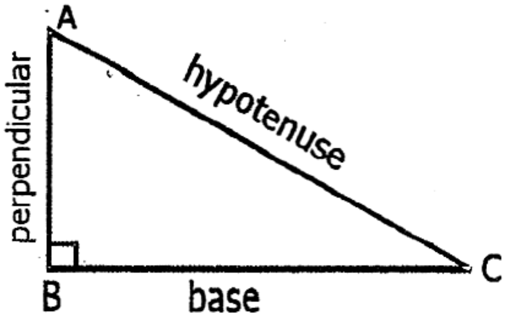
A closed figure enclosed by 3 sides is called a Triangle. ABC is a triangle. The sides AB, BC, AC are-respectively denoted by c, a, b. Please carefully note the capital and small letters.

In any triangle ABC:

- Area = $\frac{1}{2}$ x a x h = $\frac{1}{2}$ × base x perpendicular to base from opposite vertex
- Area = $\sqrt{s(s-a)(s-b)(s-c)}$, where semi perimeter s = $\frac{a+b+c}{2}$

Pythagoras theorem: In a right triangle, the square of hypotenuse is equal to the sum of the squares of the other two sides.

In triangle ABC, AB^2 + BC^2 = AC^2



$(base)^2$ + $(perpendicular)^2$ = $(hypotenuse)^2$

- You should remember some of the Pythagorean triplets (e.g. 3, 4, 5 because 5^2 = 3^2 + 4^2). Someothers are (5, 12, 13), (7, 24, 25) etc.
- Try to find out at least three other sets of Pythagorean triplets. But be clear that we are talking about distinct triplets.

- Thus, if 3, 4, is a Pythagorean triplet, then it does not mean that (3 × 10), (4 × 10), (5 ×10) is a distinct triplet.

Process of generation of Pythagorean Triplet: The first triplet is 3 4 5.

Add 2 to the first t m, 8 to the second term and 8 to the third term. So we get the 2nd triplet as,

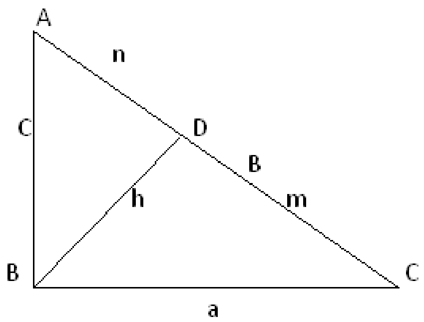
3 + 2 = 5 4 + 8 = 12 5 + 8 = 13

Again add 2 to the 1^{st} term in this triplet, (8 + 4) = 12 to the 2^{nd} and the 3^{rd} term to get the next triplet as 5 + 2 = 7 12 + 12 = 24 13 + 12 = 25

Similarly add 2 to get 1^{st} term in this triplet and (12 + 4) = 16 to the 2^{nd} and the third term to get the next triplet (as below) and so 0.

7 + 2 = 9 4 + 16 = 40 25 + 16 = 41

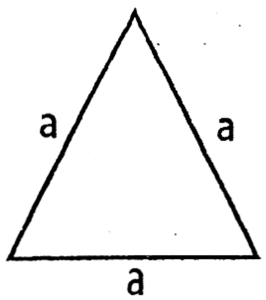
Right Angled Tangle: A triangle whose one angle is 90° is called a Right (angled) Triangle.



The above figure b is the hypotenuse and a & c the legs, called base and height respectively.

- Circum radius R = $\frac{1}{2}$ × hypotenuse
- In radius r = $\frac{A}{s} = \frac{ac}{a+b+c}$
- h^2 = mn

Equilateral Triangle: A triangle whose all sides are equal is called an equilateral triangle.



If a be the side f an equilateral triangle, then

- H = $\frac{\sqrt{3}}{2}$
- A = $\frac{\sqrt{3}}{4}a^2$

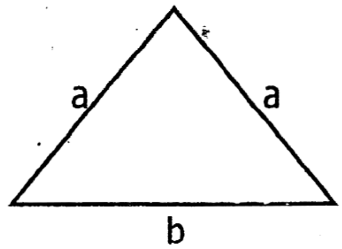
- $r = \frac{1}{3} \times H = \frac{1}{3} \times \frac{\sqrt{3}}{2} \times a = \frac{a}{2\sqrt{3}}$

Example: Find the area of a piece of card board which is in the shape of an equilateral triangle of side 12 cm.

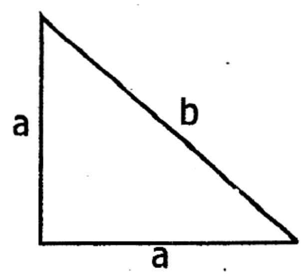
Sol. For equilateral triangle, a = 12 cm.

Area = $\frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4} \times (12)^2 = \frac{\sqrt{3}}{4} \times 144 = 36\sqrt{3}$ sq cms.

Isosceles Triangle: A triangle whose two sides are equal is an isosceles triangle. A = $\frac{b}{4}\sqrt{4a^2 - b^2}$



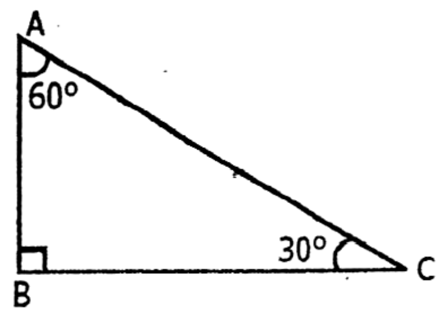
Isosceles Right Triangle: A right triangle whose two legs are equal is an isosceles right triangle



We have b = a$\sqrt{2}$ or a = $\frac{b}{\sqrt{2}}$

➔ Area = $\frac{1}{2}a^2$

30-60-90 Triangle: This is a special case of a right triangle whose angles are 30°, 60°, 90° in this triangle,



- Side opposite to $\angle 30° = \frac{HYP}{2}$
- Side opposite to $\angle 60° = \frac{\sqrt{3}}{2}$ x Hyp.

Example: Find the area of a triangle whose sides are 2.22 m, 2.46 m and 1.9 m

Sol. If the three sides a, b, c, of a triangle are given, we know that the area of the triangle can be obtained by the formula.

Area = $\sqrt{s(-a)(s - b)(s - c)}$ where s = $\frac{a+b+c}{2}$

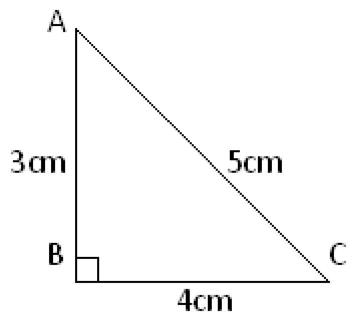
In this case

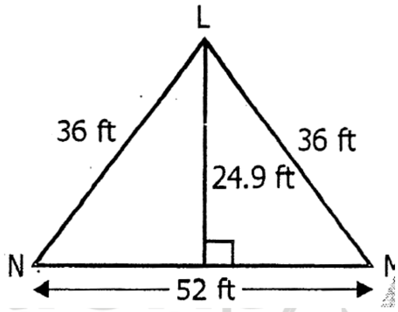
$\frac{2.2+246+1.90}{2} = \frac{6.58}{2}$ = 3.29 m = 329 cm

Therefore, Δ = $\sqrt{329\,(329 - 222)\,(329 - 246)\,(329 - 190)}\ cm^2$

➔ Area = 20160 cm^2 = 2.016m^2

Example: Find the perimeter and area of ΔABC & ΔLMN





Sol.

Perimeter ΔABC = 3 + 4 + 5 = 12 cm

Area of ΔABC = (1/2) x 4 x 3 = 6 cm^2

Perimeter *Δ*ABC = 36 + 36 + 52 = 124 ft

Area of ΔLMN = $\frac{1}{2}$ x 52 x 24.9 = 647.4 sq.ft

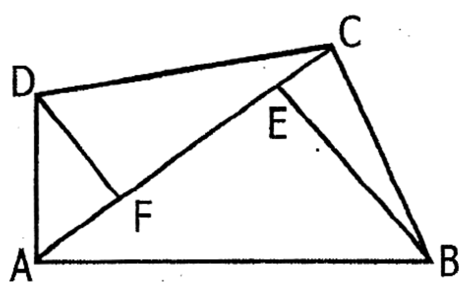
Example: Find the altitude of a right triangle with area 50 sq.ft and base 8 ft.

Sol. Area of a rig triangle = (1/2) x base x altitude 50 = (1/2) x 8 x altitude

➔ altitude = 12.5 ft.

Quadrilaterals

A closed figure (plane) bounded by four sides is called a quadrilateral



AC = d

BE = d_1

DF = d_2

Important Points:

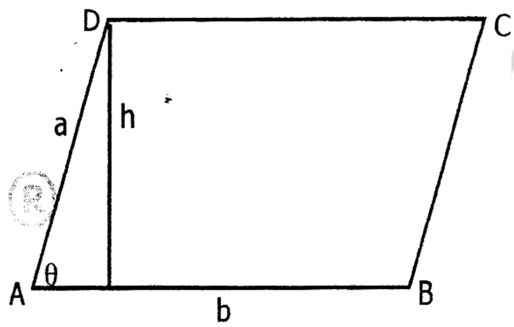
- Area = $\frac{1}{2}$ x one diagonal x (sum of perpendiculars to it from opposite vertices)

  $= \frac{1}{2} \times d(d_1 + d_2)$
- Area = $\frac{1}{2}$ x product of diagonals x sine of angle between them
- Area of a cyclic quadrilateral = $\sqrt{(s - a)(s - b)(s - c)(s - d)}$,

  where a, b, c, d are sides of quadrilateral and
- s = semi perimeter = $\frac{a + b + c + d}{2}$

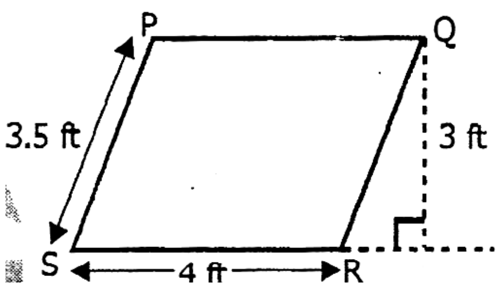
Parallelogram

A quadrilateral in which opposite sides are equal and parallel is called a parallelogram. Diagonals bisect each other.



- Area = b × h = ab sin θ
- p = 2(a + b)

Example: Find the perimeter and area of the parallelogram PQRS given below.



Sol.

Perimeter = 2(3.5 + 4) = 15cm

Area = 4× 3 = 12cm

I = length; b = breadth;

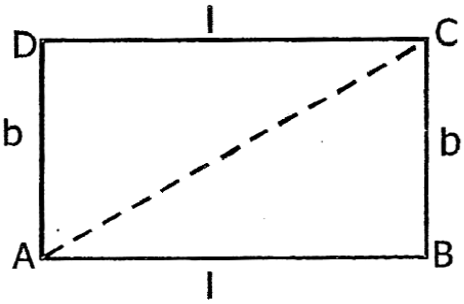
W Area = I x b;

W Perimeter P = 2(1 + b)

W Diagonal = JI2 + b2

Rectangle

A quadrilateral whose opposite sides are equal and each internal angle is equal to 90°, is called a rectangle. The diagonals are equal and bisect each other.



- l = length; b = breadth;
- Area = l x b;
- Perimeter P = 2(1 + b)
- Diagonal = $\sqrt{l^2 + b^2}$

Example: The perimeter of a rectangle is 36 cm, find its area if the length of one side is 12 cm.

Sol. Length of the rectangle = 12 cm and breadth = y cm

➔ 2 × 12 + 2y = 30

➔ 24 + 2y = 36 or 2y = 12

➔ y = 6 cm   Area = xy = 72 cm^2

Example: The length of a rectangle is 10 cm and its perimeter is 30 cm. Find the area of this rectangle.

Sol.

➔ Perimeter = 30 cm, length = 10 cm, let breadth = y cm.

➔ 2 × 10 + 2y = 30

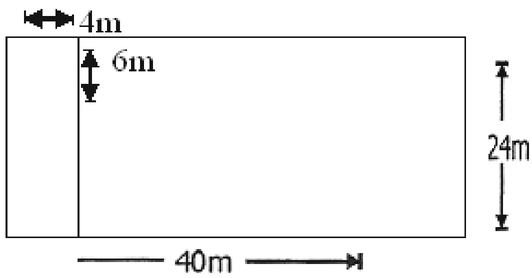
➔ 20 + 2y = 3

➔ 2y = 10 i.e. 5 cm

➔ Area > 10 15 = 50 cm^2

Example: The area of the floor of a rectangular hall of length 40 m is 960m^2. Carpets of size 6 m × 4 m are available. Find how many carpets are required to cover the hall.

Sol. Each carpet as a length 6 m and width 4 m, i.e., 24 m^2



Area of the ha = 960m^2, its lenqth = 40 m

Its breadth = $\frac{960}{40}$m = 24m

Thus, the hall as length 40 m, breadth 24 m. We have to spread the, carpets as shown in the figure, i.e., the length side of the carpets to be parallel to the width side of the hall.

Thus, we can over an area of 24 x 4 m^2 by spreading 4 carpets in a line and 10 such lines are necessary to cover the hall.

$$24 \times 4 \times 10m^2 = 960$$

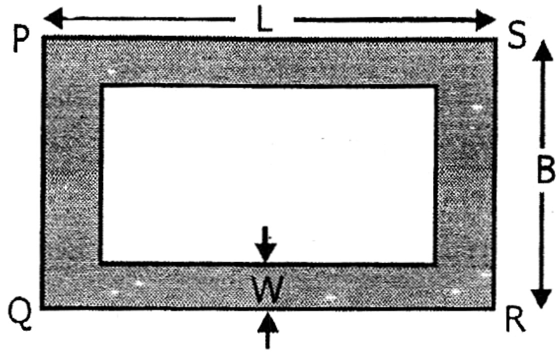
Therefore, tot number of carpets required = 4 × 10 = 40.

Pathway in a rectangle:

- Length of the rectangle = L
- Breadth of-the rectangle = B
- Width of the pathway = W

Two cases are possible for the pathway in a rectangle.

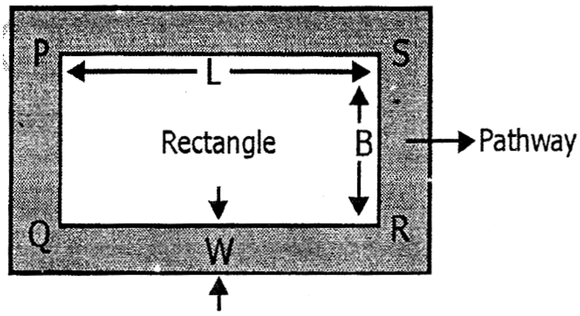
Case 1: It is inside the rectangle.



Area of the pathway = 2W (L + B - 2W)

Example: A ground is in the shape of a rectangle of length 100 m and width 80 m inside the lawn there is a footpath of uniform width 1.5 m, bordering the lawn. Find the area of the path.

Sol. Area of the pathway = 2W (L + B - 2W)



= 2 × 1.5 (100 + 80 - 2 × 1.5) = 531 sq. m.

Case 2: It is outside the rectangle.

Pathway = 2W (L + B + 2W)

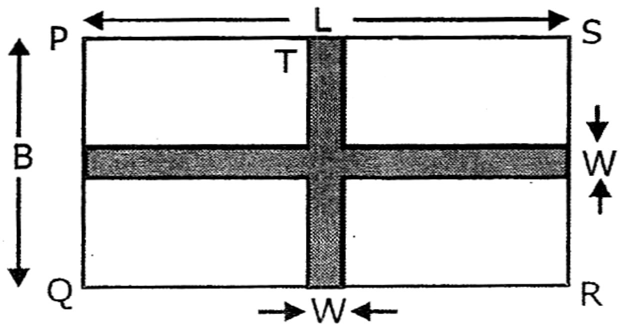
Example: A ground is in the shape of a rectangle of length 80 m and width 60 m. Outside the lawn there is a footpath of uniform, width 2, bordering the lawn. Find the area of the path.

Sol. Area of the pathway = 2W (L + B + 2W)

= 2 × 2(80 + 60 + 2 × 2) = 576 sq. m.

Parallel pathway in a rectangle:

➔ Length of the rectangle = L
➔ Breadth of the rectangle = B
➔ Width of the pathway = W



The parallel pathways run parallel to the sides of the rectangle as shown in the figure above, Let the width of the two pathways be W.

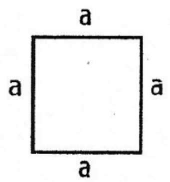
Area of the pathway = W (L + B - W)

Example: A lawn is in the shape of 1 rectangle of length 200 m and width 120 m. Inside the lawn there are two pathways of uniform width 3 m, running parallel to the sides of the lawn. Find the area of the pathways.

Sol. Area of the path way = W (L + B - W) = 3 (200 + 120 - 3) = 951 sq. m.

Square

A quadrilateral whose all sides are equal and each internal angle is 90° is called a square. The diagonals of a square cut at right angles.



- Area = $\frac{1}{2} \times d^2$ = (side)^2 where d is the length of the diagonal
- Perimeter P = 4 × side.

Example: If the area a square is 16 sq. ft., find the length of each side.

Sol.

➔ Area of a square = $(length\ of\ a\ side)^2$

➔ Length of each side = 4 ft

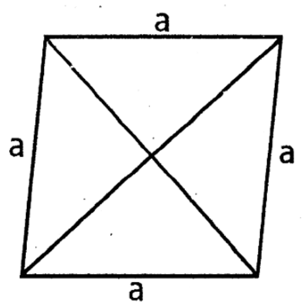
Example: Ifthe perimeter of a square = 24 inches, what is its area?

Sol. If perimeter 24 inches, length of each side = 24/4 = 6 inches.

➔ Area = (6 inches) = 36 square inches.

Rhombus

A quadrilateral whose all sides is equal and opposite sides is parallel is called rhombus. The diagonals of a rhombus cut at right angles.



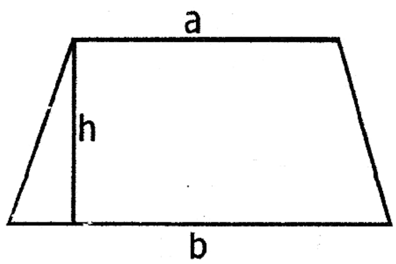
- Area = $\frac{1}{2} \times\ d_1 \times d_2 (d_1,\ d_2$ are diagonals)
- Perimeter P = 4a
- ${d_1}^2 + {d_2}^2 = 4$

Example: The diagonals of a rhombus are 48 cm and 14 cm. Find its area.

Sol. Area = (48 × 14)/2 = 336 sq. cm.

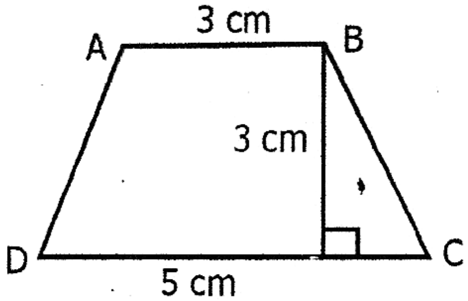
Trapezium (Trapezoid)

A quadrilateral in which only one pair of opposite sides is parallel is a trapezium.



- Area = $\frac{1}{2}$ × d = (a + b) × h

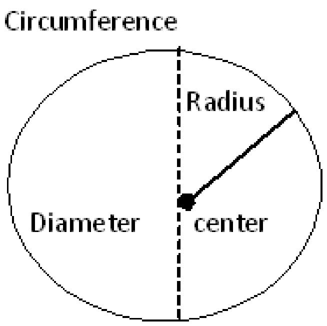
Example: Find the area of the trapezoid ABCD

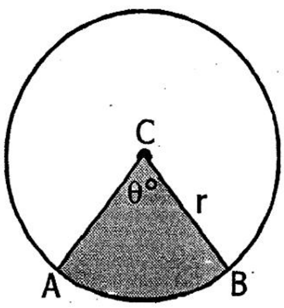


Sol. Area of the trapezoid ABCD

Circle

A circle is the travelled by a point which moves in such a way that its distance a fixed point remains constant. The fixed point is known as centre and the fixed distance is called the radius





C is called the centre and r the radius of the circle.

- Circumference = 27πr = πD.
- Area = A = πr^2 = $\frac{\pi D^2}{4}$ (D = Diameter = 2r),
- π = 22/7 or 3.14 (approx.)
- Length of arc = L = $\frac{\theta^0}{360^0}$ × 27πr

- Area of Sector ABC = $\frac{\theta^0}{360^0} \times \pi r^2$ = $\frac{1}{2}$ × r × L

Example: Find the circumference of a circle with area 25π sq. ft

Sol. Area of the circle = πr^2 = 25 π sq. ft.

➔ r = 5 ft.

➔ Circumference = 2 π r = 10 π ft.

Example: Find the area of a circle with circumference 30π cm.

Sol.

➔ Circumference = 2πr = 30π

➔ r = 15 cm

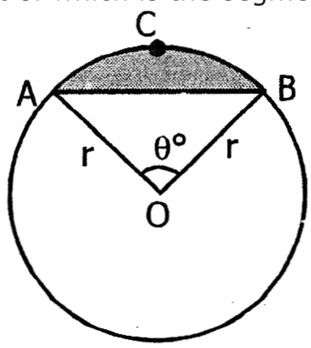
➔ Area = πr^2 = π ×152 = 225π cm^2

Area of segment of a Circle: Segment is the portion enclosed by the arc and the chord in a circle. Similar to the arcs there are two types of segments of a circle, **minor** and **major**

Draw a circle of radius r. Let the chord AB cut the circle into two segments. Suppose want to find the area of the minor segment (shaded portion in the figure). Join A, O and B.

Let ∠AOB = θ^0.

Now we have a sector, a part of which is the segment, whose area is to be calculated.



We find that,

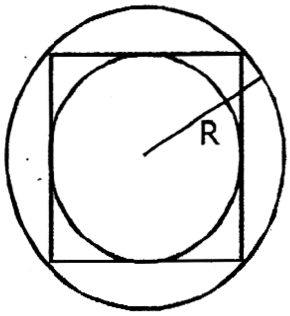
Area of the sector OACB = Area of the segment ACB + area of ΔAOB

Area of the segment ACB = Area of see r OACB - Area of ΔAOB

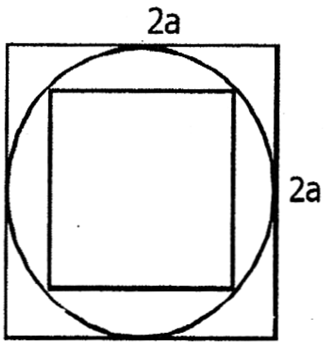
= $\frac{\theta}{360} \times \pi r^2$ - $\frac{1}{2} r^2$ sin θ = $r^2 \left[\frac{\pi\theta}{360} - \frac{\sin\,\theta}{2}\right]$

[Note that area of ΔAOB = $\frac{1}{2} r^2$ sin θ]

Ratio of the area of the two circles is = $\frac{2}{1}$



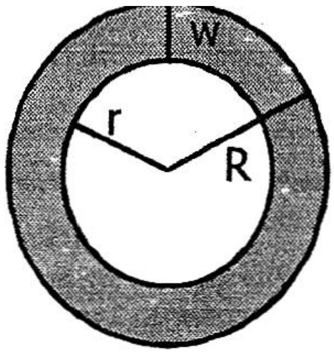
Ratio of the area of the two Squares is



$$= \frac{Area\ of\ square\ drcum\ scribing\ the\ Circle}{\text{Area of square inscribed in the Circle}} = \frac{2}{1}$$

Area of a circular path:

If outer and inner radii are given, then



➔ Area = π(R + r)(R - r) = $\pi\ (R^2 - r^2)$

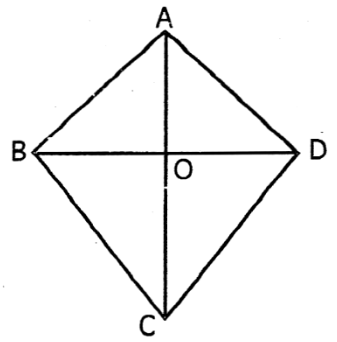
If outer radius and width of path are given, then

➔ Area = πw(2r - w)

If inner radius and width of path are given, then

➔ Area = πw(2r + w)

Kite

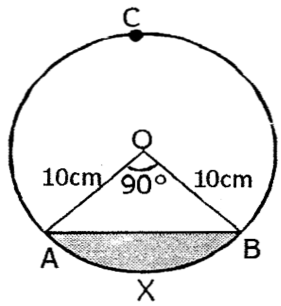


For a kite following results should be remembered.

- Diagonal AC > BD
- Side AB = AD
- Side CB = CD
- Diagonal BD is bisected by AC Thus, BO = OD
- Area of Kite = $\frac{1}{2}$ × AC × BD

Example: A chord AB a circle of radius 10 cm makes a right angle at the centre 0 the circle. Find the area of the major and the minor segments. (Take π = 3.14)

Sol. Area of the sector OAXB (in figure)



= $\frac{90}{360}$ × 3.14 × $l0^2 cm^2$ = 3.14 × 10^2- 28.5)cm^2 = (314 – 28.5) cm^2 = 285.5 cm^2

Example: A circular grass lawn of 50 meters in radius has a path 10.5 meters wide running around it on the outside. Find the area of the path.

Sol. Area of the pathway = π × w (2r + w)

= $\frac{22}{7}$ x 10.5 (2 ×50 + 10.5) = 33 × 110.5 = 3646.5 sq. m

Example: A circular grass lawn of 60 meters in radius has a path 7 meters wide running around it on the inside. Find the area of the path.

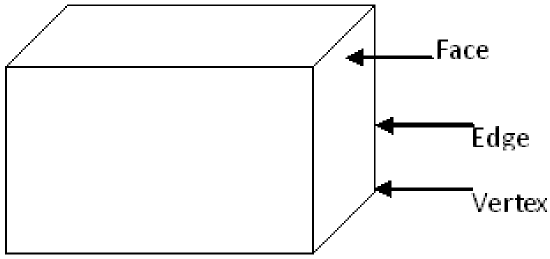
Sol. Area of the pathway = π × w (2r - w)

$\frac{22}{7}$ x $7(2 \times 60 - 7)$ = 22 × l13 = 2486 sq. m.

Mensuration (3-D)

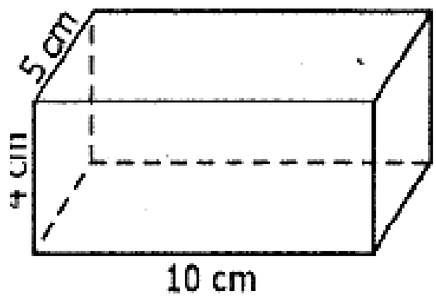
Solid Geometry

A solid is a figure bounded by one or more surfaces. Hence a solid has length, breadth and height. The plane surfaces that bind it are called its faces and the solid so generated is known as a polyhedron. A solid its faces and the solid so generated is known as a polyhedron. A solid has edges vertices and faces which are shown in the figure given below



All the geometric shapes discussed in this book till now i.e. polygons, circles etc. are planar shapes. They are called two dimensional shapes i.e. generally speaking they have only length and breadth. In the world however every object has length, breadth and height. Therefore they are called three dimensional objects. No single plane can contain such objects in totality.

Consider the simple example of a three dimensional shape – a brick.



Shown in Figure above is brick with length 10 cm, breadth 5 cm and height 4 cm. there cannot be a single plane which can contain the brick.

A brick has six surfaces and eight vertices. Each surface has an area which can be calculated. The sum of the areas of all the six surfaces is called the surface area of the brick.

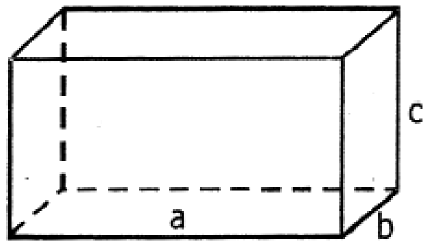
A part from surface area a brick has another measurable property, i.e., the space it occupies. This space occupied by the brick is called its volume. Every three dimensional (3-D) object occupies
A finite volume the 3-D objects or geometric solids dealt within this topic are

- Cube
- Right circular cylinder
- Pyramid

- Right circular cones and
- Sphere

The cuboid and the cube

A book, a match box, a brick are all examples of a cuboid. The definition of a cuboid is derived from that of the prism. A cuboid is a solid formed by joining the corresponding vertices of two congruent rectangles such that the lateral edges are perpendicular to the planes containing the congruent rectangles Figure shows a cuboid.



Can be seen in figure the cuboid has six surfaces and each one is congruent and parallel to the one opposite to it. Thus, there is a pair of three rectangles which goes to make cube

It is already known that the area of a rectangle is the product of its length and breadth,
a × b = Area.
If a, b, c are the edges of a box, then

- The longest diagonal = $\sqrt{a^2 + b^2 + c^2}$
- Surface area = 2(ab + bc + ac)
- Volume = abc
- If the height of a cuboid is zero it becomes a rectangle.

Example: The measurement of a box are 48 cm, 36 cm and 28 cm. Find the volume of the box. How many sq. cm of cloth is required to make a cover for the box?

Sol. Given: length a = 48 cm, breadth b = 36 cm and height = 28 cm.

➔ The required volume = 48 × 36 × 28 cm^3 = 48, 384 cm^3.

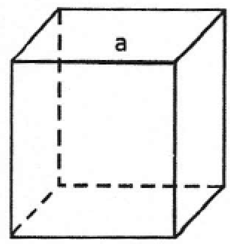
➔ The quantity of cloth required to make the cover of the box = Total surface area of the box

= 2[48 × 36 + 36 × 28 + 28 × 48] cm^2 = 2 × 48 × [1 × 36 + 3 × 7 + 28 × 1] cm^2

= 96 × 85 cm^2 = 8160 cm^2

Cube

A six-faced solid figure with all faces equal and adjacent faces mutually perpendicular is a cube. Also we can say that a cube is a square right prism with the lateral edges of the same length as that of a side of the base



If "*a*" be the edge of a cube, then

- The long diagonal = a$\sqrt{3}$
- Volume = a^3
- Total surface area = $6a^2$

Example: If the volume of a cube is 27 Find the surface area.

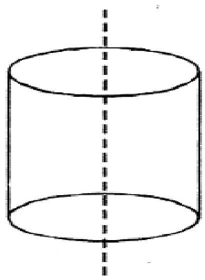
Sol. Volume of cube = cube of its length→I^3 = 27

Since I^3 = 27 $\sqrt[3]{I^3}$ = $\sqrt[3]{27}I$ = 3

Surface area of a cube = 6 I^2 = 6 $3)^2$ = 6 × 9 = 54 sq. units

Right Circular Cylinder

Pillars, pipes etc are examples of circular cylinder encountered daily.



A circular cylinder is two circles with the same radius at a Finite distance from each other with their circumferences joined. The definition of circular cylinder is a prism with circular bases. The line joining e centers of the two circles is called the axis.

- If the axis is perpendicular to the circles it is a right circular cylinder otherwise it is an oblique circular cylinder.
- If r is the radius base and h is the height of a circular cylinder, then
  - Volume = πr^2h
  - Curved surface area = 2 πrh
  - Total surface area = 2 πr(r + h)

This cylinder is generated by rotating a rectangle by fixing one of its sides.

Example: If the lateral area of a right circular cylinder is 24π and its radius is 2. What is its height?

Sol. Lateral area of a right circular cylinder = 2 πrh

Since r = 2, given that Lateral Area = 24 π

➔ 24 π = 4 πh

➔ h = 6 units.

Example: Find the total surface area of the right circular cylinder with a radius of 10 units and a height of 5 units.

Sol. Total surface area of a right circular cylinder is 2πrh + 2 πr^2

If r = 10 and h = 5

➔ Total surface area = 100 π + 200 π = 300 π

Example: What is the radius of the right circular cylinder with volume 18π cubic units and height = 2.

Sol. Volume of right circular cylinder = 7 πr^2h

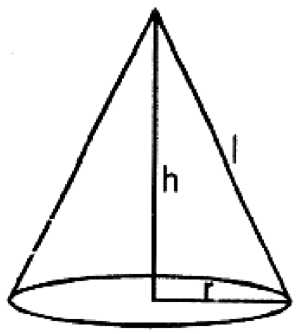
where r = radius and h = height

Given that V = *18* $\pi$and h = 2

➔ l8π = πr^2 × 2 Or r^2 = 9

➔ r = 3 units.

Right circular cone



r = radius of base h = Height;

I = slant height = $\sqrt{h^2 + r^2}$

- Volume = $\frac{1}{3} \times \pi r^2 h$
- Curved surf e Area = πrl
- Total Surface Area = πr(r + l)

Example: The radius f the base and the height of a cone are respectively 35 cm and 72 cm. Find the volume, curved surface area and total surface area of the cone. Take $\pi = \frac{22}{7}$

Sol. Volume = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 35 \times 35 \times 72\ cm^3$ = 92,400 cm^3

Curved surface area = πrl where

l = $\sqrt{h^2 + r^2}$ = $\sqrt{72^2 + 35^2}$ = $\sqrt{6409}$ = 80.05

πrl = $\frac{22}{7}$ ×35× 80.05 cm^2

➔Total surface area = $(\pi rl + \pi r^2)$ cm^2

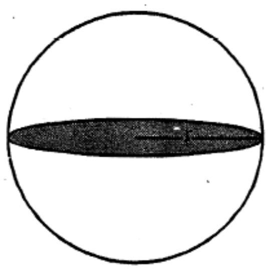
= 8,805.5 cm^2 + $\frac{22}{7}$ × 35 × 35 cm^2 = 8,805.5 cm^2 + 3,850 cm^2 = 12,655.5 cm^2

Sphere

The simplest example of a sphere is a ball. One can call here that a circle is a set of points equidistant from one point in the plane. If this is extended to the third dimension, we have all points in space equidistant from one particular point forming a sphere.

r = Radius

- Volume = $\frac{4}{3} \times \pi \times r^3$
- Surface Area = $4\pi r^2$



Example: Find the volume and surface area of a sphere of radius 4.2 cm.

Sol. Given radius = 4.2 cm

Volume = $\frac{4}{3} \times \frac{22}{7}$ × 4.2 × 4.2 × 4.2 cm^3 = 310 cm^3 approx.

And surface area = 4 × $\frac{22}{7}$ ×4.2 × 4.2 cm^2 = 222 cm^2 approx.

Example: If the volume of a sphere is 36π find its radius and surface area.

Sol. Volume of a sphere = $\frac{4\pi r^3}{3}$

Given that V = $36\ \pi$

➔ $36\pi = \frac{4\pi r^3}{3}$

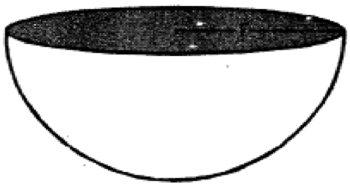
➔ $r^3 = \frac{36 \times 3}{4}$ = 27 ➔ r = 3 units.

Hemisphere

r = Radius

- Volume = $\frac{2}{3} \times \pi \times r^3$

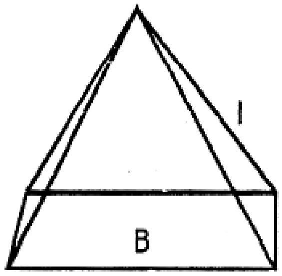
- curved surface Area = $2\pi r^2$
- Total surface Area = Area of Circle + Curved Surface Area = πr^2 + $2\pi r^2$ = $3\,\pi r^2$



Pyramid

A pyramid is a polygon with all the vertices joined to a point outside the plane of the polygon

- If the polygon is regular then the pyramid is called a regular pyramid and is named by the polygon which forms its base.



- If the base is a square the pyramid is called a regular square pyramid.
- If it is a pentagon the pyramid is called a regular pentagonal pyramid.
- The parts of the pyramid are named analogous to the geometric solids mentioned earlier in the chapter. It is a base, lateral faces, lateral edges and an altitude.
- A regular pyramid has a slant height which is the perpendicular distance between the vertex and any side of the polygon.
- The lateral area of a regular pyramid is defined using this parameter.
- Lateral area of a regular pyramid = 1/ 2p × I square units where 'p' is the perimeter and 'I' is the slant height.

Let height of side faces = slant height = I

- Volume = $\frac{1}{3}$× Base Area x height (H)
- S_c, (lateral surface area) = $\frac{1}{2}$ × perimeter of base × I
- S_r(Total surface area) = Base area + S_c

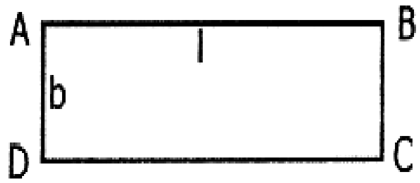
Example: For a regular square pyramid if the length of a side of base = 4 and the height = 6. Find the volume.

Sol. Since length of the base = 4

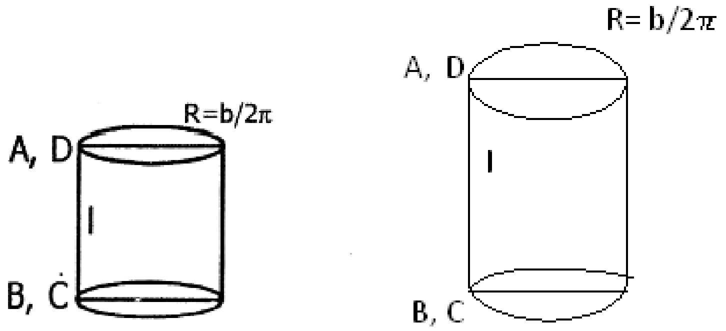
Base Area = 4^2 = 16

Volume = 1/3 × 16 × 6 = 32 cube units.

<u>Note:</u>



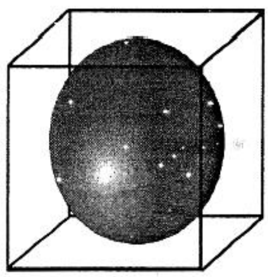
- If the given rectangular sheet of paper is rolled across its length to form a cylinder, having a height b, then the
  - Volume of the cylinder = $\frac{I^2 b}{4\,\pi}$



- If the given rectangular sheet of paper is rolled across its , form a cylinder, having a height I, then
  - Volume of the cylinder = $\frac{b^2 I}{4\,\pi}$
  - Volume of a solid ring = $\frac{\pi^2}{4}(R - r)^2$(R + r)
  - Curved s face area of a solid ring = $\pi^2(R^2 - r^2)$ (where R & r are the outer & inner radii respec.)

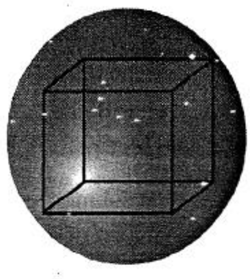
Solids inscribed/circumscribing other solids

- If a largest possible sphere is circumscribed by a cube of edge 'a' cm, then the radius of the sphere = $\frac{a}{2}$

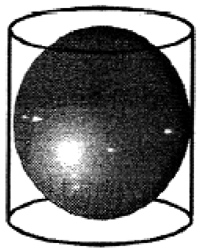


- If a largest possible cube is inscribed in a sphere of radios ‘a’ cm then

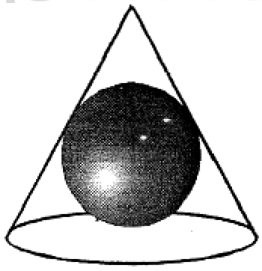
- The edge f the cube = $\frac{2a}{\sqrt{3}}$



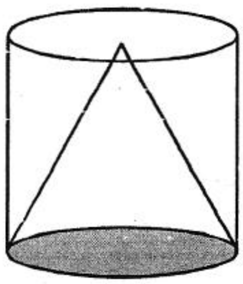
- If a largest possible sphere is inscribed in a cylinder of radius (a) cm and height 'h' cm, then for h > a,
  - The radius of the sphere = a and
  - The radius = $\frac{h}{2}$ (for a > h)



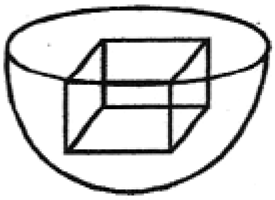
- If a largest possible sphere is inscribed in a cone of radius 'a' cm and slant height equal to the diameter of the base, then
  - The radius of the sphere = $\frac{a}{\sqrt{3}}$



- If a largest possible cone is inscribed in a cylinder of radius ‘a’ cm and height 'h' cm, then the radius of the cone = a and height = h.



- If a largest possible cube is inscribed in a of radius 'a' cm, then
  - The edge of the cube = aca .



Practice Exercise

1. In a right angled triangle, find the hypotenuse if base and perpendicular are respectively 36015 cm and 48020 cm.
   1) 69125 cm
   2) 60025 cm
   3) 91025 cm
   4) 60125 cm

2. The perimeter of an equilateral triangle is 72√3 cm. Find its height.
   1) 63 meters
   2) 24 meters
   3) 18 meters
   4) 4) 36 meters

3. The inner circumference of a circular track is 440 cm. The rack is 14 cm wide. Find the diameter of the outer circle of the track.
   1) 84 cm
   2) 168 cm
   3) 336 cm
   4) 77 cm

4. A race track is in the form of a ring whose inner and outer circumference are 352 meter and 396 meter respectively. Find the width of the track.
   1) 7 meters
   2) 14 meters
   3) 14π meters
   4) 7π meters

5. The outer circumference of a circular track is 220 meter. The track is 7 meter wide everywhere. Calculate the cost of leveling the track at the rate of 50 paise per square meter.
   1) Rs. 1556.5

2) Rs. 3113
3) Rs. 693
4) Rs. 1386

6. Find the area of a quadrant of a circle whose circumference is 44 cm.
   1) 77 cm^2
   2) 38.5 cm^2
   3) 19.25 cm^2
   4) 19.25π cm^2

7. A pit 7.5 meter long, 6 meter wide and 1.5 meter deep is dug in a field. Find the volume of soil removed in cubic meters.
   1) 135 m^3
   2) 101.25 m^3
   3) 50.625 m^3
   4) 67.5 m^3

8. Find the length of the longest pole that can be placed in an indoor stadium 24 meter long, 18 meter wide and 16 meter high.
   1) 30 meters
   2) 25 meters
   3) 34 meters
   4) $\sqrt{580}$ meters

9. The length, breadth and height of a room are in the ratio of 3:2:1. If its volume be 1296 m^3, find its breadth.
   1) 12 meters
   2) 18 meters
   3) 16 meters
   4) 24 meters

10. The volume of a cube is 216 cm^3. Part of this cube is then melted to form a cylinder of length 8 cm. Find the volume of the cylinder.
    1) 342 cm^3
    2) 216 cm^3
    3) 36 cm^3
    4) Data inadequate

11. The whole surface of a rectangular block is 8788 square cm. If length, breadth and height are in the ratio of 4:3:2, find length.
    1) 26 cm

2) 52 cm
3) 104 cm
4) 13 cm

12. Three metal cubes with edges 6 cm, 8 cm and 10 cm respectively are melted together and formed into a single cube. Find the side of the resulting cube.
1) 11 cm
2) 12 cm
3) 13 cm
4) 24 cm

13. Find curved and total surface area of a conical flask of radius 6 cm and height 8 cm.
1) 60π, 96π
2) 20π, 96π
3) 60π, 48π
4) 30π, 48π

14. Volume of a right circular cone is 100π cm^3 and its height is 12 cm. Find its curved surface area.
1) 130π cm^2
2) 65π cm^2
3) 204π cm^2
4) 65 cm^2

15. The diameters of two cones are equal. If their slant height be in the ratio 5:7, find the ratio of their curved surface areas.
1) 25:7
2) 25:49
3) 5:49
4) 5:7

16. The curved surface area of a cone is 2376 square cm and its slant height is 18 cm. Find the diameter.
1) 6 cm
2) 18 cm
3) 84 cm
4) 12 cm

17. The ratio of radii of a cylinder to that of a cone is 1:2. If their heights are equal, find the ratio of their volume?
1) 1:3
2) 2:3

3) 3:4
4) 3:2

18. A silver wire when bent in the form of a square encloses an area of 484 cm^2. Now if the same wire is bent to form a circle, the area of enclosed by it would be
    1) 308 cm^2
    2) 196 cm^2
    3) 616 cm^2
    4) 88 cm^2

19. The circumference of a circle exceeds its diameter by 16.8 cm. Find the circumference of the circle.
    1) 12.32 cm
    2) 49.28 cm
    3) 58.64 cm
    4) 24.64 cm

20. A bicycle wheel makes 5000 revolutions in moving 11 km. What is the radius of the wheel?
    1) 70 cm
    2) 135 cm
    3) 17.5 cm
    4) 35 cm

21. The volume of a right circular cone is 100π cm^3 and its height is 12 cm. Find its slant height.
    1) 13 cm
    2) 16 cm
    3) 9 cm
    4) 26 cm

22. The short and the long hands of a clock are 4 cm and 6 cm long respectively. What will be sum of distances traveled by their tips in 4 days? (Take π = 3.14)
    1) 954.56 cm
    2) 3818.24 cm
    3) 2909.12 cm
    4) None of these

23. The surface areas of two spheres are in the ratio of 1:4. Find the ratio of their volumes.
    1) 1:2
    2) 1:8
    3) 1:4
    4) 1:2

24. The outer and inner diameters of a spherical shell are 10 cm and 9 cm respectively. Find the volume of the metal contained in the shell. (Use π = 22/7)
    1) 6956 cm^3
    2) 141.95 cm^3
    3) 283.9 cm^3
    4) 478.3 cm^3

25. The radii of two spheres are in the ratio of 1:2. Find the ratio of their surface areas.
    1) 1:3
    2) 2:3
    3) 1:4
    4) 3:4

26. A sphere of radius *r* has the same volume as that of a cone with a circular base of radius r. Find the height of cone.
    1) 2*r*
    2) *r*/3
    3) 4*r*
    4) (2/3)*r*

27. Find the number of bricks, each measuring 25 cm x 12.5 cm x 7.5 cm, required to construct a wall 12 m long, 5 m high and 0.25 m thick, while the sand and cement mixture occupies 5% of the total volume of wall.
    1) 6080
    2) 3040
    3) 1520
    4) 12160

28. A road that is 7 m wide surrounds a circular path whose circumference is 352 m. What will be the area of the road?
    1) 2618 cm^2
    2) 654.5 cm^2
    3) 1309 cm^2
    4) 5236 cm^2

Answers:

1. 2	**2.** 4	**3.** 2	**4.** 2	**5.** 4
6. 2	**7.** 4	**8.** 3	**9.** 1	**10.** 4
11. 2	**12.** 2	**13.** 1	**14.** 2	**15.** 4
16. 3	**17.** 3	**18.** 3	**19.** 4	**20.** 4
21. 4	**22.** 2	**23.** 2	**24.** 2	**25.** 3
26. 3	**27.** 1	**28.** 1		