6. Average

Average of n values can be defined as equal distribution of sum of n values over "N".

Average = Sum of quantities / No. of quantities

Example: A student gets 70, 75 and 83 in three subjects. Find the average marks in three subjects?
Average = (70 + 75 + 83)/3 = 228/3 = 76 marks

Example:
Find the average of first 5 prime numbers?
Sol:(2 + 3 + 5 + 7 + 11)/3 = 28/3 = 5.6

Example
If a, b, c, d and e are five consecutive odd integers, and then their average is?
Sol: (a + a + 2 + a + 4 + a + 6 + a + 8)/5 = (5a + 20)/5 = a + 4

Example
The average of first five multiples of 3 is?
Sol: (3 + 6 + 9 + 12 + 15)/3 = 45/5 = 9

Sum of Quantities = Average x No. of quantities

Example: The average marks obtained by 60 candidates in an examination are 42. Find the total marks?
Sol: Total Marks = 60 x 42 = 2520

Example: The average of 6 numbers is 8. What is the 7^{th} number so that the average becomes 10?
Sol: {(6 x 8) + x}/7 = 10 ➔ 48 + x = 70 ➔ x = 70 – 48 = 22

Example: The average of 50 numbers is 38. If two numbers namely, 45 and 55 are discharged, the average of remaining numbers is?
Sol: {(50 x 38) – (45 + 55)} / (50 - 2) = (1900 – 100) / 48 = 1800/48 = 37.50

Example: A cricketer scored 180 runs in the first test and 258 runs in the second. How many runs should he score in the third test to make the average score 230 runs?
Sol: (180 + 258 + x) / 3 = 230 ➔ 438 + x = 690 ➔ x = 690 – 438 = 252

- If the average of 'm' quantities is 'x' and the average of 'n' quantities is 'y' then the weighted average is given by [(mx + ny) / (m + n)]

Example: The average score of cricketer in 2 matches is 27 and in 3 other matches is Find the

average score of all the matches?

Sol: Average = {(2 x 27) + (3 x 32)} / (2 + 3) = (54 + 96)/5 = 150/5 = 30

Example: In a school there are 60 boys of age 12 each, 40 of age 13 each, 50 of age 14 each and 50 of age 15 each. The average age of the boys in the school is?

Sol: {(60 x 12) + (40 x 13) + (50 x 14) + (50 x 15)}/(60 + 40 + 50 + 50)

= (720 + 520 + 700 + 750)/200 = 2690/200 = 13.45

Example: The average height of 30 girls out of 40 is 160 cm and that of the remaining girls is 156 cm. The average height of the whole class is?

Sol: {(30 x 160) + (10 x 156)}/40 = (4800 + 1560)/40 = 6360/40 = 159 cm

- If the average of 'm' quantities is 'x' and the average of 'n' quantities out of them is 'y' then the average of the rest of the quantities is (mx – ny)/(m – n)

Example: The average of 10 numbers is 12. The average of 6 out of them is 8. What is the average of remaining four numbers?

Average = {(10 x 12) – (6 x 8)}/(10 – 6) = (120 – 48)/4 = 72/4 = 18

- If the average of 'n' quantities is 'x' and when a quantity is removed the average becomes 'y'. Then the value of removed quantity is [n(x – y) + y]

Example: The average age of 24 boys and a class teacher is 15 years. If the class teacher's age is excluded the average becomes 14. Find the age of class teacher?

Sol: Age = [25(15 – 14) + 14] = 25 + 14 = 39

Example: The average age of 24 students and class teacher is 16 years. If the teacher's age is excluded then the average age reduces by 1 year. What is the age of the class teacher?

Sol: Age = [25(16 – 15) + 15] = 25 + 15 = 40

- If the average of 'n' quantities is 'x' and when a quantity is added the average becomes 'y'. Then the value of new quantity is [n(y – x) + y]

Example: The average of 30 boys of a class is 14 years. When the age of the class teacher is included the average becomes 15 years. Find the age of the class teacher?

Sol:

Method – 1:

Total age of boys = 14 x 30 = 420 years

Total age after including = 15 x 31 = 465 years

Age of class teacher = 465 – 420 = 45 years

Method – 2:

Age = [30(15 – 14) + 15] = 45

Example: The average age of 34 boys in a class is 14 years. If the teacher's age is included the average age becomes 15 years. What is teacher's age?

Sol:

Age = [34(15 – 14) + 15] = 49

- If the average of 'n' number is 'm' and if given number is multiplied to or divided by 'x' then the average of 'n' numbers becomes "mx" or "m/x" respectively.
- If the average of 'n' number is 'm' and if 'x' is added or subtracted from each given number then the average of 'n' numbers becomes "(m + x)" or "(m - x)" respectively
- The average of n numbers is x. If each of the numbers is multiplied by (n - 1), find the average of new set of numbers? ➔ (n - 1) x
- The average of x numbers is 3x. If (x - 1) is subtracted from each number. What will be the new average? ➔ 2x + 1
- The average quantity of 'n' persons is increased/decreased by 'x' when some of them whose quantity 'y' is replaced by same number of persons, then the quantity of new persons are,
  - (y + nx) ; when increased
  - (y – nx) ; when decreased

Example: The average weight of 4 men is increased by 3 kg when one of them who weigh 120 kgs is replaced by another man. What is the weight of the new man?

Sol:

Weight of new man = 120 + (4 x 3) = 120 + 12 = 132 kg

Example: The average weight of 8 persons increases by 1.5 kg. If a person weighing 65 kg is replaced by a new person, what could be the weight of the new persons?

Sol: Weight of the new person = 65 + (8 x 1.5) = 65 + 12 = 77 kg

Example: The average of 10 persons in a committee is increased by 1 year when two men aged 42 years and 38 years are substituted by two women. Find the average age of these two women?

Sol:

Average of these two women = [80 + (10 x 1)]/2 = (80 + 10)/2 = 90/2 = 45kg

Example: In a class there are 24 boys whose average age is decreased by 3 months, when 1 boy aged 20 years is replaced by a new boy. Find the age of the new boy?

Sol:

Age of the new boy = 20 – (24 x (3/12)) = 20 – (24(1/4)) = 20 – 6 = 14 years

- If the average of 'n' results (where n is an odd number) is 'a' and the average of first 'm' results is 'b' and that of last 'm' is 'c'. Then m^{th} result is [m(b + c) – na]

Example: The average of 11 results is 50. If the average of first six results is 49 and that of last six is 52, find the sixth result.

Sol:

Sixth result = [6(49 + 52) – 11(50)] = [606 – 550] = 56

Example: The average of 17 numbers is 45. The average of first 9 of these numbers is 51 and the last 9 of these numbers is 36. What is the ninth number?

Sol: Ninth number = [9(51 + 36) – 17(45)] = 9(87) – 765 = 783 – 765 = 18

Example: The average of 25 results is 18; that of first 12 is 14 and of the last 12 is 17. Thirteenth result is?

Sol:

Thirteenth result = Total of 25 results – (Total of 5 results + Total of 5 results)

= 25(18) – [12(14) + 12(17)] = 450 - 372 = 78

Miscellaneous

- The average of first 'n' natural numbers is **[(n + 1)/2]**
- The average of 'n' (where n is odd) consecutive numbers is always the **middle number**.
- The average of 'n' (where n is even) consecutive numbers is the **average of the middle two numbers**.
- The average of even numbers from 1 to 'n' is **(last even number + 2)/2**
- The average of squares of first 'n' natural numbers is **[(n + 1)(2n + 1)/6]**
- The average of cubes of first 'n' natural numbers is **[n(n + 1) 2/4]**
- The average of first 'n' consecutive even numbers is **(n + 1)**
- The average of first 'n' consecutive odd numbers is **n**
- The average of squares of first 'n' consecutive even numbers is **[2(n + 1)(2n + 1)/3]**
- The average of squares of first 'n' consecutive even numbers is **[(n + 1)(n + 2)/3]**
- The average of squares of first 'n' consecutive odd numbers is **[n(n + 2)/3]**

Practice Exercise

1. If a, b, c, d, e are five consecutive odd integers, then what is their average?
   1) a + 4 2) abcde/4 3) 5 (a + b + c + d + e) 4) a + 8 5) None of these

2. The average weight of a class having 52 students is 52 kg. Find the total weight of the class.
   1) 2504 kg 2) 2708 kg 3) 2704 kg 4) 2407 kg 5) None of these

3. The average of 6 numbers is 8. What is the 7^{th} number so that average becomes 10?
   1) 22 2) 18 3) 21 4) 20 5) None of these

4. The average age of an adult class is 40 years. 12 new students with an average age of 32 years join the class, thereby decreasing the average of the class by 4 years. The original strength of the class was:
   1) 10 2) 11 3) 12 4) 15 5) None of these

5. The average temperature of first 3 days is 27 degrees and of the next three days is 29 degrees. If the average of the whole week is 28.5 degrees then, the temperature of the last day is:
   1) 31.5 degrees 2) 10.5 degrees 3) 21 degrees 4) 42 degrees 5) None of these

6. A group of 30 girls has average age of 13 years. Average age of first 18 from the same group is 15 years. What is the average age of other 12 girls in the group?
   1) 12 years 2) 10 years 3) 16 years 4) 10.5 years 5) None of these

7. The average age of 24 students and the class teacher is 16 years. If the class teacher's age is excluded, the average reduces by 1 year. What is the age of the class teacher?
   1) 50 years 2) 45 years 3) 40 years 4) Data inadequate 5) None of these

8. The total age of 26 persons is 442 years. Out of these persons one is a teacher and others are students. If the teachers age is excluded the average reduces by 2 years. What is the age of the teacher?
   1) 50 years 2) 55 years 3) 60 years 4) 67 years 5) None of these

9. The average age of 34 boys in a class is 14 years. If the teacher's age is included the average age of the boys and the teacher becomes 15 years. What is the teacher's age?
   1) 48 years 2) 46 years 3) 49 years 4) 45 years 5) None of these

10. The average age of 30 children in a class is 9 years. If the teacher's age be included, the average age becomes 10 years. Find the teacher's age.
    1) 40 years 2) 36 years 3) 42 years 4) 39 years 5) None of these

11. The average of 13 numbers is 36. If each of the numbers is multiplied by 3, find the average of new set of numbers.

1) 108 2) 120 3) 104 4) 106 5) None of these

12. The average age of 11 persons in a committee is increased by 2 years when three men aged 32 years, 33 years and 34 years are substituted by three women. Find the average age of these three women.
    1) 40 years 2) 41 1/3 years 3) 41 years 4) 40 1/3 years 5) None

13. A batsman in his 21st innings makes a score of 88 and there by increases his average by 2) What is the average after 21 innings?
    1) 46 2) 48 3) 45 4) 44 5) None of these

14. The average age of all the students of a class is 16 years. The average age of the boys of the class is 21 years and that of the girls is 12 years. If the number of girls in the class is 10, then find the number of boys in the class.
    1) 4 2) 8 3) 12 4) 10 5) None of these

15. Find the average of first 64 consecutive odd numbers.
    1) 64 2) 63 3) 65 4) 66 5) None of these

16. Find the average of first 52 consecutive even numbers.
    1) 52 2) 53 3) 51 4) 50 5) None of these

17. Find the average of squares of consecutive odd numbers 1 to 14.
    1) 70 2) 65 3) 75 4) 66 5) None of these

18. The average age of a husband and wife was 23 years when they were married 5 years ago. The average age of the husband, the wife and a child who was born during the interval, is 20 years now. How old is the child now?
    1) 9 months 2) 1 year 3) 3 years 4) 4 years 5) None of these

19. Of the three numbers, second is twice the first and is also thrice the third. If the average of the three numbers is 44, the largest number is:
    1) 24 2) 36 3) 72 4) 108 5) None of these

20. The sum of the three numbers is 98. If the ratio between the first and second be 2:3 and that between second and third be 5:8, then the second number is:
    1) 30 2) 20 3) 58 4) 48 5) None of these

Answers:

1. 1	**2.** 3	**3.** 1	**4.** 3	**5.** 1
6. 2	**7.** 3	**8.** 4	**9.** 3	**10.** 2
11. 2	**12.** 4	**13.** 2	**14.** 2	**15.** 1
16. 2	**17.** 2	**18.** 4	**19.** 3	**20.** 1