9. Time and Distance

Speed or Velocity

Let an object move from a point A to the point B through any path, then the actual length of the path followed by the object is called the distance travelled by the object. Speed is defined as the rate at which the distance is covered.

- Speed = $\frac{\text{Distance travelled}}{\text{time taken}}$ Speed is generally measured either in KMPH or m/s

Important Points

- 1 hour = 60 minutes = 60 x 60 seconds.
- 1 kilometer = 1000 meters.
- 1 km/hr = $\frac{5}{18}$ m/sec (To convert kmph to m/s we multiply by 5/18)
- $1\ \frac{\text{m}}{\text{sec}} = \frac{18}{5}$ km/hr (To convert m/s in to kmph we multiply by 18/5)
- $1\ \frac{\text{km}}{\text{hr}} \approx \frac{5\ \text{miles}}{8\text{hr}}$
- $\frac{\text{miles}}{\text{hr}} = \frac{22}{15}\frac{\text{ft}}{\text{sec}}$
- If the distance is constant, Speed $\propto \frac{1}{\text{Time}}$, then Time = $\frac{\text{Distance}}{\text{Speed}}$
- If the time is constant, Distance $\propto$ Speed, then Distance = Time x Speed,
- If a person takes time T1 and T2 to cover the same distance with V1 and V2 speed respectively V1 x T1 = V2 x T2

Uniform Speed

If the object covers equal distances in equal time intervals, however small the interval may be, then its speed is called the uniformspeed.

Variable Speed

If theobject travels different distances in equal intervals oftimethen its speed is called a Variable speed. In this case the speed changesfrom instance to instance

Example: If a man changes his speed in the ratio m:n, then the ratio of time taken becomes n:m.

Example: Walking at 4/5 of his usual rate, a person reaches his school 20 min late. Then find his usual time taken to reach the destination.

Sol. If the person is walking at $4/5^{th}$ of his usual speed, then the time taken should become 5/4 times the original time required. Thus, we can say that the person is going to require $1/4^{th}$ more/extra time as compared to the usual time required (as $\frac{5}{4}$ T - 1T = $\frac{1}{4}$ T).

Thus, $\frac{1}{4}$ T →20 min

T → x

➔ Solve to get x = 80 min.

Example: If three men cover the same distance with speeds in the ratio a:b:c, the times taken by these three will be respectively in the ratio 1/a:1/b:1/c.

Average Speed

Average speed is defined as the total distance covered divided by total time taken.

Average Speed = $\frac{\text{Total Distance Covered}}{\text{Total Time Taken}}$

(Avg speed is not aritmetic mean of speeds.)

- If a person goes from A to B at U kmph and comes back from B to A at V kmph, then Average speed during the whole journey $\frac{2UV}{U+V}$
- When the total journey's distance is made into equal parts travelling with different speeds then the average speed is harmonic mean of the given speeds.
- If a person travels half of journey's time with U speed and remianing time with V speed the average speed is $\frac{U+V}{2}$**.** So when the total journey's time is made into equal parts average speed for the whole journey is arithmetic mean of the given speeds.

Note:

If two persons (or vehicles or trains) start at the same time from two points A & B towards each other and after crossing, they take x and y hours in reaching B & A respectively, then

$$\frac{\text{Speed of first}}{\text{Speed of second}} = \sqrt{\frac{Y}{X}}$$

Example: A man starts from A to B , another from B to a at the same time after passing each other they complete their journey sin $3\frac{1}{3}$ and $4\frac{4}{5}$ hours, respectively. If the speed of the first is 12 kmph, then find the speed of the second man

Sol. $\frac{\text{1st man's speed}}{\text{2nd man's speed}} = \sqrt{\frac{y}{x}} = \sqrt{\frac{4\frac{4}{5}}{3\frac{1}{3}}} = \sqrt{\frac{24}{5} \times \frac{3}{10}} = \sqrt{\frac{36}{25}} = \frac{6}{5}$

➔ $\frac{12}{\text{2nd man's speed}} = \frac{6}{5}$

➔ $\text{2nd man's speed} = \frac{60}{6}$ = 10 kmph

Note

If a person/moving body moves at a average speed of V_1 kmph to cover a distance of D km without stopping and moves at a average speed of V_2 kmph to cover the same distance with stoppages,then

the stoppage time per hour is given by $\frac{V_1 - V_2}{V_1}$

Example: A bus travels at a speed of 60 kmph between two stations A and B, 240 km apart, when it does not stop between anyother stations between them. But it goes at an average speedof 40 kmph when it stops. What is the average stoppage time per hour of the train?

Sol. Time taken, when it does not stop = 240/60 = 4 hrs.

Time taken, when it stops = 240/40 = 6 hrs.

Total stoppage time = 2 hrs.

Average stoppage time per hour = 2/6 = 1/3 hrs.

Short-Cut:

Using the formula directly, we get $\frac{60-40}{60} = \frac{20}{60} = \frac{1}{3}$ hrs.

Relative Speed

Suppose two trains A and B are moving with same speed in the same direction on two parallel tracks, to an observer sitting in train A, the train B appears to be stationary. This observation is expressed by saying that the relative speed of train A is zero w.r.t. train B and the relative speed of the train B is zero w.r.t. train A.

- The relative speed of a body A w.r.t body B is the rate of change of position of body A w.r.t. body B.
- The relative speed of a body A w.r.t, another bodyB whenbothare in motion, can be determined by adding to the speed of A the speed of B keeping direction in mind.

Note

The time taken by a train L meters long, travelling at a speed of x m/sec in passing a pole (or a standingman) is the same as the time taken by the train to cover a distance of L meters i.e. its own length because the length of the pole is nearly zero with respect to the length of the train though its height may be significant.

Thus T = $\frac{L}{X}$ m/s

The time taken by a train "L_1" meters long, travelling at a speed of V m/sec in passing a stationary object (a bridge or a tunnel ora train at rest or a platform) of length "L_2"meters is the same as the time taken by the train to cover a distance of (L_1 + L_2) meters.

$$T = \frac{L_1 + L_2}{V} \text{ seconds}$$

- If two trains of lengths L_1 and L_2 meters respectively, are moving in the same direction with a speed of U and V m/sec (where U > V), then (U - V) m/sec is called their Relative

speed and time taken by faster train to pass the slower train = $\frac{L_1 + L_2}{U - V}$ seconds.

- If two trains of lengths L_1 and L_2 meters respectively, are moving in opposite directions (towards each other or away from each other) with a speed of U and V m/sec, then (U + V)m/sec is called their Relative Speed and time taken by the trains to pass each other = $\frac{L_1 + L_2}{U + V}$ seconds
- If a man is running at a speed of U m/sec in the same direction in which a train of length L meters is running at a speed V m/sec then (V- U) m/sec is called the speed of the train relative to man. Then the time taken by the train to cross the man = $\frac{L}{V - U}$ seconds

If two trains are crossing each other such that

- Length of the first train = L_1 meters,
- Length of the second train = L_2 meters,
- Time taken by the two when crossing each other in oppositedirection = x sec,
- Time taken by the two when crossing each other in samedirection = y sec,
- The speed of faster train = $\frac{L_1 + L_2}{2}\left(\frac{1}{X} + \frac{1}{Y}\right)$
- Speed of the slower train = $\frac{L_1 + L_2}{2}\left(\frac{1}{X} - \frac{1}{Y}\right)$

Example: A train, 800 m long, travels at 90 kmph. How long does it take to cross

a) A pole?
b) A platform 600 m long?
c) Another train 700 m long, standing on another parallel track?
d) Another train 700 m long, running at 54 kmph in samedirection?

Sol: Since 1 kmph = $\frac{5}{18}$ m/s ➔Speed of train = 90 kmph = $90 \times \frac{5}{18}$ m/s = 25 m/s

a) Pole is a stationary object, so, the timetaken by the train is the same as the timetaken by the train to cover a distance equal to its own length

   Required time = $\frac{800}{25}$ = 40 seconds

b) The platform is stationary of length 600 m.

   Length to be covered = Length of the train + Length of the platform = 800 + 600 = 1400 m and relative speed = speed of train.

   So, required time = $\frac{1400}{25}$ = 56 seconds

c) Another train is stationary. Length to be covered = Length of the train + Length of the other train

   = 800 + 700 = 1500 m and relative speed = 25 m/s. So, required time

   = $\frac{1500}{25}$ = 60 seconds.

d) Another train is moving in same direction.

Length to be covered = Length of the train + Length of the other train = 800 + 700 = 1500 m

Relative speed = 10 m/s.

So, required time = $\frac{1500}{10}$ = 150 seconds.

Boats and Streams

Terminologies

The following terms will be used often while discussing boats and streams:

- **Stream:** It implies that the water in the river is moving orflowing.
- **Upstream:** Going against the flow of the river.
- **Downstream:** Going with the flow of the river.
- **Still water:** It implies that the speed of water is zero (generallyin a lake).

Example: Let the speed of a boat (or man) in still water be U m/sec and the speed of the stream (or current) be V m/sec:

- Speed of boat with the stream (or Downstream or D/S) = (U + V) m/sec.
- Speed of boat against the stream (or Upstream or U/S) = (U - V) m/sec.
- Speed of man/boat in still water U = $\frac{(U+V)+(U-V)}{2} = \frac{\text{Upstream} + \text{Downstream}}{2}$
- Speed of the stream = V = $\frac{(U+V)-(U-V)}{2} = \frac{\text{Downstream} - \text{Upstream}}{2}$

Example: A boat is rowed down a river 28 km in 4 hours and up a river 12 km in 6 hours. Find the speed of the boat and the river.

Sol.

Downstream speed is $\frac{28}{4}$ = 7 kmph, Upstream speed is $\frac{12}{6}$ = 2 kmph.

Speed of Boat = $\frac{1}{2}$ [downstream + upstream speed] = $\frac{1}{2}$ (7 + 2) = 4.5 kmph.

Speed of current = $\frac{1}{2}$ (downstream - upstream speed) = $\frac{1}{2}$ (7 - 2) = $\frac{5}{2}$ = 2.5 kmph.

- If a man capable of rowing at the speed of U m/sec in still water, rows the same distance up and down a stream flowing at a rate of V m/sec, then his average speed throughout the journey is

$$= \frac{(U-V)(U+V)}{U}$$

Example: When downstream distance = upstream distance, then

$$\frac{U}{V} = \frac{t\_up + t\_down}{t_{up - t_{down}}}$$

Total Journey time ($t_{up} + t_{down}$) $= \frac{U \times \text{Total distance}}{(U-V)+(U+V}$

Practice Exercise

The following exercise is to be taken by the students as a homework assignment, only after going through the theory provided. A sincere student shall stand to gain a lot of speed and conceptual clarity after having solved this exercise.

DIRECTIONS: For the followingquestions, four options are given.Choose the best option.

1. Ramesh travels 600 km to reach home partly by train and rest by car. He takes 8 hours if he travels 120 km by train and the rest by car. But he would take 20 min more if hetravels 200 km by train and the rest by car. Find the speedsof train and car.
   1) 30 kmph, 40 kmph 2) 60 kmph, 80 kmph
   3) 15 kmph, 60 kmph 4) None of these

2. A man can row upstream at a kmph and downstream at b kmph. He rows up to a certain point and then returns to his starting point and finds that his average speed is s kmph for the double journey. Express each of the letters in terms ofthe other two.
   1) s = $\frac{2ab}{a+b}$ 2)b = $\frac{sa}{2a+s}$ 3) s = $\frac{ba}{2a+b}$ 4) No relation possible

3. A boat goes up a river 20 km and down the river 24 km in 8 hours. It also goes up the river 30 km and down the river r 28km in 11 hours. Find the speed of the boat andtheriver.
   1) 6 kmph, 2 kmph 2) 3 kmph,1kmp
   3) 12 kmph, 4 kmph 4) None of these

4. Two trains, starting from the same station and travelling in the opposite directions are 227.5 km apart in 3 hours 15 min. Had they been travelling in the same direction they would have been 32.5 km apart in the same time. Find their speeds. (Ignore the length of the trains)
   1) 45 kmph, 25 kmph 2) 40 kmph, 30 kmph
   3) 55 kmph, 15 kmph 4) 80 kmph, 70 kmph

5. Excluding stoppages, the speed of a mobike is 63 kmph & including stoppages it is 36 kmph. The number of minutes per hour for which the mobike stops is
   1) 25.71 2) 26.71 3) 27.71 4) 28.71

6. For covering a distance x km, a man takes t hours. If he moves 3 kmph faster, he takes 40 min less. But if he moves2 kmph slower, he takes 40 more min. Then x is equal to
   1) 20 km 2) 36 km 3) 37.5 km 4) 40 km

7. In the above problem, the original speed of the man is
   1) 18 kmph 2) 12 m/sec 3) 3.33m/sec 4) 16 krnph

8. Two trains start at the same time from A & B and proceed towrds B & A at 36 kmph & 42 kmph respectively. When they meet, it is found that one train has moved 48 km more than the other. Then the distance between A & B is
   1) 624km 2) 636km 3) 544km 4) 460km

9. A thief spots a policeman 100 m away and takes to his heels. If the police man gives a chase immediately, then how far would the thief have run before he is overtaken? The speeds of the theif and policeman are 8 and 10 kmph respectively.
   1) 500 m 2) 200km 3) 400 m 4) 2km

10. The distance between two stations A & B is 300 km. A trainleaves A @ 30 kmph. At the same time another train departsfrom B @ 45 kmph. The distance of the point where both the trains meet, from the point A is
    1) 100 km 2) 120 km 3) 130 km 4) 200 km

11. Against a stream running at 2 km/ hr, a man can row 9 km in 3 hours. How long would he take in rowing the same distance down the stream?
    1) 9/7 hours 2) 7/9 hours 3) 1.5 hours 4) 3 hours

12. A man can swim with the stream at the rate of 3 kmph and against the stream at the rate of 2 kmph. How long will it take him to swim 7 km in still water?
    1) 3 hours 2) 2.8 hours 3) 2.6 hours 4) 3.2 hours

13. A train starts from Delhi at 1:20 pm and reaches Indore at 5:50.pm. A second train starts from Indore at 1:50 pm and reaches Delhi at 7:10 pm. The ratio of speeds of the two trains is
    1) 27:32 2) 32:37 3) 37:32 4) 32:27

14. I have to go to a certain place 7 km off in 1 hour 15 min. IfI walk some way on foot at 4 km/hr and ride some way onhorse back at 10 km/hr, I reach my destination in time. How far should I walk on foot?
    1) 14/3 km 2) 11/3 km 3) 13/3 km 4) 10/3 km

15. A student walks to school at the rate of 2.5 kmph and reaches 6 min too late. Next day he increases his speed by 2 kmph and then reaches school 10 min early. The distance of the school

from his home is

1) 1.5 km 2) 3 km 3) 6 km 4) 12 km

16. A man can swim with the stream at the rate of 10.56 kmph and 352 m against the stream in 4 min. The speed ofthe man in still water is

1) 10.56 kmph 2) 352 m/min 3) 7.92 kmph 4) 5.28 kmph

17. Two cyclists A and B ride round a circular course 60 km round. A's speed is 10km/hr and B's speed is 15 km/hr. When will they next come to the starting point supposing they ride in the same direction?

1) 4 hours 2) 6 hours 3) 10 hours 4) 12 hours

18. Sanju goes to school @ 10 kmph and reaches 6 min late.When he goes @ 12 kmph, he reaches 9 min earlier than the scheduled time. What is the distance of the school from his house?

1) 1500 m 2) 150 km 3) 20 km 4) 15 km

DIRECTIONS: Read the information given below and answer the questions that follow.

A cyclist rode at a uniform rate from A to B in 2.25hours. He started back at the same rate but after riding for an hour, he was delayed by a puncture for20 min and for the rest of the way his rate was only6 kmph.Consequently the return journey took 3 hours altogether.

19. The distance covered by the cyclist between A and B is

1) 8 km 2) 12 km 3) 18 km 4) 24 km

20. What was the initial speed of cyclist?

1) 8kmph 2) 10 kmph 3) 15kmph 4) 18 kmph

Answers:

1. 2	**2.** 1	**3.** 1	**4.** 2	**5.** 1
6. 4	**7.** 3	**8.** 1	**9.** 3	**10.** 2
11. 1	**12.** 2	**13.** 4	**14.** 2	**15.** 1
16. 3	**17.** 4	**18.** 4	**19.** 3	**20.** 1