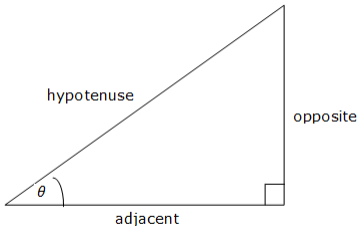
Trigonometry

Important Formulae

1. Right Triangle Definition

Assume that:

$0 < \theta < \frac{\pi}{2}$ or $0° < \theta < 90°$



$\sin\theta = \frac{opp}{hyp}$ $\quad$ $\operatorname{cosec}\theta = \frac{hyp}{opp}$

$\cos\theta = \frac{adj}{hyp}$ $\quad$ $\sec\theta = \frac{hyp}{adj}$

$\cos\theta = \frac{opp}{adj}$ $\quad$ $\cot\theta = \frac{adj}{opp}$

2. Tangent cotangent Identities

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\quad$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$

3. Reciprocal Identities

$\sin\theta = \frac{1}{\operatorname{cosec}\theta}$ $\quad$ $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

$\cos\theta = \frac{1}{\sec\theta}$ $\quad$ $\sec\theta = \frac{1}{\cos\theta}$

$\tan\theta = \frac{1}{\cot\theta}$ $\quad$ $\cot\theta = \frac{1}{\tan\theta}$

4. Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$1 + \cot^2\theta = \operatorname{cosec}^2\theta$

5. Even and Odd Formulas

$\sin(-\theta) = -\sin\theta$ $\quad \operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$

$\cos(-\theta) = \cos\theta$ $\quad \sec(-\theta) = \sec\theta$

$\tan(-\theta) = -\tan\theta$ $\quad \cot(-\theta) = -\cot\theta$

6. Double Angle Formulas

$\sin(2\theta) = 2\sin\theta\cos\theta$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$= 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta$

$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$

7. Half Angle Formulas

$\sin\theta = \pm\sqrt{\frac{1-\cos 2\theta}{2}}$

$\cos\theta = \pm\sqrt{\frac{1+\cos 2\theta}{2}}$

$\tan\theta = \pm\sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}$

8. Sum and Difference Formulas

$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$

$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$

9. Product to Sum Formulas

$\sin A\sin B = \frac{1}{2}[\cos(A - B) - \cos(A + B)]$

$\cos A\cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$

$\sin A\cos B = \frac{1}{2}[\sin(A + B) + \sin(A - B)]$

$\cos A\sin B = \frac{1}{2}[\sin(A + B) - \sin(A - B)]$

10. Sum to Product Formulas

$\sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$

$\sin A - \sin B = 2 \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$

$\cos A + \cos B = 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$

$\cos A - \cos B = -2 \sin\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$

11. Co function Formulas

$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$ $\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$

$\operatorname{cosec}\left(\frac{\pi}{2} - \theta\right) = \sec\theta$ $\sec\left(\frac{\pi}{2} - \theta\right) = \operatorname{cosec}\theta$

$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$ $\cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$

1. $\sin^2 25^\circ + \sin^2 65^\circ = ?$

a) $\frac{\sqrt{3}}{2}$ b) 1 c) 0 d) $\frac{2}{\sqrt{3}}$

2. $\cos^4\theta - \sin^4\theta = \frac{2}{5}$, $1 - 2\sin^2\theta = ?$

a) $\frac{2}{5}$ b) 1 c) 0 d) $\frac{2}{\sqrt{3}}$

3. If $\sin(60^\circ - \theta) = \cos(\Psi - 30^\circ)$, then the value of $\tan(\Psi - \theta)$ is (assume that θ and Ψ are both positive acute angles with $\theta < 60^\circ$ and $\Psi > 30^\circ$).

a) $\frac{1}{\sqrt{3}}$ b) 0 c) $\sqrt{3}$ d) 1

4. $\frac{\tan 57^\circ + \cot 37^\circ}{\tan 33^\circ + \cot 53^\circ} = ?$

a) $\tan 33^\circ \cot 53^\circ$ b) $\tan 53^\circ \cot 33^\circ$

c) $\tan 33^\circ \cot 57^\circ$ d) $\tan 57^\circ \cot 37^\circ$

5. If $\tan\theta + \cot\theta = 2$, then the value of $\tan^n\theta + \cot^n\theta = ?$ $(0^\circ < \theta < 90^\circ$, n is an integer) is

a) 2 b) 2^n c) 2n d) 2^{n+1}

6. If $\sec\theta + \tan\theta = \sqrt{3}$ $(0° \le 0 \le 90°)$, then $\tan3\theta$ is

a) Undefined b) $\frac{1}{\sqrt{3}}$ c) considered as infinity d) $\sqrt{3}$

7. If $5\tan\theta = 4$, then the value of $\left(\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 3\cos\theta}\right)$ is_

a) 1/7 b) 2/7 c) 5/7 d) 2/5

8. If $\sec\theta + \tan\theta = 2 + \sqrt{5}$, then the value of $\sin\theta + \cos\theta$ is

a) $\frac{3}{\sqrt{5}}$ b) $\sqrt{5}$ c) $\frac{7}{\sqrt{5}}$ d) $\frac{1}{\sqrt{5}}$

9. If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, then $\cos\theta - \sin\theta$ is

a) $\sqrt{2}\tan\theta$ b) $-\sqrt{2}\cos\theta$ c) $-\sqrt{2}\sin\theta$ d) $\sqrt{2}\sin\theta$

10. The value of $\frac{1}{\text{cosec}\,\theta - \cot\theta} - \frac{1}{\sin\theta}$ is

a) 1 b) $\cot\theta$ c) $\text{cosec}\,\theta$ d) $\tan\theta$

11. If $\tan\alpha = n\tan\beta$ and $\sin\alpha = m\sin\beta$, then $\cos^2\alpha$ is

a) $\frac{m^2}{n^2+1}$ b) $\frac{m^2}{n^2}$ c) $\frac{m^2-1}{n^2-1}$ d) $\frac{m^2+1}{n^2+1}$

12. If $x = \text{cosec}\theta - \sin\theta$ and $y = \sec\theta - \cos\theta$ then the value of $x^2y^2(x^2 + y^2 + 3)$ is_

a) 0 b) 1 c) 2 d) 3

13. If $\sin\theta + \sin^2\theta = 1$, then the value of $\cos^{12}\theta + 3\cos^{10}\theta + 3\cos^8\theta + \cos^6\theta - 1$ is_

a) 0 b) 1 c) -1 d) 2

14. The value of $152(\sin30^\circ + 2\cos^245^\circ + 3\sin30^\circ + 4\cos^245^\circ + \ldots\ldots + 17\sin30^\circ + 18\cos^245^\circ)$ is

a) An integer but not a perfect square b) A rational number but not an integer

c) A perfect square of an integer d) Irrational

15. Find the value of tan 1° tan2° tan 3° …… tan 89°

a) 1 b) -1 c) undefined d) 0

16. The value of cos 1° cos 2° cos 3° …. Cos 177° cos 178° cos 179° is

a) 0 b) $\frac{1}{2}$ c) 1 d) $\frac{1}{\sqrt{2}}$

17. Find the minimum value of $2\sin^2\theta + 3\cos^2\theta$

a) $-\sqrt{13}$ b) 2 c) 3 d) 5

18. The least value of $(4\sec^2\theta + 9\operatorname{cosec}^2\theta)$ is _

a) 1 b) 19 c) 25 d) 7

19. Find the minimum value of $\sin^2\theta + \cos^2\theta + \tan^2\theta + \cot^2\theta + \sec^2\theta + \operatorname{cosec}^2\theta$

a) 1 b) 3 c) 5 d) 7

20. If θ is a positive acute angle and tan2θ. tan3θ = 1, then find the value of $(2\cos^2\frac{5\theta}{2} - 1)$

a) 0 b) $\frac{1}{2}$ c) $\frac{1}{2}$ d) 1

21. If $\sin 17^\circ = \frac{x}{y}$, then $(\sec 17^\circ - \sin 73^\circ)$=?

a) $\frac{x^2}{y\sqrt{x^2-y^2}}$ b) $\frac{y^2}{x\sqrt{x^2-y^2}}$ c) $\frac{y^2}{x\sqrt{y^2-x^2}}$ d) $\frac{x^2}{y\sqrt{y^2-x^2}}$

22. If $2\sin\left(\frac{\pi x}{2}\right) = x^2 + \frac{1}{x^2}$, then what is the value of $\left(x - \frac{1}{x}\right)$

a) 1 b) 0 c) -1 d) 2

23. If x, y are positive acute angles, x + y < 90° and sin(2x - 20°) = cos(2y + 20°), then the value of sec (x + y) is_

a) √2 b) 1/√2 c) 1 d) 0

24. If tan (x + y) tan (x - y)=1, then the value of tan (2x/3) is_

a) 1/√3 b) 2/√3 c) √3 d) 1

25. If $0 \le \theta \le \pi/2$, 2y cos θ = x sin θ and 2x sec θ - y cosec θ = 3, then the value of $x^2 + 4y^2$ is_

a) 1 b) 2 c) 3 d) 4

26. If tan θ = 3/4 and θ is acute, then cosec θ is

a) 4/5 b) 5/3 c) 5/4 d) 4/3

27. The value of $\frac{1}{(1+\tan^2\theta)} + \frac{1}{(1+\cot^2\theta)}$ is

a) ¼ b) 1 c) 2 d) 1/2

28. Evaluate : 3 cos 80° cosec 10° + 2 cos 59° cosec31°

a) 1 b) 3 c) 2 d) 5

29. If tanθ + cotθ = 2, then the value of $\tan^2\theta + \cot^2\theta$ is –

a) 2 b) 1 c) √2 d) 0

30. The value of $[\frac{\cos^2 A\ (\text{Sin}\ A + \text{Cos}\ A)}{\text{cosec}^2 A(\text{Sin}\ A - \text{Cos}\ A)} + \frac{\sin^2 A\ (\text{sin}\ A - \text{Cos}\ A)}{\sec^2 A\ (\text{Sin}\ A + \text{Cos}\ A)}]$($\sec^2$ A - cosec^2 A) is –

a) 1 b) 3 c) 2 d) 4

31. If cosec 39° = x, the value of $\frac{1}{\text{cosec}^2 51°} + \sin^2 39° + \tan^2 51° - \frac{1}{\sin^2 51° \sec^2 39°}$ is

a) $\sqrt{x^2 - 1}$ b) $\sqrt{1 - x^2}$ c) x^2 - 1 d) 1 - x^2

32. The value of tan4°. tan43°. tan47°. tan86° is

a) 2 b) 3 c) 1 d) 4

33. $\frac{\tan\theta + \cot\theta}{\tan\theta - \cot\theta} = 2$, $(0 \leq \theta \leq 90°)$, then the value of $\sin\theta$ is

a) $\frac{2}{\sqrt{3}}$ b) $\frac{\sqrt{3}}{2}$ c) $\frac{1}{2}$ d) 1

34. If $\sec^2\theta + \tan^2\theta = \frac{7}{12}$, then $\sec^4\theta - \tan^4\theta$ =

a) $\frac{7}{12}$ b) $\frac{1}{2}$ c) $\frac{5}{12}$ d) 1

35. If $0 < x < \frac{\pi}{2}$ and secx = cosecy, then the value of sin(x + y) is:

a) 0 b) 1 c) $\frac{1}{2}$ d) $\frac{1}{\sqrt{3}}$

36. If A, B and C be the angles of a triangle, then of the following the incorrect relation is:

a) $\sin(\frac{A+B}{2}) = \cos\frac{C}{2}$ b) $\sin(\frac{A+B}{2}) = \sin\frac{C}{2}$

c) $\tan(\frac{A+B}{2}) = \cot\frac{C}{2}$ d) $\cot(\frac{A+B}{2}) = \tan\frac{C}{2}$

37. The measures of the angles of a triangle are in the ratio 2 : 7 : 11. Measures of angles are

a) 16°, 56°, 88° b) 18°, 63°, 99°

c) 20°, 70°, 90° d) 25°, 175°, 105°

38. If $\sin\alpha + \cos\beta = 2(0° \leq \beta < \alpha \leq 90°)$, then $\sin(\frac{2\alpha+\beta}{3})$ =

a) $\sin\frac{\alpha}{2}$ b) $\cos\frac{\alpha}{3}$ c) $\sin\frac{\alpha}{3}$ d) $\cos\frac{2\alpha}{2}$

39. If $\cos^4\theta - \sin^4\theta = \frac{2}{3}$, then the value of $2\cos^2\theta - 1$ is

a) 0 b) 1 c) $\frac{2}{3}$ d) $\frac{1}{\sqrt{3}}$

40. If $\sin\alpha \sec(30° + \alpha) = 1(0 < \alpha < 60°)$, then the value of $\sin\alpha + \cos2\alpha$ is

a) 1 b) $\frac{2+\sqrt{3}}{2\sqrt{3}}$ c) 0 d) $\sqrt{2}$

41. If $\tan\theta = 1$, then the value of $\frac{8\sin\theta + 5\cos\theta}{\sin^3\theta - 2\cos^3\theta + 7\cos\theta}$ is

a) 2 b) $2\frac{1}{2}$ c) 3 d) $\frac{4}{5}$

42. If θ be a positive acute angle satisfying $\cos^2\theta + \cos^4\theta = 1$, then the value of $\tan^2\theta + \tan^4\theta$ is

a) $\frac{3}{2}$ b) 1 c) $\frac{1}{2}$ d) 0

43. If $\tan\theta = \frac{4}{3}$, then the value of $\frac{3\sin\theta + 2\cos\theta}{3\sin\theta - 2\cos\theta}$ is

a) 0.5 b) -0.5 c) 3.0 d) -3.0

44. The simplified value of $(\sec A - \cos A)^2 + (\operatorname{cosec} A - \sin A)^2 - (\cot A - \tan A)^2$

a) 0 b) $\frac{1}{2}$ c) 1 d) 2

45. The value of $\sin^2 1° + \sin^2 5° + \sin^2 9° + \ldots. + \sin^2 89°$ is

a) $11\frac{1}{2}$ b) $11\sqrt{2}$ c) 11 d) $\frac{11}{\sqrt{2}}$

46. The numerical value of $\cot 18°(\cot 72°\cos^2 22° + \frac{1}{\tan 72°\sec^2 68°})$ is

a) 1 b) $\sqrt{2}$ c) 3 d) $\frac{1}{\sqrt{3}}$

47. If $\sin\theta - \cos\theta = \frac{7}{13}$ and $0 < \theta < 90°$, then the value of $\sin\theta + \cos\theta$ is

a) $\frac{17}{13}$ b) $\frac{13}{17}$ c) $\frac{1}{13}$ d) $\frac{1}{17}$

48. If $2\cos\theta - \sin\theta = \frac{1}{\sqrt{2}}$, $(0° < \theta < 90°)$ the value of $2\sin\theta + \cos\theta$ is

a) $\frac{1}{\sqrt{2}}$ b) $\sqrt{2}$ c) $\frac{3}{\sqrt{2}}$ d) $\frac{\sqrt{2}}{3}$

49. If $\frac{\sec\theta+\cos\theta}{\sin\theta-\cos\theta} = 3$, then the value of $\sin^4\theta - \cos^4\theta$ is

a) $\frac{1}{5}$ b) $\frac{2}{5}$ c) $\frac{3}{5}$ d) $\frac{4}{5}$

50. If $\sec^2\theta + \tan^2\theta = 7$, then the value of θ when $0° \le \theta \le 90°$, is

a) 60° b) 30° c) 0° d) 90°

Answers

1 – b	2 - a	3 - c	4 - b	5 - a	6 - a	7 - a	8 - a	9 - d	10 - b
11 - c	12 - b	13 - a	14 - c	15 - a	16 - a	17 - b	18 - c	19 - d	20 - a
21 - d	22 - b	23 - a	24 - a	25 - d	26 - b	27 - b	28 - d	29 - a	30 - c
31 - c	32 - c	33 - b	34 - a	35 - b	36 - c	37 - b	38 - b	39 - c	40 - a
41 - a	42 - b	43 - c	44 - c	45 - a	46 - a	47 - a	48 - c	49 - c	50 - a

Additional Examples

1. If $1 + cos^2\theta = 3sin\theta cos\theta$, then the integral value of $cot\theta$ is $(0 < \theta < \frac{\pi}{2})$

a) 1 b) 2 c) 0 d) 3

2. The value of the following is $3(\sin^4\theta + \cos^4\theta) + 2(\sin^6\theta + \cos^6\theta) + 12\sin^2\theta\cos^2\theta$

a) 2 b) 5 c) 3 d) 0

3. The value of the following is cos24° + cos55° + cos125° + cos204° + cos300°

a) 2 b) $\frac{1}{2}$ c) 1 d) $-\frac{1}{2}$

4. If $\frac{sec\theta + tan\theta}{sec\theta - tan\theta} = 2\frac{51}{79}$, then the value of $sin\theta$ is

a) $\frac{39}{72}$ b) $\frac{35}{72}$ c) $\frac{65}{144}$ d) $\frac{91}{144}$

5. If tanA + cotA = 2, then the value of $\tan^{10}A + \cot^{10}A$ is

a) 1 b) 4 c) 2^{10} d) 2

6. The value of $\frac{1-\sin^2(\theta+16°)}{1+\sin^2(\theta+31°)} \times \frac{\cos^2(\theta+46°)+\cos^2(\theta+16°)}{\operatorname{cosec}^2(\theta+76°)-\cot^2(\theta+76°)} \div \{\sin(\theta+46°)+\tan(\theta+16°)\}$ for $\theta = 14°$ is

a) -1 b) 0 c) $\frac{1}{2}$ d) 1

7. If $x = \frac{\cos\theta}{1-\sin\theta}$, then $\frac{\cos\theta}{1+\sin\theta}$ is equal to

a) $x-1$ b) $\frac{1}{x}$ c) $\frac{1}{x+1}$ d) $\frac{1}{1-x}$

8. If $\sin\frac{\pi x}{2} = x^2 - 2x + 2$, then the value of x is

a) 0 b) 1 c) -1 d) None of these

9. The value of $\frac{\sin 43°}{\cos 47°} + \frac{\cos 19°}{\sin 71°} - 8\cos^2 60°$ is

a) 0 b) 1 c) 2 d) -1

10. Find the minimum Value of $\text{Sec}^2\theta + \text{Sin}^2\theta.\text{Sin}^2 5\theta + \text{Cot}^2\theta + \text{Cos}^2\theta + \text{Tan}^2\theta$

a) 0 b) 4 c) 2 d) 3

11. If $0 \le \theta \le \frac{\pi}{2}$, $2y\cos\theta = x\sin\theta$ and $2x\sec\theta - y\operatorname{cosec}\theta = 3$, then the value of $x^2 + 4y^2$ is

a) 1 b) 2 c) 3 d) 4

12. If $\frac{\cos^4\alpha}{\cos^2\beta} + \frac{\sin^4\alpha}{\sin^2\beta} = 1$, then the value of $\frac{\cos^4\beta}{\cos^2\alpha} + \frac{\sin^4\beta}{\sin^2\alpha}$ is

a) 4 b) 0 c) $\frac{1}{8}$ d) 1

13. The value of $[\frac{\cos^2 A\ (\text{Sin}\ A+\text{Cos}\ A)}{\text{cosec}^2 A(\text{Sin}\ A-\text{Cos}\ A)}+\frac{\sin^2 A\ (\sin\ A-\text{Cos}\ A)}{\sec^2 A\ (\text{Sin}\ A+\text{Cos}\ A)}](\sec^2 A-\text{cosec}^2 A)$ is –

a) 1 b) 3 c) 2 d) 4

14. The eliminate of θ from $x\cos\theta - y\sin\theta = 2$ and $x\sin\theta + y\cos\theta = 4$ will give

a) $x^2 + y^2 = 20$ b) $3x^2 + y^2 = 20$

c) $x^2 - y^2 = 20$ d) $3x^2 - y^2 = 0$

15. If $cosec\theta = \frac{13}{12}$, find the value of $\frac{2sin\,\theta-3\cos\,\theta}{4sin\,\theta-9\cos\,\theta}$ =?

a) 3 b) 5 c) 7 d) 9

16. The value of $\frac{1}{\sqrt{2}}\,sin\frac{\pi}{6}.cos\frac{\pi}{4} - cot\frac{\pi}{3}.sec\frac{\pi}{6} + \frac{5tan\frac{\pi}{4}}{12sin\frac{\pi}{2}}$ is equal to

a) 0 b) 1 c) 2 d) $\frac{3}{2}$

17. Find maximum and minimum values of $11\cos^2 x + 3\sin^2 x + 6\sin x.\cos x + 5$

a) (0, 7) b) (-1, 17) c) (3, 17) d) (7, 17)

18. If $u_n = cos^n\alpha + sin^n\alpha$, then the value of $2u_6 - 3u_4 + 1$ is

a) 4 b) 6 c) 0 d)1

19. If $sin21^\circ = \frac{x}{y}$, then $sec21^\circ - sin69^\circ$ is equal to

a) $\frac{x^2}{y\sqrt{y^2-x^2}}$ b) $\frac{y^2}{x\sqrt{y^2-x^2}}$ c) $\frac{x^2}{y\sqrt{x^2-y^2}}$ d) $\frac{y^2}{x\sqrt{x^2-y^2}}$

20. If $sec\alpha + tan\alpha = 2$, then the value of $sin\alpha\ is$ (assume that $0 < \alpha < 90^\circ$)

a) 0.4 b) 0.5 c) 0.6 d) 0.8

Answers

1 – a	2 - b	3 - b	4 - c	5 - d	6 - d	7 - b	8 - b	9 - a	10 - b
11 - d	12 - d	13 - c	14 - a	15 - a	16 - a	17 - d	18 - c	19 - a	20 - c