8. Set Theory

A set is a collection of well-defined objects. The numbers of a set may be objects, persons, letters, ideas, or any set of items and are called the elements of a set.

Certain capital letters represent standard sets, e.g.

N = {1, 2, 3, 4 …}set of natural numbers

W = {0, 1, 2, 3, 4 …}set of whole numbers

I = (0, ±1, ±2, ±3 …} set of integers

If 'A' is a set and 'a' is an element of this set, we say that 'a' belongs to 'A' or a ∈ A. A set 'A' which has only a finite number of elements is called a finite set.

Operations on sets

i) Union sets

If a & B are sets, the union of A & B, denoted by A ∪ B is the set of all elements which are either in A or in B or in both A & B.

E13. If A = {1, 2, 5, 7, 9} and B = {3, 8, 9, 2, 0}, then find A ∪B

A ∪B = {0, 1, 2, 3, 5, 7, 8, 9}

ii) Intersection of sets

If A & B are sets, then the intersection of A & B denotes by A ∩ B and is the set of all elements which belong to both A and B.

E14. A = (1, 2, 3, 4, 5} and B = {3, 7, 9, 4} then find A∩B

Sol. A∩ B = {3, 4}

iii) Disjoint sets

Two sets A and B are said to be disjoint, If A∩B = ∅

If A∩B≠ ∅, then A and B are said to be intersecting sets or overlapping sets.

If A = {1, 3, 5, 7, 9}, B = {2, 4, 6, 8} and C = {2, 3, 5, 7, 11}, then A and B are disjoint sets, while A and C are intersecting.

E15. For two disjoint sets A and B, then find A∩B.

Sol. In this case A∩B = ∅.

iv) Difference of sets

The difference of two sets A & B, denoted by (A – B) is the set of elements which belong to A but not B.

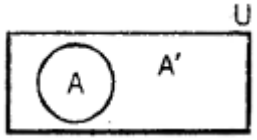
E16. A = {1, 2, 3, 4, 5} and B = {3, 7, 9, 4}, find A – B.

Sol. A – B = {1, 2, 5}

Venn Diagrams

All that has been stated about sets can be illustrated by means of diagrams. A universal set is represented by a rectangle and a subset by a circle within it. Two disjoint sets are represented by circles that do not overlap. The complement of a set will be all that is contained within the rectangle and which is not included in the circle representing that set. Non-disjoint sets are represented by overlapping circles. Such diagrams are called Venn diagrams.

The universal set U with a subset A and its complement A'

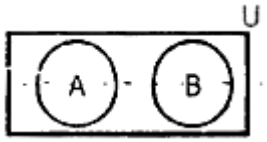


The universal set U with two disjoint sets A & B

U = Real numbers,

A = even numbers,

B = odd numbers.

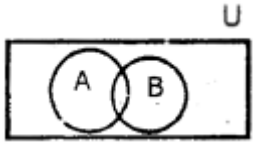


The universal set U with two overlapping sets A & B

U = Real numbers,

A = odd numbers,

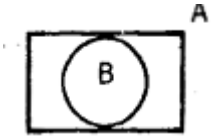
B = prime numbers.



A set A' and its subset B

A = Real numbers,

B = odd numbers.

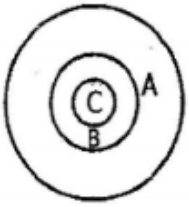


Some more important patterns

A = Men in India

B = Men in India above 35 years of age

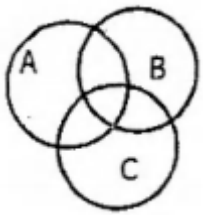
C = Indian married men who are above 35 years of age.



A = Boys in a school

B = Boys named Shyam

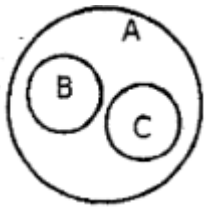
C = Boys who are 12 years old



A = Employees in a company

B = Male Employees in the same company

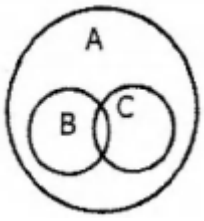
C = Female employees in the same company having children



A = Students in a college

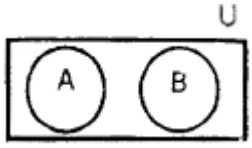
B = Students in the college playing Hockey

C = Students in the college playing Chess



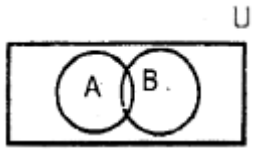
<u>Use of Venn Diagrams in Problem Solving</u>

The following method of solving problems must be very scrupulously understood by you as only then can you solve some of the problems. Remember, some problems on "set theory and Venn diagrams" are always asked in the various BANK exams.



In this venn Diagram

$n(A \cup B) = n(A) + n(B)$

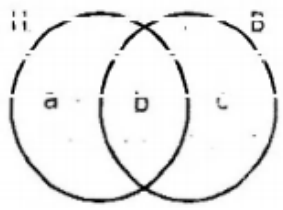


But in this diagram n(A∪B) = n(A) + n(B) – n(A∩ B)

E17. In a group of l000 persons, 760 can speak Hindi & 430 can speak Bengali, (a) How many can speak both? (b) How many can, speak Hindi only & Bengali only?

Sol. Let H denote Hindi, b denote Bengali we have a + b = 760, b + c = 430, a + b + c = 1000

=> a = 570, c = 240 or b = 190 or 190 people can speak both Hindi and Bengali. 570 speak only Hindi & 240 speak only Bengali.

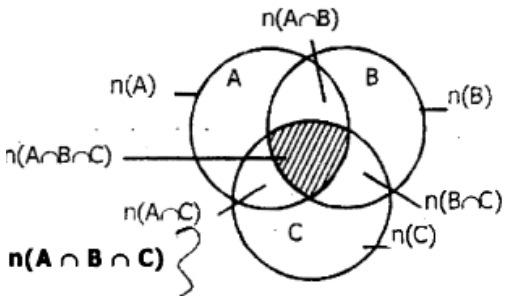


Alternate sol:

n(H∪B) = n(H) + n(B) – n(H∩B) => 1000 = 760 + 430 – n(H∩B) => n(H∩B) = 190 etc.

For three sets, the diagram given along side is valid:

n(A∪B∪C) = n(A) + n(B) + n(c) - n(A∩C) – n(A∩B) – n(B∩C) +n(A∩B∩C)



E18. In a survey of 2000 students, 48% liked coffee (C), 54% liked tea (T) 64% like to smoke (S). Of the total, 28% liked C & T, 32% liked T& S and 30% used C & S. 6% did not like any of the three Find

(i) The number of students using all three,

(ii) T & S but not C,

(iii) T but not C.

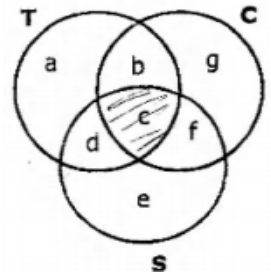
(iv) only C.

Sol. We have b + c + g + f = 960 (48% of 2000)

Similarly a + b + c + d = 1080 (54% of 2000) c + d + e + f = 1280 (64% of 2000) b + c = 560

d + c = 640, c + f = 600, a + b + c + d + e + f4- g = 1880 (94% of 2000)

Solve to find



(i) c = 360 (ii) d = 280 (iii) a + d = 520 (iv) g = 160

Alternate Sol.:

n(C∪T∪S) = n(C) + n(T) + n(S) - n(C∩T) - n(C∩S) - n(T∩S) + n(C∩T∩S)

=> 1880 = 960 + 1080 + 1280 - 560 - 640 - 600 + n(C∩T∩S)

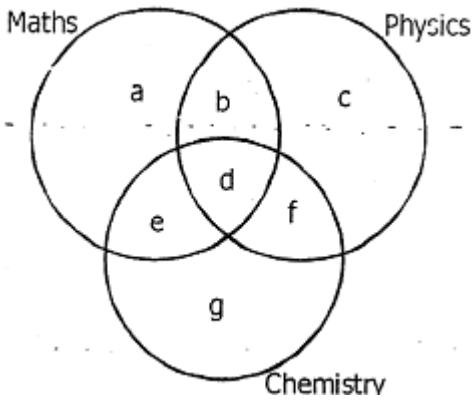
=> n(C∩T∩S) = 360. (This value is basically the "c" of the first method).

Now from the figure, all the values can be determined one by one.

In this type of DI sets, the data can be represented by Venn diagrams. Questions can then be solved using basic principles and formulae of Set Theory,

EXAMPLE # 01: Read the following data to answer the questions that follow.

In a class of 106 students, each student studies at least one of the three subjects Maths, Physics and Chemistry. 48 of them study Maths, 51 Physics and 53 Chemistry. 16 study Maths and Physics,17 study Maths and Chemistry and 18study Physics and Chemistry.



For Q1 to Q5:

We have

a + b + c + d + e + f + g = 106

a + e + d + b = 48

c + b + d + f = 51

g + e + d + f = 53

b + d = 16; d + e = 171; d + f = 18

and from the standard formula,

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

we get 106 = 48 + 51 + 53 - 17 -18 -16 + d .

=> d = 5. Now all the values can be obtained shown in the figure and all the questions can be answered.

1. The number of students who study exactly two subjects is
   1) 31 2) 32 3) 33 4) 36 5)None of these

2. The number of students who study more than one subject is
   1) 39 2) 41 3) 40 4) 42 5)None of these
   Sol. The number of students who study more than one subjects is given by the regions b, d, e, fSo, required result is = b +d + e + f = 41. Ans. 2)

3. The number of students who study all the three subjects is
   1) 5 2) 6 3) 74) 4 5)None of these
   Sol. From the venn diagram one can observe that it is d = 5. Ans. 1)

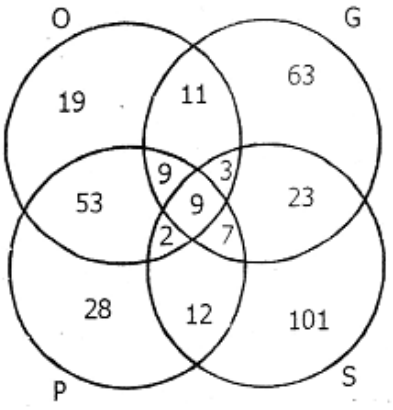
4. 4. The number of students who study, exactly one subject is
   1) 45 2) 553) 65 4) 70 5)None of these
   Sol. The number of students who study exactly one subject is given by a + c + g = 65. Ans.3)

5. The number of students who study physics and Maths but not Chemistry is
   1) 9 2) 11 3) 10 4) 12 5)None of these
   Sol. The required result is the vahre' present in the region b which is 11. Ans.2)
   Note: These type of questions and their solution, are so mechanical and direct that with proper practice, you should be able to crack it very fast

E 2: The following Venn diagram represents the results of a survey conducted by a market research firm NSD Ltd., to ascertain the profiles of a sample group. The diagram below shows the number of people who are Poets, Sportsmen, Graduates or Orators. Refer to the diagram to answer the questions that follow.

1) P= Poets, S = Sportsmen, G = Graduates, O = Orators

2)The figures in any region of the above diagram pertain to the "only" value for that region. For ex. 3 persons are only (Orators + Sportsmen + Graduates) etc.



6. Number of Sportsmen who have at least three specialties is

   1) 122) 21 3) 9 4) 30 5)None of these

   Sol. From the diagram number of sportsmen who have at least three specialties is given by 2 + 7 + 9 + 3 = 21 Ans. 2)

7. Total number of people having at least one specialty is

   1) 403 2) 321 3) 343 4) 340 5)None of these

   Sol. Adding up all the values in the diagram, we get required answer = 340

   Ans.4)

8. Number of people having only one specialty exceeded the number of people having exactly two specialties by

   1) 113 2) 111 3) 112 4) 110 5)None of these

   Sol. From the figure

   Number of people having only one specialty = 19 + 63 + 101 + 28 = 211

   Number of people having exactly two specialties = 53 + 11 + 23 + 12 = 99

   => Required answer = 211 - 99 = 112 Ans.3)

9. The number of people having at least one of the described specialities form what percentage of the total sample?

   1) 38%2) 62 %3) 44% 4) cannot be determined 5)None of these

Sol. The number of people having atleast one specialties is 340. But the total number of people surveyed is not known. Hence, percentage cannot be determined. Ans.4)

10.Orators who were neither Sportsmen nor graduates exceeded poets who were neither Orators nor Graduates by a margin of

1) 32 2) 61 3) 43 4) 27 5)None of these

Sol. Orators who were neither Sportsmen nor graduates are 53 + 19 = 72

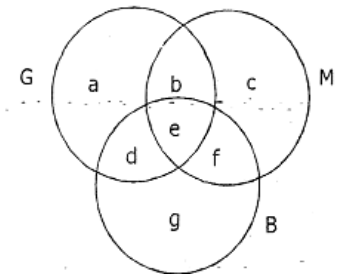
Poets who were neither Orators nor Graduates are 28 + 12 = 40

.˙. Required answer is = 72 - 40 = 32. Ans. 1)

E 3: Refer to the following information to answer the questions that follow.

New Age Consultants have three consultants Gyani, Medha and Buddhi. The sum of the number of projects handled by Gyani and Buddhi individually is equal to the number of projects in which Medha is involved. All three consultants are involved together In 6 projects. Gyani works with Medha in 14 projects. Buddhi has 2 projects with Medha but without Gyani, and 3 projects with Gyani but without Medha. The total number of projects for New Age Consultants is one less than twice the number of projects in which more than one consultant is involved.

For Q.11 to Q.13: From the data given we have the following Venn diagram.



Following equations can be formed

a + g = b + c + e + f … 1)

e = 6; b + e = 14; b = 8

f = 2 and d = 3

a + b+ c+ d + e + f + g = 2(b + d + e + f) – 1

=> a + c + g = 19 – 1 = 18 … 2)

11.What is the number fo projects in which gyani alone is involved?

1) Uniquely equal to Zero 2) Uniquely equal to 1

3) Uniquely equal to 4 4) cannot be determined Uniquely 5)None of these

Sol. The required answer will be obtained by solving equations 1) and 2) we will get c =1 and 'a' cannot be found. Ans 4).

12.What is the number of projects in which medha alone is involved?

1) Uniquely equal to Zero 2) Uniquely equal to 1

3) Uniquely equal to 4 4) cannot be determined Uniquely 5)None of these

Sol. From above we can observe that the number of projects in which medha alone is involved is 1. Ans.2)

13.What is the number of projects in which medha and gyani only are involved?

1) 82) 63) 74) 9 5)None of these

Sol. The number of projects in which Medha and Gyani only are involved = 14 - 6 = 8. Ans. 1)

Practice exercise:

Refer to the following information to answer the questions that follow.

In a pollution study of 1500 Indian rivers, the following data was reported. 520 were polluted by sulphur compounds, 335 were poliuted by phosphates, 425 were polluted by crude oil, 100 were polluted by both crude oil and sulphur compounds, 180 were polluted by both sulphur compounds and phosphates, 150 were polluted by both phosphates and crude oil and 28 were polluted by sulphur compounds, phosphates and crude oil.

1. How many of the rivers were polluted by at least one of the three impurities?
   1) 878 2) 374 3) 346 4) 890 5) None of these

2. How many of the rivers were polluted by exactly two impurities?
   1) 878 2) 374 3)346 4) 280 5) None of these

3. How many of the rivers were polluted by all the three impurities?
   1) 78 2)55 3) 65 4) 28 5) None of these

4. How many rivers were polluted by Sulphur and Phosphate compounds only?
   1) 180 2) 152 3) 72 4) 1225) None of these

Read the following information and answer the questions that follow

Observations of 500 spectators in a cricket stadium regarding their possession of hats, Sunglasses and Umbrellas are given below

45 did not have any of the three items.

25 did not have hats only.

40 did not have hats and Umbrellas only

230 had all the three items.

Those who had only Umbrellas were less then those who had none of the three items by 15.

Those who did not have only Umbrellas exceeded those who did not not have only Hats and Umbrellas by 15.

Those who did not have Sunglasses only were more than those who did not have Umbrellas only by 5.

Number of people not having Hats and Sunglasses only is double the number of people not having Umbrellas and Sunglasses only.

5. How many spectators had Sunglasses and Umbrellas?
   1) 55 2) 155 3) 255 4) 355 5) None of these

6. How many had only two of the three items?
   1) 140 2) 130 3) 120 4) 1105) None of these

7. How many had all the three items or none of the three items?
   1) 245 2) 255 3) 265 4) 275 5) None of these

8. If a hawker sells straw hats to 50% of those who do not have Hats @ ₹ 5 per hat, then what would be his collection?
   1) ₹ 150 2) ₹ 250 3) ₹ 350 4) ₹ 300 5) None of these

9. If all those having only Sunglasses leave the stadium, then what percentage of the remaining have both Hats and Umbrellas and not Sunglasses?
   1) 18 2) 13 3) 9 4) 12 5)None of these

10. Number of persons having Umbrellas only is ..... that of persons having Hats only but ..... that of persons having both Hats and Umbrellas but not Sunglasses.
    1) Twice; half 2) Thrice; half 3) Half; thrice 4) Half; twice 5) None of these

Read the information given below and answer the questions that follow.

Out of 800 tourists who visited Agra, 60% visited the Tajmahal. Number of tourists who visited all the three places were 1/3rd of those who went to see the Tajmahal. Tourists who visited only the Red Fort were equal to the total of those who visited the Tajmahal and either Red Fort or Fatehpur Sikri, but not both. Also the number of tourists visiting Red Fort and Fatehpur Sikri are same. And number of tourists visiting only Fatehpur Sikri and only the Tajmahal were 140 and 190 respectively. (Assume that each tourist visits at least one place.)

11.How many tourists visited the Red Fort?

1) 480 2) 410 3) 130 4) 140 5) None of these

12.The number of tourists visiting only one of the three places is

1) 2702) 330 3) 460 4) 410 5) None of these

13.The number of tourists who visited at least one of the Red Fort or Fatehpur Sikri is

1) 610 2) 670 3) 600 4) 640 5) None of these

14.How many tourists have visited Tajmahal and Fatehpur Sikri but not Red Fort?

1) 60 2) 220 3) 70 4) 50 5) None of these

15.The number of tourists who visited at least two places is

1) 340 2) 160 3) 180 4) 240 5) None of these

Answers:

1.1 2.3 3.4 4.2 5.3 6.1 7.4 8.3 9.2 10.1 11.2 12.3 13.1

14.1 15.1