8. Time and Work

The three major variables in this topic are work, number of people required to finish the work in given time.

- Unless otherwise specified, the amount of work done is generally taken as 1.
- Also, if it is given that a person x can finish a job in D days, then it implies that x alone can do the job in D days.
- If a man can do a piece of work in N days (or hours or any other unit of time), then the work done by him in one day will be 1/N of the total work.
- Other important concept is Work can be measured in terms of men x days
- Relation between work (W), people (P) and time (T).

  When T = Const, $W \propto P$

  P = Const, $W \propto T$

  W = Const, $P \propto 1/T$

  If all are varying the relation is $W_1/(P_1 \times T_1) = W_2/(P_2 \times T_2)$

Example: 4 men take 3 days to paint a wall. If 20 men with same efficiency are working, then how many walls can be painted in 3 days?

Sol: Since 4 men can paint the wall in three days, so 20 men can paint 20/4 i.e. 5 such walls in 3 days. T = Cosnt. So $W \propto P$, So $4/1 = 20/x \rightarrow x = 5$

Example: A man takes 3 days to paint a wall. If he works for 12 days, then how many walls can be painted by him?

Sol. Since in 3 days he can paint the wall, so in 12 days, he canpaint 12/3
i.e. 4 such walls.

Example: Ramesh can finish a piece of work by himself in 48 days. Then calculate the amount of work done by him in 12 days.

Sol: The amount of work done by Ramesh, working alone in 48 days = 1 unit of
Work. So, the amount of work done by Ramesh, working alone in 1 day = 1/48 unit of work and in 12 days = 12/48 = ¼ units of work.

If A is twice as good a workman as S, then A will take half the time S takes to finish a piece of work.

Example: Suraj can finish a piece of work by himself in 48 days. Rajesh, who is $\frac{1}{5}$ times more efficient than Suraj, requires how many days to finish the work if he works alone?

Sol: Suraj, working alone 48 days = 1 unit of work.
Rajesh is 1/5 times more efficient than Suraj. So Rajesh is 6/5 times as efficient as Suraj. Hence Rajesh shouldrequire $5/6^{th}$ of the time, the time taken by Suraj.

Therefore time taken by Rajesh = 5/6 x 48 = 40 days.

Measurement of work

Work can be measured by many units. One of them is Man-days

For example, 8 men can finish a piece of work in 12 days. This implies that the work is of 8 × 12 = 96men days, i.e., if one man alone works to finish the given work then he is going to take 84 days to finish it and if 96 men work together, then the work would be finish in 1 day.

Man-hours

If people work for differnet number of hours perday then we have to measure work in man hours.

Example: 8 men of equal efficiency can finish a piece of work byworking 10 hrs a day in 7 days. How many men will berequired to finish the same work if they work 4 hrs a dayand the work is to be completed in 10 days?

Sol. The total work to be done = 8 x 10 x 7 = 560 man hours.

Let x men be required to finish this job, then 560 = x x 4 x 10 ➔ x = 14 . So, 14 men are required.

One day's work

- If a man can do a piece of work in N days (or hours or any other unit of time), then the work done by him in one day will be 1/N of the total work.
- If A and B can do a piece of work in x & y days respectively while working alone, they will together take $\frac{X \times Y}{(X+Y)}$daysto complete it.

  It can be explained as follows:

  A' s one day work = $\frac{1}{x}$, B's one day work = $\frac{1}{Y}$

  ➔ (A + B)'s total one day work = $\frac{1}{x} + \frac{1}{y}$ of the total.

  Let the total work be 1, Now $\frac{1}{x} + \frac{1}{y}$ of 1 can be finished in one day.

  ➔ Total work can be finished in $\frac{1}{\frac{1}{X}+\frac{1}{y}} = \frac{X \times Y}{x+Y}$ days.

Example: A can finish a piece of work by working alone in 12 days. B while working alone can finish the same work in 24 days. If both of them work together, then in how many days, the work will be finished?

Sol: Applying formula, $\frac{12 \times 24}{12+24} = \frac{288}{36} = 8$

The work will be finished in 8 days.

If A, B, C can do a piece of work in x, y, z days respectively while working alone, they will together take $\frac{X \times Y \times Z}{x \times y + y \times z + z \times x}$ days to finish it.

It can be explained as follows:

A's one day work = $\frac{1}{x}$, B's one day work = $\frac{1}{y}$ andC's one day work = $\frac{1}{Z}$.

➔ Their total one day work = $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$ of the total work.

Let the total work be 1 (unity).

Now $\frac{1}{X} + \frac{1}{Y} + \frac{1}{Z}$ of 1 can be finishedin one day

➔ Total work can be finished in $\frac{1}{\frac{1}{x}+\frac{1}{y}+\frac{1}{z}} = \frac{X \times Y \times Z}{X \times y + y \times z + z \times x}$ days.

Example: Ganesh, Ramesh and Mohan can do a job in 15,20 and 30days respectively. In how many days can the job be finishedif they work together?

Sol: Ganesh's 1 day work = 1/15 of the total.

➔ Ramesh's 1 day work = 1/20 of the total.

Mohan's 1 day work = 1/30 of the total.

They can do ($\frac{1}{15} + \frac{1}{20} + \frac{1}{30}$) of the total work in 1 day.

➔ Total work can be finished in $\frac{1}{\frac{1}{15}+\frac{1}{20}+\frac{1}{30}} = \frac{15 \times 20 \times 30}{15 \times 20 + 20 \times 30 + 30 \times 15} = \frac{9000}{1350}$

= 20/3 days.

Example: A and B require 10 days to complete a job. Band C require12 days to complete the same job. A and C require 15 days to complete the same job. Find the number of days required, if all are at work to complete the same job.

Sol: A + B = $\frac{1}{10}$, B + C = $\frac{1}{12}$ and C + A = $\frac{1}{15}$

2 (A + B + C) = $\frac{1}{10} + \frac{1}{12} + \frac{1}{15} = \frac{6+5+4}{60} = \frac{15}{60} = \frac{1}{4}$

A + B + C = 1/8.

Hence if A, B and C all work together, they will need total 8days to complete the work.

- If A,B and C can finish a piece of work in x, y and z days respectively, while working alone and together they requireM days to finish the work, then the amount of work done by A is M/x, B is M/y and C is M/z.

Example: A contractor undertook to do a certain work in 75 days and employed 60 men to do it. After 25 days he found that only $\frac{1}{4}$th of the work was done. How many more men must beemployed

so that the work may be finished in time?

Sol. 60 men in 25 days can do 1/4 of work. For 3/4th of work in 50 days, men required = 60 x (3/2) 90. Additional men = 90 - 60 = 30.

Note

- Two persons A and B, working together, can finish a piece of work in M days. If A can finish the job in x days by working alone, then B will finish the job by working alone in $\frac{M \times X}{X - M}$ days

Example: Ram and Shyam working together can finish a work in 12 days. If Ram alone can do the same in 20 days, then in howmanydays, Shyam alone can complete the work?

Sol: Let Shyam take x days to complete the work.

Work done by both in 1 day = $\frac{1}{20} + \frac{1}{x}$

Number of days required = $\frac{20 \times x}{20 + x}$ 12 (given).

$20x$ - $12x$ = 240 ➔ x = 30

➔ So shyam alone can do the work in 30 days.

Short-cut

Using formula we can get the answer as $\frac{12 \times 20}{20 - 12}$ = 30 days

➔ A can complete $\frac{a}{b}$ part of a work in x days, then $\frac{c}{d}$ part of the work will be done in $\frac{b \times c \times X}{a \times d}$ days.

Example: If A can complete a work in 16 days, then in how many days can he complete 3/4th of the work?

Sol: Using formula we get $\frac{1}{1}$ x $\frac{3}{4}$ x 16 = 12 days.

- If A can finish a work in x days and B is K times as efficient then the time required by both A and B working together to finish the job will be $\frac{X}{1 + K}$ days.

Example: Mohan can finish a work in 10 days. Rajesh is 2 times as efficient as Mayank. If they work together, in how many days will the work be finished?

Sol: Time taken by Mohan to do the work = 10 days.

➔Time taken by Rajesh to do the work = 5 days.(He is twice as efficient)

1 day's work = $\frac{1}{10} + \frac{1}{5} = \frac{3}{10}$

➔Time required to complete the job = $\frac{10}{3}$ days.

Short-cut

Using formula, we can get the answer as $\frac{10}{1 + 2} = \frac{10}{3}$ days.

A is K times as good a worker as B and takes x days less than B to finish the work. Then the amount of time required by A and B working together is $\frac{k \times x}{k^2-1}$ days.

Example: Ram is three times as good a worker as Rahim and takes 8 days less than Rahim to finish the work. Find the amount of time required by Ram and Rahim working together, to do the work.

Sol: Let Rahim take x days to complete the work and so Ram

Takes x - 8 days = $\frac{x}{3}$ days (Ram is 3 times efficient).

➔ x = 12andso x - 8 = 4.

Time required completing the work, working together $\frac{1}{\frac{1}{12}+\frac{1}{4}} = \frac{12}{4}$ = 3 days.

Short-cut:

Using formula we get $\frac{3 \times 8}{3^2-1}$ = 3 days.

Concept of Work Equivalence

Work eqivalence is used when the same work is finished with different number of people for different number of days. This is just using the concept of measuring work in terms of people x days. So Total work for one combination should be same as other combination.

Example: A group of soldiers can completely destroy an enemy bunker in 7 days. However 12 soldiers fell ill. The remaining soldiers now can do the job in 10 days. Find the original group strength.

Sol. Here, first of all, let us see how WORK can be defined. It is obvious that work can be measured as "destruction of theenemy bunkers.

In the first case, let us say that there were S numbersoldiers in the group. So they had to work for 7 days for the work which we call W.

➔ S x 7 = W ... (1)

Now 12 fell ill and the remaining did the work in 10 days. Hence the new equation is

(S - 12) x 10 = W... (2)

Just compare the two equations to get the answer.

S x 7 = (S - 12) x 10

➔ 7s = 10s - 120

➔ 120 = 3S

➔ S = 40 soldiers. Hence, there were 40 soldiers in the group initially.

Pipes and Cisterns

The same principle of Time & Work is employed to solve the problems on pipes & Cisterns. The only

difference being that in this case, the work done is in terms of filling or emptying a cistern (tank) and the time taken is the time taken by a pipe or a leak (crack) tofillor emptya cistern respectively.

Unlessotherwise specified, the amount of work done, i.e., filling or emptying a cistern is generally taken as 1. Also, if it is given that a pipe can fill a cistern in 10 min, then it impliesthat the pipe alone can fill the cistern in 10min.

Generally, the time taken to fill a cistern is taken as positive andthe time taken to empty a cistern is taken as negative.

A pipe connected with a cistern is called an inlet pipe or an outlet pipe, accordingly as it fills it or empties it respectively.

Some important formulae

- If an inlet pipe can fill a cistern in x hours, the part filled in 1 hour is $\frac{1}{X}$
- If an inlet pipe can fill a tank in x hours and an outlet pipe empties the full tank in y hours, then the net part filled in 1 hour when both the pipes are opened = $\frac{1}{X} - \frac{1}{Y}$
- In 1 hour, the part filled (or emptied) = $\frac{1}{X} - \frac{1}{Y}$
- Time required to fill or empty the tank = $\frac{X \times Y}{X \sim Y}$ hours.
- ($x \sim y$ indicates [x - y] or [y - x], whichever is positive).

Note

If $x > y$, then an empty tank can never be filled. Similarly if $x < y$, then a full tank can never be emptied.

Example: If an inlet pipe can fill a tank in 4 hours and an outlet pipes empties the full tank in 5 hours, then what is the net part filled in 1 hour when both the pipes are opened?

Sol. The net part filled in 1 hour when both the pipes are opened = (1/4) - (1/5)

= 1/20.

So, time required to fill the tank = 1/(1/20) = 20 hrs.

Example: An inlet pipe can fill the tank in 6 hours and an outlet pipecan empty the tank in 4 hours. When the tank is full andboth the pipes are open, find the net part emptied in 1hour.

Sol. The net part emptied in 1 hour = $\frac{1}{4}$ - $\frac{1}{6}$ = 12 {:. Time required to empty the full tank is 12 hours.}

- Two pipes A and B can fill a cistern in x and y hours respectively, while working alone. If both the pipes are opened together, then the time taken to fill the cistern will be $\frac{X \times Y}{X + Y}$

Example: Two pipes A and B can fill a cistern in 4 and 5 hoursrespectively, while working alone. If

both the pipes areopened together, then find the time taken to fill the cistern.

Sol. Pipe Acan fill the cistern in 4 hours and,

Pipe B can fill the cistern in 5 hours. When the pipes are opened together, the time taken to fill the cistern will be $\frac{4 \times 5}{4+5}$ = 9 hours.

- Three pipes A, B and C can fill a cistern in x, y and z hoursrespectively, while working alone. If all the three pipes areopened together, the time taken to fill the cistern will be $\frac{X \times Y \times Z}{XY + YZ + ZX}$

Example: Three pipes A, B and C can fill a cistern in 10, 12 and 15 hours respectively, while working alone. If all the three pipesare opened together, then find the time taken to fill thecistern.

Sol: The pipes can fill the cistern in 10, 12 and 15 hours. When the three pipes are opened together, the time taken to fill the cistern

$$= \frac{10 \times 12 \times 15}{10 \times 12 + 12 \times 15 + 10 \times 15} = 4\text{hours}$$

Example: A cistern has a leak which can empty it in 4 hours. A pipe which admits 20 litres of water per hour into the cistern is turned on and now the cistern is emptied in 6 hours. What is the capacity of the cistern?

Sol. The leak can empty the cistern in 4 hours. A pipe admits 20 litres of water per hour into the cistern. When the pipe is turned on, the cistern is emptied in 6 hours.

So, the capacity of the cistern is $\frac{4 \times 6 \times 20}{6-4}$ = 240 litres. The expression for filling pipe which can fill P_1 part of cisternin t_1 min and p_2 part in t_2 minis $\frac{t_1}{p_1} = \frac{t_2}{p_2}$.

Example: A pipe can fill 3/4 of the cistern in 12 min. In how many minutes, it can fill 1/2 of the cistern?

Sol. Using the formula, $\frac{t_1}{p_1} = \frac{t_2}{p_2} => \frac{12}{\frac{3}{4}} = \frac{t_2}{\frac{1}{2}} => t_2 = 8\,\text{min}$

The expression for emptying pipe which can empty e_1 part of cistern in t_1 min and e_2 part in t_2 min is $\frac{t_1}{e_1} = \frac{t_2}{e_2}$

Practice Exercise

DIRECTIONS: For the following questions, four options are given. Choose the best option.

1. A can do a piece of work in 15 days and B in 20 days. They finished the work with the assistance

of C in 5 days and got Rs. 45 as their wages. The share of each in wages (in Rs.) is
1) 22.5,12,10.5 2) 10.5,12,22.5 3) 15,11.25,18.75 4) 12.5,13,19.5

2. 5 men and 6 boys finish a piece of work in 4 days; 4 men and 3 boys in 6 days. In how many days would 3 men and 6 boys finish the same work?
1) 5 days 2) 36/7 days 3) 4 days 4) 29/7 days

3. If 144 men can dig a trench 132 yds. long, 5ft wide and 2ft deep in 5 days of 11 hours each, then in how many days can 56 men dig a trench 210 yds. Long, 8ft. wide an 13ft. deep working 9 hours per day?
1) 260 days 2) 250 days 3) 290 days 4) 300 days

4. Five people A, B, C, D, E do a certain job. A, B and C together complete the job in 7.5hrs A, C and E together complete it in 5 hours; A, C and D together complete it in 6 hours; B, D, E together complete it in 4 hours. If al 1 the five people work together, then how much time will be required to complete the job?
1) 3.5 hours 2) 3 hours 3) 2.5 hours 4) 1.5 hours

5. When firewood was selling at Rs. 40 per ton, I bought firewood worth Rs. 78.75 to last for 6 months. What is the value of the quantity necessary for 2 months when a ton of firewood sells at Rs. 48?
1) Rs. 31.50 2) Rs. 63 3) Rs. 13.125 4) RS. 35.50

6. A, Band C together can do a piece of work in 10 days; B and C together work thrice as much as A and A and B together work 4 times as much as C. In what time can each do it alone (in days)?
1) 45, 22, 52 2) 40,18,50 3) 40, 200/11, 50 4) 30, 200/11, 55

7. A, Band C individually can finish a work in 6, 8 and 15 hours respectively. They started the work together and after completing the work got RS. 94.60. Find their individual shares (in Rs.)
1) 19,26,49.60 2) 44,33,17.60 3) 49,26,19.60 4) None of these

8. A can do a piece of work in 24 days and B in 30 days. A worked for 6 days and then B also joined him. In how many days will the whole work be completed?
1) 12 days 2) 14 days 3) 15 days 4) 16 days

9. If x bulbs consume x^2 units of electricity in x^3 days, then find the time in which y bulbs will consume y units of electricity. ($x > y$)
1) x^1 days 2) x^2 days 3) x^3 days 4) x^0 days

10. If a family of 9 men in Indore spends RS. 16380 in a year, what must be the expenses of 8 men in Calcutta who live in the same style for 7 months, assuming that the prices at Calcutta are 3/5th of what they are in Indore?

1) Rs. 5096 2) Rs. 5460 3) RS. 4879 4)Rs. 5224

11. 25 men were employed to do a piece of work in 24 days. After 15 days, 10 more men were engaged and the work was finished a day too soon. In what time could they finish the work if extra men were not employed?
1) 29.5 days 2) 24.6 days 3) 26.2 days 4) 21.7 days

12. If 12 men or 18 women can reap a field in 7 days, then in what time can 4 men and 8 women reap the same field?
1) 4 days 2) 8 days 3) 7 days 4) 9 days

13. 70 cattle can graze a piece of land for 28 days. How many cattle can graze a field three times as large in 70 days?
1) 168 2) 84 3) 64 4) 252

14. A, Band C can do a piece of work in 12, 18 and 24 days respectively. They work at it together, A stops the workafter 4 days & B is called off 2 days before the work is finished. In how many days was the work finished?
1) 7 days 2) 11 days 3) 8 days 4) 9 days

15. 10 horses and 15 cows eat grass of 5 acres in a certain time. How many acres will feed 15 horses and 10 cows for the same time, supposing a horse eats as much as 2 cows?
1) 40/7 acres 2) 39/8 acres 3) 40/11acres4) 25/9 acres

16. If 15 boys can finish a piece of work in 12 days of 8 hours a day, then how long will it take for 16 boys to do a piece ofwork 4/6 as great, working 9 hours a day?
1) 3.33 days 2) 6.66 days 3) 10 days 4) 15 days

17. 6 men and 5 boys can do a piece of work in 7 days. They work at it till they have-completed 3/4 of it. Then two of the men leave and two more boys come in. How long will thework take to complete if a boy does half as much work as aman?
1) 213/30 days 2) 198/31 days 3) 211/25 days 4) 217/30 days

18. 3 men or 7 women can do a piece of work in 32 days Find the number of days required by 5 men and 7 women to do apiece of work twice as large.
1) 29 days 2) 31 days 3) 24 days 4) 19 days

19. 36 workmen are employed to finish a certain work in 48 days. But it is found that in 24 days only 1/3 of the work was done How many more men must be taken in order to do the piece of work in time?
1) 16 men 2) 18 men 3) 20 men 4) 12 men

20. 8 men with 2 boys can do a piece of work in 7 days, which 25 men with 13 boys can do in 2 days. What fraction of the whole work, can be done by 11 men with 20 boys in 1 day?
    1) 1/2 2) 1/3 3) 1/4 4) 1/5

21. If 7 men or 8 women or 10 boys can finish a piece of work in 24 days each of 9' hours, then find how many men with the help of 4 women and 5 boys can finish the work in 18 days each of 6 hours?
    1) 10 men 2) 12 men 3) 7 men 4) 6 men

22. Pipes A and B can fill a cistern in 10 and 12 hours respectively and pipe C can empty it in 6 hours. If all the three are simultaneously opened, then the time required for the tankto be full is
    1) 20 hours 2) 60 hours 3) 80 hours 4) 40 hours

23. A water tub can be filled by two taps in 8 min. One tap is closed after 3 min; the other tap fills the remaining tub in 15min. How much time the faster tap will take to fill the tub?
    1) 10 min 2) 11 min 3) 12 min 4) 15 min

24. An electric pump can fill a tank in 3 hours. Because of a leak at the bottom of tank it is taking 3.5 hours to fill the tank. Find the time the leak can drain all the water of the tank when it is full.
    1) 15 hours 2) 21 hours 3) 19 hours 4) 29 hours

Answers:

1. 3	**2.** 2	**3.** 1	**4.** 2	**5.** 1
6. 3	**7.** 2	**8.** 4	**9.** 2	**10.** 1
11. 3	**12.** 4	**13.** 2	**14.** 3	**15.** 1
16. 2	**17.** 4	**18.** 3	**19.** 2	**20.** 2
21. 3	**22.** 2	**23.** 3	**24.** 2	