

## Algebra

### Important Formulae

$$1. (a+b)^2 = a^2 + 2ab + b^2$$

$$2. (a-b)^2 = a^2 - 2ab + b^2$$

$$3. (a+b)^2 = (a-b)^2 + 4ab$$

$$4. (a-b)^2 = (a+b)^2 - 4ab$$

$$5. (a+b)^3 = a^3 + b^3 + 3ab(a+b) = a^3 + b^3 + 3a^2b + 3ab^2$$

$$6. (a-b)^3 = a^3 - b^3 - 3ab(a-b) = a^3 - b^3 - 3a^2b + 3ab^2$$

$$7. a^3 + b^3 = (a+b)^3 - 3ab(a+b)$$

$$8. a^3 - b^3 = (a-b)^3 + 3ab(a-b)$$

$$9. a^2 - b^2 = (a-b)(a+b)$$

$$10. a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$11. a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$12. a^m \times a^n = a^{m+n}$$

13.  $a^m / a^n = a^{m-n}$

14.  $(a/b)^{(m/n)} = (b/a)^{-(m/n)}$

15.  $a^m / b^{-n} = a^m \times b^n$

1. If  $x + y + z = 6$  and  $x^2 + y^2 + z^2 = 20$ , then find the value of  $x^3 + y^3 + z^3 - 3xyz$ 

- a) 64                      b) 70                      c) 72                      d) 76

2. If  $a^2 + b^2 + c^2 = 2(a - b - c) - 3$ , then the value of  $2a - 3b + 4c$  is \_

- a) 3                      b) 1                      c) 2                      d) 4

3. If  $a^2 + b^2 + 4c^2 = 2(a + b - 2c) - 3$  and  $a, b, c$  are real, then the value of  $(a^2 + b^2 + c^2)$  is

- a) 3                      b)
- $3\frac{1}{4}$
- c) 2                      d)
- $2\frac{1}{4}$

4. If  $x^2 = y + z$ ,  $y^2 = z + x$  and  $z^2 = x + y$ , then the value of  $\frac{1}{1+x} + \frac{1}{1+y} + \frac{1}{1+z}$  is

- a) -1                      b) 1                      c) 2                      d) 0

5. If  $\frac{a}{1-a} + \frac{b}{1-b} + \frac{c}{1-c} = 1$ , then the value of  $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c}$  is -

- a) 1                      b) 3                      c) 4                      d) 0

6. If  $\frac{4x-3}{x} + \frac{4y-3}{y} + \frac{4z-3}{z} = 0$ , then the value of  $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$  is

- a) 9                      b) 3                      c) 4                      d) 6

7. If  $x^2 - 3x + 1 = 0$ , then the value of  $x^2 + x + \frac{1}{x} + \frac{1}{x^2}$  is

- a) 10                      b) 2                      c) 6                      d) 8

8. If  $5a + \frac{1}{3a} = 5$ , then the value of  $9a^2 + \frac{1}{25a^2}$  is \_



- a)  $34/5$                       b)  $39/5$                       c)  $42/2$                       d)  $52/2$

9. If  $x + \frac{1}{x} = 7$ , find  $x^4 + \frac{1}{x^4}$



- a) 2207                      b) 2209                      c) 2723                      d) 2203

10. If  $x + \frac{1}{x} = 2$ , find  $x^8 + \frac{1}{x^8}$



- a) 1                      b) 0                      c) 2                      d) 3

11. If  $2x - \frac{1}{2x} = 6$ , then the value of  $x^2 + \frac{1}{16x^2}$  is \_



- a)  $19/2$                       b)  $17/2$                       c)  $18/3$                       d)  $15/2$

12. If  $a^2 + \frac{1}{a^2} = 98$  ( $a > 0$ ), then the value of  $a^3 + \frac{1}{a^3}$  will be \_



- a) 535                      b) 1030                      c) 790                      d) 970

13. If  $x + \frac{1}{x} = 5$ , then the value of  $\frac{x^4 + \frac{1}{x^2}}{x^2 - 3x + 1}$  is \_



- a) 70                      b) 50                      c) 110                      d) 55

14. If  $x = 2 + \sqrt{3}$ ,  $y = 2 - \sqrt{3}$ , then the value of  $\frac{x^2 + y^2}{x^3 + y^3}$  is \_



- a)  $7/38$                       b)  $7/40$                       c)  $7/19$                       d)  $7/26$

15. If  $x = 1 + \sqrt{2} + \sqrt{3}$  then the value of  $(2x^4 - 8x^3 - 5x^2 + 26x - 28)$  is \_



- a)  $6\sqrt{6}$                       b) 0                      c)  $3\sqrt{6}$                       d)  $2\sqrt{6}$

16. What number must be added to the expression  $16a^2 - 12a$  to make it a perfect square?



- a)  $\frac{9}{4}$                       b)  $\frac{11}{2}$                       c)  $\frac{13}{2}$                       d) 16

17. If  $(n^r - tn + \frac{1}{4})$  be a perfect square, then the values of t are



- a)  $\pm 2$                       b) 1, 2                      c) 2, 3                      d)  $\pm 1$

18. Find the minimum value of  $(x - 2)(x - 9)$



- a)  $-\frac{11}{4}$                       b)  $\frac{49}{4}$                       c) 0                      d)  $-\frac{49}{4}$

19. If  $4x = 18y$ , then the value of  $(\frac{x}{y} - 1)$  is \_

- a)  $\frac{1}{3}$                       b)  $\frac{7}{2}$                       c)  $\frac{2}{3}$                       d)  $\frac{3}{2}$

20. If  $(x - c)^2 + (y - 5)^2 + (z - d)^2 = 0$  then the value of  $\frac{x^2}{9} + \frac{y^2}{25} + \frac{z^2}{16}$  is -

- a) 12                      b) 9                      c) 3                      d) 1

21. If  $\frac{4x}{3} + 2P = 12$  for what value of P,  $x = 6$ ?

- a) 6                      b) 4                      c) 2                      d) 1

22. The value of  $\frac{4 + 3\sqrt{3}}{7 + 4\sqrt{3}}$  is -

- a)  $5\sqrt{3} - 8$                       b)  $5\sqrt{3} + 8$                       c)  $8\sqrt{3} + 5$                       d)  $8\sqrt{3} - 5$

23. If  $x(3 - \frac{2}{x}) = \frac{3}{x}$ , then the value of  $x^2 + \frac{1}{x^2}$  is -

- a)  $2\frac{1}{9}$                       b)  $2\frac{4}{9}$                       c)  $3\frac{1}{9}$                       d)  $3\frac{4}{9}$

24. If  $(\frac{3}{4})^3 \times (\frac{4}{3})^{-7} = (\frac{3}{4})^{2x}$  then x is

- a) -2                      b) 2                      c) 5                      d)  $2\frac{1}{2}$

25. If  $p - 2q = 4$ , then the value of  $p^3 - 8q^3 - 24pq - 64$  is

- a) 2                      b) 0                      c) 3                      d) -1

26. If  $\frac{x}{a} = \frac{1}{a} - \frac{1}{x}$ , then the value of  $x - x^2$  is

- a)  $-a$                       b)  $\frac{1}{a}$                       c)  $-\frac{1}{a}$                       d)  $a$

27. If  $(x + \frac{1}{x}) = 4$ , then the value of  $x^4 + \frac{1}{x^4}$  is

- a) 64                      b) 194                      c) 81                      d) 124

28. If  $\frac{x}{x^2 - 2x + 1} = \frac{1}{3}$ , then the value of  $x^3 + \frac{1}{x^3}$  is

- a) 81                      b) 110                      c) 125                      d) 27

29. If  $\frac{4 + 3\sqrt{3}}{2 + \sqrt{3}} = A + \sqrt{B}$ , then  $B - A$  is

- a) -13                      b)  $2\sqrt{13}$                       c) 13                      d)  $3\sqrt{3} - \sqrt{7}$

30. If the expression  $x^2 + x + 1$  is written in the form  $(x + \frac{1}{2})^2 + q^2$ , then the possible values of  $q$  are

- a)  $\pm \frac{1}{3}$                       b)  $\pm \frac{\sqrt{3}}{2}$                       c)  $\pm \frac{2}{\sqrt{3}}$                       d)  $\pm \frac{1}{2}$

31. If  $a^2 - 4a - 1 = 0$ , then value of  $a^2 + \frac{1}{a^2} + 3a - \frac{3}{a}$  is

- a) 25                      b) 30                      c) 35                      d) 40

32. If  $x = \sqrt[3]{a + \sqrt{a^2 + b^3}} + \sqrt[3]{a - \sqrt{a^2 + b^3}}$ , then  $x^3 + 3bx$  is equal to

- a) 0                      b)  $a$                       c)  $2a$                       d) 1

33. If  $\frac{1}{\sqrt[3]{4} + \sqrt[3]{2} + 1} = a\sqrt[3]{4} + b\sqrt[3]{2} + c$  and  $a, b, c$  are rational numbers, then  $a + b + c$  is equal to

- a) 0                      b) 1                      c) 2                      d) 3

34.  $\frac{1}{1 + 2^{a-b}} + \frac{1}{1 + 2^{b-a}}$  is

- a)  $a-b$                       b)  $b-a$                       c) 1                      d) 0

35. If  $\frac{a}{b} = \frac{4}{5}$  and  $\frac{b}{c} = \frac{15}{16}$ , then  $\frac{18c^2 - 7a^2}{45c^2 + 20a^2}$  is equal to


- a)  $\frac{1}{3}$                       b)  $\frac{2}{5}$                       c)  $\frac{3}{4}$                       d)  $\frac{1}{4}$

### Answers:


|        |        |        |        |        |        |        |        |        |        |
|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 1 - c  | 2 - b  | 3 - d  | 4 - b  | 5 - c  | 6 - c  | 7 - a  | 8 - b  | 9 - a  | 10 - c |
| 11 - a | 12 - d | 13 - d | 14 - d | 15 - a | 16 - a | 17 - d | 18 - d | 19 - b | 20 - c |
| 21 - c | 22 - a | 23 - b | 24 - c | 25 - b | 26 - d | 27 - b | 28 - b | 29 - c | 30 - b |
| 31 - b | 32 - c | 33 - a | 34 - c | 35 - d |        |        |        |        |        |

### Additional Examples


1. If  $x = 2 - 2^{1/3} + 2^{2/3}$ , then the value of  $x^3 - 6x^2 + 18x + 18$  is

-  a) 22                      b) 33                      c) 40                      d) 45


2. If  $x = \frac{4ab}{a+b}$  ( $a \neq b$ ) the value of  $\frac{x+2a}{x-2a} + \frac{x+2b}{x-2b}$  is

-  a)  $a$                       b)  $b$                       c)  $2ab$                       d) 2

3. If  $x = b + c - 2a$ ,  $y = c + a - 2b$ ,  $z = a + b - 2c$ , then the value of  $x^2 + y^2 - z^2 + 2xy$  is

-  a) 0                      b)  $a+b+c$                       c)  $a-b+c$                       d)  $a+b-c$


4. If  $a^2 + b^2 = 2$  and  $c^2 + d^2 = 1$ , then the value of  $(ad - bc)^2 + (ac + bd)^2$  is

-  a)  $\frac{4}{9}$                       b)  $\frac{1}{2}$                       c) 1                      d) 2


5. If  $x + \frac{1}{x} = 5$ , then  $\frac{2x}{3x^2 - 5x + 3}$  is equal to

- a) 5                      b)  $\frac{1}{5}$                       c) 3                      d)  $\frac{1}{3}$

6. If  $x - y = \frac{x+y}{7} = \frac{xy}{4}$ , the numerical value of  $xy$  is

-  a)  $\frac{4}{3}$                       b)  $\frac{3}{4}$                       c)  $\frac{1}{4}$                       d)  $\frac{1}{3}$

7. If  $3x + \frac{1}{2x} = 5$ , then the value of  $8x^3 + \frac{1}{27x^3}$  is:


-  a)  $118\frac{1}{2}$                       b)  $30\frac{10}{27}$                       c) 0                      d) 1

8. Two numbers  $x$  and  $y$  ( $x > y$ ) are such that their sum is equal to three times their difference.

The value of  $(\frac{3xy}{2(x^2 - y^2)})$  will be:

- a)  $\frac{2}{3}$                       b) 1                      c)  $1\frac{1}{2}$                       d)  $1\frac{2}{3}$

9. If  $p = 99$ , then value of  $p(p^2 + 3p + 3)$  is

-  a) 999                      b) 9999                      c) 99999                      d) 999999


10. If  $p = 101$ , then the value of  $\sqrt[3]{p(p^2 - 3p + 3)} - 1$  is

-  a) 100                      b) 101                      c) 102                      d) 1000


11. If  $a = \sqrt{7 + 2\sqrt{12}}$  and  $b = \sqrt{7 - 2\sqrt{12}}$ , then  $(a^3 + b^3)$  is equal to

- a) 40                      b) 44                      c) 48                      d) 52


12. If  $\frac{\sqrt{x+4} + \sqrt{x-4}}{\sqrt{x+4} - \sqrt{x-4}} = 2$  then  $x$  is equal to

-  a) 2.4                      b) 3.2                      c) 4                      d) 5

13. If  $1.5x = 0.04y$ , then the value of  $\frac{y^2 - x^2}{y^2 + 2xy + x^2}$  is

-  a)  $\frac{730}{77}$       b)  $\frac{73}{77}$       c)  $\frac{73}{770}$       d)  $\frac{74}{77}$


14. The value of the expression  $x^4 - 17x^3 + 17x^2 - 17x + 17$  at  $x = 16$  is

-  a) 0      b) 1      c) 2      d) 3

15. If  $x = \frac{\sqrt[3]{m+1} + \sqrt[3]{m-1}}{\sqrt[3]{m+1} - \sqrt[3]{m-1}}$  value of  $x^3 - 3mx^2 + 3x - m$  is

- a) 0      b)  $m - \frac{1}{m}$       c)  $m + \frac{1}{m}$       d) 1


16. The simplest form of the expression  $\frac{p^2 - p}{2p^3 + 6p^2} \div \frac{p^2 - 1}{p^2 + 3p} \div \frac{p^2}{p + 1}$  is

-  a)  $2p^2$       b)  $\frac{1}{2p^2}$       c)  $p + 3$       d)  $\frac{1}{p + 3}$


17. If  $a + b + c = 4\sqrt{3}$  and  $a^2 + b^2 + c^2 = 16$ , then the ratio  $a:b:c$  is

- a) 1:1:1      b)  $1:\sqrt{2}:\sqrt{3}$       c) 1:2:3      d) None

18. If  $2^x = 4^y = 8^z$  and  $xyz = 288$ , the value of  $\frac{1}{2x} + \frac{1}{4y} + \frac{1}{8z}$  is

-  a)  $\frac{11}{12}$       b)  $\frac{11}{96}$       c)  $\frac{29}{96}$       d)  $\frac{27}{96}$

19. If  $x = \sqrt{a^3 \sqrt{b \sqrt{a^3 \sqrt{b}}}} \dots \infty$ , then the value of  $x$  is

-  a)  $\sqrt[3]{a^3 b}$       b)  $\sqrt[5]{a^3 b}$       c)  $\sqrt[3]{a^5 b}$       d)  $\sqrt[5]{a b^2}$

20. If  $\sqrt{\frac{x-a}{x-b}} + \frac{a}{x} = \sqrt{\frac{x-b}{x-a}} + \frac{b}{x}$ ,  $b \neq a$ , then the value of  $x$  is

- a)  $\frac{ab}{a+b}$       b) 1      c)  $\frac{a}{a+b}$       d)  $\frac{b}{a+b}$



21. If  $x = \frac{2\sqrt{24}}{\sqrt{3}+\sqrt{2}}$ , then the value of  $\frac{x+\sqrt{8}}{x-\sqrt{8}} + \frac{x+\sqrt{12}}{x-\sqrt{12}}$  is



- a) 2                      b) 0                      c) -2                      d) 1

22. If  $x = 1 + \sqrt{2} + \sqrt{3}$ , then the value of  $(2x^4 - 8x^3 - 5x^2 + 26x - 28)$  is

- a)  $6\sqrt{6}$                       b) 0                      c)  $3\sqrt{6}$                       d)  $2\sqrt{6}$

23. If  $\frac{m-a^2}{b^2+c^2} + \frac{m-b^2}{c^2+a^2} + \frac{m-c^2}{a^2+b^2} = 3$ , then the value of m is

- a)  $a^2 + b^2 + c^2$                       b)  $a^2 - b^2 - c^2$   
c)  $a^2 + b^2 - c^2$                       d)  $a^2 + b^2$

24. If  $m - 5n = 2$ , then the value of  $(m^3 - 125n^3 - 30mn)$  is

- a) 9                      b) 7                      c) 6                      d) 8

25. If  $x = \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$  and  $y = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ , then the value of  $\frac{x^2+xy+y^2}{x^2-xy+y^2} = ?$

- a)  $\frac{67}{65}$                       b)  $\frac{69}{67}$                       c)  $\frac{65}{63}$                       d)  $\frac{63}{61}$

### Answers:

|        |        |        |        |        |        |        |        |        |        |
|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 1 - c  | 2 - d  | 3 - a  | 4 - d  | 5 - b  | 6 - a  | 7 - b  | 8 - b  | 9 - d  | 10 - a |
| 11 - d | 12 - d | 13 - b | 14 - b | 15 - a | 16 - b | 17 - a | 18 - b | 19 - b | 20 - a |
| 21 - a | 22 - a | 23 - a | 24 - d | 25 - d |        |        |        |        |        |