# 12. Probability

## Experiment

An operation which results in some well-defined outcomes is called an experiment.

## Random Experiment

An experiment whose outcome cannot be predicted with certainty is called a random experiment. In other words, if an experiment is performed many times under similar conditions and the outcome of each time is not the same, then this experiment is called a random experiment.

**Example:**

- Tossing of a fair coin
- Throwing of an unbiased die
- Drawing of a card from a well shuffled pack of 52 playing cards

## Sample Space

The set of all possible outcomes of a random experiment is called the sample space for that experiment. It is usually denoted by S.

**Example:**

- When a die is thrown, any one of the numbers 1, 2, 3, 4, 5, 6 can come up. Therefore. Sample space S = {1, 2, 3, 4, 5, 6}
- When a coin is tossed either a head or tail will come up, then the sample space w.r.t. the tossing of the coin is S = {H, T}
- When two coins are tossed, then the sample space is {H,T,H,T}

**Sample point / Event point:** Each element of the sample spaces is called a sample point or an event point.

**Example:** When a die is thrown, the sample space is S = {1, 2, 3, 4, 5, 6} where 1, 2, 3, 4, 5 and 6 are the sample points.

**Discrete Sample Space:** A sample space S is called a discrete sample if S is a finite set.

## Event

A subset of the sample space is called an event.

## Problem of Events:

- Sample space S plays the same role as universal set for all problems related to the particular experiment.
- $\phi$ is also the subset of S and is an impossible Event.
- S is also a subset of S which is called a sure event or a certain event.

**Types of Events**

**Simple Event/Elementary Event:** An event is called a simple Event if it is a singleton subset of the sample space S.

**Example:**

- When a coin is tossed, then the sample space is
  S = {H, T}
  Then A = {H} occurrence of head and B = {T} occurrence of tail are called Simple events.
- When two coins are tossed, then the sample space is
  S = {(H,H); (H,T); (T,H); (T,T)}
  Then A = {(H,T)} is the occurrence of head on $1^{st}$ and tail on $2^{nd}$ is called a Simple event.

**Mixed Event or Compound Event or Composite Event:** A subset of the sample space S which contains more than one element is called a mixed event or when two or more events occur together, their joint occurrence is called a Compound Event.

**Example:** When a dice is thrown, then the sample space is
S = {1, 2, 3, 4, 5, 6}
Then let A = {2, 4 6} is the event of occurrence of even and B = {1, 2, 4} is the event of occurrence of exponent of 2 are Mixed events

Compound events are of two type:

a) Independent Events, and
b) Dependent Events

**Equally likely events:** Outcomes are said to be equally likely when we have no reason to believe that one is more likely to occur than the other.

**Example:** When an unbiased die is thrown all the six faces 1, 2, 3, 4, 5, 6 are equally likely to come up.

**Exhaustive Events:** A set of events is said to be exhaustive if one of them must necessarily happen every time the experiments is performed.

**Example:** When a die is thrown events 1, 2, 3, 4, 5, 6 form an exhaustive set of events.

**Note:** We can say that the total number of elementary events of a random experiment is called the exhaustive number of cases.

**Mutually Exclusive Events:** Two or more events are said to be mutually exclusive if one of them occurs, others cannot occur. Thus if two or more events are said to be mutually exclusive, if

not two of them can occur together. Hence, $A_1$, $A_2$, $A_3$,…, An are mutually exclusive if and only if $A_i \cap A_j = \phi \forall i \neq j$

**Example:**

- When a coin is tossed the event of occurrence of a head and the event of occurrence of a tail are mutually exclusive events because we cannot have both head and tail at the same time.
- When a die is thrown, the sample space is S = {1, 2, 3, 4, 5, 6}
  Let,
  A is an event of occurrence of number greater than 4 i.e., {5, 6}
  B is an event of occurrence of an odd number {1, 3, 5}
  C is an event of occurrence of an even number {2, 4, 6}
  Here, events B and C are Mutually Exclusive but the event A and B or A and C are not Mutually Exclusive.

**Independent Events or Mutually Independent events:** Two or more event are said to be independent if occurrence or non-occurrence of any of them does not affect the probability of occurrence of or non-occurrence of their events. Thus, two or more events are said to be independent if occurrence or non-occurrence of any of them does not influence the occurrence or non-occurrence of the other events.

**Example:** Let bag contains 3 Red and 2 Black balls. Two balls are drawn one by one with replacement.

Let,

- A is the event of occurrence of a red ball in first draw.
- B is the event of occurrence of a black ball in second draw.

Then probability of occurrence of B has not been affected if A occurs before B. As the ball has been replaced in the bag and once again we have to select one ball out of 5(3R + 2B) given balls for event B.

**Dependent Events:** Two or more events are said to be dependent, if occurrence or non-occurrence of any one of them affects the probability of occurrence or non-occurrence of others.

**Example:** Let a bag contains 3 Red and 2 Black balls. Two balls are drawn one by one without replacement.

Let,

- A is the event of occurrence of a red ball in first draw
- B is the event of occurrence of a black ball in second draw.

In this case, the probability of occurrence of event B will be affected. Because after the occurrence of event A i.e. drawing red ball out of 5(3R + 2B), the ball is not replaced in bag. Now, for the event B, we will have to draw 1 black ball from the remaining 4(2R + 2B) balls which gets affected due to the occurrence of event A.

**Complementary Events:** Let S be the sample space for a random experiment and let E be the event. Also, Complement of event E is denoted by E' or $\bar{E}$, where E' means non occurrence of event E. Thus E' occurs if and only if E does not occur.

➔ $n(E) + n(E') = n(S)$

**Occurrence of an Event:** For a random experiment, let E be an event

Let E = {a, b, c}. If the outcome of the experiment is either a or b or c then we say the event has occurred.

- Sample Space: The outcomes of any type
- Event : The outcomes of particular type

**Probability of Occurrence of an event:** Let S be the same space, then the probability of occurrence of an event E is denoted by p(E) and is defined as

- P(E) = n(E)/n(S) = number of elements in E/number of elements in S
- P(E) = $\frac{number\ \ of\ favourable\ or\ particular\ \ cases}{Total\ number\ \ of\ cases}$

**Example:**

- When a coin is tossed, then the sample space is S = {H, T}
  Let E is the event of occurrence of a head

  ⇒ E = {H}
- When a die is tossed, sample space S = {1, 2, 3, 4, 5, 6}
  Let A is an event of occurrence of an odd number. And B is an event of occurrence of a number greater than 4

  ⇒ A = {1, 3, 5} and B = {5, 6}

➔ P(A) = Probability of occurrence of an odd number = n(A)/n(S)
= 3/6 = ½

➔ P(B) = Probability of occurrence of a number greater than 4 = n(B)/n(S)
= 2/6 = 1/3

## Practice Exercise

**DIRECTION:** For the following questions-four options are given. Choose the best

1. In rolling two dice, find the probability that the sum is 5.
   1) 1/9 2) 2/9 3) 4 / 9 4) 3/9

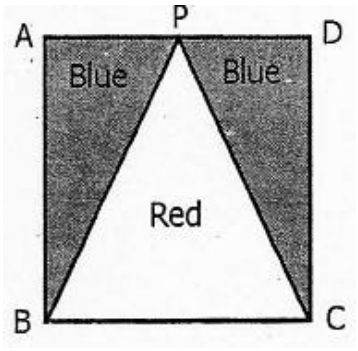
2. The probability that a man will be alive for 25 years is envelopes 3/5 and the probability that his wife will be alive for 25 years is 2/3. Find the probability that only the man will be alive for years.
   1) 2/5 2) 1/5 3) 3/5 4) 4/5

3. In the above question, what is the probability that at least one will b alive for 25 years?
   1) 3/15 2) 11/15 3) 13/15 4) 1/15

4. Five persons entered the lift cabin on the ground floor of an 8-floor house. Suppose that each of them independently and with equal probability can leave the cabin at any floor beginning with the first, find out the probability of all five persons leaving at different floors.
   1) $1/7^5$ 2) $7_{C_5}/\ 7^5$ 3) $3/7^5$ 4) $7_{P_s}/7^5$

5. There are 5 balls in a bag. One ball is picked and it is found to be red. What is the probability that all the balls in the bag are red?
   1) 0.3 2) 0.2 3) 0.1 4) 0.5

6. A dart board is square in shape and it is painted as shown. What is the probability of hitting somewhere in the area painted Red?
   1) 1/2 2) 1/3 3) 1 4) ¼




7. What is the probability that the three cards drawn from a pack of 52 cards are all red?
   1) 1/17 2) 2/17 3) 3/17 4) 4/17

8. There are 10 envelopes and 10 letters to go inside them. Each letter is meant for a specified envelope only. What is the probability that exactly 9 of them are in the right envelopes?

1) 1/10! 2) 1 3) 0 4) None of these

9. A point is selected at random from the interior of a circle. Find the probability that the point is closer to the centre than the boundary of the circle.
   1) 1 2) 1/2 3) 1/3 4) 1/4

10. There are four letters and four addressed envelopes. What is the probability that all letters are not placed in the right envelopes?
    1) 1/24 2) 1/6 3) 23/24 4) 5/24

**Directions (Q.13-17): Study the given information carefully and answer the questions that follow**:

An urn contains 6 red, 4 blue, 2 green and 3 yellow marbles.

11. If four marbles are picked at random, what is the probability that at least one is blue?
    1) $\frac{4}{15}$ 2) $\frac{69}{91}$ 3) $\frac{11}{15}$ 4) $\frac{22}{91}$ 5) None of these

12. If two marbles are picked at random, what is the probability that both are red?
    1) $\frac{1}{6}$ 2) $\frac{1}{3}$ 3) $\frac{2}{15}$ 4) $\frac{2}{5}$ 5) None of these

13. If three marbles are picked at random, what is the probability that two are blue and one is yellow?
    1) $\frac{3}{91}$ 2) $\frac{1}{5}$ 3) $\frac{18}{455}$ 4) $\frac{7}{15}$ 5) None of these

14. If four marbles are picked at random, what is the probability that one is green, two are blue and one is red?
    1) $\frac{24}{455}$ 2) $\frac{13}{35}$ 3) $\frac{11}{15}$ 4) $\frac{7}{91}$ 5) None of these

15. If two marbles are picked at random, what is the probability that either both are green or both are yellow?
    1) $\frac{5}{91}$ 2) $\frac{1}{35}$ 3) $\frac{1}{3}$ 4) $\frac{4}{105}$ 5) None of these

**Answers:**

| | | | | |
|---|---|---|---|---|
| **1.** 1 | **2.** 2 | **3.** 3 | **4.** 4 | **5.** 2 |
| **6.** 1 | **7.** 2 | **8.** 3 | **9.** 4 | **10.** 3 |
| **11.** 2 | **12.** 4 | **13.** 3 | **14.** 1 | **15.** 4 |