3. Percentage

The term percentage means parts per 100 or "for every hundred". Thus, when we say a man made a profit of 20 percent we mean to say that he gained Rs. 20 for every hundred rupees he invested in the business, i.e., 20/100 rupees for each Rupee.

The abbreviation of percent is p.c. and it is generally denoted by %.

1. A Percentage can be expressed as a Fraction.

10% can be expressed as 10/100 or 1/10.

To express a percentage as a fraction divide it by 100 a% = a/100.

Example. Express the following as fraction

a.25% b. $33\frac{1}{3}\%$

Sol. a. 25% = $\frac{25}{100}$ (since % means $\frac{1}{100}$) = $\frac{1}{4}$ b. $33\frac{1}{3}\% = \frac{100}{3}\% = \frac{100}{3 \times 100} = \frac{1}{3}$

2. To express a fraction as a percent multiply it by 100.

➔ $\frac{a}{b}$ = [($\frac{a}{b}$)× 100]%

Example. Express 1/8 as a percentage.

Sol. $\frac{1}{8} = \frac{1}{8} \times \frac{100}{8}\% = \frac{25}{2}\% = 12\frac{1}{2}\%$

3. A Percentage can be expressed as a Decimal. To express percentage as a decimal we remove the symbol % and shift the decimal point by two places to the left. For example, 10% can be expressed as 0.1.

Example. Express $6\frac{1}{2}\%$ as a decimal.

Sol. $6\frac{1}{2}\% = \frac{13}{2}\% = 6.5\% = \frac{6.5}{100} = 0.065$

4. To express decimal as a percentage we shift the decimal point by two places to the right and write the number obtained with the symbol % or simply we multiply the decimal with 100.

Example. Express 0.7 as a percentage.

Sol. 0.7 = 0.7 × 100% = (7/10) × 100 = 70%

5. If x is R% of a given number N, then N **=** $\frac{X \times 100}{R}$

Example. 25% of a number is 60. What is the number?

Sol. Let the number be x. According to the given condition

$\frac{25}{100} \times x = 60 \rightarrow x = \frac{60 \times 100}{25} = 240$

6. If A's income is r % more than that of B, then B's income is $\frac{r\ \%}{(100+r)\ \%} \times 100$ less than that of A.

Exapmle. The income of A is 50% more than that of B. Then B's income is less than A by what percent?

Sol. Let income of B be Rs. 100. Hence, income of A is Rs. 150. B's income is Rs. 50 less than that of A,

In percentage, $= \left(-\frac{\mathbf{50}}{\mathbf{150}}\right) \times 100 = -33.33\%$. (-ve sign shows that income of B is less than A).

Hence, B's income is 33.33% less than that of A.

7. If A's income is r % less than that of B, then B's income is $\frac{r\%}{(100-r\ )\%} \times 100$ more than that of A.

Example. If A's income is 20% less than that of B, then B's income is how much percent more than that of A?

Sol. Let income of B be Rs. 100. Hence, income of A is Rs. 80. B's income is Rs. 20 more than that of A. In percentage, $= \frac{20}{80} \times 100 = 25\%$

Hence, B's income is 25% more than that of A.

8. An increase of, say 25%, means that for each 100 units in the original value, there is an increase of 25 units, making the new value 125 units. Therefore, Increase % $= \frac{\text{increase}}{\text{originalvalue}} \times 100$

If x is increased by r% Then new value = x + r% of x

Also, New value = Original Value × (1 + Increase).

The increase given is not to be taken in percentage, it should be taken as a fraction.

Example. The present population of a city is 14,00,000. The population of the city 3 years ago was 10,00,000. What is the percentage increase in the population of the city over the given period?

Sol. Applying the formula given above, we get

Percentage Increase $= \frac{(1400000-1000000)}{1000000} \times 100 = \frac{300000}{1000000} \times 100 = 30\ \%$

9. A decrease of, say 25%, means that for each 100 units in the original value, there is a decrease of 25 units, making the new value 75 units. Therefore,

Decrease% $= \frac{Decrease}{origin\ alValue} \times 100$

If x is decreased by r% Then new value = x - r% x

Also, New value = Original Value × (1 - Decrease)

The decrease given is not to be taken in percentage; it should be taken as a fraction.

Example. The number of books published by a group dropped from 375000 units to 250000 units. What is the percentage decrease in the number of books published by the group?

Sol. Applying the formula given above, we get

Percentage decrease = $\frac{(375000-250000)}{375000} \times 100 = \frac{125000}{375000} \times 100 = 33.3\%$.

10. A change of, say + 25% (implies an increase) or -25% (implies a decrease), means that for each 100 units in the original value, there is a change of $\pm$ 25 units, making the new value 125 or 75 units respectively. Therefore, change% = $\frac{change}{originalvalue} \times 100$

Hence the ratio of the new value to the original value = 125:100 or 75:100.
Also, change should be taken positive if it is an increase and negative if it is a decrease

Example. A company manufacturing television sets observed that over the last three years there was an increase in the number of units sold in the local market. It was found that in the year 2000 the company was able to sell 90,000 units as against only 55,000 units sold by them in the year 1997. This increase could be mostly attributed to the decrease in the price per television set from Rs. 15,000 to Rs. 12,000 over the same period. What has been the percentage change in the price per television set and the number of units sold by the company

Sol. The percentage change in the PRICE of the television set and second, the percentage change in the NUMBER of units sold by the company have to be found

Answer I: percentage change in the price of the television set, Applying the formula, we get

Percentage change = $\frac{12000-15000}{15000} \times 100 = \frac{-3000}{15000} \times 100$ = -20%there is a decrease of 20% in the price of the television set.

Answer II: percentage change in the number of television sets sold, Applying the formula discussed above for percentage change we get

Percentage change = $\frac{9000-55000}{55000} \times 100 = \frac{35000}{55000} \times 100$ = 64% Hence, both the answers can be found.

11. The net percentage change when two variables are increased / decreased by given percentages, say a% and b% will be a + b + $\frac{a \times b}{100}$

Example: If the length of a rectangle is decreased by 40% and the breadth is increased by 30%, then find the percentage change in the area of the rectangle.

Sol. Area of rectangle = length x breadth Here, both length and breadth are changed.

So, using the formula,

➔ net percentage change in area = - 40 + 30 + $\frac{(-40)(30)}{100}$

The above formula can be used to find out the following

(1) Percentage effect on expenditure.

(2) Percentage effect on area of rectangle/square.

12. For fixed total expenditure

Price goes up by …%	Consumption comes down by …%
20	16.66
2	20
33.33	25
50	33.33
100	50
…	…
…	…

13. For fixed total expenditure

Price come down by …%	Consumption goes up by …%
20	25
25	33.33
33.33	50
50	100
75	300
…	…
…	…

Example: The decrease in the price of sugar in the market by 20% lead Satish to increase his consumption of sugar by so much that his total expenditure on sugar did not change. What was the percentage increase?

Sol. Let E be the expenditure, x be the original price of sugar and Y be the original consumption. Then E = xy... (1)

The new price of petrol is 20% lower than original value, hence new price = 0.8x. The expenditure should remain the same and so let us assume that the new consumption is N such that E = (0.8x) x N ... (2) Equating (1) and (2), we get P xy = (0.8x) x N

Thus, N = $1.25Y$ Thus, he should increase his consumption by 25%.

Points to remember

- $x\% = x/100$
- If a = m% then b is equal to $\frac{b}{a}$ x m%
- If a% is m then b% is equal to $\frac{b}{a}$ x m
- A% B = B% A
- x is what % of $y = \frac{X}{Y}$ x 100%
- x is what % more than $y = \frac{X-Y}{Y}$ x 100 %

Practice Exercise

1. If the price of 1kg of rice increased by 25%, the increase is Rs. 12. Find the new price of rice per Kg.
   1) Rs. 48 2) Rs. 60 3) Rs. 72 4) Rs. 36 5) None of these

2. If the price of a pencil is decreased by 16 2/3 %and the decrease is Rs. 3, find the new price of pencil.
   1) Rs. 18 2) Rs. 21 3) Rs. 24 4) Rs. 16 5) None of these

3. One type of liquid contains 14% of milk, the other contains 24% of milk. A can filled by the 5 parts of the first liquid and 5 parts of the second liquid. Find the percentage of milk in the new mixture.
   1) 19 2) 29 3) 20 4) 21 5) None of these

4. The daily wage is increased by 25% and a person now gets Rs. 25 per day. What was his daily wage before the increase?
   1) Rs. 22 2) Rs. 24 3) Rs. 21 4) Rs. 20 5) None of these

5. Two numbers are respectively 25% and 20% more than a third. What percentage is the first of the second?
   1) 104% 2) 104.16% 3) 104.26% 4) 105% 5) None of these

6. Two numbers are respectively 15% and 84% more than a third. What percentage is the first of the second?
   1) 64 ½ % 2) 65 ½% 3) 63 ½% 4) 62 ½% 5) None of these

7. Two numbers are respectively 28% and 25% less than a third number. What percent is the first of the second?
   1) 120% 2) 96% 3) 84% 4) 108% 5) None of these

8. A man spends 30% of his income in board and lodging, 25% of the remainder in other personal necessities and 20% of the rest in Charity. If his income is Rs. 25000, find the amount left by

him at the end.

1) Rs. 8500 2) Rs. 9500 3) Rs. 10500 4) Rs. 10000 5) None of these

9. A man loses 25% of his money and after spending 75% of the remainder, how much is he left with if initial money is Rs. 3200?

1) Rs. 800 2) Rs. 400 3) Rs. 900 4) Rs. 600 5) None of these

10. If the annual increase in the population of a town be 4% and the present population be 16224, what was the population two years ago?

1) 15000 2) 14000 3) 15500 4) 16000 5) None of these

11. The population of a town is 12500. It increases by 10% during the first year. During the second year, it decreases by 15% and increased by 20% during the third year. What is the population after 3 years?

1) 14025 2) 14625 3) 15025 4) 14035 5) None of these

12. The population of a town is 6250. It decreases by 10% during the first year, 20% during the second year and 30% during the third year. What will be the population after 3 years?

1) 3250 2) 3150 3) 3510 4) 3100 5) None of these

13. If the price of a commodity is raised by 12 ½%, find how much percent must a house holder reduce his consumption of that commodity, so as not to increase his expenditure.

1) 9 1/9% 2) 10 1/9% 3) 11 1/9% 4) 9 1/11% 5) None of these

14. If A's salary is 20% more than that of B, then how much percent is B's salary less than that of A?

1) 16 2/3% 2) 20% 3) 40% 4) 10% 5) None of these

15. If A's salary is 10% more than that of B, then how much percent of B's salary less than that of A?

1) 9 1/11% 2) 11 1/9% 3) 10% 4) 20% 5) None of these

16. A number is 50% more than the other. Then how much percent is the second number less than the first?

1) 50% 2) 25% 3) 33 2/3% 4) 33 1/3% 5) None of these

17. The salary of a worker is first increased by 15% and thereafter it was reduced by 5%. What was the change in his salary?

1. Increase in his salary, increase % is 0.25
2. Decrease in his salary, decrease % is 0.25
3. Increase in his salary, increase % is 4
4. Decrease in his salary, decrease % is 0.5

5. None of these

18. A shopkeeper marks the prices of his goods at 5% higher than the original price. Due to increase in demand he again increases by 5%. What profit did he get?
1) 10% 2) 10 ¼% 3) 5% 4) 12% 5) None of these

19. A shopkeeper marks the prices at 5% higher than the original price. Due to increase in demand, he further increases the price by 10%. How much percent profit will he get?
1) 15% 2) 15.25% 3) 15.5% 4) 16% 5) None of these

20. A student has to secure 30% marks to get through. If he gets 40 marks and fails by 20 marks, find the maximum marks set for the examination.
1) 600 2) 200 3) 100 4) 300 5) None of these

21. A student has to secure 16% marks to get through. If he gets 55 marks and fails by 25 marks, find the maximum marks set for the examination.
1) 400 2) 500 3) 550 4) 450 5) None of these

22. In a library 5% of the books are in Hindi, 10% of the remaining are in English and 15% of the remaining are in French. The remaining 5814 books are in regional languages. What is the total number of books in the library?
1) 8000 2) 8140 3) 6000 4) 8500 5) None of these

23. In a library 20% of the books are in Hindi, 25% of the remaining are in English and 30% of the remaining are in French. The remaining 29400 books are in regional languages. What is the total number of books in the library?
1) 35000 2) 70000 3) 45000 4) 90000 5) None of these

24. In a library 8% of the books are in Hindi, 12% of the remaining are in English and 72% of the remaining are in French. The remaining 3542 books are in regional languages. What is the total number of books in the library?
1) 16525 2) 15625 3) 12655 4) 16625 5) None of these

25. What quantity of water should be added to reduce 5 liters of 45% acidic liquid to 25% acidic liquid?
1) 3 Liters 2) 2 Liters 3) 4 Liters 4) 5 Liters 5) None of these

26. What quantity of water should be added to reduce 16 liters of 25% acidic liquid to 20% acidic liquid?
1) 5 Liters 2) 4 Liters 3) 12 Liters 4) 8 Liters 5) None of these

27. What quantity of water should be taken out to concentrate 12 liters of 30% acidic liquid to 40% acidic liquid.

1) 4 Liters 2) 6 Liters 3) 3 Liters 4) 8 Liters 5) None of these

28. What quantity of water should be taken out to concentrate 29 liters of 17% acidic liquid to 29% acidic liquid.

1) 12 Liters 2) 13 Liters 3) 12.5 Liters 4) 13.5 Liters 5) None of these

29. In 2 kg mixture of water and milk 30% is milk. How much water should be added so that the proportion of milk becomes 15%?

1) 4 kg 2)0.5 kg 3) 2 kg 4) 1 kg 5) None of these

Solutions

1. 2	**2.** 2	**3.** 1	**4.** 4	**5.** 2
6. 4	**7.** 2	**8.** 3	**9.** 4	**10.** 1
11. 1	**12.** 2	**13.** 3	**14.** 1	**15.** 1
16. 4	**17.** 2	**18.** 2	**19.** 3	**20.** 2
21. 2	**22.** 1	**23.** 2	**24.** 2	**25.** 3
26. 2	**27.** 3	**28.** 1	**29.** 3	