1. Number Systems

Numbers

Numbers are building blocks of Mathematics and in all competitive exams questions related to number systems are very common. Also this is one topic where all skills of aptitude like concept, reasoning and simplification skills are checked.

Numeral

In Hindu Arabic system, we use ten symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 called 'digits' to represent any number. A group of digits, denoting a number is called a 'numeral'**.** We represent a number, say 12321 as shown below: Decimal system is the one which we use to denote the numbers in general.

Ten Thousands (10^4)	Thousands (10^3)	Hundreds (10^2)	Tens (10^1)	Units (10^0)
1	2	3	2	1

We read it as Twelve thousand three hundred and twenty one only.

Example: The number obtained by interchanging the two digits of two digit number is 45 less than the actual number and sum of the digits is 9, what is the actual number?

Ans: Let ab be the number. When digits are interchanged it becomes ba.

ba - ab = 45 (ab stands for a x 10 + b in decimal system), So

b x 10 + a – (a x 10 + b) = 45 ➔ 9b - 9a = 45 ➔ a - b = 9

and it is given that a + b = 9. Solving above we get a = 7 & b = 2, So the actual number is 72

Classification of Numbers

All the numbers that we see or use on a regular basis can be classified as given in the chart below.

Natural or Counting numbers

To count objects we use counting numbers like 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 etc. The set of positive counting numbers is called natural numbers. These are also at times called positive integers.

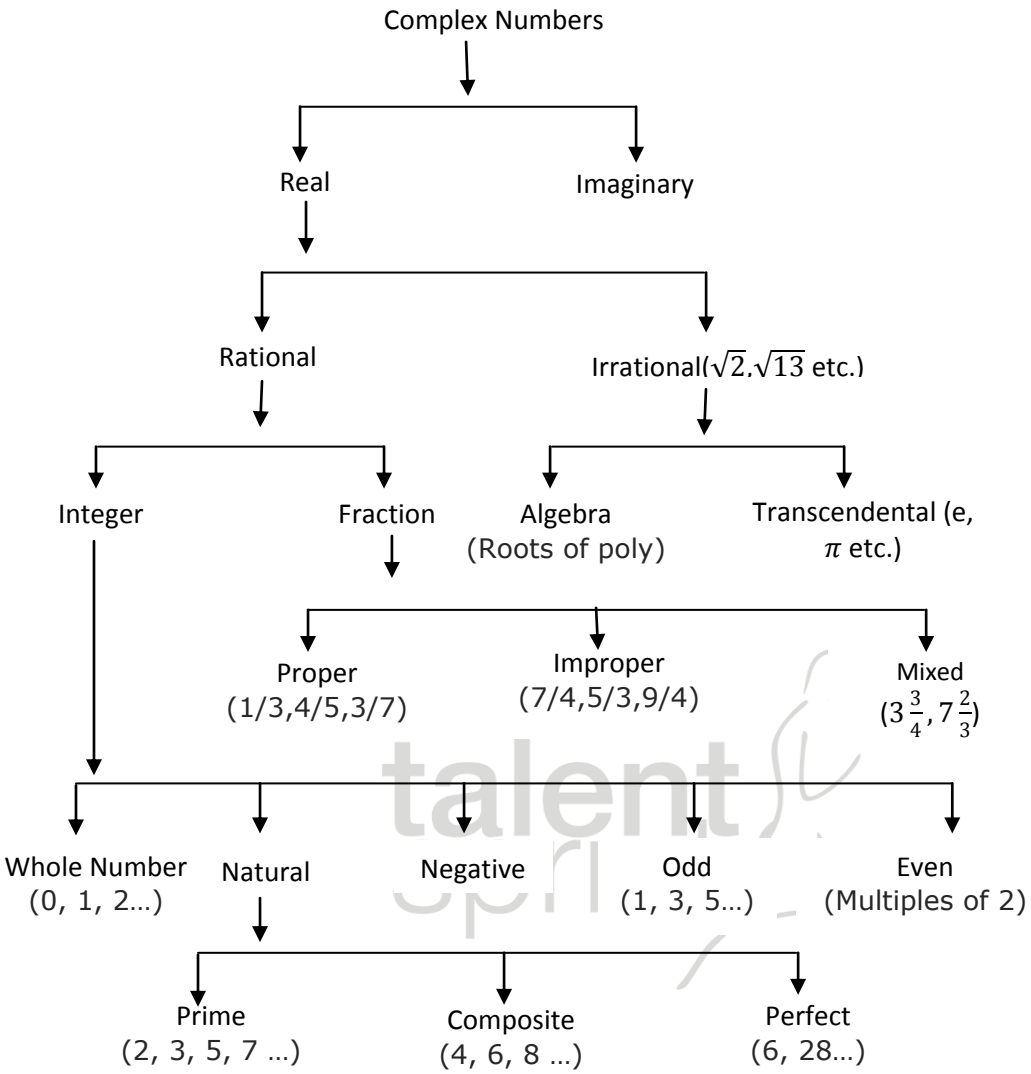
N = {1, 2, 3, 4 ....}

Whole Numbers

The set of natural taken along with 0, gives us the set of whole numbers. Thus, the numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, etc. set of whole numbers.

W = {0, 1, 2, 3…}

Numbers Classification Table



Integers

All the counting numbers (positive and negative) including zero are called the integers.

For example, -100, -99, -50, -40, 0, 13, 17 are all integers.

I/Z = {O, ±1, ±2, ±3 ...} Integer Algebraic

Rational Numbers

This is the set of real numbers that can be written in the form $\frac{a}{b}$, where a and b are integers and b is not equal to zero (b # 0). All integers and all fractions are rational numbers including the finite decimal numbers (i.e. terminating). The numbers -4, $\frac{2}{3}$, $\frac{50}{7}$, - $\frac{10}{3}$, - $\frac{1}{4}$, 0, 145 are examples of rational numbers.

Q = $\{\frac{a}{b}$: a, b∈ I& b≠ 0$\}$

Irrational Numbers

The numbers which are not rational are called irrational numbers, such as $\sqrt{2}$, π. These numbers give an approximate answer in terms of decimals. Also the digits after the decimal are non-terminating and non-recurring. Thus,

$\sqrt{2}$ = 1.4142135 ... π = 3.141592 ... etc.

Real numbers:

Real numbers represent actual physical quantities in a meaningful way e.g. length, height, density etc. Real number scan be further divided into subgroups. For example, rational/irrational, odd/even, prime/composite etc.

Classification Based On Properties

Even Numbers

The set of Even Numbers is the set of integers which are divisible by 2 like 2, 4, 6, 8, 10.... Even numbers are expressed in the form 2n, where n is an integer. Thus, 0, -2, -6 etc. are also even numbers.

Odd Numbers

The set of Odd Numbers is the set of integers which are not divisible by 2 like 1, 3, 5, 7, 9…. Odd numbers are expressed in the form (2n + 1), where n is an integer (not necessarily prime). Thus, -1, -3, -9 … etc. are all odd numbers.

Note

- The smallest natural number is 1.
- The smallest whole number is 0.

Factors and Multiples

If a number x divides another number y exactly, we say that x is a factor of y. In this case, y is called a multiple of x.

Example: Factors for 12 are 1, 2, 3, 4, 6 & 12 and multiple for 12 are 12, 24, 36, 48 etc.

Prime Numbers

A natural number which does not have any other factors besides itself and unity (1) is a prime number. For example 2, 3, 5, 7, 11, 13 etc. The set of such numbers is the set of prime numbers.

Note

- 1 is neither prime nor composite.
- The only even prime number is 2.
- Two numbers are said to be relatively prime to each other or co-prime when their HCF (i.e. Highest common factor) is 1.

  Ex: (i) 9 and 28, (ii) 32 and 15, (iii) 14 and 29 etc.

- If a number has no prime factor equal to or less than its square root, then the number is prime. This is a test to judge whether a number is prime or not.
- Every prime number is in the form of $6x + 1$ but the converse is not true.

Composite Numbers

The set of Composite Numbers is the set of natural numbers which have other factors also, besides itself and unity like 8, 12, 39 etc. Alternatively we might say that a natural number (except 1) which isn't prime is a composite number.

System of Real Numbers

The system of real numbers as we know it today is a result of gradual progress, as the following indicates.

- Natural numbers 1, 2, 3, 4 ... used in counting are also known as **positive integers**. If two such numbers are added or multiplied; the result is always a natural number.
- Numbers that can be expressed in the form p/q, where p and q are co-prime integers, are known as **rational numbers**. Ex :2/3, 8/5, 121/17, -3/2 etc.
- **Irrational numbers** are numbers which cannot be expressed in the above form. Such as $\sqrt{2}$, π etc.
- **Zero** helps us enlarge the number system so as to permit such operations as 8 - 8, 12 x 0. Zero has the properties that
  - Any number multiplied by zero is zero.
  - Zero divided by any number (≠ 0) is zero.
  - Any number divided by zero is undefined.
- When no sign is placed before a number, **a plus sign** is understood. Thus, 5 is + 5, $\sqrt{2}$ is + $\sqrt{2}$

Note

- Zero is a rational number without any sign.
- The real number system consists of the collection of Positive& Negative Rational & irrational Numbers and Zero.

Absolute Value

The absolute value or numerical value of a number means the distance of the number from the origin on the number line. Thus |-6| = 6, | + 4| = 4, |-3/4| = 3/4. In other words, absolute value of a number means the value of the number without its sign i.e. the magnitude only.

Example: Write the absolute values of the following

(1) -51 (2) 25 3) 439 4) -854 (5) 0

Sol: The absolute values of the given integers are

(1) 51 (2) 25 3) 439 4) 854 (5) 0

Rules of Sign

- To add two numbers with similar signs, add their absolute values and prefix the common sign. Thus, 3 + 4 = 7, (-3) + (-4) = -7
- **To add two numbers with opposite signs, find the difference between their absolute values and prefix the sign of the number with greater absolute value.** For example, 17 + (-8) = 9, (-6) + 4 = -2, (-18) + 15 = -3.
- To subtract one number b from another number a, change the operation to addition and replace b by its opposite,-b. Thus, 12 - (7) = 12 + (-7) = 5,
- To multiply (or divide) two numbers having similar Signs, multiply (or divide) their absolute values and prefix a plus sign (or no sign). Ex: (5) 3) = 15, (-5) (-3) = 15, $\frac{-216}{-36} = 6$.
- To multiply (or divide) two numbers having opposite signs, multiply (or divide) their absolute values and Prefix a minus sign. Ex: (-3) (6) = -18 3) (-6) = -18, $\frac{-12}{4}$ = -3.

Other special numbers

Perfect Numbers

If the sum of the divisors or factors of N excluding N itself but including unity is equal to N, then N is called a perfect number like 6, 28 etc.

- 6 = 1 + 2 + 3, where 1, 2 and 3 are divisors of 6.
- 28 = 1 + 2 + 4 + 7 + 14.

Fibonacci Numbers

Fibonacci numbers form a sequence in which each term is the sum of the two terms immediately preceding it. It is named for its discoverer, Leonardo Fibonacci (Leonardo Pisano). The Fibonacci Sequence that has 1 as its first term is 1, 1, 2, 3, 5, 8, and 13, 21, 34, 55…. These numbers are referred to as Fibonacci numbers.

Some important results on numbers:

- If the sum of two positive quantities is given, their product is greatest. When they are equal. Ex: Give $x + y = 30$ → Possible (x, y) are (1, 29), (2, 28), (3, 27) ... and so on. Out of all these, the pair that gives the maximum product will be (15, 15).
- If the product of two positive quantities is given, their sum is least when they are equal. Ex: Given $x\,y = 100$ → Possible (x, y) are (1, 100), (2, 50), (4, 25) ... and so on. Out of all these, the pair that gives the minimum sum will be (10, 10).
- The sum of a positive number and its reciprocal is always greater than or equal to 2, i.e., (a/b + b/a) ≥ 2, (a/b + b/c + c/d + d/a) ≥ 4, $[(x) + (1/x)] \geq 2$.

Base System

The numerals we use today are 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. These numbers are a part of decimal system, because there are only 10 basic symbols.

Expressing the decimal number 63472 in the expanded form; 2 is at the unit's digit and has a place value of 1 i.e. 2 ones, 7 is at the ten's digit and has a place value of 10 i.e. 7 tens, 4 is at the hundred's digit and has a place value of 100 i.e. 4 hundreds, 3 is at the thousand's digit and has a place value of 1000 i.e. 3 thousands, 6 is at the ten thousand's digit and has a place value of 10000 i.e. 6 ten thousands. Thus, the number can be represented as, 63472 = $6 \times 10^4 + 3 \times 10^3 + 4 \times 10^2 + 7 \times 10^1 + 2 \times 10^0 = 60000 + 3000 + 400 + 70 + 2$.

Since there are 10 symbols, this system of representation of numbers is known as the Decimal System (base 10). In a similar way, a system in which only 0 and 1 exist is known as Binary System (base 2).

Similarly, other systems can also be established like Hexadecimal (base 16), Octal (base 8) etc. In each of these systems, the number of symbols used is restricted to the base number. Thus an octal system has only 8 symbols (0 to 7) and hexadecimal system has 16 symbols (0 to 9, A to F).

Conversions

Decimal to Binary Conversion: To convert the decimal number to binary we begin by dividing decimal number by 2 and then dividing each resulting quotient by 2 until there is a 1 quotient.

Example: Convert the decimal number 50 to a binary number.

Sol: Remainder

2	50	0
2	25	1
2	12	0 ↑
2	6 0	
2	3	1
	1 →	1

Hence the binary Equivalent of 50 is 110010.

Example: Convert the binary number 110010 to a decimal number.

Sol: $1 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$

$= 32 + 16 + 0 + 0 + 2 + 0 = 50$.

Decimal to Octal Conversion: The method of converting a decimal number into an octal number is repeated division by 8, similar to the method used in conversion of decimal to binary. Each successive division by 8 yields a remainder that becomes a digit in the equivalent octal

number.

Example: Convert the decimal number 20579 to an octal number.

Sol: Remainder

	20579	3
8	2572	4
8	321	1 ↑
8	400	
	5 →	5

Hence the octal number is 50143.

Decimal to Hexadecimal Conversion: Repeated division of a decimal number by 16 will give the equivalent hexadecimal number which is formed by the remainders of each division. It is similar to the method used in conversion of decimal to binary.

Example: Convert $(650)_{10}$ to a hexadecimal number.

Sol:

16	650	A
16	40	8
	2 →	2

$(650)_{10} = (28A)_{16}$

Octal to Binary Conversion: To convert an octal number to a binary number, simply replace each octal digit by the appropriate three digits binary equivalent.

Example: Change octal $(3574)_8$ to its binary equivalent.

Sol:

$$\overbrace{011}^{3}\ \overbrace{101}^{5}\ \overbrace{111}^{7}\ \overbrace{100}^{4}$$

Hence $(3574)_{\mathbf{8}} = (011\ 101\ 111\ 100)_2 = (011101111100)_2$

Binary to Octal Conversion: Conversion of a binary to an octal number is also a straightforward process. Beginning from the least significant position, simply break the binary number into groups of three digits from right and convert each group into its equivalent octal digit.

Example: Convert $(110101)_2$ to an octal number

Sol:

$$\underbrace{110}_{6}\ \underbrace{101}_{5}$$

$(110101)_2 = (65)_8$

Binary to Hexadecimal Conversion: Converting a binary number to hexadecimal is a straight forward procedure. Simply break the binary number into four bit groups starting from the least

significant position and replace each group with the equivalent hexadecimal number.

Example: Convert $(1100101001010111)_2$ to a hexadecimal number

Sol: $\underbrace{1100}\underbrace{1010}\underbrace{0101}\underbrace{0111}$

C A 5 7

$(1100101001010111)_2 = (CA57)_2$

Hexadecimal to Binary Conversion: To convert from a hexadecimal number to a binary number, reverse the process and replace each hexadecimal number with, their equivalent four digits binary number.

Example: Convert $(10A4)_{16}$ to a binary number.

Sol: 1 0 A 4

$\overbrace{0001}\ \overbrace{0000}\ \overbrace{1010}\ \overbrace{0100}$

Or $(10A4)_{16} = (1100101001010111)_2$

Divisibility Tests

Following are important divisibility tests.

Divisible by	Test	Examples	
2	The unit's digit should be even or, 0 (i.e. in the given number at the units place we should have 2, 4, 6, 8, 0)	26, 64 etc.	6 ÷ 2 = 3, 4 ÷ 2 = 4
3	The sum of the digits of the number should be divisible by 3	15789	(1 + 5 + 7 + 8 + 9 = 27), 27 ÷ 3 = 7
4	The number formed by the last two digits (units and tens) of the given number should be divisible by 4	21964	64 ÷ 4 = 16
5	The unit's digit should be 0 or 5.	865,17840	Last digits are 5 and 0 respectively

6	The sum of the digits of the 6 number should be divisible by 3, Number is and the number should be even	4242	(4 + 2 + 4 + 2 = 12), 12 ÷ 3 = 4, number is even	
8	The number formed by the last three digits (units', tens' and hundreds') of the given number should be divisible by 8	52612	672 ÷ 8 = 84	
9	The sum of the digits of the Number should be divisible by 9	127296	(1 + 2 + 7 + 2 + 9 + 6 = 27), 27 ÷ 9 = 3	
10	The unit's digit should be 0	96520	Unit's digit is zero	
11	The difference between the sums of the digits in the even and odd places should be zero or a multiple of 11	543543	5 + 4 + 3 = 5 + 4 + 3 = 12, Hence difference is zero	
12	The sum of the digits of the number should be divisible by 3 and the number should also be divisible by 4	1728	(1 + 7 + 2 + 8 = 18), 18 ÷ 3 = 6 Also 28 ÷ 4 = 7	
15	The sum of the digits of the number should be divisible by 3 and unit's digit of the number should be 0 or 5	810645	(8 + 1 + 0 + 6 + 4 + 55 = 24), 24 ÷ 3 = 8 also last digit is 5	
16	The number formed by the last Four digits (units', tens', hundreds' and thousands') of the given number should be divisible by 16	12978320	8320 ÷ 16 = 520	

25	The last two digits of the number should be 25, 50, 75 or 00	7125, 1875, 23500, 50	The last two digits areas required
125	The last three digits of the number should be 125, 250, 375, 500, 625, 750, 875 or 000	5125, 1875, 145500, 85625, 76375, 29250, 92750, 90000	The last three digits are as required

Some other important tests

Divisibility test for 7

The test holds good only for numbers with more than three digits and is applied as follows:

1. Group the numbers in sets of three digits from the unit's digit.
2. Add the odd groups and the even groups separately.
3. The difference of the odd and the even groups should beeither 0 or divisible by 7.

Example: Is 85437954 divisible by 7?

Sol: 85 437954

- Adding up the first and the third sets, we get 85 + 954 = 1039.
- Now their difference is 1039 - 437 = 602.
- Since 602 ÷ 7 = 86, hence the number is divisible by 7.

Divisibility test for 13

The test holds good only for numbers with more than three digits. The test to be applied is as follows:

1. Group the numbers in sets of three from the unit's digit.
2. Add the odd groups and the even groups separately.
3. The difference of the odd and the even groups should be either 0 or divisible by 13.

Example: Is 136999005 divisible by 13?

Sol: 136 999 005

- Adding up the first and the third sets, we get 136 + 5 = 141
- Now their difference is 999 - 141 = 858
- Since 858 + 13 = 66, so the number is divisible by 13

Example: Find x & y when

(1) $14x9351y$ is divisible by 72.

Sol: (1) Since 72 = 8 x 9, so the number must be divisible by 8 and 9.

- The last three digits of the number should be divisible by 8. Hence 51 y/8 must be an integer (last 3 digits), i.e. y = 2. Now, the given number should be divisible by 9.
- 1 + 4 + x + 9 + 3 + 5 + 1 + 2 = 25 + x should be divisible by 9. Thus x = 2. Hence the number is 14293512.

Note

- If two numbers, say x and y, are divisible by a third number, say z then (x - y) and (x + y) are also divisible by z. For example, 20 and 64 are divisible by 4. Also, (64 - 20) & (64 + 20) are divisible by 4.
- When any number with even number of digits is added to its reverse, the sum is always divisible by 11. For example, 2341 + 1432 = 3773 which is divisible by 11
- When any number with odd number of digits is subtracted from its reverse, the absolute difference is always divisible by 11 & 9. For example, 23411 - 11432 = 11979, which is divisible by 11 & 9
- If x is a prime number, then for any whole number "a", (a^x - a) is divisible by x. For example, Let x = 3 and a = 5. Then according to our rule 5^3 - 5 should be divisible by 3. Now (5^3 - 5) = 120 which is divisible by 3.

Fractions

A fraction denotes a part or parts of a unit. The different types of fractions are as follows:

- **Common Fractions:** Fractions whose denominator is not 10 or a multiple of 10. E.g. 2/3, 17/18 etc.
- **Decimal Fractions:** Fractions whose denominator is 10 or a multiple of 10.
- **Proper Fractions:** Fractions whose numerator < denominator. E.g. 2/10, 0, 6/7, 8/9 …. Hence its value < 1
- **Improper Fractions:** Fractions whose numerator > denominator. E.g. 10/2, 7/6, 8/7 …. Hence its value > 1
- **Mixed Fractions:** In these types of fractions, there are two parts, an integral part and a fractional part. E.g. $1\frac{3}{8}$, $5\frac{1}{6}$ etc. are all mixed fraction.
- **Compounded Fraction:** A fraction of a fraction is known as compounded fraction E.g. $\frac{4}{5}$ of $\frac{1}{6}$ etc are compounded fractions.
- **Complex Fraction:** If the numerator or the denominator or both of a fraction are

fractions, then the fraction is called a complex fraction. E.g. $\frac{1}{\frac{4}{7}}$, $\frac{\frac{2}{3}}{\frac{4}{9}}$, $\frac{8}{\frac{2}{9}}$ are all complex fractions.

Operations with Fractions

Operations with fractions may be performed according to the following rules:

- The value of a fraction remains the same if its numerator and denominator are both multiplied or divided by the same number provided the number is not zero.
  For example, $\frac{3}{4} = \frac{3 \times 2}{4 \times 2} = \frac{6}{8}$, $\frac{15}{18} = \frac{15 \div 3}{18 \div 3} = \frac{5}{6}$
- Changing the sign of either the numerator or the denominator of a fraction changes the sign of the fraction.
  For example, $\frac{-3}{5} = -\frac{3}{5} = \frac{3}{-5.}$
- Adding two fractions with a common denominator yields a fraction whose numerator is the sum of the numerators of the given fractions and whose denominator is the common denominator.
  For example, $\frac{3}{5} + \frac{4}{5} = \frac{3+4}{5} = \frac{7}{5}$.
- The sum or difference of two fractions having different denominators may be found by converting the fractions to a common denominator.
  For example, $\frac{1}{4} + \frac{2}{3} = \frac{3}{12} + \frac{8}{12} = \frac{11}{12}$.
- The product of two fractions is a fraction whose numerator is the product of the numerators of the given fractions and whose denominator is the product of the denominators of the fractions.
  For example, $\frac{2}{3} \times \frac{4}{5} = \frac{2 \times 4}{3 \times 5} = \frac{8}{15}$

Addition of mixed Fractions

Addition of mixed fractions can be done by separating the integer part from fraction for the given fractions and doing the addition for respective parts.

Example : $3\frac{1}{4} + 2\frac{2}{3} = 3 + 2 + \frac{1}{4} + \frac{2}{3} = 5\frac{11}{12}$

Multiplication of mixed Fractions

Finding the product of mixed fractions is easier by converting them into improper fractions.

Decimal Fractions

Fractions in which the denominators are the powers of 10 are called decimal fractions. In general,

the decimal fractions are of the following types

Recurring Decimals: If in a decimal fraction, a figure or a set of figures is repeated continually, then such a number is called recurring decimal fraction.

If a single figure is repeated, it is shown by putting a dot on it. But if a set of figures is repeated; we express it either by putting one dot at the starting digit and one dot at the last digit of the repeating digits or by placing a bar or a vinculum on the repeating digit(s).

- 2/3 = 0.6666... = $0.\dot{6}$ = $0.\overline{6}$
- 22/7 = 3.142857142857 = $\overline{3.142857}$

Pure Recurring Decimals: A decimal in which all the figures after the decimal point repeat is called a pure recurring decimal.

Example: $0.\dot{6}$, $3.\dot{1}42857$ etc.

Mixed Recurring Decimals: A decimal in which some figures do not repeat and some of them repeat is called a mixed recurring decimal.

Example :1$5.8\dot{3}$ etc

- **Conversion of a Pure Recurring Decimal into fraction:**
  Write the recurring figures only once in the numerator and take as many nines in the denominator as the number of repeating figures.
  - 0.6 = 6/9 = 2/3.
  - 16.6 = 16 + 0.6 = 16 + 6/9 = 16 + 2/3 = 50/3.
- **To convert a Mixed Recurring Decimal into fraction:**
  In the numerator, write the difference between the number formed all the digits after decimal point (taking repeated digits only once) and that formed by the digits which are not repeated.

  In the denominator, write the number formed by as many nines as there are repeating digits followed by as many zeroes as in non-repeating digits.
  - $0.1\dot{7}$ = (17 -1)/90 = 16/90 = 8/45.
  - $0.12\dot{5}\dot{4}$ = (1254 - 12)/9900 = 69/550.
  - $253\dot{6}$ = 2 = (536 – 53)/900 = $2\frac{161}{300}$

Example: Let D be a decimal of the form, D = $0.a_1a_2a_1a_2a_1a_2\ldots$, where digits a_1 & a_2 lie between 0 and 9. Then which of thefollowing numbers necessarily produces an integer, whenmultiplied by D?

(1) 18 (2) 108 3) 198 4) 208

Sol. It is recurring decimal and can be written as D = 0. $\overline{a_1a_2}$. To convert this to fraction, we can

write it as $a_1a_2/99$. Thus when the number is multiplied by 99 or a multiple of it, we shall necessarily get an integer. Of the given options, only 3) is a multiple of 99, hence Ans. 3)

Exponents

The product 10 x 10 x 10 can be written as 10^3 and is read as 10 raised to the third power. In general, a x a x a.... a (n times) is written asa^n. The base is raised to the nth power and n is called the exponent or the index.

Examples:

- 3^2 = 3 x 3 read as "3 squared"
- 2^3 = 2 x 2 x 2 read as "2 cubed"
- 5^4 = 5 x 5 x 5 x 5 read as "5 to the fourth power".

If the exponent is 1, it is usually understood and not written thus, a^1 = a.

Laws

- $a^m \times a^n = a^{m+n}$
- $a^m/a^n = a^{m-n} = 1/a^{n-m}$ (if $a \neq 0$)
- $(a^m)^n = a^{mn}$
- $a^{-m} = 1/a^m$
- $a^0 = 1$
- (Any number with zero exponents is equal to 1)
- $(a \times b)^m = a^m \times b^m$
- $(a \div b)^m = a^m \div b^m$ (if $b \neq 0$)
- $\sqrt[m]{a} = a^{1/m}$
- $a^{p/q} = \sqrt[q]{a^p}$

Example: Simplify $\frac{(3^4)^3 \times (3^2)^4}{(-3)^{15} \times 3)^4}$

Sol: $\frac{(3^4)^3 \times (3^2)^4}{(-3)^{15} \times 3)^4} = \frac{3)^{12} \times 3)^8}{(-3)^{15} \times 3)^4} = \frac{3^{20}}{3^{19}} = (-3)^1 = -3$

Example: Simplify $\frac{3^8}{3^5} - \frac{4^4 \times 2^4}{2^6} + 3(-2)^3$

Sol: $\frac{3^8}{3^5} - \frac{4^4 \times 2^4}{2^6} + 3(-2)^3 = 3^{8-5} - \frac{4^2}{2^{6-4}} + 3(-8)$

$3^3 - \frac{4^2}{2^2} + 3(-8) = 27 - 4 - 24 = -1$

Squares & Square Roots

By the square of a number, we mean the product of number by the number itself. If a^2 = b, we say that square root of b is a and we write here square of b = a^2.

From the above discussion, it is clear that

- A square of a natural number cannot end with 2, 3, 7, 8 and an odd number of zeroes.
- The square of an odd number is odd and that of an even number is even.
- Every square number is a multiple of 3 or exceeds a multiple of 3 by unity.
- Every square number is a multiple of 4 or exceeds a multiple of 4 by unity.
- If a square number ends in 9, the preceding digit is even.

Methods for finding Square Roots

Factorization: When a given number is a perfect square, we resolve it into prime factors and take the product of prime factors choosing one out of every pair of the same primes. For example :4624 = 2 x 2 x 2 x 2 × 17 × 17 ➔ 4624 = 2^2 x 2^2×17^2

So, the square root of 4624 = $\sqrt{4624}$ = 2 x 2 x 17 = 68.

Long division method: The nature of the Banking Recruitment exams Entrance test precludes the possibility of the student ever gainfully exploiting the long division method. In simple terms, what we mean is, DON'T use the long division method. The following example will illustrate the use of this method.

$$\sqrt{45369}$$

2		1	3
2	4	53	69
2	4		
41	0	53	
1		41	
423		12	69
3		12	69

$\sqrt{45369}$ = 213

In this method, we pair two numbers from the units place. Like in this example, 69, 53 and 4 are the pairs. Now we find the greatest divisor of 4 which when multiplied by itself gives a number less than or equal to 4. Here we get 2. We get the new divisor, by adding 2 (divisor) by 2 only. Our dividend now is 53. Now we have to find a number which when put with 4 (1 here) and the result (41 here) multiplied with the new number (1 here), we get (41) a number less than or equal to 53. Now the dividend is 1269 (12 is the remainder) and the divisor is 41 + 1 = 42_. As discussed earlier, we can put 3(in place of _) to get 423 as divisor which when multiplied by 3, gives 1269 and hence the square root will be 213.

By approximation: In order to find the square root in the actual test you should use the method of approximation. In order to use the method of approximation effectively you must know the:

- Squares of numbers up to 30 or more.
- Rapid multiplication techniques etc.

Suppose you want to find$\sqrt{75}$, you know that $\sqrt{64}$ = 8 and $\sqrt{81}$ = 9. Hence $\sqrt{75}$ will lie between 8& 9. Now 75 is closer to 81 than 64, hence $\sqrt{75}$ will be between 8.5 and 9 and now you can approximate that its value is somewhere around 8.7

Cube and cube roots

The cube root of a number x is the number whose cube is the number x. We denote the cube root of x by $\sqrt[3]{x}$ or $x^{1/3}$. We resolve the given number into prime factors and take the product of prime numbers choosing one out of three of each prime number.

Thus,

$\sqrt[3]{8} = \sqrt[3]{(2 \times 2 \times 2)} = (2^{3^{1/3}} = 2^{3 \times 1/3} = 2^1 = 2$

$\sqrt[3]{729} = \sqrt[3]{(9 \times 9 \times 9)} = (9^3)^{1/3} = 9^{3 \times 1/3} = 9^1 = 9$

$\sqrt[3]{9261} = \sqrt[3]{(3 \times 3 \times 3 \times 7 \times 7 \times 7)} = \sqrt[3]{3^3 \times 7^3} = 3 \times 7 = 21$

Remember Squares of 1 to 30

Numbers	Squares	Numbers	Squares
1	1	16	256
2	4	17	289
3	9	18	324
4	16	19	361
5	25	20	400
6	36	21	441
7	49	22	484
8	64	23	529
9	81	24	576
10	100	25	625
11	121	26	676
12	144	27	729
13	169	28	784
14	196	29	841
15	225	30	900

Remember Cubes of 1 to 10

Numbers	Cubes
1	1
2	8
3	27
4	64
5	125
6	216
7	343
8	512
9	729
10	1000

Practice Exercise– 01

What should be in place of question mark (?) in the following questions?

1. $[(144)^2 \div 48 \times 18] \div 36 = \sqrt{?}$

1) 23328 2) 36 3) 216 4) 46656 5) none of these

2. $(27)^{18} \div (27)^3 = ?$

1) $(27)^{54}$ 2) $(27)^{21}$ 3) $(27)^{15}$ 4) $(27)^6$ 5) none of these

3. $5\frac{1}{4} + 6\frac{2}{3} + 7\frac{1}{6} = ?$

1) 19.5 2) $19\frac{11}{12}$ 3) $19\frac{1}{12}$ 4) 19 5) none of these

4. $4895 + 364 \times 0.75 - 49 = ?$

1) 5119 2) 3895 3) 3907 4) 5210 5) none of these

5. $24336 \div ? = 78 \times 24$

1) 6 2) 13 3) 11 4) 17 5) none of these

6. $156 + 16 \times 1.5 - 21 = ?$

1) 126 2) 258 3) 237 4) 159 5) none of these

7. $(98)^{45} \times (98)^{-35} = ?$

1) 98 2) $(98)^2$ 3) $(98)^{-5}$ 4) $(98)^{10}$ 5) none of these

8. $434.43 + 43.34 + 3.44 + 4 + 0.33 = ?$

1) 421.45 2) 455.54 3) 485.54 4) 447.45 5) none of these

9. (23.6% of 1254) - (16.6% of 834) = ?

1) 159.5 2) 157.5 3) 155.5 4) 153.5 5) none of these

10. $(78.95)^2$ - $(43.35)^2$ = ?

1) 4148 2) 4353.88 3) 4235.78 4) 4148 5) none of these

Answers:

1. 4	**2.** 3	**3.** 3	**4.** 1	**5.** 2
6. 4	**7.** 4	**8.** 3	**9.** 2	**10.**2

What should come in place of the question mark (?) in the following?

1. $\frac{2}{7} \times \frac{5}{6} \times \frac{3}{8} \times ?$ = 790

1) 8848 2) 8246 3) 8484 4) 8868 5) none of these

2. 968 × 445 ÷ ? = 17230.4

1) 60 2) 40 3) 25 4) 35 5) none of these

3. (0.05 × 6.25) ÷ 2.5 = ?

1) 0.105 2) 0.95 3) 0.115 4) 1.25 5) none of these

4. $(41)^2$ + $(38)^2$ × $(0.15)^2$ = ?

1) 3125.0225 2) 1713.49 3) 3125.15 4) 59204.0225 5) none of these

5. 1728 ÷ $\sqrt[3]{262144}$ ×? – 288 = 4491

1) 148 2) 156 3) 173 4) 177 5) none of these

ANS:

1. 1	**2.** 3	**3.** 5	**4.** 2	**5.** 4

What should be come in the place of question mark (?) in the following?

1. [$(58)^2$ × $(48)^2$] ÷ ? = 2152.96

1) 60 2) 2500 3) 50 4) 3600 5) none of these

2. 7432 ÷ 92.9 × 18.5 = ?

1) 1450 2) 1600 3) 1480 4) 1560 5) none of these

3. 99 × 21 - $\sqrt[3]{?}$ = 1968

1) 1367631 2) 111 3) 1366731 4) 1367 5) none of these

4. 9634 × $\frac{3}{8}$÷ ? = 28.902

1) 115 2) 95 3) 110 4) 120 5) none of these

5. 19.99 × 9.9 + 99.9 = ?

1) 129.79 2) 297.801 3) 1009 4) 296.91 5) none of these

ANS:

1. 4	**2.** 3	**3.** 1	**4.** 5	**5.** 2

What should come in the place of question mark (?) in the following?

1. (47 × 588) ÷ (28 × 120) = ?

   1) 6.284 2) 7.625 3) 8.225 4) 8.285 5) none of these

2. 45% of 224 × ?% of 120 = 8104.32

   1) 67 2) 62 3) 15 4) 71 5)none of these

3. $\sqrt{7921} \times 51 + 374 = (?)^3$

   1) 16 2) 19 3) 15 4) 21 5) none of these

4. $6573 \div 21 \times (0.2)^2 = ?$

   1) 7825 2) 62.6 3) 1565 4) 12.52 5) none of these

5. 74156 - ? – 321 – 20 + 520 = 69894

   1) 3451 2) 4441 3) 5401 4) 4531 5) none of these

ANS:

1. 3	**2.** 1	**3.** 5	**4.** 4	**5.** 2

What should come in the place of question mark (?) in the following?

1. 18.5 × 22.5 × ? = 5161.5

   1) 13.5 2) 11.4 3) 16.5 4) 12.4 5) none of these

2. $(9)^2 + (12)^2 = (?)^2$

   1) 21 2) 15 3) 25 4) 18 5) none of these

3. $(23)^3 = ?$

   1) 12167 2) 17576 3) 2197 4) 5832 5) none of these

4. 15 ÷ 5 ÷ 5 = ?

   1) 1 2) 15 3) 0 4) 3 5) none of these

5. $18^{1.3} \times 18^{4.4} = 18^{?}$

   1) 3.1 2) 4 3) 5.7 4) 6 5) none of these

ANS:

1. 4	**2.** 4	**3.** 1	**4.** 5	**5.** 3

What should come in the place of question mark(?) in the following?

1. $(12)^{20} \times (12)^{5} = ?$

   1) $(12)^{100}$ 2) $(12)^{4}$ 3) $(12)^{25}$ 4) $(12)^{15}$ 5) None of these

2. 34.667 - 15.597 - 8.491 – 0.548 = ?

   1) 14.403 2) 10.031 3) 18.301 4) 21.043 5) None of these

3. $[(140)^{2} \div 70 \times 16] \div 8 = 14 \times ?$

   1) 38 2) 22 3) 55 4) 40 5) None of these

4. 456 + 24 × 0.75 – 12 = ?

   1) 462 2) 672 3) 348 4) 624 5) None of these

5. $\sqrt{\sqrt{(17956} + \sqrt{24025}} = ?$

   1) 256 2) 289 3) 155 4) 19 5) None of these

6. ?% of 398 + 31% of 993 = 403.35

   1) 46 2) 24 3) 18 4) 32 5) None of these

7. $7\frac{1}{2} + 3\frac{2}{3} + 3\frac{5}{6} = ?$

   1) 17 2) 16.75 3) 15 4) 32 5) None of these

8. $\{(56)^{2} + (44)^{2}\} \div ? = 16$

   1) 329 2) 335 3) 343 4) 317 5) None of these

9. (15.6% of 1296) – (10.8% of 87) = ?

   1) 109.218 2) 108.216 3) 107.214 4) 106.212 5) None of these

10. $(55.25)^{2}$ - 637.5625 = ?

    1) 25.25 2) 625 3) 2415 4) 1375 5) None of these

ANS:

1. 3	**2.** 2	**3.** 4	**4.** 1	**5.** 5
6. 2	**7.** 3	**8.** 4	**9.** 2	**10.**3

What should come in the place of question mark(?) in the following?

1. $\frac{1}{2} + \frac{1}{4} + \frac{3}{4} + \frac{2}{3} = ?$

   1) $2\frac{1}{5}$ 2) $\frac{1}{16}$ 3) $2\frac{1}{16}$ 4) $2\frac{1}{6}$ 5) None of these

2. $4)^{?} = 1024$

   1) 1 2) 2 3) 3 4) 4 5) None of these

3. 22.5 × 0.05 = ?

   1) 11.25 2) 1.125 3) 22.55 4) 112.5 5) None of these

4. 999 + 111 × 0.5 = ?

   1) 555 2) 500 3) 1054.5 4) 1110.5 5) None of these

5. 40% of 250 = 50% of ?

   1) 200 2) 100 3) 150 4) 4005 5) None of these

ANS:

1. 4	**2.** 5	**3.** 2	**4.** 3	**5.** 1

What should come in the place of question mark(?) in the following?

1. $\{(52)^2 + (45)^2\} \div ? = 8$

   1) 611.345 2) 487.225 3) 591.125 4) 372.425 5) None of these

2. 72% of 752.6 = 48% of?

   1) 1128.9 2) 1332.7 3) 1536.5 4) 1742.3 5) None of these

3. ?% of 658 + 40% of 845 = 568.3

   1) 46 2) 42 3) 38 4) 35 5) None of these

4. (15.5% of 1245) – (12.5% of 1458) = ?

   1) 10.725 2) 10.735 3) 10.745 4) 10.755 5) None of these

5. $(64)^2 \div (8)^2 = ?$

   1) $(8)^{12}$ 2) $(8)^8$ 3) $(8)^4$ 4) $(8)^2$ 5) None of these

6. 99.75^2 - 2250.0625 = ?

   1) 9900.625 2) 7700 3) 6545.625 4) 8875 5) None of these

7. $12.25^2 - \sqrt{625} = ?$

   1) 235.1625 2) 125.0625 3) 375.2625 4) 465.3625 5) None of these

8. 383 × 38 × 3.8 = ?

   1) 58305.8 2) 57305.6 3) 56305.4 4) 55305.2 5) None of these

9. 43.231 – 12.779 - 6.542 – 0.669 = ?

   1) 27.341 2) 25.242 3) 23.241 4) 21.342 5) None of these

10. 572 + 38 × 0.50 – 16 = ?

    1) 289 2) 305 3) 448 4) 565 5) None of these

ANS:

1. 3	**2.** 1	**3.** 4	**4.** 1	**5.** 5
6. 2	**7.** 2	**8.** 4	**9.** 3	**10.**5

What should come in the place of question mark (?) in the following?

1. 8451 + 793 + 620 - ? = 6065 + 713

   1) 4912 2) 4712 3) 4312 4) 4512 5) None of these

2. 81 + 20 × 0.75 – 9 = ?

   1) 97 2) 107 3) 87 4) 77 5) None of these

3. 811.81 + 88.11 - 0.88 + 1.88 + 8 = ?

   1) 910.68 2) 912.56 3) 904.88 4) 902.67 5) None of these

4. 224400 ÷ $\sqrt{?}$ = 34 × 12

   1) 55 2) 3136 3) 65 4) 3025 5) None of these

5. 3402 ÷ ?$\sqrt{26244}$

   1) 162 2) 21 3) 441 4) 42 5) None of these

ANS:

1. 5	**2.** 3	**3.** 1	**4.** 4	**5.** 2

What should come in the place of question mark (?) in the following?

1. $\sqrt{571536}$ ÷ 42 ×? = 5850

   1) 420 2) 240 3) 315 4) 325 5) None of these

2. $(34)^{56} \times (34)^{-53}$ = ?

   1) 39304 2) 1156 3) 170504 4) 102 5) None of these

3. 378.35 + 478 ÷ 12.5 = ?

   1) 508.268 2) 416.59 3) 425.28 4) 68.508 5) None of these

4. (550% of 250) ÷ 275 = ?

   1) 15 2) 1.5 3) 0.5 4) 25 5) None of these

5. 334.41 + 47.26 + 1.25 + 5 + 0.66 = ?

   1) 411.24 2) 396.15 3) 388.58 4) 376.85 5) None of these

6. 74844 ÷? = 54 × 63

   1) 34 2) 42 3) 22 4) 54 5) None of these

7. $(21.35)^2$ - $(12.25)^2$ = ?

1) 171.4125 2) 305.75 3) 604.085 4) 463.8125 5) None of these

8. 124 + 56 × 1.5 – 12 = ?

1) -1890 2) 252 3) 230 4) 196 5) None of these

9. $\sqrt[3]{1092727}$ = ?

1) 108 2) 99 3) 97 4) 107 5) None of these

10. (46351 – 36418 – 4505) ÷ ? = 1357

1) 4 2) 6 3) 3 4) 2 5) None of these

ANS:

1. 4	**2.** 1	**3.** 2	**4.** 5	**5.** 3
6. 3	**7.** 2	**8.** 4	**9.** 5	**10.**1

What should come in the place of question mark (?) in the following?

1. 74 + 12 × 0.75 – 6 = ?

1) 72 2) 67 3) 62 4) 77 5) None of these

2. 2432 ÷? = $\sqrt{23104}$

1) 12 2) 14 3) 18 4) 16 5) None of these

3. 8888 + 848 + 88 - ? = 7337 + 737

1) 1750 2) 1650 3) 1550 4) 1450 5) None of these

4. 515.15 – 15.51 – 1.51 – 5.11 – 1.11 = ?

1) 491.91 2) 419.91 3) 499.19 4) 411.19 5) None of these

5. $(?)^2$ + $(123)^2$ = $(246)^2$ – $(99)^2$ - 2462

1) 184 2) 186 3) 182 4) 180 5) None of these

6. 414 ×? × 7 = 127512

1) 36 2) 40 3) 44 4) 48 5) None of these

7. [$(84)^2$ ÷ 28 × 12] ÷ 24 = 7 ×?

1) 15 2) 17 3) 19 4) 21 5) None of these

8. (7.9% of 134) – (3.4% of 79) = ?

1) 8.1 2) 7.8 3) 8.6 4) 7.3 5) None of these

9. $(3)^8$ × $(3)^4$ = ?

1) $(27)^3$ 2) $(729)^2$ 3) $(27)^5$ 4) $(729)^3$ 5) None of these

10. 24.424 + 50656 + 1.131 + 0.089 = ?

1) 31.003 2) 31.3 3) 31.03 4) 31.0003 5) none of these

ANS:

1. 4	**2.** 4	**3.** 1	**4.** 1	**5.** 3
6. 3	**7.** 5	**8.** 5	**9.** 2	**10.**2

What should come in the place of question mark (?) in the following?

1. $\sqrt[3]{?}$ = (756 × 67) ÷ 804

1) 195112 2) 250047 3) 226981 4) 274625 5) None of these

2. 0.3 + 3 + 3.33 + 3.3 + 3.03 + 333 = ?

1) 375.66 2) 345.99 3) 375.93 4) 355.96 5) None of these

3. (73425 – 33267 – 22418 – 17650) × $\sqrt{11025}$ = ?

1) 10165 2) 9785 3) 8370 4) 9450 5) None of these

4. -76 × 33 + 221 = ?

1) -2287 2) -19304 3) 2287 4) 19304 5) None of these

5. $(34.12)^2$ - $\sqrt{7396}$ = ?

1) 1080.1744 2) 1078.1474 3) 1078.1744 4) 1080.1474 5) None of these

ANS:

1. 2	**2.** 5	**3.** 4	**4.** 1	**5.** 3

What should come in the place of question mark (?) in the following?

1. $66^2 - 34^2$ = ?

1) 3600 2) 3200 3) 2146 4) 2466 5) None of these

2. 185% of 400 + 35% of 240 = ?% of 1648

1) 85 2) 75 3) 125 4) 50 5) None of these

3. $\frac{3}{8}$ of $\frac{4}{9}$ of 1092 = ?

1) 182 2) 728 3) 364 4) 218 5) None of these

4. 12.28 × 1.5 – 36 ÷ 2.4 = ?

1) 3.24 2) 7.325 3) 6.42 4) 4.32 5) None of these

5. $\sqrt{24^4}$ + 224 = ? × 20^2

1) 20 2) 4 3) 2 4) 16 5) None of these

ANS:

1. 2	**2.** 4	**3.** 1	**4.** 5	**5.** 3

What should come in the place of question mark (?) in the following?

1. 3895 – 1563 + 1089 = ?

   1) 13.6 2) 12.6 3) 12.8 4) 13.8 5) None of these

2. 45 × 390 ÷ 26 = ?

   1) 645 2) 675 3) 765 4) 745 5) None of these

3. 2.2 × 5.6 + 17.8 = ?

   1) 30.12 2) 30.012 3) 31.12 4) 31.012 5) None of these

4. 358.085 + 42.91 + 25.55 = ?

   1) 425.565 2) 426.545 3) 426.555 4) 425.545 5) None of these

5. 140% of 56 + 56% of 140 = ?

   1) 78.4 2) 158.6 3) 156.6 4) 87.4 5) None of these

ANS:

1. 5	**2.** 2	**3.** 1	**4.** 2	**5.** 5

What should come in the place of question mark (?) in the following?

1. 58.621 – 13.829 – 7.302 – 1.214 = ?

   1) 37.281 2) 35.272 3) 36.276 4) 31.254 5) None of these

2. ?% of 450 + 46% of 285 = 257.1

   1) 34 2) 32 3) 21 4) 28 5) None of these

3. $(81)^4 \div (9)^5$ = ?

   1) 6561 2) 729 3) 81 4) 9 5) None of these

4. 618 + 62 × 0.50 – 29 = ?

   1) 625 2) 660 3) 640 4) 655 5) None of these

5. 282 × 82 × 0.2 = ?

   1) 4624.8 2) 4734.6 3) 46044.4 4) 4324.2 5) None of these

ANS:

1. 3	**2.** 4	**3.** 3	**4.** 5	**5.** 1

What should come in the place of question mark (?) in the following?

1) 2.8 × 1.5 + 8% of 250 = ?

   1) 24.2 2) 24.02 3) 242.2 4) 2.42 5) none of these

2) $3\frac{2}{7} - 2\frac{1}{14} - 1\frac{1}{14} - 1\frac{1}{7} = ? + 2\frac{1}{13}$

1) $1\frac{1}{7}$ 2) $1\frac{2}{7}$ 3) $\frac{1}{7}$ 4) $\frac{3}{7}$ 5) None of these

3) $\frac{3}{8}$ of $\frac{4}{5}$ of $\frac{2}{3}$ of 730 = ?

1) 86 2) 146 3) 156 4) 93 5) None of these

4) $8^7 \times 2^6 \div 8^{2.4} = 8^?$

1) 10.6 2) 9.6 3) 8.6 4) 6.6 5) None of these

5) 160% of 250 + ? = 120% of 400

1) 160 2) 40 3) 80 4) 120 5) None of these

ANS:

1. 1	**2.** 5	**3.** 2	**4.** 4	**5.** 3