Quadratic Eq.

$\boxed{ax^2 + bx + c = 0}$ variable $\to$ x $\boxed{x^2}$

$ax^2+bx+c=0$ when $\boxed{a, b, c}$ are real number

$\boxed{a \neq 0}$

$0 \times x^2 + bx + c = 0$

$0 + bx + c = 0$

$\boxed{bx + c = 0} \Rightarrow x^2$ quadr—

$\boxed{ax^2 + bx + c = 0}$ $\boxed{ax^2 + 6 = 0}$

$\boxed{ax^2 = 0}$ $\boxed{ax^2 + 5x = 0}$

Three way to solve any quadratic Eq.

(i) By Factorisation

(ii) By Completing the Square

(iii) By Median

(I) Factorization

$ax^2+bx+c=0$

$\boxed{2x^2 - 5x + 3 = 0}$ $\Rightarrow$ sol^n = 2

$\boxed{x = 1,\ x = 1}$ $x=1$

$a \times c = 2 \times 3$

$= 6$

$\Rightarrow$ b

$b_1 + b_2 = b$

$b_1 \times b_2$

$ax^2+bx+c=0$

$\boxed{b = b_1+b_2}$

$\boxed{b_1 \times b_2 = ac}$

$2x^2-5x+3=0$

$\Rightarrow 2x^2-2x-3x+3=0$ $\boxed{+6}$

$\boxed{-5}$

$\Rightarrow 2x^2-2x-3x+3=0$

1 2

$\boxed{-2+-3} \times 6$

$=-5$

$\Rightarrow 2\cdot x\cdot x-2x-3\cdot x+3=0$

$\Rightarrow 2x(x-1)-3(x-1)=0$

$\Rightarrow (x-1)(2x-3)=0$

$\Rightarrow (x-1)\times(2x-3)=0$

$\Rightarrow x-1=\frac{0}{2x-3}=0$

$x-1=0 \Rightarrow \boxed{x=1}$

$2x-3=\frac{0}{x-1}=0$

$\Rightarrow 2x-3=0 \Rightarrow 2x=3$

$\Rightarrow 2\times x=3$

$\Rightarrow x=3\div 2=3/2=1\frac{1}{2}$

$\boxed{\therefore x=1,\ x=3/2}$ Ans

$6x^2\boxed{-x}-2=0$

$6\times 2 = 12$

$\begin{array}{r|l} 2 & 12 \\ 2 & 6 \\ & 3 \end{array}$ $\boxed{2\times2\times3}=12$

$4\times3=12$

$6x^2-x-2=0$

$\Rightarrow 6x^2-4x+3x-2=0$ $\boxed{3-4=-1}$

$\Rightarrow 2\cdot3\cdot x\cdot x-2\cdot2\cdot x+3\cdot x-2=0$

$\Rightarrow 2x(3x-2)+\boxed{3x-2}=0$

$\Rightarrow 2x(3x-2)+1\cdot(3x-2)=0$

$\Rightarrow (3x-2)(2x+1)=0$

$\Rightarrow$ $3x-2=0$, $2x+1=0$

$3x=2$, $2x=-1$

$\boxed{x=2/3,\ x=-1/2}$

(3) $3x^2-2\sqrt{6}x+2=0$

$b_1\times b_2$

6

$\begin{array}{r|l} 2 & 6 \\ & 3 \end{array}$ $\boxed{2\times3}$

$(\sqrt{6})^2$

$(6^{1/2})^2$ $\sqrt{6}$

$\boxed{\sqrt{6}\times\sqrt{6}=6}$

2×6

$\boxed{\sqrt{6}+\sqrt{6}=2\sqrt{6}}$

$\sqrt{6}+\sqrt{6}=2\sqrt{6}$

$$3x^2 - 2\sqrt{6}x + 2 = 0$$

$$\Rightarrow 3x^2 - \sqrt{6}x - \sqrt{6}x + 2 = 0$$

$$\Rightarrow \sqrt{3}\sqrt{3}x \cdot x - \sqrt{2} \cdot \sqrt{3}x - \sqrt{2} \cdot \sqrt{3} \cdot x + \sqrt{2} \cdot \sqrt{2} = 0$$

$$\Rightarrow \sqrt{3}x(\sqrt{3}x - \sqrt{2}) - \sqrt{2}(\sqrt{3} \cdot x - \sqrt{2}) = 0$$

$$\Rightarrow (\sqrt{3}x - \sqrt{2})(\sqrt{3}x - \sqrt{2}) = 0$$

$$\Rightarrow \sqrt{3}x - \sqrt{2} = 0, \quad \sqrt{3}x - \sqrt{2} = 0$$

$$\sqrt{3}x = \sqrt{2} \qquad \sqrt{3}x = \sqrt{2}$$

$$x = \frac{\sqrt{2}}{\sqrt{3}}, \qquad x = \frac{\sqrt{2}}{\sqrt{3}}$$

$$\boxed{x = \sqrt{\frac{2}{3}}, \quad x = \sqrt{\frac{2}{3}}}$$

(4) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$10 \quad \sqrt{2} \times 5\sqrt{2}$

5×2

10

$5 + 2$

5×2

$$\sqrt{2}x^2 + 5x + 2x + 5\sqrt{2} = 0$$

$$\Rightarrow \sqrt{2}x^2 + 2x + 5x + 5\sqrt{2} = 0$$

$$\Rightarrow \sqrt{2} \cdot x \, x + \sqrt{2} \cdot \sqrt{2} \cdot x + 5x + 5\sqrt{2} = 0$$

$$\Rightarrow \sqrt{2}x(x + \sqrt{2}) + 5(x + \sqrt{2}) = 0$$

$$\Rightarrow (x + \sqrt{2})(\sqrt{2}x + 5) = 0$$

$$\Rightarrow x + \sqrt{2} = 0, \quad \sqrt{2}x + 5 = 0$$

$$\Rightarrow x = -\sqrt{2}, \quad \sqrt{2}x = -5$$

$$x = \frac{-5}{\sqrt{2}} = \frac{-5\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{-5\sqrt{2}}{2}$$

$$\Rightarrow \boxed{x = -\sqrt{2}, \ -\frac{5\sqrt{2}}{2}} \quad \text{Ans}$$

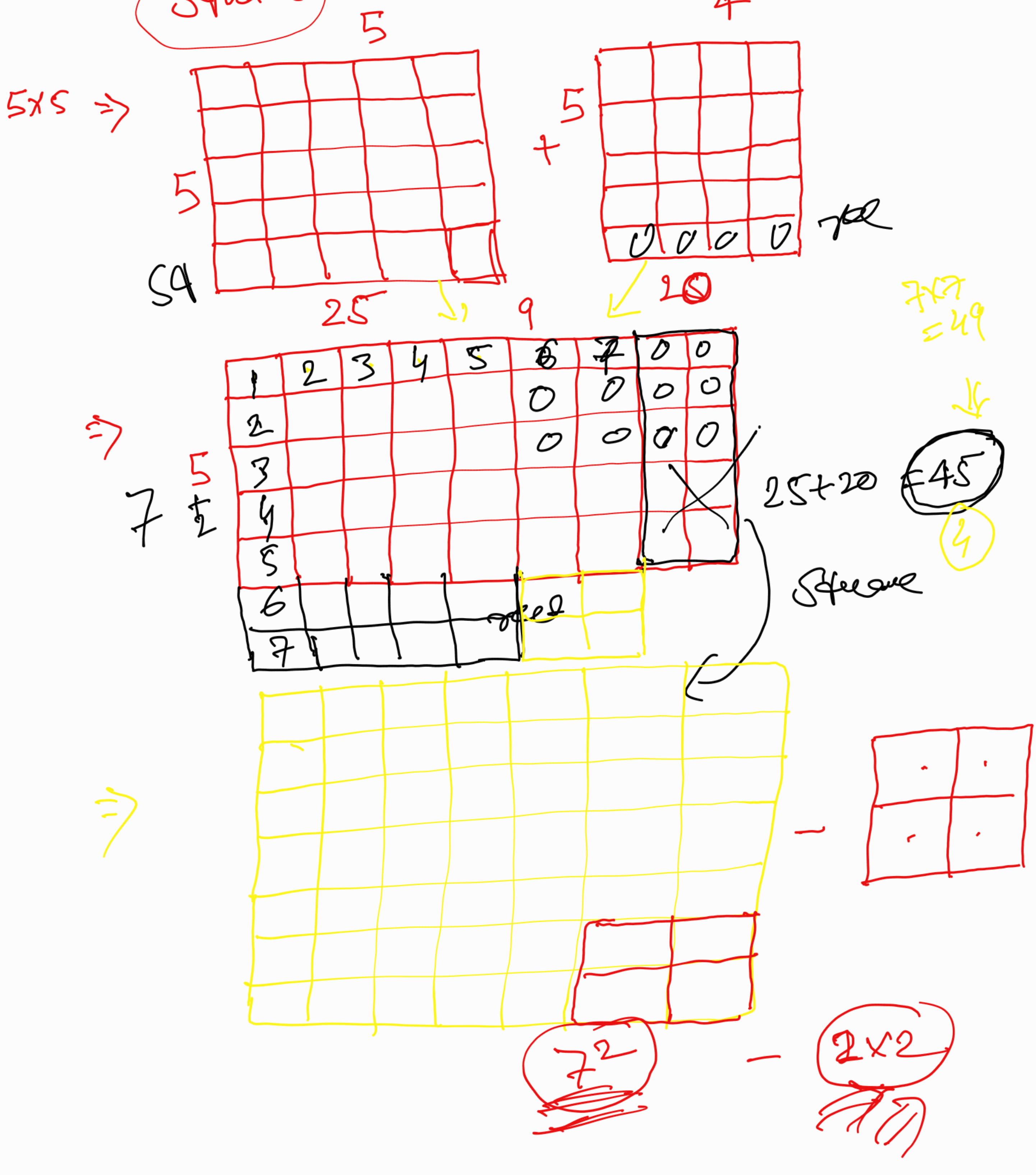
By Completing the Square

$(5\times5) + \boxed{4\times5}$

5^2

Square + rectangle

Square



$$x^2 + 4x - 5 = 0$$

$$\Rightarrow x^2 + 2 \cdot x \cdot 2 + 2^2 - 5 = 0$$

$$\Rightarrow x^2 + 2 \cdot x \cdot 2 + 2^2 - 5 = 0$$

$$\Rightarrow (x+2)^2 - 5 = 0$$

$$\Rightarrow (x+2)^2 = 5$$

$$\Rightarrow x + 2 = \pm\sqrt{5}$$

$$x = -2 \pm \sqrt{5}$$

$$\Rightarrow \boxed{x = -2+\sqrt{5},\ -2-\sqrt{5}}$$

$$(a+b)^2 = a^2 + b^2 + 2ab$$

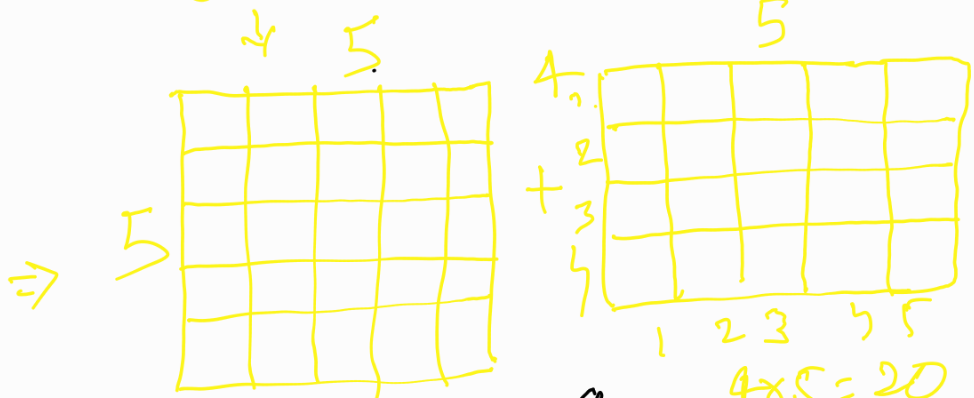
$$\Rightarrow \boxed{a^2 + 2 \cdot a \cdot b + b^2}$$

$$\begin{array}{r} a+b \\ a+b \\ \hline ab + b^2 \\ a^2 + ab \\ \hline a^2 + 2ab + b^2 \end{array}$$

$$(a+b)^2$$

By Completing the Square

$\boxed{5\times5} + \boxed{4\times5} \Rightarrow \boxed{\text{Square}}$?

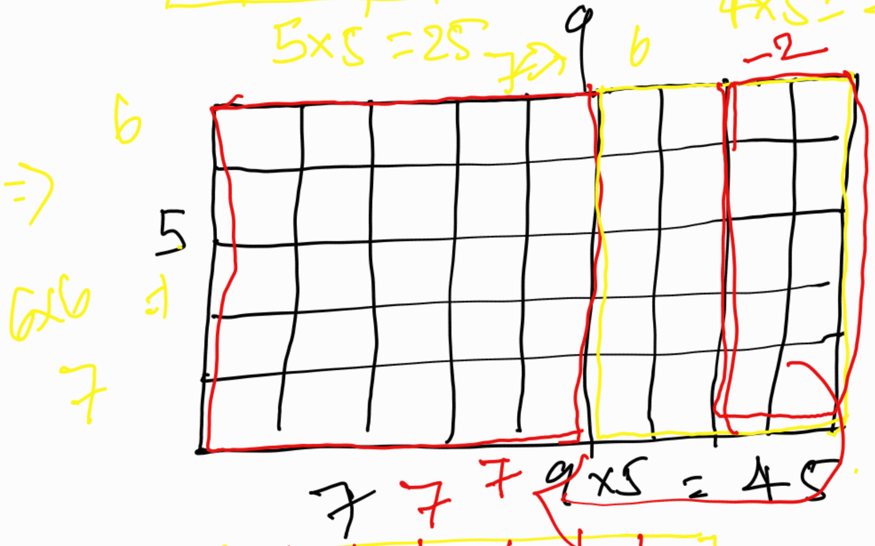


$5\times5 = 25$

$4\times5 = 20$

$\times$ 5×4

$\boxed{25+20 = 45}$



$9\times5 = 45$

rectangle

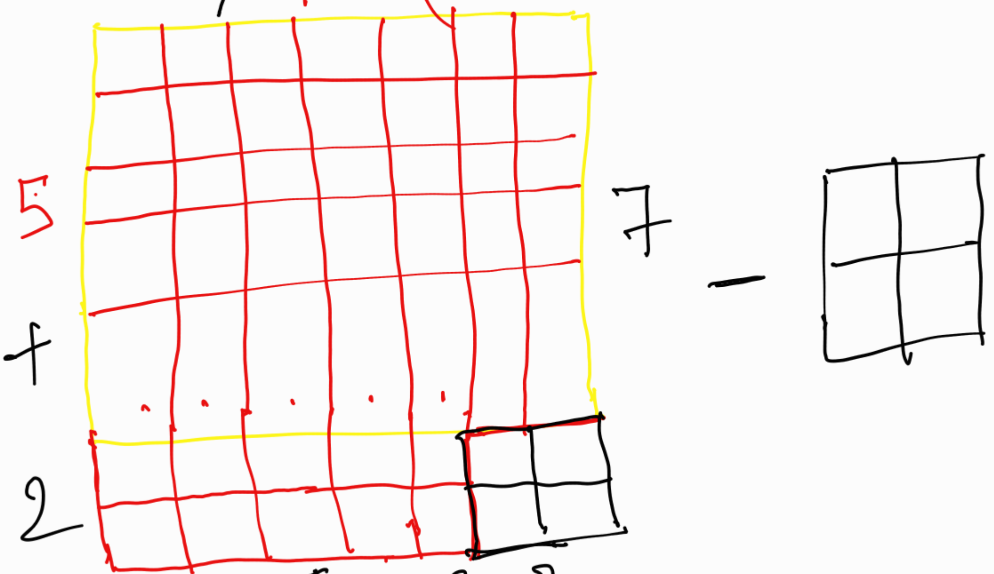
$6\times6 = 36$

$\boxed{45-36 = 9} \Rightarrow$

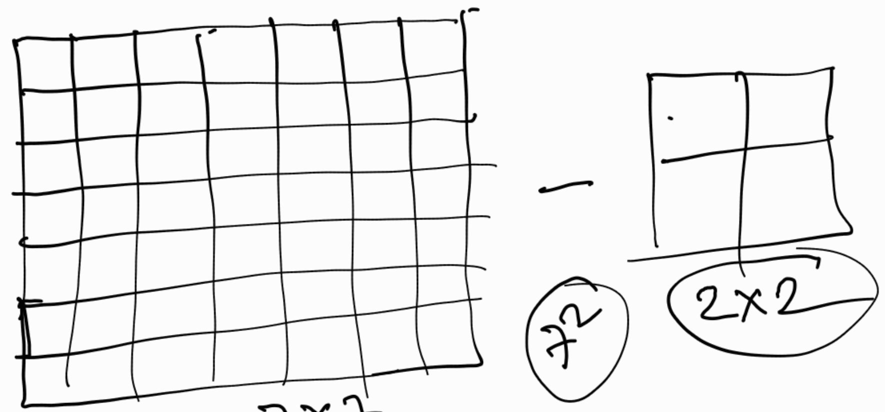
$7\times7 = 49$

45

-4



$5. + 2 = 7$



7×7

7^2

2×2

$$x^2+4x-5=0$$

$$\Rightarrow x\cdot x+2\cdot 2\cdot x-5=0$$

$$\Rightarrow \underset{a\cdot a}{x\cdot x}+\underset{2\cdot a\cdot b}{2\cdot x\cdot 2}+2^2-2^2-5=0$$

$$\Rightarrow (x+2)^2-4-5=0$$

$$\Rightarrow (x+2)^2-9=0$$

$$\Rightarrow (x+2)^2=9$$

$$\Rightarrow x+2=\pm\sqrt{9}=\pm 3$$

$$x=-2\pm 3$$

$$=-2+3,\ -2-3$$

$$=+1,\ -5$$

$$\boxed{x=1,\ x=-5}\ \text{Ans}$$

$$5\times 5+\boxed{4\times 5}=\cancel{}\ 2\times 2$$

$$\cancel{5\times}\ 2\cdot 2\cdot 5=(5+2)^2-2\times 2$$

$$5\times 5+2\cdot 2\cdot 5=(5+2)^2-2^2$$

$$\Rightarrow \underset{a}{5^2}+\underset{2\cdot b\cdot a}{2\cdot 2\cdot 5}+\underset{b}{2^2}=\underset{a+b}{(5+2)^2}$$

$$\Rightarrow (a+b)^2\Rightarrow$$

$$(a+b)^2=a^2+2ab+b^2$$

$$(3x^2-5x+2)=0/3$$

$$a^2+2\cdot a\cdot b+b^2=0$$

$$(a-b)^2=a^2-2ab+b^2$$

$$\Rightarrow x^2-\frac{5}{3}x+\frac{2}{3}=0$$

$$x^2-2\cdot x\cdot\frac{5}{3}\cdot\frac{1}{2}+\frac{2}{3}=0$$

$$5/3=2$$

$$\Rightarrow \underset{a}{x^2}-\underset{2\cdot a\cdot b}{2\cdot x\cdot\frac{5}{6}}+\frac{2}{3}=0$$

$$\Rightarrow x^2-2\cdot x\cdot\frac{5}{6}+\frac{25}{36}-\frac{25}{36}+\frac{2}{3}=0$$

$$x^2-2\cdot x\cdot\frac{5}{6}+\left(\frac{5}{6}\right)^2-\frac{25}{36}+\frac{2}{3}=0$$

$$\Rightarrow \left(x-\frac{5}{6}\right)^2-\frac{25-24}{36}=0$$

$$\Rightarrow \left(x-\frac{5}{6}\right)^2-\frac{1}{36}=0$$

$$\Rightarrow \left(x-5/6\right)^2=1/36$$

$$x-5/6=\pm\sqrt{1/36}$$

$$x-\frac{5}{6}=\pm 1/6$$

$$x=\frac{5}{6}\pm\frac{1}{6}$$

$$x=\frac{5}{6}+1/6,\ 5/6-1/6$$

$$\Rightarrow \frac{6}{6},\ 4/6=2/3$$

$$\Rightarrow (1,\ 2/3)$$

$$\boxed{x=1,\ x=2/3}$$

$$\alpha = -\frac{b}{2a} + \frac{\sqrt{b^2-4ac}}{2a}$$
$$\beta = -\frac{b}{2a} - \frac{\sqrt{b^2-4ac}}{2a}$$

$$ax^2 + bx + c = 0 \quad \text{— (1)}$$

$$\Rightarrow \frac{1}{a}\left(ax^2 + bx + c\right) = \frac{0}{a} = 0$$

$$\Rightarrow \frac{ax^2}{a} + \frac{bx}{a} + \frac{c}{a} = 0$$

$$\Rightarrow x^2 + \frac{b}{a}\cdot x + \frac{c}{a} = 0 \qquad a^2+2ab+b^2$$

$$x^2 + 2\cdot x\cdot \frac{b}{a} + b^2$$

$$\Rightarrow x^2 + 2\cdot x\cdot \frac{b}{a}\cdot\frac{1}{2} + \frac{c}{a} = 0$$

$$\Rightarrow x^2 + 2\cdot x\cdot \frac{b}{2a} + \frac{c}{a} = 0$$

$$\Rightarrow x^2 + 2\cdot x\cdot \frac{b}{2a} + \frac{b^2}{4a^2} - \frac{b^2}{4a^2} + \frac{c}{a} = 0$$

$$a \quad 2\cdot a\cdot b + b^2$$

$$\Rightarrow \left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{c}{a} = 0$$

$$\Rightarrow \left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a} = \frac{b^2-4ac}{4a^2}$$

$$\Rightarrow x + \frac{b}{2a} = \pm\sqrt{\frac{b^2-4ac}{4a^2}} = \pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$\Rightarrow x = -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$$

$$\alpha = -\frac{b}{2a} + \frac{\sqrt{b^2-4ac}}{2a} \Rightarrow \frac{-b+\sqrt{b^2-4ac}}{2a}$$

$$\beta = \frac{-b}{2a} - \frac{\sqrt{b^2-4ac}}{2a} \Rightarrow \frac{-b-\sqrt{b^2-4ac}}{2a}$$

Solve Quadratic Eq. by Median Method

1. By completing the square
2. factorization
3. by Median Method:

Basic Structure $ax^2+bx+c=0$, where $a \neq 0$

$\Rightarrow \frac{1}{a}(ax^2+bx+c)=0$

$\Rightarrow \frac{\cancel{a}x^2}{\cancel{a}}+\frac{bx}{a}+\frac{c}{a}=0/a$

$\Rightarrow \boxed{x^2+\left(\frac{b}{a}\right)\cdot x+\left(\frac{c}{a}\right)=0}$ — (1)

$\boxed{b/a=b', \quad c/a=c'}$

$\Rightarrow \boxed{x^2+b'x+c'=0}$ — (A)

2 sol^n: $\boxed{\alpha, \beta}$

$(\alpha, \beta) \Rightarrow \boxed{(x-\alpha)(x-\beta)=ax^2+bx+c}$

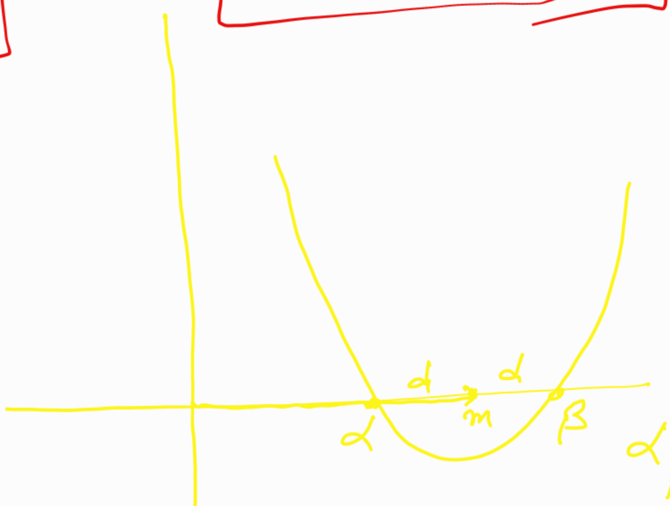
$(x-\alpha)(x-\beta)$

$= x^2-\alpha x-\beta x+\alpha\beta$

$\Rightarrow x^2-x(\alpha+\beta)+\alpha\beta = x^2+b'x+c'$

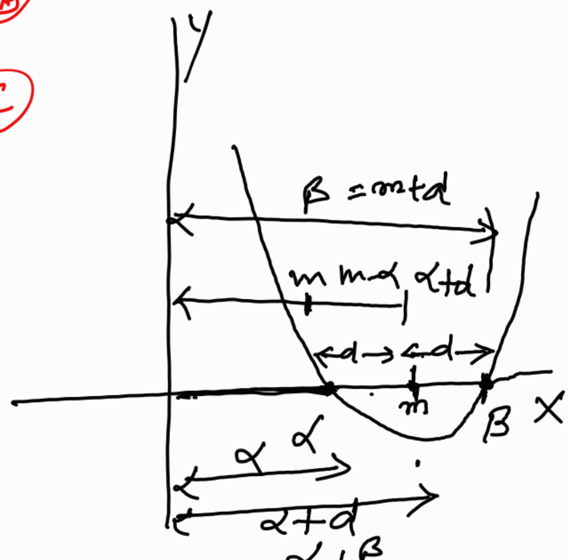
$\boxed{\begin{aligned} b' &= -(\alpha+\beta) \quad \text{— (B)} \\ c' &= \alpha\beta \quad \text{— (C)} \end{aligned}}$

$\boxed{\alpha, \beta}$



$\alpha = m-d$

$\beta = m+d$



$m=\frac{\alpha+\beta}{2}$

$\alpha = m - d$
$\beta = m + d$

$b' = -(\alpha+\beta)$
$c' = \alpha\beta$

$c' = (m-d)(m+d) = m^2 - d^2$

$\therefore \boxed{c' = m^2 - d^2} \to F_1$

$b' = -(\alpha+\beta)$
$c' = \alpha\beta$
$c' = m^2 - d^2$

$b' = -(\alpha+\beta)$

$\Rightarrow \quad = -\boxed{\frac{1}{2}(\alpha+\beta)} \times 2$

$\frac{\alpha+\beta}{2} = m$

$b' = -m \times 2$

$-m = \frac{b'}{2}$

$\boxed{m = -\frac{b'}{2}}$

$c' = m^2 - d^2$
$m = -\frac{b'}{2}$ $\longrightarrow F$

$x^2 + b'x + c' = 0$

(1) $2x^2 - 5x + 3 = 0$

$\Rightarrow x^2 - \frac{5}{2}x + \frac{3}{2} = 0$

$b' = -\frac{5}{2}, \; c' = \frac{3}{2}$

$m = -\frac{b'}{2} = -\frac{\left(-\frac{5}{2}\right)}{2} = +\frac{5}{4}$

$c' = m^2 - d^2 \Rightarrow c' = \left(\frac{5}{4}\right)^2 - d^2$

$\Rightarrow \frac{3}{2} = \frac{25}{16} - d^2$

$\Rightarrow d^2 = \frac{25}{16} - \frac{3}{2} = \frac{25-24}{16} = \frac{1}{16}$

$\Rightarrow \boxed{d = \pm\frac{1}{4}}$

$\alpha = m - d$
$\beta = m + d$

$\alpha, \beta = m \pm d$
$= \frac{5}{4} \pm \frac{1}{4} \Rightarrow \frac{5}{4} + \frac{1}{4}, \frac{5}{4} - \frac{1}{4}$
$\Rightarrow \frac{6}{4}, \frac{4}{4} = \boxed{\frac{3}{2}, 1}$

$\boxed{\alpha, \beta = 3/2, 1}$ Ans

(2) $x^2+4x-5=0$ $\qquad \boxed{x^2+b'x+c'=0}$

$b'=4,\ c'=-5$

$m=-\frac{b'}{2}=-\frac{4}{2}=-2$

$\therefore \boxed{m=-2}$

$c'=m^2-d^2$

$-5=(-2)^2-d^2=4-d^2$

$d^2=4+5=9$

$d=\pm 3$

$m=-2,\ d=\pm 3$

$\alpha,\beta=m\pm d$

$=-2\pm 3$

$=-2+3,\ -2-3$

$=1,-5$

$\therefore \boxed{\alpha,\beta\ (1,-5)}$

Ans

(3) $3x^2-4x+5=0$ $\qquad \boxed{x^2+b'x+c'=0}$

$x^2-\frac{4}{3}x+\frac{5}{3}=0$

$b'=-\frac{4}{3},\ c'=5/3$

$m=-\frac{b'}{2}=+\frac{4}{3\times 2}=+\frac{2}{3}$

$c'=m^2-d^2$

$\Rightarrow \frac{5}{3}=(2/3)^2-d^2$

$\Rightarrow d^2=\frac{4}{9}-\frac{5}{3}$

$\Rightarrow d^2=\frac{4-15}{9}=\frac{-11}{9}$

$d=\pm\sqrt{\frac{-11}{9}}$

$\alpha,\beta=m\pm d$

$=\frac{2}{3}\pm\sqrt{\frac{-11}{9}}$ Ans

$ax^2+bx+c=0$

$x^2+\left(\frac{b}{a}\right)x+c/a=0$

$\boxed{m=-\frac{b}{2a}}$

$m=-\frac{b'}{2}\Rightarrow m=-\frac{b/a}{2}=-\frac{b}{2a}$

$c'=m^2-d^2$

$c/a=\left(-\frac{b}{2a}\right)^2-d^2\Rightarrow d^2=\left(\frac{b}{2a}\right)^2-c/a$

$\Rightarrow d^2=\frac{b^2}{4a^2}-c/a=\frac{b^2-4ac}{4a^2}$

$\Rightarrow d=\pm\sqrt{\frac{b^2-4ac}{4a^2}}=\pm\frac{\sqrt{b^2-4ac}}{2a}$

$\alpha, \beta = m \pm d$

$= -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$

$= \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\boxed{\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a} \quad \beta = \frac{-b-\sqrt{b^2-4ac}}{2a}}$

$x^2 \quad \boxed{(x+\alpha)(x-\alpha)} = x^2 - \alpha^2$ $\quad \alpha = 1$

$\Rightarrow \boxed{(x+1)(x-1) = x^2-1}$

$(2+1)\times(2-1)$	3×1	$\leftarrow$	$2^2 = 4-1 = 3$
$(3+1)\times(3-1)$	4×2	$\leftarrow$	$3^2 = 9-1 = 8$
$(4+1)\times(4-1)$	5×3	$\leftarrow$	$4 = 16-1 = 15$
$(5+1)\times(5-1)$	6×4	$\leftarrow$	$5 = 25-1 = 24$
$(6+1)\times(6-1)$	7×5	$\leftarrow$	$6 = 36-1 = 35$
$(7+1)\times(7-1)$	8×6	$\leftarrow$	$7 = 49-1 = 48$

$\overset{20}{400} - 1 = 399$

$(20+1)(20-1) = 21\times19$

900 → 899 → $\boxed{31\times29}$

$624 = 25^2$

$(25+1)(25-1)$ $\quad \boxed{(x-1)(x+1)}$

$= 26\times24$

$1000\times1000 = \boxed{1000000}$

$= \boxed{999999}+1$

$\boxed{1001\times999}$

$$(a+b)(a+b)$$
$$= a\times(a+b) + b(a+b)$$
$$= a\times a + a\times b + b\times a + b\times b$$
$$= a^2 + ab + ab + b^2$$
$$= a^2 + 2ab + b^2$$

$$\begin{array}{r} 12 \\ \times\ 12 \\ \hline 24 \\ 12 \\ \hline \boxed{144} \end{array}$$

$$\boxed{(a+b)^2 = a^2 + 2ab + b^2}$$ (I)

$$\Rightarrow (a-b)(a-b)$$
$$a\times(a-b) - b\times(a-b)$$
$$= a\times a - a\times b - b\times a + b^2$$
$$= a^2 - ab - ab + b^2$$
$$= a^2 - 2ab + b^2$$

$$\boxed{(a-b)^2 = a^2 - 2ab + b^2}$$ (II)

$$(a+b)(a+b)(a+b)$$
$$= (a+b)\left[a(a+b) + b(a+b)\right]$$
$$= (a+b)\left[a^2 + ab + ab + b^2\right]$$
$$= (a+b)\left[a^2 + 2ab + b^2\right]$$
$$= a(a^2 + 2ab + b^2) + b(a^2 + 2ab + b^2)$$
$$= a^3 + 2a^2b + ab^2 + ba^2 + 2ab^2 + b^3$$
$$= a^3 + 2a^2b + ab^2 + a^2b + 2ab^2 + b^3$$
$$= a^3 + 3a^2b + 3ab^2 + b^3$$
$$= a^3 + 3ab(a+b) + b^3$$

$$(a+b)^3$$
$$= a^3 + b^3 + 3ab(a+b)$$
$$\Rightarrow a^3 + 3ab(a+b) + b^3$$ (III)

$$(a-b)(a-b)(a-b)$$

$$(a-b)^3 = a^3 - 3ab(a-b) - b^3$$

$$= (a-b)(a-b)^2$$

$$= (a-b)\left(a^2 - 2ab + b^2\right)$$

$$= a(a^2-2ab+b^2) - b(a^2-2ab+b^2)$$

$$= a^3 - 2a^2b + ab^2 - a^2b + 2ab^2 - b^3$$

$$= a^3 - 3a^2b + 3ab^2 - b^3$$

$$= a^3 - 3ab(a-b) - b^3 \Rightarrow$$

$$(6x^3 + 5x^2 + 5abx + 6)(5x^2 + 6abx + 7)$$

$$\Rightarrow \quad 6x^3 + 5x^2 + 5abx + 6$$

$$\times \quad + 5x^2 + 6abx + 7$$

$$42x^3 + 35x^2 + 35abx + 42$$

$$36abx^4 + 30abx^3 + 30a^2b^2x^2 + 36abx$$

$$30x^5 + 25x^4 + 25abx^3 + 30x^2$$

$$+42x^3 + 55abx^3 + 30a^2b^2x^2 + 65x^2 + 71abx + 42$$

$$30x^5 + 25x^4 + 36abx^4$$