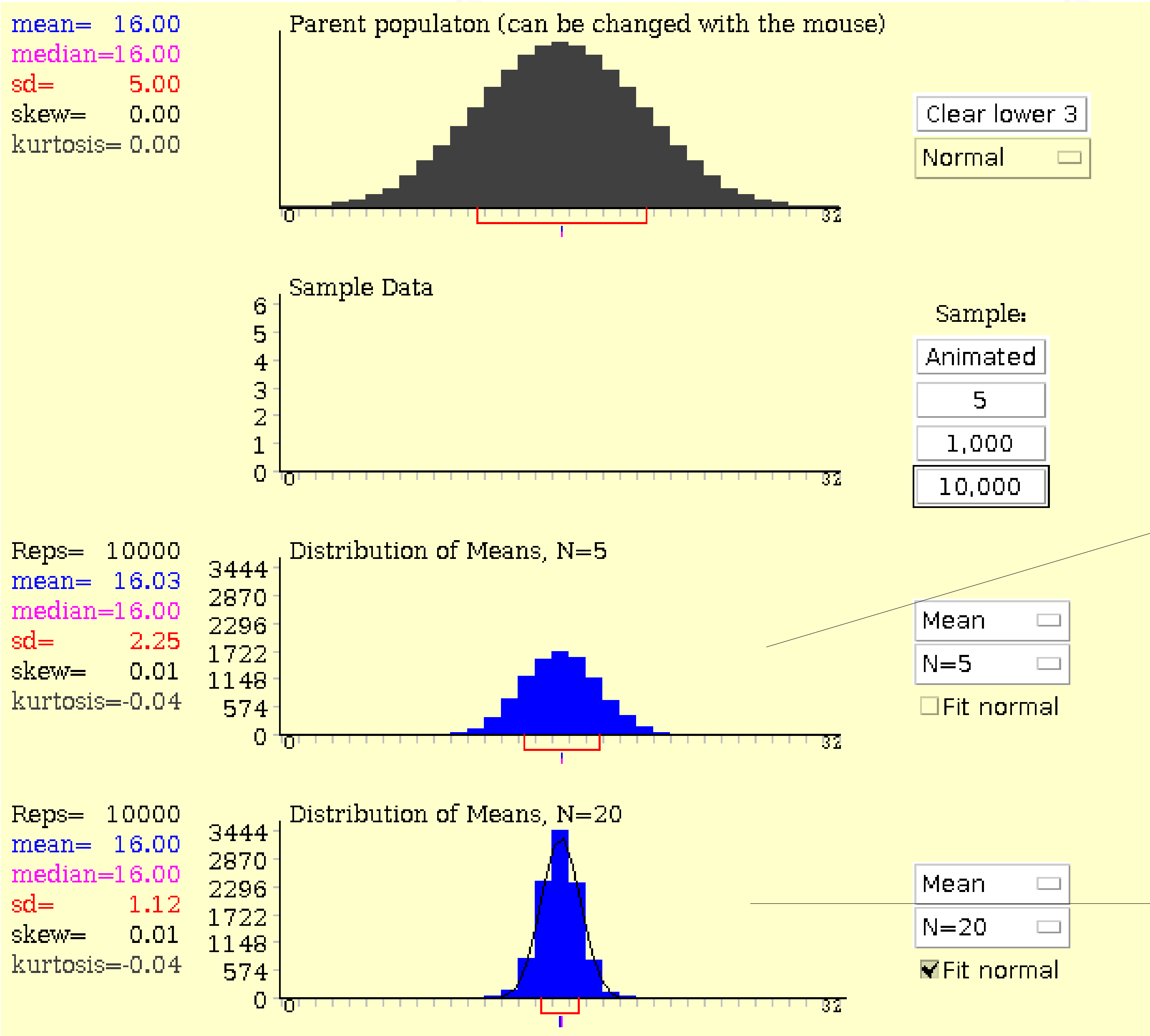
Central Limit Theorem And Confidence Interval

The central limit theorem (CLT) is a statistical theory that states that given a sufficiently large sample size from a population with a finite level of variance, the mean of all samples from the same population will be approximately equal to the mean of the population.

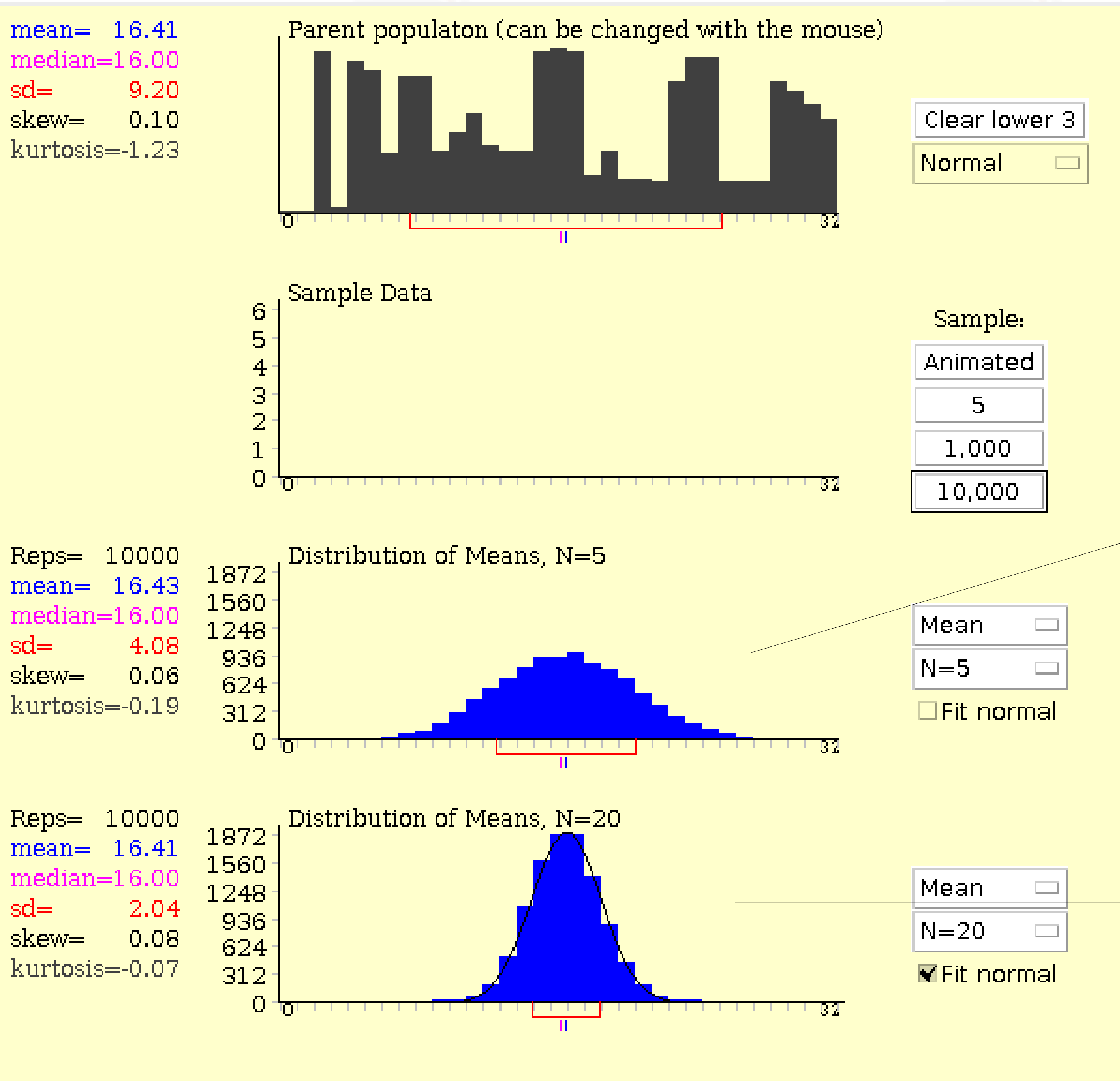
The central limit theorem states that: Given a population with a finite mean μ and a finite non-zero variance σ2, the sampling distribution of the mean approaches a normal distribution with a mean of μ and a variance of σ2/N as N, the sample size, increases.

Confidence Interval: A range of values that defined there is a specified probability that the value of a parameter lies within it.



Population SD = 5
Standard Error(SD) = 5/sqrt(5)
= 2.23606
= 2.24
Almost equal to sampling dist mean SD – 2.25
(vice-versa is not true)

Population SD = 5
Standard Error(SD) = 5/sqrt(20)
= 1.118033
= 1.12
Equal to sample dist mean SD – 1.12
(vice-versa is not true)



Population SD = 9.20
Standard Error(SD) = 9.20/sqrt(5)
= 4.11436
= 4.11
Almost equal to sample dist mean SD – 4.08
(vice-versa is not true)

Population SD = 9.20
Standard Error(SD) = 9.20/sqrt(20)
= 2.05718
= 2.06
Almost equal to sample dist mean SD – 2.04
(vice-versa is not true)

Example: Sample 36 apples form your farm's harvest of over 2,00,000 apples. The mean weight of the sample is 112 grams (with a 40 gram sample standard deviation). What is the probability that the mean weight of all 2,00,000 apples is within 100 and 124 grams ?

Useful information:
The Sample Size(36)
Sample Mean(112 grams)
Standard Deviation(40) – best estimator of population SD

What to find?
* Probability of *the mean weight of population within 100 and 124 grams (I.e.,* confidence interval of the sampling distribution of mean*)*

* Think that we took many samples of size 36 (sampling distribution), the sampling distribution mean is 112 grams.
* From Central Limit Theorem – the sampling distribution mean (112) is same as the population mean.
* Now we have to calculate standard deviation of the *sampling distributions of means* as we have to find probability of *mean weight of the population between 100 and 124*

* From Central Limit Theorem :- sample standard error = Population SD/Sqrt(sample size)
= 40/Sqrt(36)
= 40/6 = 6.66667
= 6.67
* **12** is **1.8** Standard Deviations away form 112. (100, 124 are -12, +12 away from 112. 124-112/6.6667 = 12/6.6667 = 1.799)

* **Confidence Interval:** Lets find out probability of the ***mean apple wait*** is between **mean** and **1.8 standard deviations above the mean** by looking at Z-Table. This value need to be doubled to find confidence interval.

Standard Normal Probabilities

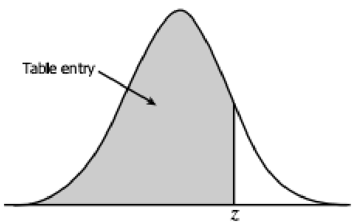
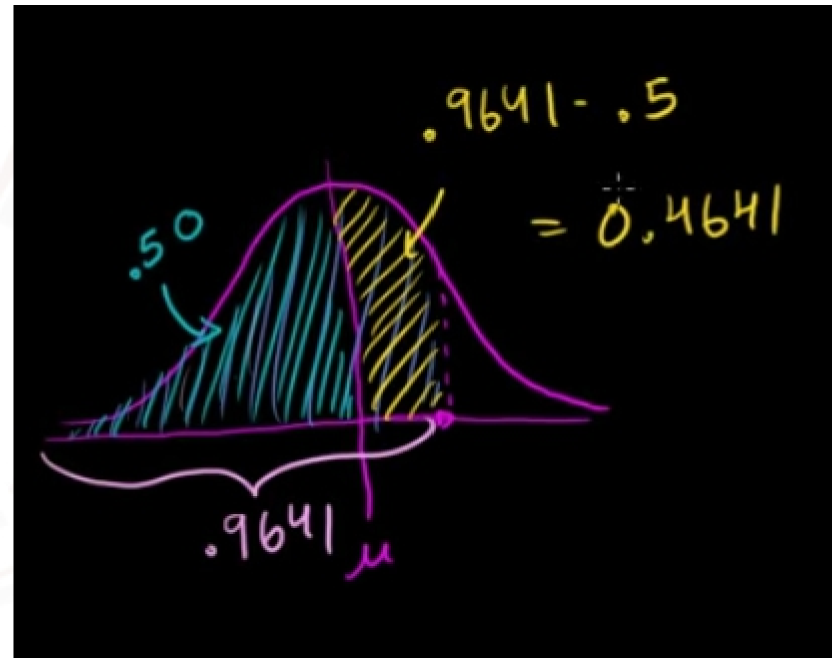


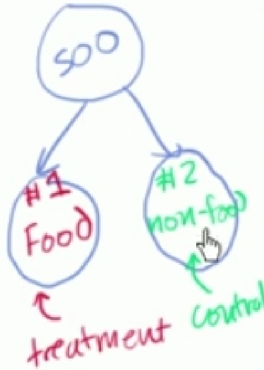
Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998



* The confidence interval is **2 x 0.4642 = 0.9282.**
* We are **92.82% confident that the population mean is between 100 and 124 range.**

Statistical Significance

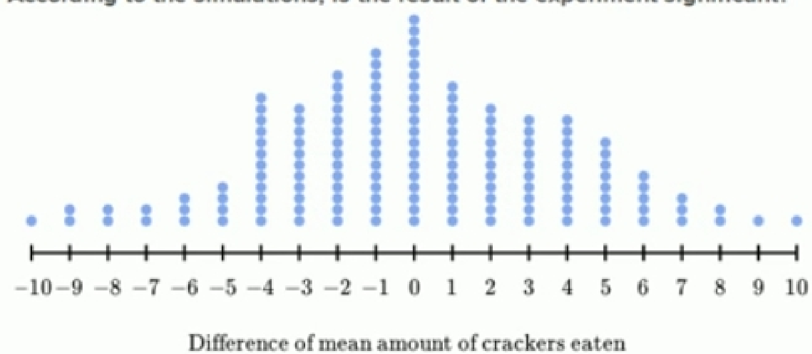


In an experiment aimed at studying the effect of advertising on eating behavior in children, a group of 500 children, 7-11 years old, were randomly assigned to two different groups. After randomization, each child was asked to watch a cartoon in a private room, containing a large bowl of goldfish crackers. The cartoon included 2 commercial breaks.

The first group watched food commercials (mostly snacks), while the second group watched non-food commercials (games and entertainment products). Once the child finished watching the cartoon, the conductors of the experiment weighed the crackers bowl to measure how many grams of crackers the child ate. They found that the mean amount of crackers eaten by the children who watched food commercials is 10 grams greater than the mean amount of crackers eaten by the children who watched non-food commercials.

Using a simulator, they re-randomized the results into two new groups and measured the difference between the means of the new groups. They repeated this simulation 150 times, and plotted the resulting differences, as given below.

According to the simulations, is the result of the experiment significant?



Significance Test: The probability of some thing **randomly happening** is less than (significance level – alpha) 5% then a we will consider the experiment as statistically significant.

Here the above experiment states that the kids who watched food commercials eat 10 grams more that the other group. The dot-plot of the group means after re-randomization shows that only twice the difference in mean is 10 grams. Only one it is positive 10 grams.

This states that the kids who watch food commercials eat 10 grams more that the other group at random is only 1.33%. Which is less than 5%.

Hence the experiment is significant. That is the kids who watched food commercials eat 10 grams more than the other group.