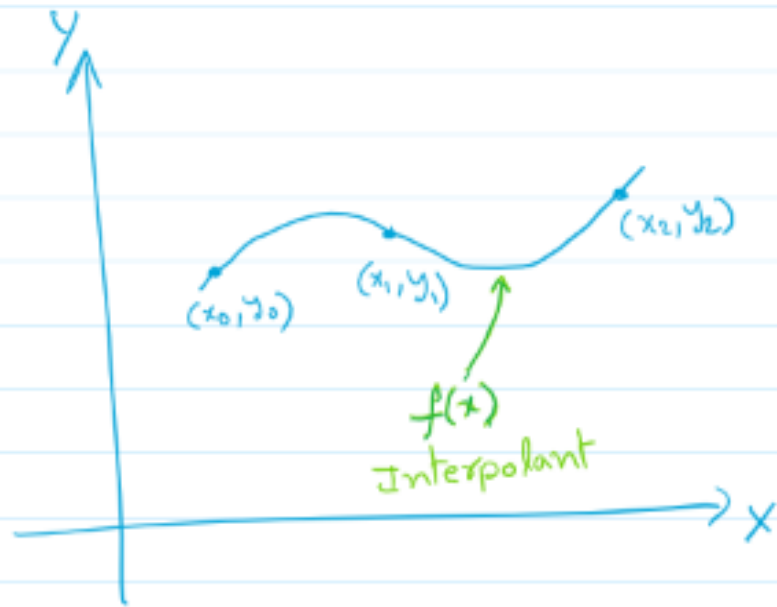
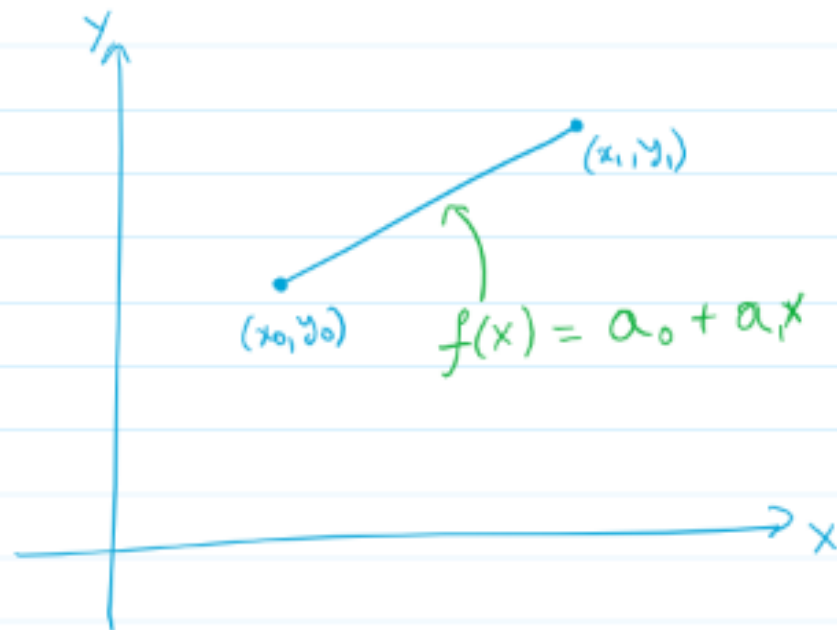


Interpolation



Linear Interpolation



time(s)	velocity(m/s)
0	0
10	227.04
15	362.78
20	517.35
22.5	602.97
30	901.67

$$v(t) = a_0 + a_1 t$$

→ we have to find two unknowns (a_0, a_1) , hence we have to take two equations $362.78 = a_0(1) + a_1(15)$

$$517.35 = a_0(1) + a_1(20)$$

Note: (1) Inverse of a 2×2 matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$(2) \text{ Identity matrix } I = A A^{-1} = A^{-1} A$$

→ Solution 1:

$$\begin{bmatrix} 362.78 \\ 517.35 \end{bmatrix} = \begin{bmatrix} 1 & 15 \\ 1 & 20 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 15 \\ 1 & 20 \end{bmatrix}^{-1} \begin{bmatrix} 362.78 \\ 517.35 \end{bmatrix} = \begin{bmatrix} 1 & 15 \\ 1 & 20 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 15 \\ 1 & 20 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$$

$$\frac{1}{1 \times 20 - 1 \times 15} \begin{bmatrix} 20 & -15 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 362.78 \\ 517.35 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$$

$$\frac{1}{5} \begin{bmatrix} 20 & -15 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 362.78 \\ 517.35 \end{bmatrix} = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -3 \\ -1/5 & 1/5 \end{bmatrix} \begin{bmatrix} 362.78 \\ 517.35 \end{bmatrix} = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$$

$$\begin{bmatrix} 4 \times 362.78 - 3 \times 517.35 \\ -1/5 \times 362.78 + 1/5 \times 517.35 \end{bmatrix} = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$$

$$\begin{bmatrix} 1451.12 - 1552.05 \\ -72.56 + 103.47 \end{bmatrix} = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$$

$$\begin{bmatrix} -100.93 \\ 30.91 \end{bmatrix} = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$$

Hence $a_0 = -100.93$

$$a_1 = 30.91$$

$$\Rightarrow V(t) = -100.93 + 30.91t$$

To find velocity at 16 seconds

$$\begin{aligned} V(16) &= -100.93 + 30.91 \times 16 \\ &= -100.93 + 494.56 \\ &= 393.63 \text{ m/s} \end{aligned}$$

→ Solution 2:

$$\text{step 1) } a_0 + a_1 \cdot 15 = 362.78 \rightarrow \textcircled{\text{eq1}}$$

$$a_0 + a_1 \cdot 20 = 517.35 \rightarrow \textcircled{\text{eq2}}$$

step 2) Subtract $\textcircled{\text{eq1}}$ from $\textcircled{\text{eq2}}$

$$\begin{array}{r} a_0 + a_1 \cdot 20 = 517.35 \\ \cancel{a_0 + a_1 \cdot 15} = \cancel{362.78} \\ \hline a_1 \cdot 5 = 154.57 \end{array}$$

$$\begin{aligned} \text{step 3) } a_1 &= 154.57/5 \\ &= 30.914 \end{aligned}$$

step 4) Substitute a_1 in $\textcircled{\text{eq1}}$ or $\textcircled{\text{eq2}}$

$$a_0 + 30.914 \times 15 = 362.78$$

$$a_0 = 362.78 - 463.71$$

$$= -100.93$$