

main.tex

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1 Introduction

2 Theory

The Navier-Stokes equations are the fundamental equations of fluid mechanics. Using standard Einstein summation convention in cartesian coordinates, they are given as

$$\rho \frac{\partial u_i}{\partial t} + \rho \left(u_j \frac{\partial u_i}{\partial x_j} \right) = -\frac{\partial p}{\partial x_i} + \mu \frac{\partial}{\partial x_j} \frac{\partial u_i}{\partial x_j} + f_i, \quad (1)$$

with u being the velocity, x is position, p is pressure, and f is external forces on the body.

So then, the average of the momentum equations become

$$\begin{aligned} \rho \left(\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} \right) &= -\frac{\partial P}{\partial x_i} + \frac{\partial T_{ij}}{\partial x_j} + \frac{\partial R_{ij}}{\partial x_j} + \langle f_i \rangle \\ &= \frac{\partial}{\partial x_j} (-p\delta_{ij} + T_{ij} + R_{ij}) + \langle f_i \rangle \end{aligned} \quad (2)$$

with

$$T_{ij} = \mu \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) \quad (3)$$

$$R_{ij} \equiv -\langle \rho u'_i u'_j \rangle. \quad (4)$$