main.tex

Henrik Linder

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1 Introduction

2 Theory

The Navier-Stokes equations are the fundamental equations of fluid mechanics. Using standard Einstein summation convention in cartesian coordinates, they are given as

$$\rho\frac{\partial u_i}{\partial t} + \rho\left(u_j\frac{\partial u_i}{\partial x_j}\right) = -\frac{\partial p}{\partial x_i} + \mu\frac{\partial}{\partial x_j}\frac{\partial u_i}{\partial x_j} + f_i, \tag{1}$$

with u being the velocity, x is position, p is pressure, and f is external forces on the body.

So then , the averga of the momentum equations become

$$\begin{aligned}\rho\left(\frac{\partial U_i}{\partial t} + U_j\frac{\partial U_i}{\partial x_j}\right) &= -\frac{\partial P}{\partial x_i} + \frac{\partial T_{ij}}{\partial x_j} + \frac{\partial R_{ij}}{\partial x_j} + \langle f_i\rangle \\ &= \frac{\partial}{\partial x_j}\left(-p\delta_{ij} + T_{ij} + R_{ij}\right) + \langle f_i\rangle\end{aligned} \tag{2}$$

with

$$T_{ij} = \mu\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) \tag{3}$$

$$R_{ij} \equiv -\langle\rho u_i' u_j'\rangle. \tag{4}$$