Spring 2021
PHYS 377 Advanced Computational Physics
HW # 5a

<u>Problem 1:</u> Wien's displacement constant

Planck's radiation law tells us that the intensity of radiation per unit area and per unit wavelength λ from a black body at temperature T is

$$I(\lambda) = \frac{2\pi hc^2\lambda^{-5}}{e^{hc/\lambda k_B T} - 1}$$

where h is Planck's constant, c is the speed of light, and k_B is Boltzmann's constant.

(a) (5 points) **Show by differentiating** that the wavelength λ at which the emitted radiation is strongest is the solution of the equation

$$5e^{-hc/\lambda k_B T} + \frac{hc}{\lambda k_B T} - 5 = 0$$

Make the substitution $x = hc/\lambda k_B T$ and hence show that the wavelength of maximum radiation obeys the Wien displacement law:

$$\lambda = \frac{b}{T}$$

where the so-called ***Wien displacement constant*** is

$$b = \frac{hc}{k_B x}$$

and x is the solution to the nonlinear equation

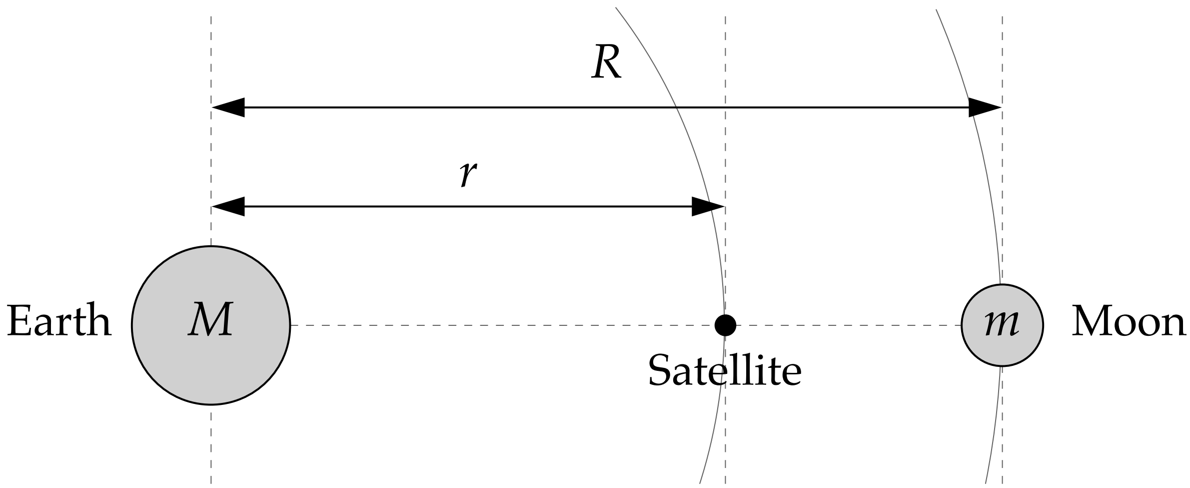
$$5e^{-x} + x - 5 = 0$$

(b) (45 points) **Write a program** to solve this equation to an accuracy of 10^{-6} using the **binary search** method, and hence find a value for the displacement constant.

(c) (5 points) The displacement law is the basis for the method of *optical pyrometry*, a method for measuring the temperatures of objects by observing the color of the thermal radiation they emit. The method is commonly used to estimate the surface temperatures of astronomical bodies, such as the Sun. The wavelength peak in the Sun's emitted radiation falls at λ = 502 nm. From the equations above and your value of the displacement constant, estimate the surface temperature of the Sun.

Problem 2: The Lagrange point

(45 points) There is a magical point between the Earth and the Moon, called the L_1 Lagrange point, at which a satellite will orbit the Earth in perfect synchrony with the Moon, staying always in between the two. This works because the inward pull of the Earth and the outward pull of the Moon combine to create exactly the needed centripetal force that keeps the satellite in its orbit. Here's the setup:



Assuming circular orbits, and assuming that the Earth is much more massive than either the Moon or the satellite, show that the distance r from the center of the Earth to the L_1 point satisfies

$$\frac{GM}{r^2} - \frac{Gm}{(R-r)^2} = \omega^2 r$$

where M and m are the Earth and Moon masses, G is Newton's gravitational constant, and ω is the angular velocity of both the Moon and the satellite.

The equation above is a *fifth-order polynomial equation* in r (also called a quintic equation). Such equations cannot be solved exactly in closed form, but it's straightforward to solve them numerically. **Write a program** that uses the **secant method** to solve for the distance r from the Earth to the L_1 point. Compute a solution accurate to at least **four significant figures**.

The values of the various parameters are:

$G = 6.674 \times 10^{-11}$ m^{3}kg^{-1}s^{-2}
$M = 5.974 \times 10^{24}$ kg
$m = 7.348 \times 10^{22}$ kg
$R = 3.844 \times 10^{8}$ m
$\omega = 2.662 \times 10^{-6}$ s^{-1}

You will also need to choose two suitable starting values for r in order to use the **secant method**.