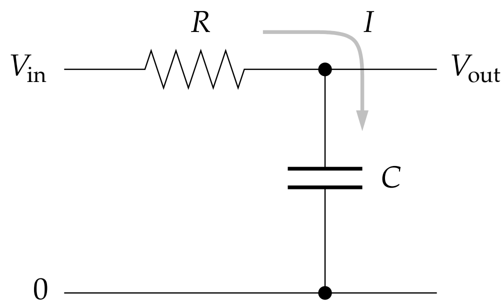
Spring 2021
PHYS 377 Advanced Computational Physics
HW # 3b

Problem 1: A low-pass filter

Here is a simple electronic circuit with one resistor and one capacitor:



This circuit acts as a low-pass filter: you send a signal in on the left and it comes out filtered on the right.

Using Ohm's law and the capacitor law and assuming that the output load has very high impedance, so that a negligible amount of current flows through it, we can write down the equations governing this circuit as follows. Let *I* be the current that flows through *R* and into the capacitor, and let *Q* be the charge on the capacitor. Then:

$$IR = V_{in} - V_{out}\ , \quad Q = CV_{out}\ , \quad I = \frac{dQ}{dt}$$

Substituting the second equation into the third, then substituting the result into the first equation, we find that

$$V_{in} - V_{out} = RC\left(\frac{dV_{out}}{dt}\right)$$

or

$$\frac{dV_{out}}{dt} = \frac{1}{RC}(V_{in} - V_{out})$$

(a) **Write a program** (or modify a previous program) to solve this equation for $V_{out}(t)$ using the ***4^{th}-order Runge–Kutta*** method when the input signal is a square-wave with frequency 1 and amplitude 1:

$$V_{in}(t) = \begin{cases} 1 & if\ \lfloor 2t \rfloor\ is\ even \\ -1 & if\ \lfloor 2t \rfloor\ is\ odd \end{cases}$$

where $\lfloor x \rfloor$ means *x* rounded down to the next lowest integer. Use the program to make plots of the output of the filter circuit from $t = 0$ to $t = 10$ when $RC = 0.01$, 0.1, and 1, with initial condition $V_{out}(0) = 0$. You will have to make a decision about what value of *h* to use in your calculation.

Small values give more accurate results, but the program will take longer to run. Try a variety of different values and choose one for your final calculations that seems sensible to you.
(b) Based on the graphs produced by your program, describe what you see and explain what the circuit is doing.

A program similar to the one you write is running inside most stereos and music players, to create the effect of the "bass" control. In the old days, the bass control on a stereo would have been connected to a real electronic low-pass filter in the amplifier circuitry, but these days there is just a computer processor that simulates the behavior of the filter in a manner similar to your program.

<u>Problem 2:</u> The Lotka–Volterra equations

The **Lotka–Volterra** equations are a mathematical model of predator–prey interactions between biological species. Let two variables x and y be proportional to the size of the populations of two species, traditionally called "rabbits" (the prey) and "foxes" (the predators). You could think of x and y as being the population in thousands, say, so that $x = 2$ means there are 2000 rabbits. Strictly the only allowed values of x and y would then be multiples of 0.001, since you can only have whole numbers of rabbits or foxes. But 0.001 is a pretty close spacing of values, so it's a decent approximation to treat x and y as continuous real numbers so long as neither gets very close to zero.

In the Lotka–Volterra model the rabbits reproduce at a rate proportional to their population, but are eaten by the foxes at a rate proportional to both their own population and the population of foxes:

$$\frac{dx}{dt} = \alpha x - \beta xy$$

where α and β are constants. At the same time the foxes reproduce at a rate proportional the rate at which they eat rabbits—because they need food to grow and reproduce—but also die of old age at a rate proportional to their own population:

$$\frac{dy}{dt} = \gamma xy - \delta y$$

where γ and δ are also constants.

(a) Write a program to solve these equations using the ***4^th^-Order Runge–Kutta*** method for the case $\alpha = 1$, $\beta = \gamma = 0.5$, and $\delta = 2$, starting from the initial condition $x = y = 2$. Have the program make a graph showing both x and y as a function of time on the same axes from $t = 0$ to $t = 30$.

Hint: Use your understanding of solving simultaneous differential equations. You may want to take a look at lecture notes (**Lecture 3b**) on myCourses. Notice that the differential equations in this case do not depend explicitly on time t—in vector notation, the right-hand side of each equation is a function $f(\mathbf{r})$ with no t dependence. You may nonetheless find it convenient to define a *Python* function f(r,t) including the time variable. You don't have to do it that way, but it can avoid some confusion.

(b) Describe in words what is going on in the system, in terms of rabbits and foxes.