Spring 2021
PHYS 377 Advanced Computational Physics
HW # 8b

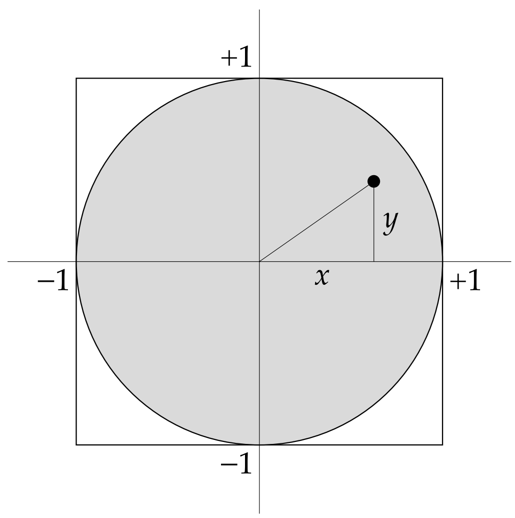
<u>Problem 1 (100 points):</u> Mean value method

$$I = \int_0^2 sin^2\left[\frac{1}{x(2-x)}\right]dx$$

Estimate the above integral using the mean value method with 10000 points (90 points). Also, evaluate the error (10 points).

<u>Extra Credit Problem (40 points):</u> Volume of a hypersphere

This exercise asks you to estimate the volume of a sphere of unit radius in ten dimensions using a Monte Carlo method. Consider the equivalent problem in two dimensions, the area of a circle of unit radius:



The area of the circle, the shaded area above, is given by the integral

$$I = \int\int_{-1}^{+1} f(x,y)dxdy$$

where $f(x,y) = 1$ everywhere inside the circle and zero everywhere outside. In other words,

$$f(x,y) = \begin{cases} 1 & if\ x^2+y^2 \le 1 \\ 0 & otherwise \end{cases}$$

So, if we didn't already know the area of the circle, we could calculate it by Monte Carlo integration. We would generate a set of N random points (x, y), where both x and y are in the range from −1 to 1. Then the two-dimensional version of the below equation

$$I \cong \frac{V}{N} \sum_{i=1}^{N} f(\mathrm{r}_i)$$

for this calculation would be

$$I \cong \frac{4}{N} \sum_{i=1}^{N} f(x_i, y_i)$$

Generalize this method to the ten-dimensional case and **write a program** to perform a **Monte Carlo calculation** of the volume of a sphere of unit radius in ten dimensions.

If we had to do a ten-dimensional integral the traditional way, it would take a very long time. Even with only 100 points along each axis (which wouldn't give a very accurate result) we'd still have $100^{10} = 10^{20}$ points to sample, which is impossible on any computer. But using the Monte Carlo method we can get a pretty good result with a million points or so.