Spring 2021
PHYS 377 Advanced Computational Physics
HW # 7a

Problem 1 (100 points): Thermal diffusion in the Earth's crust

A classic example of a **diffusion problem** with a time-varying boundary condition is the diffusion of heat into the crust of the Earth, as surface temperature varies with the seasons. Suppose the mean daily temperature at a particular point on the surface varies as:

$$T_0(t) = A + Bsin\frac{2\pi t}{\tau}$$

where $\tau = 365$ days, $A = 10°C$ and $B = 12°C$. At a depth of 20 m below the surface almost all annual temperature variation is ironed out and the temperature is, to a good approximation, a constant 11°C (which is higher than the mean surface temperature of 10°C—temperature increases with depth, due to heating from the hot core of the planet). The thermal diffusivity of the Earth's crust varies somewhat from place to place, but for our purposes we will treat it as constant with value $D = 0.1\ m^2\ day^{-1}$.

Write a program, or modify an earlier program, to calculate the **temperature profile** of the crust as a function of depth up to 20 m and time up to 10 years. Start with temperature everywhere equal to 10°C, except at the surface and the deepest point, choose values for the number of grid points and the time-step h, then run your program for the first nine simulated years, to allow it to settle down into whatever pattern it reaches. Then for the tenth and final year plot four temperature profiles taken at 3-month intervals on a single graph to illustrate how the temperature changes as a function of depth and time.