Spring 2021
PHYS 377 Advanced Computational Physics
HW # 4a

<u>Problem 1:</u> The driven pendulum

A **pendulum** (like the one in lecture notes) can be driven by, for example, exerting a small oscillating force horizontally on the mass. Then the equation of motion for the pendulum becomes

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l} sin\theta + Ccos\theta sin\Omega t$$

where C and Ω are constants.

(a) **Write a program** to solve this equation for θ as a function of time with $l = 10$ cm, $C = 2\text{s}^{-2}$ and $\Omega = 5\text{s}^{-1}$ and **make a plot** of θ as a function of time from $t = 0$ to $t = 100$ s. Start the pendulum at rest with $\theta = 0$ and $d\theta/dt = 0$.

(b) Now change the value of Ω, while keeping C the same, to find a value for which the pendulum resonates with the driving force and swings widely from side to side. **Make a plot** for this case also.

<u>Problem 2:</u> Harmonic and anharmonic oscillators

The **simple harmonic oscillator** (SHO) arises in many physical problems, in mechanics, electricity and magnetism, and condensed matter physics, among other areas. Consider the standard **oscillator** equation

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

(a) Using the methods described class, turn this **second-order equation** into **two coupled first-order equations**. Then **write a program** to solve them for the case $\omega = 1$ in the range from $t = 0$ to $t = 50$. A second-order equation requires two initial conditions, one on x and one on its derivative. For this problem use $x = 1$ and $dx/dt = 0$ as initial conditions. Have your program make a graph showing the value of x as a function of time.

(b) Now **increase the amplitude** of the oscillations by making the initial value of x bigger— say $x = 2$—and confirm that the period of the oscillations stays roughly the same.

(c) **Modify your program** to solve for the motion of the **anharmonic oscillator** described by the equation

$$\frac{d^2x}{dt^2} = -\omega^2 x^3$$

Again take $\omega = 1$ and initial conditions $x = 1$ and $dx/dt = 0$ and make a plot of the motion of the oscillator. Again increase the amplitude. You should observe that the oscillator oscillates faster at higher amplitudes. (You can try lower amplitudes too if you like, which should be slower.)

(d) **Modify your program** so that instead of plotting x against t, it plots dx/dt against x, i.e., the "*velocity*" of the oscillator against its "*position*." Such a plot is called a *phase space* plot.

EXTRA CREDIT:

(e) The *van der Pol oscillator*, which appears in electronic circuits and in laser physics, is described by the equation

$$\frac{d^2x}{dt^2} - \mu(1 - x^2)\frac{dx}{dt} + \omega^2 x = 0$$

Modify your program to solve this equation from $t = 0$ to $t = 20$ and hence make a phase space plot for the *van der Pol oscillator* with $\omega = 1$, $\mu = 1$, and initial conditions $x = 1$ and $dx/dt = 0$. Try it also for $\mu = 2$ and $\mu = 4$ (still with $\omega = 1$). Make sure you use a small enough value of the time interval h to get a smooth, accurate phase space plot.