Spring 2021
PHYS 377 Advanced Computational Physics
HW # 4b

<u>Problem 1:</u> Leapfrog method

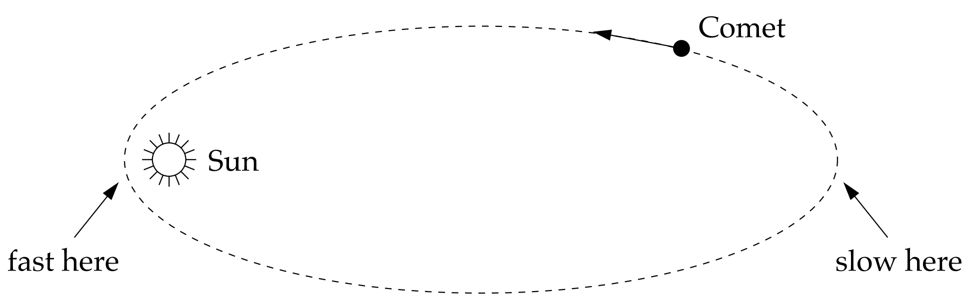
Write a program to solve the differential equation given below using the **leapfrog method**.

$$\frac{d^2x}{dt^2} - \left(\frac{dx}{dt}\right)^2 + x + 5 = 0$$

Solve from $t = 0$ to $t = 50$ in steps of $h = 0.001$ with initial condition $x = 1$ and $dx/dt = 0$. Make a plot of your solution showing x as a function of t.

<u>Problem 2:</u> Orbit of the earth

Many comets travel in highly elongated orbits around the Sun. For much of their lives they are far out in the solar system, moving very slowly, but on rare occasions their orbit brings them close to the Sun for a fly-by and for a brief period of time they move very fast indeed:



The differential equation obeyed by a comet is straightforward to derive. The force between the Sun, with mass M at the origin, and a comet of mass m with position vector $\mathbf{r}$ is GMm/r^2 in direction $-\mathbf{r}/r$ (*i.e.* the direction towards the Sun), and hence Newton's second law tells us that

$$m\frac{d^2\mathbf{r}}{dt^2} = -\left(\frac{GMm}{r^2}\right)\frac{\mathbf{r}}{r}$$

Canceling the m and taking the x and the y component we have

$$\frac{d^2x}{dt^2} = -GM\frac{x}{r^3} \qquad and \qquad \frac{d^2y}{dt^2} = -GM\frac{y}{r^3}$$

where $r = \sqrt{x^2 + y^2}$

Note that we can throw out one of the coordinates because the comet stays in a single plane as it orbits. If we orient our axes so that this plane is perpendicular to the z-axis, we can forget about the z coordinate and we are left with just two second-order equations to solve (given above).

(a) **Turn these** two second-order equations into four first-order equations, using the methods you have learned. No need to explicitly program the equations, just give the breakdown at the start of the program and comment them out.

(b) Use the **Verlet method** to calculate the orbit of the Earth around the Sun. The equations of motion for the position $\mathbf{r} = (x, y)$ of the planet in its orbital plane are the same as those for any orbiting body as given earlier. In vector form, they are

$$\frac{d^2\mathbf{r}}{dt^2} = -GM\frac{\mathbf{r}}{r^3}$$

where $G = 6.6738 \times 10^{-11}$ m^{3}kg^{-1}s^{-2} is Newton's gravitational constant and $M = 1.9891 \times 10^{30}$ kg is the mass of the Sun.

The orbit of the Earth is not perfectly circular, the planet being sometimes closer to and sometimes further from the Sun. When it is at its closest point, or *perihelion*, it is moving precisely tangentially (*i.e.* perpendicular to the line between itself and the Sun) and it has distance 1.4710×10^{11} m from the Sun and linear velocity 3.0287×10^4 ms^{-1}

Write a program to calculate the orbit of the Earth using the **Verlet method**, with a time-step of $h = 1$ hour. Make a plot of the orbit, showing several complete revolutions about the Sun. The orbit should be very slightly, but visibly, non-circular. You can directly use the equations given in the lecture notes.

(c) The gravitational potential energy of the Earth is $-GMm/r$, where $m = 5.9722 \times 10^{24}$ kg is the mass of the planet, and its kinetic energy is ½ mv^2 as usual. **Modify your program** to calculate both of these quantities at each step, along with their sum (which is the total energy), and make a plot showing all three as a function of time on the same axes. You should find that the potential and kinetic energies vary visibly during the course of an orbit, but the total energy remains constant.

(d) Now plot the total energy alone without the others and you should be able to see a slight variation over the course of an orbit. Because you're using the **Verlet method**, however, which *conserves energy* in the long term, the energy should always return to its starting value at the end of each complete orbit.