

**Spring 2021**  
**PHYS 377 Advanced Computational Physics**  
**HW # 8a**

**Problem 1 (20 points): Rolling dice**

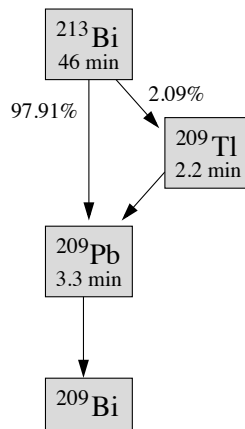
(a) Write a program that generates and prints out two random numbers between 1 and 6, to simulate the rolling of two dice.

(b) Modify your program to simulate the rolling of two dice a million times and count the number of times you get a double six. Divide by a million to get the *fraction* of times you get a double six. You should get something close to, though probably not exactly equal to  $1/36$ .

**Problem 2 (40 points): Radioactive decay chain**

This HW problem looks at a more advanced version of the simple radioactive decay simulation in class notes.

The isotope  $^{213}\text{Bi}$  decays to stable  $^{209}\text{Bi}$  via one of two different routes, with probabilities and half-lives thus:



(Technically,  $^{209}\text{Bi}$  isn't really stable, but it has a half-life of more than  $10^{19}$  years, a billion times the age of the universe, so it might as well be.)

Starting with a sample consisting of 10000 atoms of  $^{213}\text{Bi}$ , simulate the decay of the atoms as shown in the example problem in notes by dividing time into slices of length  $\delta t = 1\text{s}$  each and on each step doing the following:

(a) For each atom of  $^{209}\text{Pb}$  in turn, decide at random, with the appropriate probability, whether it decays or not. The probability can be calculated from (class notes)

$$P(t) = 1 - 2^{-t/\tau}$$

Count the total number that decay, subtract it from the number of  $^{209}\text{Pb}$  atoms, and add it to the number of  $^{209}\text{Bi}$  atoms.

(b) Now do the same for  $^{209}\text{Tl}$ , except that decaying atoms are subtracted from the total for  $^{209}\text{Tl}$  and added to the total for  $^{209}\text{Pb}$ .

(c) For  $^{213}\text{Bi}$  the situation is more complicated: when a  $^{213}\text{Bi}$  atom decays you have to decide at random with the appropriate probability the route by which it decays. Count the numbers that decay by each route and add and subtract accordingly.

Note that you have to work up the chain from the bottom like this, not down from the top, to avoid inadvertently making the same atom decay twice on a single step. Keep track of the number of atoms of each of the four isotopes at all times for 20000 seconds and make a **single graph showing the four numbers** as a function of time on the same axes.

### **Problem 3 (40 points): Brownian motion**

Brownian motion is the motion of a particle, such as a smoke or dust particle, in a gas, as it is buffeted by random collisions with gas molecules. Make a simple computer simulation of such a particle in two dimensions as follows. The particle is confined to a square grid or lattice  $L \times L$  squares on a side, so that its position can be represented by two integers  $i, j = 0 \dots L - 1$ . It starts in the middle of the grid. On each step of the simulation, choose a random direction—up, down, left, or right—and move the particle one step in that direction. This process is called a **random walk**. The particle is not allowed to move outside the limits of the lattice—if it tries to do so, choose a new random direction to move in.

**Write a program** to perform a million steps of this process on a lattice with  $L = 101$  and make an animation on the screen of the position of the particle. (We choose an odd length for the side of the square so that there is one lattice site exactly in the center.)

Note: The visual package doesn't always work well with the **random** package, but if you import functions from visual first, then from random, you should avoid problems.