Spring 2021
PHYS 377 Advanced Computational Physics
HW # 2b

Problem 1: ***The diffraction limit of a telescope***

Our ability to resolve detail in astronomical observations is limited by the diffraction of light in our telescopes. Light from stars can be treated effectively as coming from a point source at infinity. When such light, with wavelength λ, passes through the circular aperture of a telescope (which we'll assume to have unit radius) and is focused by the telescope in the focal plane, it produces not a single dot, but a circular diffraction pattern consisting of central spot surrounded by a series of concentric rings. The intensity of the light in this diffraction pattern is given by

$$I(r) = \left(\frac{J_1(kr)}{kr}\right)^2$$

where r is the distance in the focal plane from the center of the diffraction pattern, $k = 2\pi/\lambda$, and $J_1(x)$ is a **Bessel function**. The Bessel functions $J_m(x)$ are given by

$$J_m(x) = \frac{1}{\pi}\int_0^{\pi} \cos(m\theta - x sin\theta)\, d\theta$$

where m is a nonnegative integer and $x \geq 0$.

(a) Write a **Python function** J(m,x) that calculates the value of $J_m(x)$ using Simpson's rule with $N = 1000$ points. Use your function in a program to make a plot, on a single graph, of the Bessel functions J_0, J_1, and J_2 as a function of x from $x = 0$ to $x = 20$.
(b) Write a second program that makes a **density plot** of the intensity of the circular diffraction pattern of a point light source with $\lambda = 500$ nm, in a square region of the focal plane, using the formula given above. Your picture should cover values of r from zero up to about 1 μm.

Please use appropriate comments in the program.

Hint 1: You may find it useful to know that $\lim_{x\to 0} J_1(x)/x = ½$

Hint 2: The central 2 spot in the diffraction pattern is so bright that it may be difficult to see the rings around it on the computer screen. If you run into this problem a simple way to deal with it is to use one of the other color schemes for density plots. The "*hot*" scheme works well. For a more sophisticated solution to the problem, the *imshow* function has an additional argument *vmax* that allows you to set the value that corresponds to the brightest point in the plot. For instance, if you say "*imshow*(*x,vmax*=0.1)", then elements in *x* with value 0.1, or any greater value, will produce the brightest (most positive) color on the screen. By lowering the *vmax* value, you can reduce the total range of values between the minimum and maximum brightness, and hence increase the sensitivity of the plot, making subtle details visible. (There is also a *vmin* argument that can be used to set the value that corresponds to the dimmest (most negative) color.) For this exercise a value of *vmax*=0.01 appears to work well.

For instance, the diffraction pattern produced by a point source of light when viewed through a telescope is given below: