Spring 2021
PHYS 377 Advanced Computational Physics
HW # 1b

Problem 1: Quadratic Equations

(a) **Write a program** that takes as input three numbers, *a, b*, and *c,* and prints out the two solutions to the quadratic equation $ax^2 + bx + c = 0$ using the standard formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Use your program to compute the solutions of $0.001x^2 + 1000x + 0.001 = 0$

(b) There is another way to write the solutions to a quadratic equation. Multiplying top and bottom of the solution above by $-b \mp \sqrt{b^2 - 4ac}$, show that the solutions can also be written as

$$x = \frac{2c}{-b \mp \sqrt{b^2 - 4ac}}$$

Add further lines to your program to print these values in addition to the earlier ones and again use the program to solve $0.001x^2 + 1000x + 0.001 = 0$. What do you see? How do you explain it?

Problem 2: Calculating Derivatives

Suppose we have a function $f(x)$ and we want to calculate its derivative at a point x. We can do that with pencil and paper if we know the mathematical form of the function, or we can do it on the computer by making use of the definition of the derivative:

$$\frac{df}{dx} = \lim_{\delta \to 0} \frac{f(x + \delta) - f(x)}{\delta}$$

On the computer we can't actually take the limit as δ goes to zero, but we can get a reasonable approximation just by making δ small.

(a) **Write a program** that defines a function $f(x)$ returning the value $x(x - 1)$, then calculates the derivative of the function at the point $x = 1$ using the formula above with $\delta = 10^{-2}$. Calculate the true value of the same derivative analytically and compare with the answer your program gives. The two will not agree perfectly. Why not?

(b) Repeat the calculation for $\delta = 10^{-4}$, 10^{-6}, 10^{-8}, 10^{-10}, 10^{-12}, and 10^{-14}. You should see that the accuracy of the calculation initially gets better as δ gets smaller, but then gets worse again. Why is this?

<u>Problem 3:</u> Wave Interference

Suppose we drop a pebble in a pond and waves radiate out from the spot where it fell. We could create a simple representation of the physics with a sine wave, spreading out in a uniform circle, to represent the height of the waves at some later time. If the center of the circle is at x_1, y_1 then the distance r_1 to the center from a point x, y is

$$r_1 = \sqrt{(x - x_1)^2 + (y - y_1)^2}$$

and the sine wave for the height is

$$\xi_1(x, y) = \xi_0 \sin kr_1$$

where ξ_0 is the amplitude of the waves and k is the wavevector, related to the wavelength λ by $k = 2\pi/\lambda$.

Now suppose we drop another pebble in the pond, creating another circular set of waves with the same wavelength and amplitude but centered on a different point x_2, y_2:

$$\xi_2(x, y) = \xi_0 \sin kr_2 \quad \text{and} \quad r_2 = \sqrt{(x - x_2)^2 + (y - y_2)^2}$$

Then, assuming the waves add linearly (a reasonable assumption for water waves, provided they are not too big), the total height of the surface at a point x, y is

$$\xi(x, y) = \xi_0 \sin kr_1 + \xi_0 \sin kr_2$$

Suppose the wavelength of the waves is $\lambda = 5$ cm, the amplitude is 1 cm, and the centers of the circles are 20 cm apart. **Write a program** to make an image of the height over a 1 m square region of the pond. To make the image, create an array of values representing the height ξ at a grid of points and then use that array to make a density plot. Use a grid of 500 × 500 points to cover the 1 m square, which means the grid points have a separation of 100/500 = 0.2 cm.