The Earth Temperature

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Contents

1 Introduction

In this assignment, we will use a radiative transfer model in order to compute the average temperature of the Earth's surface by modeling the atmosphere using a multilayer approximation, then we will compute the absorption, transmission and emission of visible and infrared radiation from each layer.

2 Theory

2.1 Simplified Models of Earth's temperature

To get a better understanding of how Earth's surface temperature is modeled, we will consider different approaches to calculate it:

2.1.1 Model 1: No atmosphere

The first model neglects the atmosphere completely and treats the Earth as a perfect blackbody. By considering the incoming solar flux averaged over the Earth surface, and that the Earth has an albedo of $\alpha \approx 0.34$, the absorbed energy is calculated as follow:

$$I_{\text{Sun}} = 344 \ \ \text{W/m}^2 \quad \rightarrow \quad A_{\text{Earth}} = (1-\alpha) \cdot I = 227 \ \ \text{W/m}^2 \tag{1}$$

Using the Stefan-Boltzmann law, we get a surface temperature of:

$$A_{\text{Earth}} = \sigma T^4 \quad \rightarrow \quad T = \left(\frac{A_{\text{Earth}}}{\sigma}\right)^{\frac{1}{4}} \approx -21^\circ \text{ C} \tag{2}$$

Which, as expected, severely underestimates the actual average temperature as it omits the atmospheric greenhouse effect.

2.1.2 Model 2: Atmosphere with a perfect greenhouse Layer

This model improves upon the previous one by (partially) taking into account the greenhouse effect, it introduces a single-layer atmosphere that absorbs IR radiation emitted by the surface and re-emits it both upward and downward.
Assuming that the atmosphere emits a flux E_1, and only absorbs a fixed amount of short-wave solar radiation, A_1, and that all IR radiation emitted from the surface is absorbed by the atmosphere (Expect for a small leakage, T_3), then we can calculate the Flux, E_2, radiated by the earth's surface and re-entering the atmosphere as:

$$E_2 = E_1 - A_1 + T_3 \tag{3}$$

Using the Stefan–Boltzmann law, this gives a surface temperature of:

$$T = \left(\frac{E_2}{\sigma}\right)^{\frac{1}{4}} \approx 36^\circ \text{ C} \tag{4}$$

As we can see, the model yields a higher temperature than the previous one by capturing the warming effect of the atmosphere, but it is still an overestimation since it assumes perfect IR absorption and neglects latent heat transport.

3 Numerical Method

In an attempt to calculate the average temperature in a slightly better way, we will in this assignment model the atmosphere by dividing it up in N segments, each with length $dh = H/N$, where H is the height of the atmosphere. We will then consider the radiation balance by following the absorption, transmission and reflection in each segment.
We start by calculating the air density at different altitudes using the barometric formula:

$$\rho(z) = \rho_0 \cdot e^{\left(-\frac{g_0 M_r z}{R T_0}\right)} \tag{5}$$

Incoming solar radiation is attenuated exponentially as it travels downward with an attenuation coefficient α_{VR}. Similarly, infrared radiation is absorbed and re-emitted at each atmospheric layer with, α_{IR} (which will be a varying parameter in our code), we then model this radiative transfer iteratively as follows:

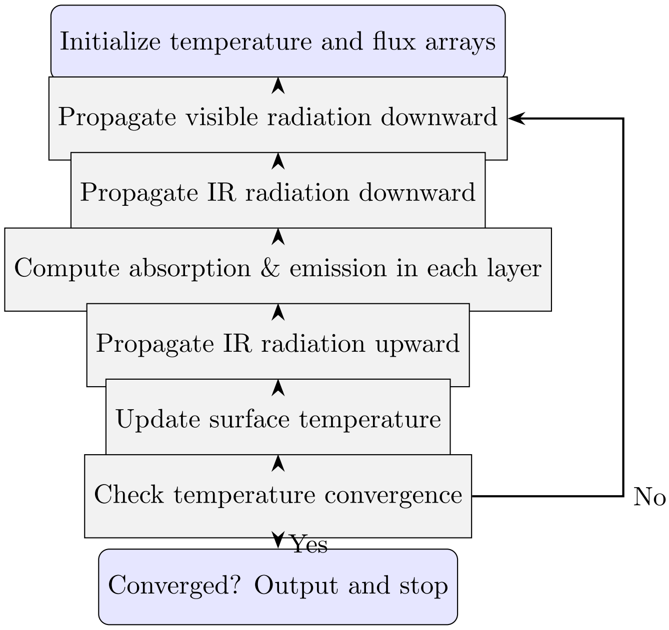


Figure 1: Iterative radiative transfer loop used to compute surface temperature.

Assuming that each layer absorbs a portion of the incoming flux and re-emits accordingly (to ensure radiative balance), the total emission in each layer is adjusted to equal the absorbed energy. The following formulas are used through the iterative procedure:

$$\begin{cases} T_{\mathrm{vis}}^{\downarrow}(i) & = T_{\mathrm{vis}}^{\downarrow}(i-1) \cdot e^{(-\alpha_{\mathrm{VR}} \cdot \Delta z)} \\ T_{\mathrm{IR}}^{\downarrow}(i) & = T_{\mathrm{IR}}^{\downarrow}(i-1) \cdot e^{(-\alpha_{\mathrm{IR}} \cdot \Delta z)} \\ T_{\mathrm{IR}}^{\uparrow}(i) & = \left(T_{\mathrm{IR}}^{\uparrow}(i-1) - \frac{1}{2}E(i-1)\right) \cdot e^{(-\alpha_{\mathrm{IR}}\rho(i-1) \cdot \Delta z)} \\ T_{\mathrm{surface}} & = \left(\frac{F_{\mathrm{absorbed}}}{\sigma}\right)^{1/4} \end{cases}$$

By doing this we are taking into account the atmospheric back-radiation, and by testing different values of the IR-absorption coefficient we will be able examine its effect on the final absorbed flux and average surface temperature.

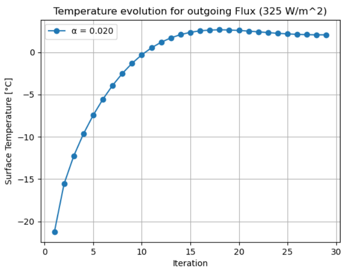
4 Results

To investigate the effect of infrared opacity on surface temperature, we ran the code for six different values of the infrared attenuation coefficient:

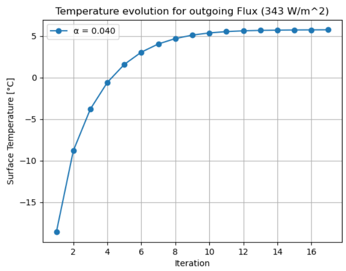
α_{IR} (m^{-1})	T_{surf} (°C)	Flux Out ($\mathrm{W/m}^2$)
0.020	2.05	325.2
0.040	5.74	343.0
0.060	10.66	367.9
0.080	15.23	392.1
0.100	19.30	414.8
0.120	22.93	435.7

Table 1: Final surface temperatures and fluxes for different α_{IR}.

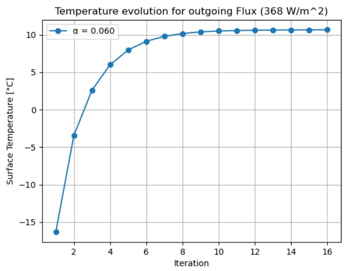
Figure 1 shows the convergence behavior for the different α_{IR} values:



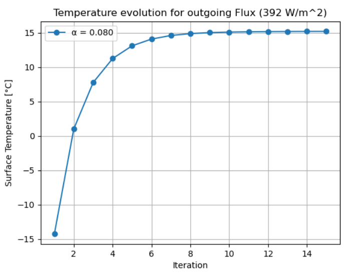
(a) $\alpha_{\mathrm{IR}} = 0.020$



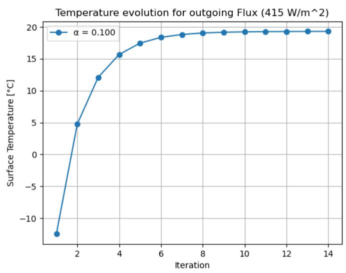
(b) $\alpha_{\mathrm{IR}} = 0.040$



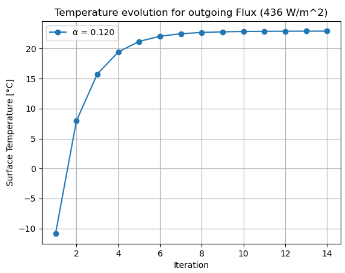
(c) $\alpha_{\mathrm{IR}} = 0.060$



(d) $\alpha_{\mathrm{IR}} = 0.080$



(e) $\alpha_{\mathrm{IR}} = 0.100$



(f) $\alpha_{\mathrm{IR}} = 0.120$

Figure 2: Convergence of surface temperature for different α_{IR}.

As expected, increasing α_{IR} enhances the absorption and re-emission of infrared radiation by the atmosphere layers and hence increases the surface temperature, while lower values allow for the escape of more IR radiation to outer space, resulting in a lower average temperature.

5 Discussion

Despite its simplicity, the iterative radiative transfer model successfully captured the greenhouse effect of the atmosphere and gave us an average temperature within the reasonable range.